Q.02) teorema da divergência: $\oint D\,ds = \int \nabla D\,dv$

$$\nabla D = \frac{\partial D_x}{dx} + \frac{\partial D_y}{dy} + \frac{\partial D_z}{dz}$$

$$\begin{cases} \dfrac{\partial D_x}{dx} = \dfrac{\partial\, yz \cos y^2}{dx} = 0 \\[2ex] \dfrac{\partial D_y}{dy} = \dfrac{\partial\, xz \cos x^2}{dy} = 0 \\[2ex] \dfrac{\partial D_z}{dz} = \dfrac{\partial 1}{dz} = 0 \end{cases}$$

$\nabla D = 0$

→ o fluxo é nulo, a quantidade que entra no filtro é igual a que sai

→ a água no interior do filtro é constante.

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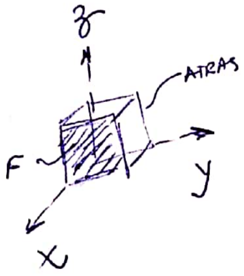
Q.03) 1º lado:

4 lados na origem



$$\oint D\,ds = \int_{FRENTE} + \int_{ATRÁS} + \int_{ESQ} + \int_{DIR}$$

$$\oint_{FRENTE} = \int_0^2 \int_0^2 xy^2 \; a_x \; dy\,dz \; a_x \qquad p/ \; x=2$$

$$= \int_0^2 \int_0^2 2y^2 \; dy\,dz$$

$$\oint_F = \frac{2y^3}{3}\Big|_0^2 \; z\Big|_0^2 = 2 \cdot \frac{8}{3} \cdot 2 = \frac{32}{3}$$

$$\oint_{ATRÁS} = \int_0^2 \int_0^2 xy^2 \; a_x \; dy\,dz \; (-a_x) \qquad p/ \; x=0$$

$$\oint_A = 0$$

$$\oint_{ESQ} = \int_0^2 \int_0^2 y x^2 \; a_y \; dx\,dz \; (-a_y) \qquad p/ \; y=0$$

$$\oint ESQ = 0$$

$$\phi_{DIR} = \int_0^2 \int_0^2 y x^2 \, a_y \, dx \, dz \, (a_y) \qquad p/ \; y = 2$$

$$= \int_0^2 \int_0^2 2x^2 \, dx \, dz$$

$$= 2 \left. \frac{x^3}{3} \right|_0^2 \left. z \right|_0^2 = 2 \cdot \frac{8}{3} \cdot 2 = \frac{32}{3}$$

$$\oint D \, ds = \frac{32}{3} + 0 + 0 + \frac{32}{3} = \frac{64 \, C}{3}$$

2° lado:

$$\nabla \vec{D} = \frac{\partial Dx}{dx} + \frac{\partial Dy}{dy} + \frac{\partial Dz}{dz}$$

$$\nabla \vec{D} = \underbrace{\frac{\partial x y^2}{dx}}_{y^2} + \underbrace{\frac{\partial y x^2}{dy}}_{x^2}$$

$$\nabla \vec{D} = x^2 + y^2$$

$$\int_V \nabla \vec{D} \, dv = \int_0^2 \int_0^2 \int_0^2 x^2 + y^2 \, dx \, dy \, dz$$

$$\int \nabla \vec{D} \, dv = 2 \left[\left. \frac{x^3}{3} \right|_0^2 + \left. \frac{y^3}{3} \right|_0^2 \right]$$

$$= 2 \left[\frac{2 \cdot 8}{3} + \frac{8 \cdot 2}{3} \right] = \frac{64 \, C}{3}$$

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Q.04) FRENTE — $yz\ (ax)$
ATRÁS — $yz\ (-ax)$
TOPO — $xy\ (az)$
BASE — $xy\ (-az)$ $\Big\}$ $D=0$, $\theta = 90°$
LD — $xz\ (ay)$
LE — $xz\ (-ay)$

aresta = 2m

$$\boxed{D = 10x^2\, a_x + xy^3\, a_y}$$

FRENTE:

$$\oint_F D\,ds = \int_{-1}^{1}\int_{-1}^{1} 10x^2\, a_x + \cancel{xy^3 a_y}\; dy\,dz\,(a_x)\Big|_{p/\,x=1}$$

