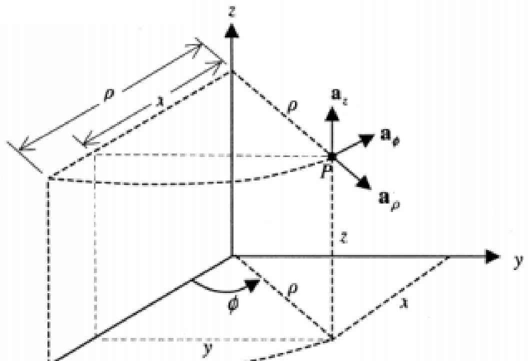
Vetores unitários

Coordenadas retangulares – $\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$

Coordenadas cilíndricas – $\mathbf{a}_\rho$, $\mathbf{a}_\phi$, $\mathbf{a}_z$

Coordenadas esféricas – $\mathbf{a}_r$, $\mathbf{a}_\theta$, $\mathbf{a}_\phi$



Coordenadas cilíndricas

Transformação de coordenadas

$$x = \rho\cos\phi = r\,\mathrm{sen}\,\theta\cos\phi$$

$$y = \rho\,\mathrm{sen}\,\phi = r\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\phi$$

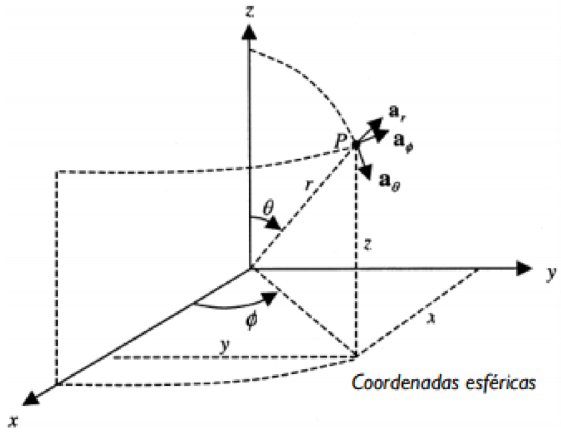
$$z = r\cos\theta$$

$$\rho = \sqrt{x^2 + y^2} = r\,\mathrm{sen}\,\theta$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{\rho^2 + z^2}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right) = \tan^{-1}\left(\frac{\rho}{z}\right)$$



Coordenadas esféricas

Transformação de coordenadas de componentes vetoriais

$$A_x = A_\rho\cos\phi - A_\phi\,\mathrm{sen}\,\phi = A_r\,\mathrm{sen}\,\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\,\mathrm{sen}\,\phi$$

$$A_y = A_\rho\,\mathrm{sen}\,\phi + A_\phi\cos\phi = A_r\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\phi + A_\theta\cos\theta\,\mathrm{sen}\,\phi + A_\phi\cos\phi$$

$$A_z = A_r\cos\theta - A_\theta\,\mathrm{sen}\,\theta$$

$$A_\rho = A_x\cos\phi + A_y\,\mathrm{sen}\,\phi = A_r\,\mathrm{sen}\,\theta + A_\theta\cos\theta$$

$$A_\phi = -A_x\,\mathrm{sen}\,\phi + A_y\cos\phi$$

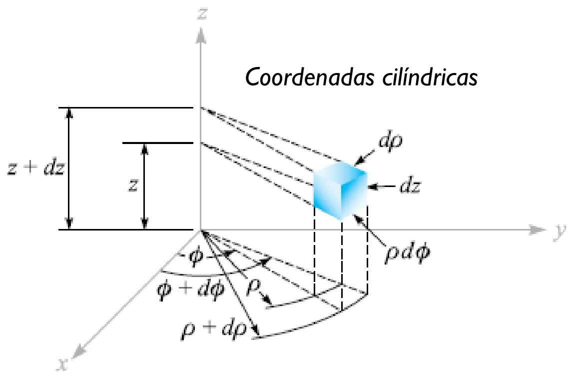
$$A_r = A_x\,\mathrm{sen}\,\theta\cos\phi + A_y\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\phi + A_z\cos\theta = A_\rho\,\mathrm{sen}\,\theta + A_z\cos\theta$$

$$A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\,\mathrm{sen}\,\phi - A_z\,\mathrm{sen}\,\theta = A_\rho\cos\theta - A_z\,\mathrm{sen}\,\theta$$

