Lista3

November 6, 2020

1 Lista 3

1.1 Q1

traçando um cilindro para manter a simétria com a linha infinita, podemos descrever que a carga interna da gaussiana é dada por:

$$q_{int} = \int \rho_L dz$$

$$q_{int} = \rho_L z$$

já na lei de gauss

$$\oint \vec{E}\vec{n}dA = \frac{q_{int}}{\epsilon_0}$$

sabendo que a area do cilindro é $A = 2\pi r\ z$ podemos concluir que

$$EA = \frac{\rho_L z}{\epsilon_0}$$

logo

$$E = \frac{\rho_L}{2\pi\epsilon_0 r}$$

uma vez que o fluxo elétrico é definido por $\vec{D} = \epsilon_0 \vec{E}$

$$D = \frac{\rho_L}{2\pi r}$$

2 Q2

relembrando o teorema da divergência

$$\iint_S \vec{D} \cdot \vec{n} dS = \iiint_V \vec{\nabla} \cdot \vec{D} dV$$

$$\vec{\nabla} \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

logo desenvolvendo

$$\begin{cases} \dfrac{\partial D_x}{\partial x} & = \dfrac{\partial yz\cos y^2}{\partial x} = 0 \\ \\ \dfrac{\partial D_y}{\partial y} & = \dfrac{\partial xz\cos x^2}{\partial y} = 0 \\ \\ \dfrac{\partial D_z}{\partial z} & = \dfrac{\partial 1}{\partial z} = 0 \end{cases}$$

como $\vec{\nabla} \cdot \vec{D} = 0$ diz que temos um fluxo nulo, a quantidade que entra é igual ao que sai a corrente é constante.

2.1 Q3

Calculando face a face do cubo pode-se observar que para

$$\begin{cases} \\ x = 0 & \to \displaystyle\int_{atras} = 0 \\ x = 2 & \to \displaystyle\int_{frente} = \int_0^2 \int_0^2 xy^2(ax \cdot ax)dydz = \frac{32}{3} \\ y = 0 & \to \displaystyle\int_{esquerda} = 0 \\ y = 2 & \to \displaystyle\int_{direita} = \int_0^2 \int_0^2 yx^2(ay \cdot ay)dxdz = \frac{32}{3} \end{cases}$$

logo a soma

$$\oint Dds = \frac{64}{3}\ C$$

já pelo outro lado

$$\nabla D = \frac{\partial xy^2}{\partial x} + \frac{\partial yx^2}{\partial y}$$

Resolvendo a integral:

$$\int \nabla D dv = \int_0^2 \int_0^2 \int_0^2 (x^2 + y^2)dxdydz$$

$$\int \nabla D dv = \frac{64}{3}$$

3 Q4

Calculando cada face do cubo

$$\begin{cases} x=-1 & \to \int_{atras} = \int_{-1}^{1}\int_{-1}^{1} 10x^2(ax\cdot ax)dydz = 40 \\ x=1 & \to \int_{frente} = \int_{-1}^{1}\int_{-1}^{1} 10x^2(ax\cdot(-ax))dydz = -40 \\ y=-1 & \to \int_{esquerda} = 0 \\ y=1 & \to \int_{direita} = 0 \end{cases}$$

Logo pode-se ver que o fluxo é nulo

já do outro lado do teorema

$$\nabla D = \frac{\partial 10x^2}{\partial x} + \frac{\partial xy^3}{\partial y} = 20x + 3xy^2$$

$$\int \nabla D dv = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 20x + 3xy^2 dxdydz = 0$$

[1]:
```
from math import *
import numpy as np
import sympy as sp

x = sp.Symbol('x')
y = sp.Symbol('y')
z = sp.Symbol('z')

f = 20*x+3*x*y**2

Dv = sp.integrate(sp.integrate(sp.integrate(f,(x,-1,1)),(y,-1,1)),(z,-1,1))
Dv
```