$$= \int_{-1}^{1}\int_{-1}^{1} 10\,(1)^2\, dy\,dz$$

$$= 10 \cdot 2 \cdot 2 = 40$$

ATRÁS: p/ $x=-1$

$$\oint_A D\,ds = -10\,(-1)^2\,(2)(2) = -40$$

LD:
$$\oint_{LD} D\,ds = \int_{-1}^{1}\int_{-1}^{1} \cancel{10x^2 a_x} + xy^3\, a_y\, dx\,dz\,a_y\Big|_{y=1}$$

$$= \int_{-1}^{1}\int_{-1}^{1} xy^3\, dx\,dz$$

$$= \frac{x^2}{2}\Big|_{-1}^{1}\, y^{3 \cdot 1} = 0$$

LE: $\oint_{LE} = 0$

$$\oint D\,ds = \int_F + \int_A + \int_{LE} + \int_{LD} = 0$$

$$\int \nabla D \, dv$$

$$\nabla \vec{D} = \frac{\partial 10x^2}{dx} + \frac{\partial xy^3}{dy}$$

$$\nabla \vec{D} = 20x + 3xy^2$$

$$\int \nabla D \, dv = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 20x + 3xy^2 \, dx\, dy\, dz$$

$$\int \nabla D \, dv = 2\left(\int_{-1}^{1}\int_{-1}^{1} 20x \, dy\, dx + \int_{-1}^{1}\int_{-1}^{1} 3xy^2 \, dx\, dy\right)$$

$$= 2\left(20\frac{x^2}{2}\Big|_{-1}^{1} \cdot 2 + \frac{3x^2}{2}\Big|_{-1}^{1} \frac{y^3}{3}\Big|_{-1}^{1}\right)$$

$$= 2\left(40\left(\frac{1}{2} - \frac{(-1)^2}{2}\right) + 3\left(\frac{1}{2} - \frac{1}{2}\right)\cdot\left(\frac{1}{3} + \frac{1}{3}\right)\right)$$

$$\int \nabla D \, dv = 0$$

Q.05) $\begin{cases} \vec{D} = 5\rho^3 \vec{a_\rho} \\ \rho = 3m \end{cases}$

a) $\rho_v = \nabla \vec{D} = \frac{1}{\rho}\frac{\partial \rho D_\rho}{\partial \rho} + \underbrace{\frac{1}{\rho}\frac{\partial D_\phi}{d\phi}}_{0} + \underbrace{\frac{\partial D_z}{dz}}_{0}$

$\rho_v = \frac{1}{\rho}\frac{\partial \rho . 5\rho^3}{d\rho}$

$\rho_v = \frac{1}{\rho}\frac{\partial 5\rho^4}{d\rho} = \frac{1}{\rho} 20\rho^3 = 20\rho^2$

p/ $\rho = 3 \rightarrow 20 . 3^2 \rightarrow \rho_v = 180\ C/m^3$

b) $\vec{D} = 5\rho^3 \vec{a_\rho}$ p/ $\rho = 3m \rightarrow \vec{D} = 5.3^3$

$\vec{D} = 135\, \vec{a_\rho}\ C/m^2$

c) $\Psi = \oint D ds = Q_{interna} = \int_{vol} \rho_v dv$

$\Psi = \oint D ds \begin{cases} \vec{D} = 5\rho^3 \vec{a_\rho} \\ \vec{ds} = \rho d\phi dz\, \vec{a_\rho} \end{cases}$

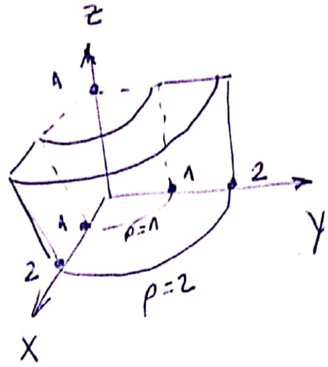
$\Psi = \int_{-2,5}^{2,5} \int_0^{2\pi} 5\rho^3\, \rho d\phi dz = \int_{-2,5}^{2,5} \int_0^{2\pi} 5\rho^4\, d\phi dz$