Cálculo Vetorial

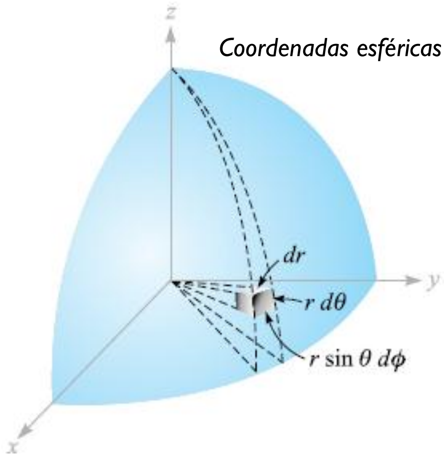
Elementos vetoriais diferenciais de comprimento

$$d\mathbf{L}=\begin{cases}\mathbf{a}_x dx+\mathbf{a}_y dy+\mathbf{a}_z dz\\ \mathbf{a}_\rho d\rho+\mathbf{a}_\phi \rho\, d\phi+\mathbf{a}_z dz\\ \mathbf{a}_r dr+\mathbf{a}_\theta r\, d\theta+\mathbf{a}_\phi r \operatorname{sen}\theta\, d\phi\end{cases}$$



Elementos vetoriais diferenciais de área

$$d\mathbf{S}=\begin{cases}\mathbf{a}_x dy\,dz+\mathbf{a}_y dx\,dz+\mathbf{a}_z dx\,dy\\ \mathbf{a}_\rho \rho\, d\phi\, dz+\mathbf{a}_\phi d\rho\, dz+\mathbf{a}_z \rho\, d\rho\, d\phi\\ \mathbf{a}_r r^2 \operatorname{sen}\theta\, d\theta\, d\phi+\mathbf{a}_\theta r\operatorname{sen}\theta\, dr\, d\phi+\mathbf{a}_\phi r\, dr\, d\theta\end{cases}$$



Elementos diferenciais de volume

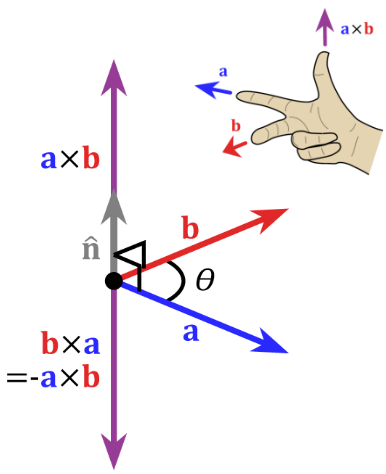
$$dV=\begin{cases}dx\,dy\,dz\\ \rho\, d\rho\, d\phi\, dz\\ r^2\operatorname{sen}\theta\, dr\, d\theta\, d\phi\end{cases}$$

Operações multiplicativas vetoriais
(válidas para qualquer sistema de coordenadas)

$$\mathbf{A}\cdot\mathbf{B}=|\mathbf{A}||\mathbf{B}|\cos(A\sphericalangle B)=A_xB_x+A_yB_y+A_zB_z$$

$$\mathbf{A}\times\mathbf{B}=|\mathbf{A}||\mathbf{B}|\operatorname{sen}(A\sphericalangle B)\mathbf{a}_n=$$

$$=\left(A_yB_z-A_zB_y\right)\mathbf{a}_x+\left(A_zB_x-A_xB_z\right)\mathbf{a}_y+\left(A_xB_y-A_yB_x\right)\mathbf{a}_z$$



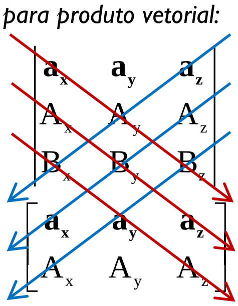
Coordenadas retangulares – $\begin{cases}\mathbf{a}_x\times\mathbf{a}_y=\mathbf{a}_z\\ \mathbf{a}_y\times\mathbf{a}_z=\mathbf{a}_x\\ \mathbf{a}_z\times\mathbf{a}_x=\mathbf{a}_y\end{cases}$

Coordenadas cilíndricas – $\begin{cases}\mathbf{a}_\rho\times\mathbf{a}_\phi=\mathbf{a}_z\\ \mathbf{a}_\phi\times\mathbf{a}_z=\mathbf{a}_\rho\\ \mathbf{a}_z\times\mathbf{a}_\rho=\mathbf{a}_\phi\end{cases}$

Coordenadas esféricas – $\begin{cases}\mathbf{a}_r\times\mathbf{a}_\theta=\mathbf{a}_\phi\\ \mathbf{a}_\theta\times\mathbf{a}_\phi=\mathbf{a}_r\\ \mathbf{a}_\phi\times\mathbf{a}_r=\mathbf{a}_\theta\end{cases}$