[1]:

$$0$$

4 Q5

Sabendo que

$$\rho_v = \vec{\nabla}\vec{D}$$

4.0.1 a)

$$\rho_v = \frac{1}{\rho}\frac{\partial(\rho D_\rho)}{\partial \rho}$$

[2]:
```python
p = sp.Symbol('p')

Dp = 53*p**3
pv = (1/p)*sp.diff(p*Dp,p)

pv.subs(p,3)
```

[2]:

$$1908$$

$$\rho_v = 1908 \; C/m^3$$

4.0.2 b)

[3]:
```python
Dp.subs(p,3)
```

[3]:

$$1431$$

$$\vec{D} = 1431\vec{a}_\rho \; C/m^3$$

4.1 c)

$$\psi = \oint \vec{D}dA$$
$$\psi = \int_{-2.5}^{2.5}\int_{-2.5}^{2.5} 53\rho^3 \; \rho d\phi dz$$

[4]:
```python
phi = sp.Symbol("phi")
Dv = 53*p**3

psi = sp.integrate(sp.integrate(Dv*p,(p,-2.5,2.5)),(phi,0,2*pi))
psi
```

[4]:

$$13008.1570812702$$

$$\psi = 13008.157 \; C$$

4.1.1 d)

$$Q_{int} = \psi = 13008.157 \; C$$

4.2 Q6

Sabendo que

$$Q_{int} = \oint \vec{\nabla} \cdot \vec{D} dv$$

primeiro calculo do divergente

$$\vec{\nabla} \cdot \vec{D} = \frac{1}{\rho}\frac{\partial(\rho D_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial(D_\phi)}{\partial \phi} + \frac{\partial(D_z)}{\partial z}$$

[5]:
```
z = sp.Symbol("z")
phi = sp.Symbol("phi")
p = sp.Symbol("rho")

Dp = (-20/p**2)*(sp.sin(phi))**2
Dphi = (20/p**2)*(sp.sin(2*phi))

D = (1/p)*sp.diff(p*Dp,p) + (1/p)*sp.diff(Dphi,phi)
D.simplify()
```

[5]:

$$\frac{20\left(2 - 3\sin^2(\phi)\right)}{\rho^3}$$

tendo o divergente, pode-se qualquer a carga

$$Q_{int} = \int_0^1 \int_0^{\pi/2} \int_1^2 \frac{20(2 - 3\sin^2(\phi))}{\rho^3} \rho d\rho d\phi dz$$

[6]:
```
f = 20*(2-3*(sp.sin(phi))**2)/p**2

Q = sp.integrate(sp.integrate(sp.integrate(f,(p,1,2)),(phi,0,sp.pi/2)),(z,0,1))
Q
```

[6]:

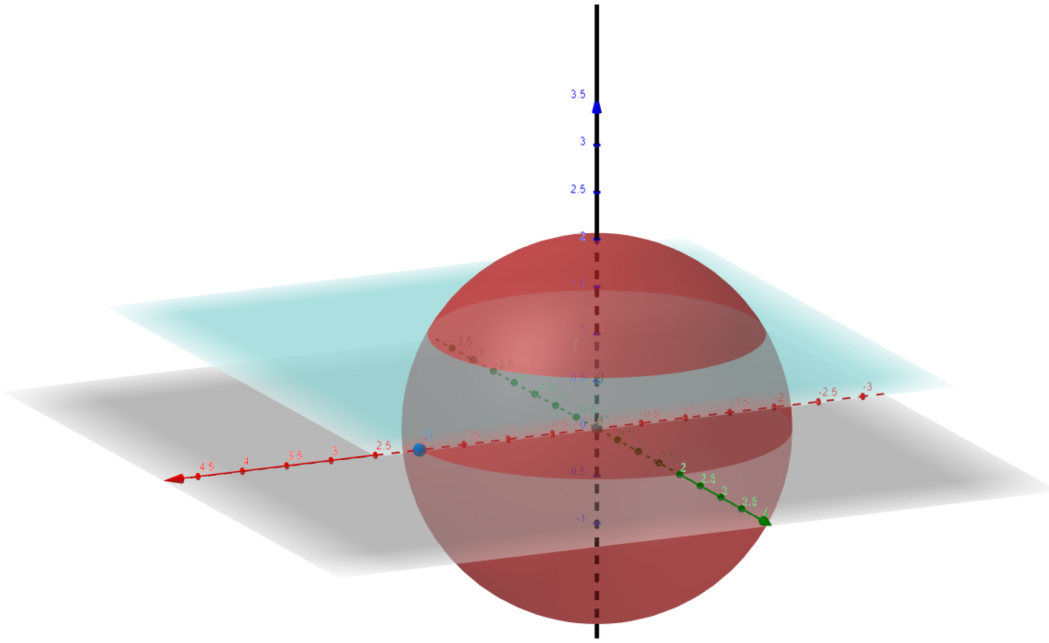
$$\frac{5\pi}{2}$$

$$Q_{int} = \frac{5\pi}{2}\ C$$

4.3 Q7

[7]:
```
from IPython.display import Image
Image("esfera.png")
```

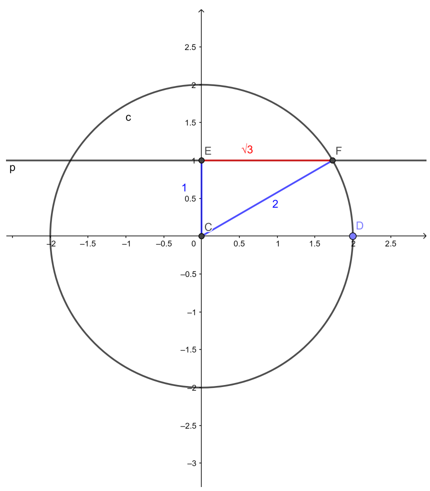
[7]:



Planificando para achar os limites de integração, aplicando geometria básica consegue-se achar o limite de $\sqrt{3}$

[8]:
```
from IPython.display import Image
Image("plano.png")
```

[8]:



Sabe-se que $\psi = Q\_\{int\} = Q\_\{linha\} + Q\_\{plano\}$ e

$$
\begin{cases}
Q_{linha} = \int_{-2}^{2} \rho_L dl = 16\pi \ C \\
\\
Q_{plano} = \iint \rho_s dS = \int_0^{2\pi} \int_0^{\sqrt{3}} 20\rho d\rho d\phi = 60\pi \ C
\end{cases}
$$

logo o fluxo é

$$\psi = Q_{int} = 76\pi \ C$$

4.4 Q8

$$\psi = \oint \vec{D} \ d\vec{s}$$

$$
\begin{cases}
\vec{D} = \dfrac{Q}{4\pi r^2}\vec{a}_r \\
\\
d\vec{s} = r^2 \sin\theta \ d\phi \ d\theta \ \vec{a}_r
\end{cases}
$$

$$\psi = \int_\alpha^\beta \int_0^\pi \frac{Q}{4\pi r^2} r^2 \sin\theta \ d\theta \ d\phi$$

[9]:
```
B = sp.Symbol("beta")
a = sp.Symbol("alpha")
r = sp.Symbol("r")
o = sp.Symbol("theta")
Q = sp.Symbol("Q")
phi = sp.Symbol("phi")

f = (sp.sin(o)*Q)/(4*sp.pi)

psi = sp.integrate(sp.integrate(f,(o,0,sp.pi)),(phi,a,B))
psi.simplify()
```

[9]:

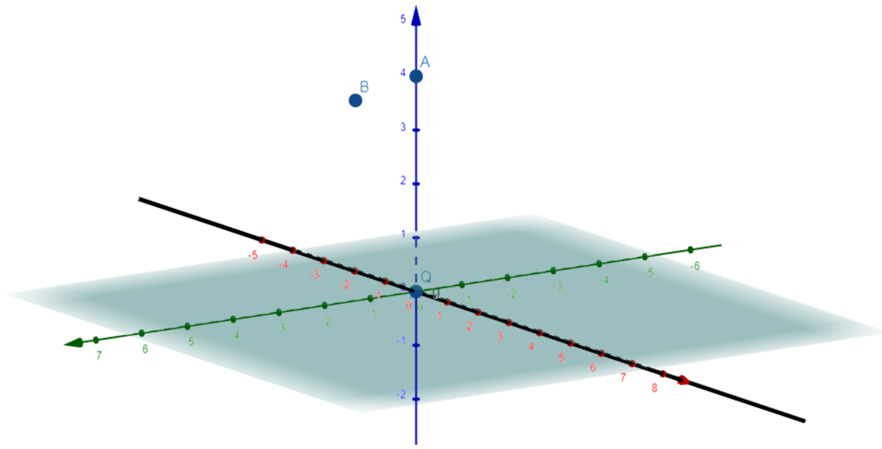
$$\frac{Q\left(-\alpha + \beta\right)}{2\pi}$$

4.5 Q9

4.5.1 a)

[10]:
```
from IPython.display import Image
Image("A.png")
```

[10]:



Considerou-se o fluxo de desindade pelos três elementos de distribuição, tal que :

$$\vec{D} = \vec{D}_{carga} + \vec{D}_{linha} + \vec{D}_{plano}$$

$$\begin{cases} \vec{D}_{carga} = \dfrac{Q}{4\pi r^2}\vec{a}_z \\ \vec{D}_{linha} = \dfrac{\rho_L}{2\pi\rho}\vec{a}_z \\ \vec{D}_{plano} = \dfrac{\rho_s}{2}\vec{a}_z \end{cases}$$

[11]:
```
Q=6e-9
rhoL = 180e-9
rhoS = 25e-9
r = 4
rho = 4

Dq = Q/(4*sp.pi*r**2)

Dl = rhoL/(2*sp.pi*rho)

Dp = rhoS/2
```

```
D = Dq+Dl+Dp

D.evalf(4)
```

[11]:

$$1.969 \cdot 10^{-8}$$

$$\vec{D} = 0.1969\ \vec{a}_z$$

4.5.2 b)

precisa-se arrumar a parte vetorial, temos

$$\begin{cases} \text{Carga} \ \rightarrow \vec{a}_r = \dfrac{\vec{a}_x + 2\vec{a}_y + 4\vec{a}_z}{\sqrt{21}} \\ \\ \text{Linha} \ \rightarrow \vec{a}_\rho = \dfrac{2\vec{a}_y + 4\vec{a}_z}{\sqrt{20}} \\ \\ \text{Plano} \ \rightarrow \vec{a}_n = \vec{a}_z \end{cases}$$

[12]:
```
r = np.array([1,2,4])
R = np.sqrt(sum(r**2))
ar = r/R

p = np.array([0,2,4])
P = np.sqrt(sum(p**2))
ap = p/P

Dq = (Q/(4*pi*R**2))*ar

Dl = (rhoL/(2*pi*P))*ap

Dp = (rhoS/2)*np.array([0,0,1])

D = Dq+Dl+Dp

print("Dq =  {:.3e}ax \t {:.3e}ay \t {:.3e}az ".format(*Dq))
print("Dl = {:.3e}ax \t {:.3e}ay \t {:.3e}az ".format(*Dl))
print("Dp = {:.3e}ax \t {:.3e}ay \t {:.3e}az \n\n".format(*Dp))

print("D = {:.3e}ax \t {:.3e}ay \t {:.3e}az ".format(*D))
```

```
Dq =  4.961e-12ax        9.923e-12ay     1.985e-11az
Dl = 0.000e+00ax         2.865e-09ay     5.730e-09az
Dp = 0.000e+00ax         0.000e+00ay     1.250e-08az
```

```
D = 4.961e-12ax          2.875e-09ay      1.825e-08az
```