$\Psi = 5 . 3^4\, 2\pi . 5 \rightarrow \Psi = 4050\pi\ C$

d) $Q_{interna} = 4050\pi\ C$

↳ por Gauss $\oint D ds = \int \rho_v dv \rightarrow Q_{int} = \Psi$

P6)



$$\vec{D} = \frac{20}{\rho^2}\left(-\operatorname{sen}^2\phi\, \vec{a}_\rho + \operatorname{sen}2\phi\, \vec{a}_\phi\right)$$

$$D_\rho = -\frac{20\operatorname{sen}^2\phi}{\rho^2} \quad e \quad D_\phi = \frac{20\operatorname{sen}2\phi}{\rho^2}$$

$$Q_{int} = \int \vec{D}\,ds = \int \nabla\vec{D}\,dv$$

$$\nabla\vec{D} = \frac{1}{\rho}\frac{\partial \rho D_\rho}{d\rho} + \frac{1}{\rho}\frac{D\phi}{d\phi} + \frac{\partial D_z}{dz}$$

$$= \frac{1}{\rho}\frac{\partial}{d\rho}\left(-\frac{20\operatorname{sen}^2\phi}{\rho}\right) + \frac{1}{\rho}\frac{\partial}{d\phi}\left(\frac{20\operatorname{sen}2\phi}{\rho^2}\right)$$

$$= -\frac{20\operatorname{sen}^2\phi}{\rho}\cdot\left(-\frac{1}{\rho^2}\right) + \frac{1}{\rho}\cdot\frac{20}{\rho^2}\,2\cos 2\phi$$

$$= \frac{20\operatorname{sen}^2\phi}{\rho^3} + \frac{40\cos 2\phi}{\rho^3}$$

$\operatorname{sen}^2\phi + \cos^2\phi = 1$

$$= \frac{20}{\rho^3}\left[\frac{1}{2} - \frac{\cos 2\phi}{2} + 2\cos 2\phi\right]$$

Dados:

$$\operatorname{sen}^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{10}{\rho^3} + \frac{30\cos 2\phi}{\rho^3}$$

$$= \frac{10}{\rho^3}\left(1 + 3\cos 2\phi\right)$$

$$Q_{int} = \int_0^1\int_0^{\pi/2}\int_1^2 \left[\frac{10}{\rho^3}\left(1+3\cos 2\phi\right)\right]\rho\,d\rho\,d\phi\,dz$$

$$= \left[-\frac{10}{\rho}\right]_1^2 \cdot \left[\phi + 3\,\frac{\operatorname{sen}2\phi}{2}\right]_0^{\pi/2} \left[z\right]_0^1$$

$$= \left(-\frac{10}{2} + 10\right)\cdot\frac{\pi}{2}\cdot 1 \quad \rightarrow \quad \boxed{Q_{int} = \frac{5\pi}{2}\, C}$$

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Q07) $Q_{int} = Q_{linha} + Q_{plano}$

$$\begin{cases} Q_{linha} = \int \rho_l \, dl = \int_{-2}^{2} 4\pi \, dz \\ = 4\pi \, [z]_{-2}^{2} \rightarrow Q_{linha} = 16\pi C \end{cases}$$

$$Q_{plano} = \int \rho_s \, ds = \int_0^{2\pi} \int_0^{\sqrt{3}} 20 \rho \, d\rho \, d\phi$$

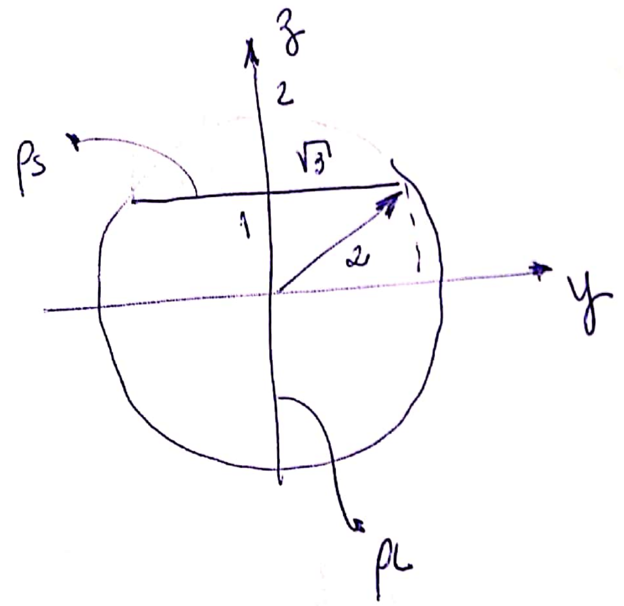
$$= 20 \cdot \left[\frac{\rho^2}{2} \right]_0^{\sqrt{3}} [\phi]_0^{2\pi} \rightarrow Q_{plano} = 60\pi C$$