Regra de Sarrus para produto vetorial:

$+\mathbf{a}_x A_y B_z$
$+\mathbf{a}_z A_x B_y$
$+\mathbf{a}_y A_y B_x$
$-\mathbf{a}_z A_y B_x$
$-\mathbf{a}_x A_z B_y$
$-\mathbf{a}_y A_x B_z$



Cálculo Vetorial

Operadores vetoriais diferenciais

$\nabla\psi$ → **gradiente** de um campo escalar ψ (retorna um campo vetorial)

$\nabla\cdot\mathbf{A}$ → **divergente** te um campo vetorial $\mathbf{A}$ (retorna um campo escalar)

$\nabla\times\mathbf{A}$ → **rotacional** de um campo vetorial $\mathbf{A}$ (retorna um campo vetorial)

$\nabla^2\psi$ → **Laplaciano** de um campo escalar ψ (retorna um campo escalar)

$\nabla^2\mathbf{A}$ → **Laplaciano vetorial** de um campo vetorial $\mathbf{A}$ (retorna um campo vetorial)

Coordenadas retangulares:

$$\nabla\psi = \frac{\partial\psi}{\partial x}\mathbf{a_x} + \frac{\partial\psi}{\partial y}\mathbf{a_y} + \frac{\partial\psi}{\partial z}\mathbf{a_z}$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\mathbf{a_x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\mathbf{a}_z$$

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \nabla\cdot\nabla\psi$$

$$\nabla^2\mathbf{A} = \nabla^2 A_x\mathbf{a_x} + \nabla^2 A_y\mathbf{a_y} + \nabla^2 A_z\mathbf{a_z} = \nabla(\nabla\cdot\mathbf{A}) - \nabla\times(\nabla\times\mathbf{A})$$

Coordenadas cilíndricas:

$$\nabla\psi = \frac{\partial\psi}{\partial\rho}\mathbf{a_\rho} + \frac{1}{\rho}\frac{\partial\psi}{\partial\phi}\mathbf{a}_\phi + \frac{\partial\psi}{\partial z}\mathbf{a_z}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right)\mathbf{a}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)\mathbf{a}_\phi + \frac{1}{\rho}\left(\frac{\partial(\rho A_\phi)}{\partial\rho} - \frac{\partial A_\rho}{\partial\phi}\right)\mathbf{a}_z$$

$$\nabla^2\psi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\phi^2} + \frac{\partial^2\psi}{\partial z^2}$$

$$\nabla^2\mathbf{A} = \left(\nabla^2 A_\rho - \frac{2}{\rho^2}\frac{\partial A_\phi}{\partial\phi} - \frac{A_\rho}{\rho^2}\right)\mathbf{a_\rho} + \left(\nabla^2 A_\phi - \frac{2}{\rho^2}\frac{\partial A_\rho}{\partial\phi} - \frac{A_\phi}{\rho^2}\right)\mathbf{a}_\phi + \nabla^2 A_z\mathbf{a_z}$$

Cálculo Vetorial

Coordenadas esféricas:

$$\nabla\psi = \frac{\partial\psi}{\partial r}\mathbf{a_r} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}\mathbf{a}_\theta + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial\psi}{\partial\phi}\mathbf{a}_\phi$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial(A_\theta\,\mathrm{sen}\,\theta)}{\partial\theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\begin{aligned}\nabla\times\mathbf{A} &= \frac{1}{r\,\mathrm{sen}\,\theta}\left(\frac{\partial(A_\phi\,\mathrm{sen}\,\theta)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right)\mathbf{a}_r + \\ &+ \frac{1}{r}\left(\frac{1}{\mathrm{sen}\,\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial(rA_\phi)}{\partial r}\right)\mathbf{a}_\theta + \\ &+ \frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)\mathbf{a}_\phi\end{aligned}$$