4.5.3 c)

$$\begin{cases} \text{Linha } \rightarrow Q_L = \int_{-4}^{4} \rho_L dx = 1440 \ nC \\ \\ \text{Plano } \rightarrow Q_p = \int_{0}^{2\pi} \int_{0}^{4} \rho_s \ \rho d\rho d\phi = 400\pi \ nC \end{cases}$$

$$Q = Q_L + Q_p = 8.7 \ \mu C$$

4.6 Q10

Calculo do divergente

$$\vec{\nabla} \cdot \vec{D} = \frac{1}{\rho}\frac{\partial(\rho D_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial(D_\phi)}{\partial \phi} + \frac{\partial(D_z)}{\partial z}$$

[13]:
```
z = sp.Symbol("z")
p = sp.Symbol("rho")

Dp = 30*sp.exp(-p)
Dz = -2*z

D = (1/p)*(sp.diff(p*Dp,p)) + sp.diff(Dz,z)
D
```

[13]:

$$-2 + \frac{-30\rho e^{-\rho} + 30e^{-\rho}}{\rho}$$

integrando:

$$\int \vec{\nabla} \cdot \vec{D} dv = \int_0^5 \int_0^{2\pi} \int_0^2 \vec{\nabla} \cdot \vec{D} \ \rho d\rho d\phi dz$$

[ ]:
```
```

[14]:
```
d=sp.integrate(sp.integrate(sp.integrate(D*p,(p,0,2)),(phi,0,2*sp.pi)),(z,0,5))
d
```

[14]:

$$10\pi\left(-4 + \frac{60}{e^2}\right)$$

[15]:
```
d.evalf(6)
```

[15]:

129.437

$$\int \vec{\nabla} \cdot \vec{D} dv = 129.437\ C$$

4.7 Q11

4.7.1 a)

[16]:
```python
x = sp.Symbol("x")
z = sp.Symbol("z")
y = sp.Symbol("y")


D = 3*x**2 + 2*z + x**2 *z

Div = sp.Derivative(D,x) + sp.Derivative(D,z)
Div
```

[16]:

$$\frac{\partial}{\partial x}\left(x^2 z + 3x^2 + 2z\right) + \frac{\partial}{\partial z}\left(x^2 z + 3x^2 + 2z\right)$$

[17]:
```python
Div.doit()
```

[17]:

$$x^2 + 2xz + 6x + 2$$

[18]:
```python
Int = sp.Integral(sp.Integral(sp.Integral(Div,(x,-2,2)),(y,-2,2)),(z,-2,2))
Int
```

[18]:

$$\int\limits_{-2}^{2}\int\limits_{-2}^{2}\int\limits_{-2}^{2}\left(\frac{\partial}{\partial x}\left(x^2 z + 3x^2 + 2z\right) + \frac{\partial}{\partial z}\left(x^2 z + 3x^2 + 2z\right)\right) dx\, dy\, dz$$

[19]:
```python
Int.doit().evalf(6)
```

[19]:

$$213.333$$

4.7.2 b)

[20]:
```python
IntB = sp.Integral(sp.Integral(sp.Integral(Div,(x,0,4)),(y,0,4)),(z,0,4))
IntB
```

[20]:

$$\int\limits_{0}^{4}\int\limits_{0}^{4}\int\limits_{0}^{4}\left(\frac{\partial}{\partial x}\left(x^2 z + 3x^2 + 2z\right) + \frac{\partial}{\partial z}\left(x^2 z + 3x^2 + 2z\right)\right) dx\, dy\, dz$$

[22]:

```
IntB.doit().evalf(6)
```

[22]:

$$1749.33$$