$$\Psi = Q_{int}$$

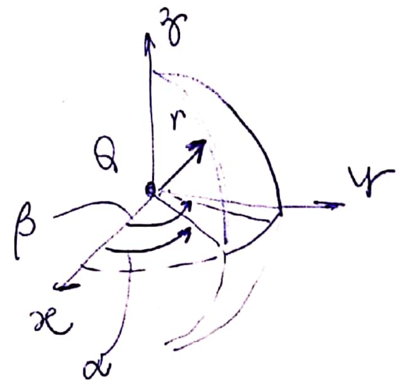
$$= Q_{linha} + Q_{plano}$$

$$= 16\pi + 60\pi$$

$$\Psi = 76\pi C$$



Q.08)



MÉTODO 1:

$$\Psi = \oint_S D\,ds \quad \begin{cases} \vec{D} = \dfrac{Q}{4\pi r^2}\,\vec{a_r} \\ \vec{ds} = r^2 \operatorname{sen}\theta\, d\phi\, d\theta\, \vec{a_r} \end{cases}$$

$$\Psi = \int_\alpha^\beta \int_0^\pi \frac{Q}{4\pi r^2} \cdot r^2 \operatorname{sen}\theta\, d\phi\, d\theta$$

$$\Psi = \frac{Q}{4\pi} \int_\alpha^\beta d\phi \int_0^\pi \operatorname{sen}\theta\, d\theta$$

$$\Psi = \frac{Q}{4\pi} (\beta - \alpha) \cdot (-\cos\pi + \cos 0^\circ)$$

$$\Psi = \frac{Q}{2\pi} (\beta - \alpha) \; [C]$$

MÉTODO 2:

$\Psi_{esf} = Q$

Área esfera $= S = 4\pi r^2$

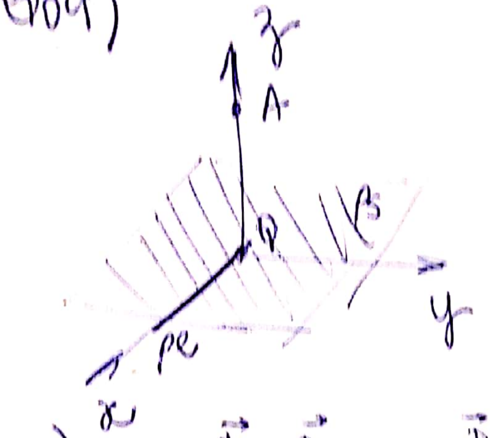
$$S_{casca}\ ds_{casca} = \int_\alpha^\beta \int_0^\pi r^2 \operatorname{sen}\theta\, d\phi\, d\theta$$

$$S_{casca} = [\phi]_\alpha^\beta \; r^2 [-\cos\theta]_0^\pi$$

$$S_{casca} = 2r^2 (\beta - \alpha)$$

$$\Psi_{casca} = \Psi_{esf} \frac{S_{casca}}{S_{esf}} \rightarrow \Psi_{casca} = Q \frac{2r^2(\beta-\alpha)}{4\pi r^2} \rightarrow \Psi_{casca} = \frac{Q}{2\pi}(\beta - \alpha)$$