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\,\mathrm{sen}\,\theta}\frac{\partial}{\partial\theta}\left(\mathrm{sen}\,\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\,\mathrm{sen}^2\,\theta}\frac{\partial^2\psi}{\partial\phi^2}$$

$$\begin{aligned}\nabla^2\mathbf{A} &= \left[\nabla^2A_r - \frac{2A_r}{r^2} - \frac{2}{r^2\,\mathrm{sen}\,\theta}\left(\frac{\partial(A_\theta\,\mathrm{sen}\,\theta)}{\partial\theta} + \frac{\partial A_\phi}{\partial\phi}\right)\right]\mathbf{a_r} + \\ &+ \left[\nabla^2A_\theta - \frac{A_\theta}{r^2\,\mathrm{sen}^2\,\theta} + \frac{2}{r^2}\frac{\partial A_r}{\partial\theta} - \frac{2\cos\theta}{r^2\,\mathrm{sen}^2\,\theta}\frac{\partial A_\phi}{\partial\phi}\right]\mathbf{a}_\theta + \\ &+ \left[\nabla^2A_\phi - \frac{A_\phi}{r^2\,\mathrm{sen}^2\,\theta} + \frac{2}{r^2\,\mathrm{sen}\,\theta}\frac{\partial A_r}{\partial\phi} + \frac{2\cos\theta}{r^2\,\mathrm{sen}^2\,\theta}\frac{\partial A_\theta}{\partial\phi}\right]\mathbf{a}_\phi\end{aligned}$$

Propriedades:

$$\nabla(\psi+\phi) = \nabla\psi + \nabla\phi$$
$$\nabla\cdot(\mathbf{A}+\mathbf{B}) = \nabla\cdot\mathbf{A} + \nabla\cdot\mathbf{B}$$
$$\nabla\times(\mathbf{A}+\mathbf{B}) = \nabla\times\mathbf{A} + \nabla\times\mathbf{B}$$

$$\nabla\times\nabla\psi = 0$$
$$\nabla\cdot\left(\nabla\times\mathbf{A}\right) = 0$$

Identidades:

$$\nabla\cdot\nabla\psi = \nabla^2\psi$$
$$\nabla\times\left(\nabla\times\mathbf{A}\right) = \nabla\left(\nabla\cdot\mathbf{A}\right) - \nabla^2\mathbf{A}$$
$$\nabla\cdot(\psi\mathbf{A}) = \left[\mathbf{A}\cdot\nabla\psi\right] + \left[\psi\nabla\cdot\mathbf{A}\right]$$
$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = (\nabla\times\mathbf{A})\cdot\mathbf{B} - \mathbf{A}\cdot(\nabla\times\mathbf{B})$$
$$\nabla\times(\psi\mathbf{A}) = \left[\psi(\nabla\times\mathbf{A})\right] + \left[(\nabla\psi)\times\mathbf{A}\right]$$
$$\nabla^2(\phi\,\psi) = \phi\nabla^2\psi + 2(\nabla\phi\cdot\nabla\psi) + \psi\nabla^2\phi$$

Cálculo Vetorial

Teoremas de cálculo vetorial

Teorema da Divergência:

$$\oiint_S \mathbf{A}\cdot d\mathbf{S} = \iiint_V \nabla\cdot\mathbf{A}\,dV$$

Teorema de Stokes:

$$\oint_C \mathbf{A}\cdot d\mathbf{L} = \iint_S \nabla\times\mathbf{A}\cdot d\mathbf{S}$$

Circulação de um campo conservativo F (campo irrotacional):

Se $\int_P \mathbf{F}\cdot d\mathbf{L}$ independe do caminho P, então $\oint_C \mathbf{F}\cdot d\mathbf{L} = 0$ e $\nabla\times\mathbf{F} = 0$.

Para $\oint_C \mathbf{F}\cdot d\mathbf{L} = 0$ e $\nabla\times\mathbf{F} = 0$, $\mathbf{F} = \nabla\psi$ (pois $\nabla\times\nabla\psi = 0$).

Álgebra complexa e fasores

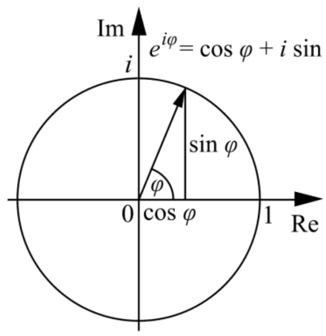
Unidade imaginária: $j^2 = -1$

Complexos conjugados:

$$(\alpha + j\beta)^* = (\alpha - j\beta)$$

$$(\alpha + j\beta)(\alpha + j\beta)^* = \alpha^2 + \beta^2$$

$$\frac{(\sigma + j\omega)}{(\alpha + j\beta)} = \frac{(\sigma\alpha + \omega\beta) - j(\sigma\beta - \omega\alpha)}{\alpha^2 + \beta^2}$$



Fórmula de Euler: $e^{j\varphi} = \cos\varphi + j\,\mathrm{sen}\,\varphi$

Forma polar (fasores):

$$z = \sigma + j\omega = Z e^{j\varphi}$$

$$\begin{cases} \sigma = Z\cos\varphi \\ \omega = Z\,\mathrm{sen}\,\varphi \\ Z = |z| = \sqrt{\sigma^2 + \omega^2} \\ \varphi = \tan^{-1}\left(\omega/\sigma\right) \end{cases}$$

se $\begin{cases} z = (\sigma + j\omega) = Z e^{j\theta} \\ p = (\alpha + j\beta) = P e^{j\phi} \end{cases}$

$$z\cdot p = (\alpha + j\beta)(\sigma + j\omega) = MP\,e^{j(\theta+\phi)}$$

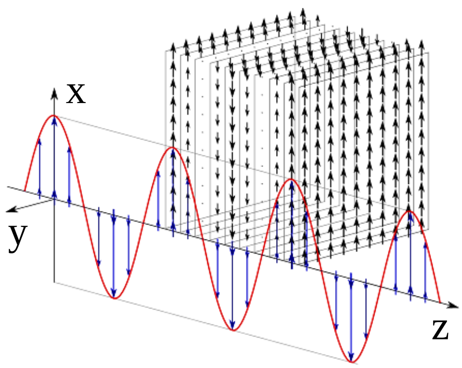
$$\frac{z}{p} = \frac{(\alpha + j\beta)}{(\sigma + j\omega)} = \frac{M}{P}e^{j(\theta-\phi)}$$

Equação de onda

Unidimensional: $$\frac{\partial^2 u(z,t)}{\partial t^2} = c^2\frac{\partial^2 u(z,t)}{\partial z^2}$$

Geral: $$\frac{\partial^2\psi}{\partial t^2} - c^2\nabla^2\psi = 0$$

Solução: qualquer função do tipo $\psi = \psi(z \pm ct)$
(onda viajante em z com velocidade c)



Eletromagnetismo

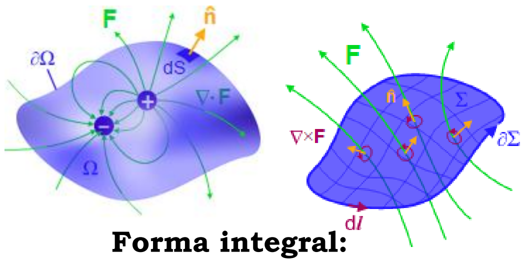
Relações constitutivas

Densidade de corrente de condução (lei de Ohm vetorial): $\mathbf{J} = \sigma \mathbf{E}$ (A/m²)

Densidade de fluxo elétrico (vetor deslocamento elétrico): $\mathbf{D} = \varepsilon \mathbf{E}$ (C/m²)

Densidade de fluxo magnético (vetor indução magnética): $\mathbf{B} = \mu \mathbf{H}$ (T = Wb/m²)

Equações de Maxwell



	Forma diferencial (pontual):	**Forma integral:**
Lei de Gauss: *(cargas elétricas geram campos elétricos)*	$\nabla \cdot \mathbf{D} = \rho_v$	$\oiint_{S=\partial\Omega} \mathbf{D} \cdot d\mathbf{S} = Q$
Lei de Gauss para o magnetismo: *(não existem monopolos magnéticos)*	$\nabla \cdot \mathbf{B} = 0$	$\oiint_{S=\partial\Omega} \mathbf{B} \cdot d\mathbf{S} = 0$
Lei de Faraday-Maxwell: *(campos magnéticos variantes no tempo induzem campos elétricos)*	$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint_{C=\partial\Sigma} \mathbf{E} \cdot d\mathbf{L} = -\dfrac{\partial}{\partial t} \iint_{\Sigma} \mathbf{B} \cdot d\mathbf{S}$
Lei de Ampère-Maxwell: *(campos magnéticos são gerados por corrente elétrica e campos elétricos variantes no tempo)*	$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint_{C=\partial\Sigma} \mathbf{H} \cdot d\mathbf{L} = I + \dfrac{\partial}{\partial t} \iint_{\Sigma} \mathbf{D} \cdot d\mathbf{S}$

nas quais Ω é um volume fechado de fronteira $S = \partial\Omega$ (superfície fechada)

e Σ é uma superfície aberta de fronteira $C = \partial\Sigma$ (curva fechada).

- Carga elétrica total contida em um volume Ω: $Q = \iiint_{\Omega} \rho_v dV$
- Corrente elétrica de condução através de uma superfície Σ: $I = \iint_{\Sigma} \mathbf{J} \cdot d\mathbf{S}$

"Campos elétricos e magnéticos variantes no tempo que não satisfazem a estas equações não podem existir. Eletricidade, magnetismo e luz são todas manifestações dum mesmo fenômeno – o fenômeno eletromagnético."