Q09) 06



$$\begin{cases} Q = 6\,\mu C \\ \rho_\ell = 180\,nC/m \\ \rho_s = 25\,nC/m^2 \\ A = (0,0,4) \end{cases}$$

a) $\vec{D} = \vec{D}_{carga} + \vec{D}_{linha} + \vec{D}_{plano}$

$$\vec{D}_Q = \frac{Q}{4\pi r^2}\vec{a_r} = \frac{6 \cdot 10^{-6}}{4\pi \cdot 4^2}\vec{a_z} = \frac{3}{32\pi}\vec{a_z}\ \mu C/m^2$$

$$\vec{D}_\ell = \frac{\rho_\ell}{2\pi\rho}\vec{a_\rho} = \frac{180 \cdot 10^{-9}}{2\pi \cdot 4}\vec{a_z} = \frac{45}{2\pi}\vec{a_z}\ nC/m^2$$

$$\vec{D}_p = \frac{\rho_s}{2}\vec{a_n} = \frac{25 \cdot 10^{-9}}{2} = \frac{25}{2}\vec{a_z}\ nC/m^2$$

$$\boxed{\vec{D} = 0{,}0495\ \mu C/m^2}$$

b) $B = (1,2,4)$

$$\vec{D}_Q = \frac{6 \cdot 10^{-6}}{4\pi(\sqrt{21})^2}\left(\frac{\vec{a_x} + 2\vec{a_y} + 4\vec{a_z}}{\sqrt{21}}\right)$$

$$= 4{,}96\,\vec{a_x} + 9{,}92\,\vec{a_y} + 19{,}84\,\vec{a_z}\ nC/m^2$$

$$\vec{D}_\ell = \frac{180 \cdot 10^{-9}}{2\pi \cdot \sqrt{20}}\ \frac{2\vec{a_y} + 4\vec{a_z}}{\sqrt{20}} = 2{,}86\,\vec{a_y} + 5{,}73\,\vec{a_z}\ nC/m^2$$

$$\vec{D}_p = \frac{25}{2}\vec{a_z}\ nC/m^2$$

$$\boxed{\vec{D} = 4{,}96\,\vec{a_x} + 12{,}78\,\vec{a_y} + 38{,}07\,\vec{a_z}\ nC/m^2}$$

c)

$$\Psi = Q + Q_L + Q_P$$

$$Q_L = \int_L \rho_L \, dl$$

$$= \rho_L [x]_{-4}^{4} = 8\rho_L = 8 \cdot 180n$$

$$Q_L = 1440 \, nC$$

$$Q_P = \int_S \rho_S \, ds$$

$$= \int_S \rho_S \, \rho \, d\rho \, d\phi$$

$$= \rho_S \left[\frac{\rho^2}{2} \right]_0^4 [\phi]_0^{2\pi}$$

$$= 16.2\pi \rho_S = 2\pi . 16 . 25$$

$$= 400 \pi \, nC$$

$$\Psi = 6 . 10^{-6} + 1440 . 10^{-9} + 400\pi . 10^{-9}$$

$$\boxed{\Psi = 8{,}7 \, \mu C}$$

Q.10) $D = 30e^{-\rho}\vec{a_\rho} - 2z\vec{a_z}$ cilíndricas

$$\begin{cases} r = 2 \\ z = 0 \text{ e } 5m \end{cases}$$

$$\int \nabla\vec{D}\, dv = \int_0^5 \int_0^{2\pi} \int_0^2 \nabla D \, \rho \, d\rho \, d\phi \, dz$$

$$\nabla\vec{D} = \frac{1}{\rho} \partial\rho \frac{D_\rho}{d\rho} + \frac{1}{\rho} \frac{\partial D_\phi}{d\phi} + \frac{\partial D_z}{dz}$$

$$\nabla\vec{D} = \frac{1}{\rho} \partial\rho \frac{30e^{-\rho}}{d\rho} + \frac{\partial(-2z)}{dz}$$

$$\nabla\vec{D} = \frac{1}{\rho}\Big(\underbrace{30e^{-\rho} - \rho 30e^{-\rho}}_{u'v - uv'}\Big) + (-2)$$

$$\nabla.\vec{D} = \frac{30e^{-\rho}(1-\rho)}{\rho} - 2\rho$$

$$= \int_0^5 \int_0^{2\pi} \int_0^2 \left(\frac{30e^{-\rho}(1-\rho)}{\rho} - 2\cancel{\rho}\right) \rho \, d\rho \, d\phi \, dz$$

$$= \underbrace{2\pi . 5}_{10\pi} \underbrace{\int_0^2 30e^{-\rho} d\rho}_{A} - \underbrace{\int_0^2 30e^{-\rho} \rho \, d\rho}_{B} - \underbrace{\int_0^2 2\rho \, d\rho}_{C}$$