Princípio da conservação de carga

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho_v}{\partial t} = 0$$

(eq. continuidade, LKC vetorial)

Constantes no vácuo:

$\varepsilon_0 = 8{,}8541878176 \times 10^{-12}\ F/m$

$\mu_0 = 4\pi\ \times 10^{-7}\ H/m$

Eletrostática

Lei de Coulomb

$$\mathbf{E} = \frac{q}{4\pi\varepsilon} \frac{(\mathbf{r_m} - \mathbf{r_q})}{|\mathbf{r_m} - \mathbf{r_q}|^3} \qquad (V/m)$$

Fluxo elétrico e Lei de Gauss

$$\Psi = \oiint_S \mathbf{D} \cdot d\mathbf{S} = Q \qquad (C)$$

(1ª eq. Maxwell)

Energia e Potencial Elétrico

$$W_{AB} = qV_{AB} \qquad (J)$$

$$ddp = V_{AB} = -\int_B^A \mathbf{E} \cdot d\mathbf{L} \qquad (V = J/C)$$

Como $\oint_C \mathbf{E} \cdot d\mathbf{L} = 0$ (campo conservativo, LKT vetorial),

então existe um campo escalar V tal que

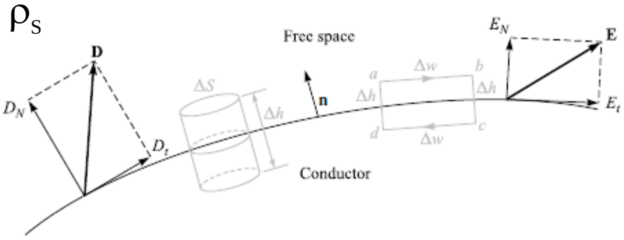
$$\mathbf{E} = -\nabla V$$

(consequência da 3ª eq. Maxwell para $\frac{\partial \mathbf{B}}{\partial t} = 0$)

Condições de fronteira

Condutor-espaço livre – $\begin{cases} D_t = E_t = 0 \\ D_n = \varepsilon_0 E_n = \rho_S \end{cases}$

Dielétrico-dielétrico – $\begin{cases} E_{t1} = E_{t2} \\ D_{n1} = D_{n2} \end{cases}$



Capacitância

$$C = \frac{Q}{V_{AB}} = \frac{\oiint_S \mathbf{D} \cdot d\mathbf{S}}{-\int_{B(-)}^{A(+)} \mathbf{E} \cdot d\mathbf{L}} = cte. \qquad (F = C/V)$$

Equações de Poisson & Laplace

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

$$\nabla^2 V = 0$$

Magnetostática

Lei de Biot-Savart

$$d\mathbf{H} = \frac{I\,d\mathbf{L} \times (\mathbf{r_m} - \mathbf{r_I})}{4\pi |\mathbf{r_m} - \mathbf{r_I}|^3}$$

Superposição: $$\mathbf{H} = \int_C \frac{I\,d\mathbf{L} \times \mathbf{a_R}}{4\pi R^2} \qquad (A/m)$$

Lei Circuital de Ampère

$$\oint_C \mathbf{H} \cdot d\mathbf{L} = I$$

(4ª eq. Maxwell para $\frac{\partial \mathbf{D}}{\partial t} = 0$)

Fluxo magnético e Lei de Gauss para o magnetismo

$$\Phi = \iint_S \mathbf{B} \cdot d\mathbf{S} \qquad (Wb = V \cdot s)$$

$$\oiint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

(2ª eq. Maxwell)

Potencial magnético vetorial

$$\mathbf{A} = \int_C \frac{\mu I\,d\mathbf{L}}{4\pi R} \qquad (V \cdot s/m)$$

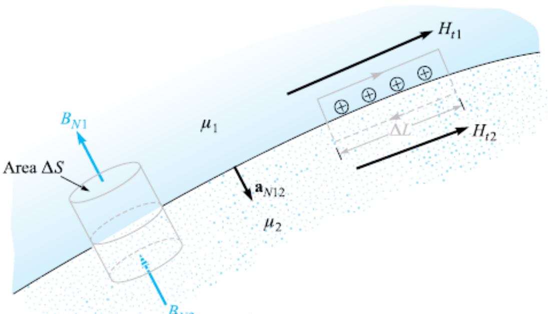
$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\Phi = \oint_{\mathbf{C}} \mathbf{A} \cdot d\mathbf{L}$$

(consequência da 2ª eq. Maxwell)

Condições de fronteira

Entre dois meios – $$\begin{cases} B_{n1} = B_{n2} \\ H_{t1} - H_{t2} = K_n \end{cases}$$



Indutância

$$L = \frac{\Phi}{I} = \frac{\iint_S \mathbf{B} \cdot d\mathbf{S}}{\oint_C \mathbf{H} \cdot d\mathbf{L}} = \text{cte.} \qquad (H = Wb/A)$$

Equação vetorial de Poisson

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

Eletrodinâmica Clássica

Força de Lorentz

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \qquad \text{(N)}$$

Lei de Faraday-Lenz

$$\text{f.e.m.} = -N\frac{d}{dt}\Phi \qquad \text{(V)}$$

Equações de Maxwell no vácuo

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

Acoplamento fundamental entre os campos magnético e elétrico:

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Equações da onda eletromagnética

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - \frac{1}{\mu_0 \varepsilon_0}\nabla^2 \mathbf{E} = 0$$

$$\frac{\partial^2 \mathbf{H}}{\partial t^2} - \frac{1}{\mu_0 \varepsilon_0}\nabla^2 \mathbf{H} = 0$$

Em 3D: $$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\mathbf{E}(x,y,z,t) = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2}\mathbf{E}(x,y,z,t)$$

Onda eletromagnética plana no espaço livre

(campos ctes. em x e y, $k = \omega\sqrt{\mu_0\varepsilon_0} = 2\pi/\lambda$ e $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$, propaga em z com $c_0 = \omega/k = \sqrt{1/\mu_0\varepsilon_0}$)

$$\mathbf{E}(z,t) = E_{x0}e^{j(\omega t - kz)}\,\mathbf{a_x} = E_x(z,t)\,\mathbf{a_x}$$

$$c_0 = 1/\sqrt{\mu_0\varepsilon_0} \approx 3\times 10^8 \text{ m/s}$$

Como $$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} = \frac{\partial E_x}{\partial z}\mathbf{a_y},$$

$$\eta_0 = \sqrt{\mu_0/\varepsilon_0} = \mu_0 c_0 \approx 120\pi\ \Omega$$

$$\mathbf{H}(z,t) = \frac{1}{\eta_0}E_{x0}e^{j(\omega t - kz)}\,\mathbf{a_y} = \frac{1}{\eta_0}E_x(z,t)\,\mathbf{a_y} = H_y(z,t)\,\mathbf{a_y}$$

Vetor de Poynting

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} \qquad (\text{W}/\text{m}^2)$$

$$P_{med} = \frac{1}{T}\int_0^T \left[\iint_S \mathbf{S} \cdot d\mathbf{S}\right] dt$$

Efeitos Eletromagnéticos em Alta Frequência

Efeito pelicular

$$\mathbf{J}(r) = \mathbf{J}_s e^{-r/\delta}$$

Profundidade de penetração – $\delta = \sqrt{\dfrac{2}{\omega\mu\sigma}} = \sqrt{\dfrac{1}{\pi f \mu \sigma}}$

Linhas de transmissão

Equações do telégrafo (p/ correntes e tensões senoidais, LT ao longo de z):

$$\begin{cases} \dfrac{d}{dz}V(z,t) = -R\,I(z,t) - L\dfrac{d}{dt}I(z,t) \\ \dfrac{d}{dz}I(z,t) = -G\,V(z,t) - C\dfrac{d}{dt}V(z,t) \end{cases} = \begin{cases} \dfrac{\partial^2}{\partial z^2}V(z) - \gamma^2 V(z) = 0 \\ \dfrac{\partial^2}{\partial z^2}I(z,t) - \gamma^2 I(z) = 0 \end{cases}$$

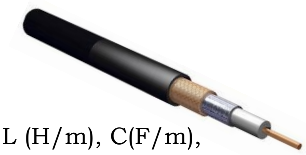
Solução – $V(z) = V_o^{+}e^{-\gamma z} + V_o^{-}e^{+\gamma z}$

Cte. propagação – $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \cong j\omega\sqrt{LC}$

Impedância Característica – $Z_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}} = \dfrac{R + j\omega L}{\gamma} \cong \sqrt{\dfrac{L}{C}}$