$$\int \nabla D dv = 10\pi \, (A - B - C)$$

$$A = \int_0^2 30e^{-\rho}d\rho - 30\int_0^2 e^{-\rho}d\rho$$

$$A = 30\frac{e^{-\rho}}{-1}\Big|_0^2 = -30e^{-2} - (-30e^{0})$$

$$A = -30(e^{-2} - e^{0}) = -30(-0{,}86) = 25{,}94$$

$$B = \int_0^2 30e^{-\rho}\rho\, d\rho$$

$$\begin{cases} u = \rho & dv = e^{-\rho}d\rho \\ du = d\rho & v = -e^{-\rho} \end{cases}$$

$$B = 30\int e^{-\rho}\rho\, d\rho$$

$$= 30\left(\rho(-e^{-\rho}) - \int -e^{-\rho}d\rho\right)$$

$$= 30\left(-\rho e^{-\rho} + \int e^{-\rho}d\rho\right)$$

$$= 30\left(-2e^{-2} - (e^{-2} - e^{0})\right)$$

$$= 30\left(-0{,}27 - (-0{,}86)\right) = 30 \cdot 0{,}59 = 17{,}82$$

$$C = \int_0^2 2\, d\rho$$

$$= 2 \cdot 2 = 4$$

$$\int \nabla D\, dv = 10\pi\,(\underbrace{25{,}94 - 17{,}82}_{4{,}12} - 4)$$

$$\int \nabla D\, dv = 129{,}44\ C$$

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$\int D\,ds$

BASE: $= \int_0^{2\pi}\int_0^{2} 30\cancel{e^{-\rho}a_\rho} - 2z\,a_z \;(-\rho\, d\rho\, d\phi\, a_z) \Big|_{\text{para } z=0}$

$= \int_0^{2\pi}\int_0^{2} 2.(0)\; \rho\, d\rho\, d\phi = 0$

CORPO: $\int_0^{5}\int_0^{2\pi} 30e^{-\rho}a_\rho - 2z\,a_z \;\; \rho\, d\phi\, dz\; a_\rho \Big|_{\text{para } \rho=2}$

$= \int_0^{5}\int_0^{2\pi} 30e^{-\rho}\,\rho\, d\phi\, dz$

$= \int_0^{5}\int_0^{2\pi} 30\,e^{-2}.2\; d\phi\, dz$

$= 8{,}12 \int_0^{5}\int_0^{2\pi} = 255{,}10$

TOPO: $= \int_0^{2\pi}\int_0^{2} -2z\;\; \rho\, d\rho\, d\phi \Big|_{\text{para } z=5}$

$= \int_0^{2\pi}\int_0^{2} -10\;\rho\, d\rho\, d\phi$

$= 2\pi.\,10\frac{\rho^2}{2}\Big|_0^2 = 2\pi.\,\frac{10.4}{2} = -40\pi$
$-125{,}66$

$\int_{TOPO} + \int_{BASE} + \int_{CORPO} = 255{,}10 + 0 - 125{,}66 = 129{,}44\ C$

Q.11) $\nabla D = \frac{\partial 3x^2}{dx} + \frac{\partial 2z}{dy} + \frac{\partial x^2 z}{dz}$

$= 3(2x) + 0 + x^2$

$= 6x + x^2$

a) $= \int_{-2}^{2}\int_{-2}^{2}\int_{-2}^{2} 6x + x^2 \, dx\,dy\,dz$

$= \left(\frac{6x^2}{2} + \frac{x^3}{3}\right)_{-2}^{2} \cdot \underbrace{(y)_{-2}^{2}}_{4} \cdot \underbrace{(z)_{-2}^{2}}_{4}$

$= \left(\frac{24}{2} + \frac{8}{3} - \left(-\frac{24}{2} - \frac{8}{3}\right)\right) \cdot 4 \cdot 4$

$= \frac{176}{\cancel{6}_{3}} \cdot \cancel{16}^{8} = \frac{1408}{3} = 469{,}3\ C$

b) $= \int_0^4\int_0^4\int_0^4 = \left(6 \cdot \frac{4^2}{2} + \frac{4^3}{3}\right) \cdot 4 \cdot 4$

$\left(48 + \frac{64}{3}\right) \cdot 16 = \frac{3328}{3}$

$= 1109{,}33\ C$