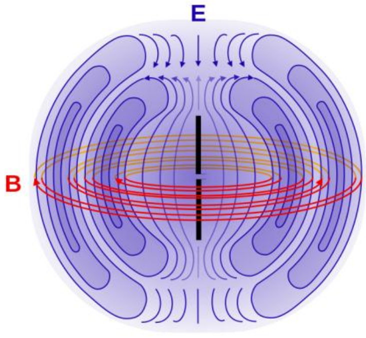
Velocidade de fase – $v_p = \dfrac{\omega}{\beta} \cong \dfrac{1}{\sqrt{LC}}$

Coeficiente de reflexão – $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$



L (H/m), C(F/m), R(Ω/m), G (S/m)

Antenas

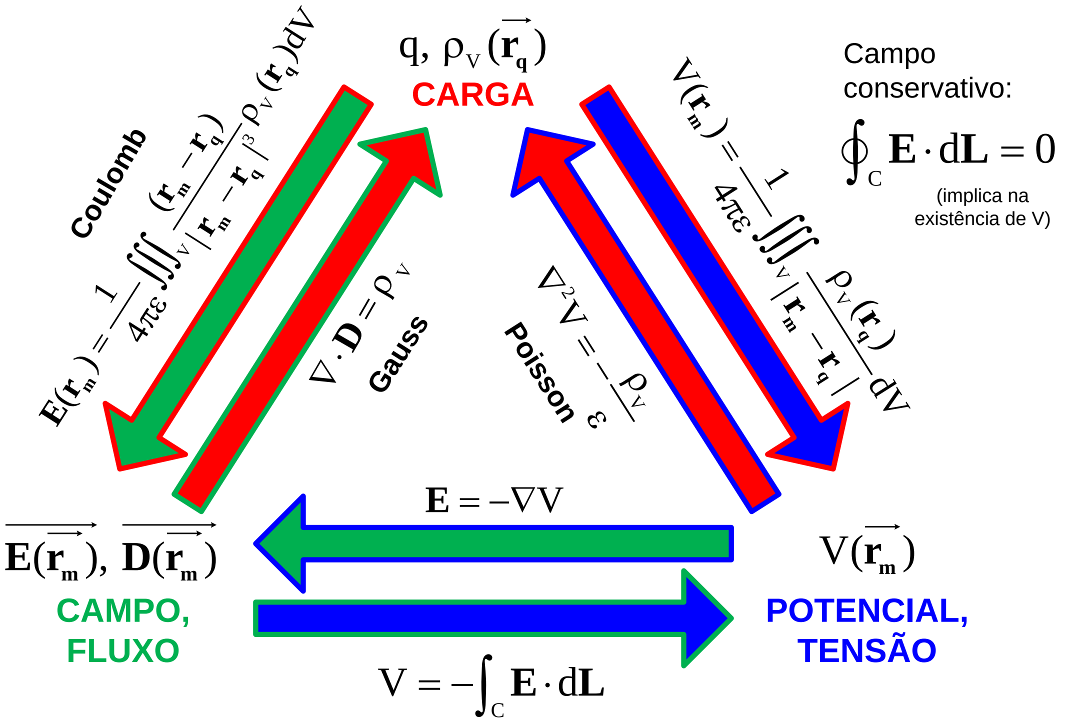


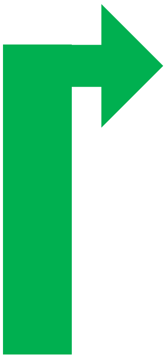
Dipolo Hertziano ($d \ll \lambda$) – $\mathbf{A} = \dfrac{\mu I d}{4\pi R}\mathbf{a}_z$

Solução para campo distante – $\begin{cases} H_\phi = j\dfrac{I\,d\,k^2}{4\pi k r}e^{-jkr}\,\text{sen}\,\theta \\ E_\theta = j\dfrac{I\,d\,k^2}{4\pi k r}\sqrt{\dfrac{\mu_0}{\varepsilon_0}}e^{-jkr}\,\text{sen}\,\theta \end{cases}$

Resistência de radiação – $R_{rad} = 2\dfrac{P_{med}}{I^2} = \eta_0 \dfrac{2\pi}{3}\dfrac{d^2}{\lambda^2}$

Triângulo de Leis da Eletrostática em R^3





$$\nabla\cdot\mathbf{J}=-\frac{\partial\rho_v}{\partial t}$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$$

$$\text{f.e.m.}=-\frac{\partial}{\partial t}\iint_S \mathbf{B}\cdot d\mathbf{S}$$

$$\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}$$

$$\text{f.m.m.}=I+\frac{\partial}{\partial t}\iint_S \mathbf{D}\cdot d\mathbf{S}$$

Triângulo de Leis da Magnetostática em R^3