lista 4

Eletromagnetismo: Potêncial Elétrico

1 $Q=-20\mu C \longrightarrow P(4,2,0)$ $\qquad W=-Q\cdot E\cdot l$

$E = 2x+8y\,\vec{a_x} + 8x\,\vec{a_y}\ \frac{V}{m} \qquad x^2=8y$

I) Até o ponto:

$dw = 20\times10^{-6}(2x+8y, 8x)\,dx\,\vec{a_x}$

$w = 20\times10^{-6}\left[x^2+8yx\right]\Big|_0^4 = 20\times10^{-6}(16+32y)$

$y=2$

$w = 20\times10^{-6}(\underbrace{16+64}_{80}) \longrightarrow 1{,}600\cdot10^{\cancel{-6}\,-3} = \boxed{1{,}6\,mJ}$

II) na Trajetória:

$x^2=8y \longrightarrow y=\frac{x^2}{8} \longrightarrow dy=\frac{x}{4}\,dx$

$dw = 20\mu(2x+8y, 8x)(dx, \frac{x}{4}dx)$

$dw=20\mu(2x+3x^2)\,dx$

$w=20\mu\int_0^4 2x+3x^2\,dx = 20\mu(\underbrace{16+64}_{80})$

$w=\boxed{1{,}6\,mJ}$

④ $V = \frac{10}{r^2} \operatorname{sen}(\theta)\cos(\phi)$

$\vec{D} = \varepsilon_0 . \vec{E} = \varepsilon_0 (-\nabla V)$

$-\nabla V = -\left(\frac{\partial V}{\partial r}, \frac{1}{r}\frac{\partial V}{\partial \theta}, \frac{1}{r \operatorname{sen}\theta}\frac{\partial V}{\partial \phi}\right) \rightarrow (r, \theta, \phi)$

$-\nabla V = -\left(\frac{-20}{r^3} \operatorname{sen}(\theta)\cos(\phi), \frac{10}{r^3}\cos(\theta)\cos(\phi), \frac{-10}{r^3}\frac{\cancel{\operatorname{sen}(\theta)}\operatorname{sen}(\phi)}{\cancel{\operatorname{sen}(\theta)}}\right)$

$-\nabla V = \left(\frac{20}{r^3}\operatorname{sen}(\theta)\cos(\phi), \frac{-10}{r^3}\cos(\theta)\cos(\phi), \frac{10}{r^3}\operatorname{sen}(\phi)\right)$

a) $\vec{D}$ em $(2, \frac{\pi}{2}, 0)$

$\vec{D} = \varepsilon_0 . \left(\frac{20}{2^3}\operatorname{sen}\left(\frac{\pi}{2}\right)\cos(0), \cancel{\frac{-10}{2^3}\cos\left(\frac{\pi}{2}\right)\cos(0)}, \cancel{\frac{10}{2^3}\operatorname{sen}(0)}\right)$

$\vec{D} = \varepsilon_0 . \left(\frac{\cancel{20}^{5}}{\cancel{8}_{2}}\right) = 8{,}85 . 10^{-12} . \frac{5}{2}\,\vec{a_r}$

$\vec{D} = \boxed{22{,}1\ \vec{a_r}\ pC/m^2}$

B) $Q = 10\mu C \rightarrow A(1, 30°, 120°) \quad B(4, 90°, 60°)$

$W = -Q\int_A^B E\, dl = V_{AB} \longrightarrow W = Q . (V_B - V_A)$

$V = \frac{10}{r^2}\operatorname{sen}(\theta)\cos(\phi)$

④ B)

$V_A = \frac{10}{1^2} \operatorname{sen}(30^\circ)\cos(120^\circ)$

$V_A = 10 \cdot \left(\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right)$

$V_A = -\frac{5}{2} V$

$V_B = \frac{10}{4^2} \operatorname{sen}(90^\circ)\cos(60^\circ)$

$V_B = \frac{10}{16} \cdot 1 \cdot \frac{1}{2}$

$V_B = \frac{5}{16} V$

$W = Q\left(\frac{5}{16} - \left(-\frac{5}{2}\right)\right) =$ $10\mu\,(2,81)$

$W = \boxed{28,1 \mu J}$

⑤ $P_1 = -5\vec{a_z}\ \frac{mC}{m} \longrightarrow (0,0,-2)$

$P_2 = 9\vec{a_z}\ \frac{mC}{m} \longrightarrow (0,0,3)$

V em (0,0,0) $\quad V = \frac{P\,\vec{a_r}}{4\pi\varepsilon_0 r^3}$

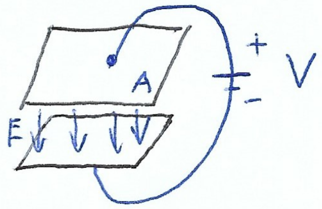
$r_1 = (0,0,0) - (0,0,-2) = 2\vec{a_z}$

$r_2 = (0,0,0) - (0,0,3) = -3\vec{a_z}$

$V = V_1 + V_2 = \frac{\cancel{+}5 \cdot \cancel{2} \cdot 10^{-9}}{4\pi\varepsilon_0 \cdot (\cancel{+}2)^{\cancel{3}2}} + \frac{9 \cdot (\cancel{-3}) \cdot 10^{-9}}{4\pi\varepsilon_0\, 3^{\cancel{3}2}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{5}{4} - \frac{\cancel{9}1}{\cancel{3^2}}\right) \cdot 10^{-9}$

$V = \frac{1}{4\pi\varepsilon_0}(2,25) \cdot 10^{-9} \longrightarrow V = \boxed{20,13\,V}$

(6)



$$C = \frac{\varepsilon_0 A}{d}$$

$$E = \frac{Q}{\varepsilon_0 A} \rightarrow V = E \cdot d \rightarrow E = \frac{V}{d}$$

$$W = \frac{1}{2}\int \varepsilon_0 E^2 dv = \frac{1}{2}\varepsilon_0 \cdot \int \left(\frac{V}{d}\right)^2 dv$$

$$W = \frac{1}{2}\varepsilon_0 \left(\frac{V}{d}\right)^2 \int dv \qquad dv = A \cdot d$$

$$W = \frac{1}{2}\varepsilon_0 \left(\frac{V^2}{d^{\cancel{2}}}\right) \cdot A \cdot \cancel{d} = \frac{1}{2}\frac{\varepsilon_0 V^2 \cdot A}{d} \qquad C = \frac{\varepsilon_0 A}{d}$$

$$W = \boxed{\frac{1}{2} C V^2}$$

(8) $r_A = 5m$ $\quad Q = 500pC$ $\quad (0,0,0)$

$r_B = 15m$ $\quad r(\infty) \rightarrow V = 0$

$$V(r=5) = \frac{Q}{4\pi\varepsilon_0 r} = \frac{500p}{4\pi \cdot 8{,}85 \cdot 5} = 0{,}89V$$

$$V(r=15) = \frac{Q}{4\pi\varepsilon_0 r} = \frac{500p}{4\pi \cdot 8{,}85 \cdot 15} = 0{,}30V$$

$$V_{AB} = V_A - V_B = 0{,}89 - 0{,}30 \approx \boxed{0{,}6\,V}$$

9. $\phi = 0 \quad 0 \leqslant \phi \leqslant \frac{\pi}{6}$

$\phi = \frac{\pi}{6} \quad V = -\frac{60\phi}{\pi}$

$0{,}1 \leqslant r \leqslant 0{,}6\ m$

$0 \leqslant z < 1\ m$

$$\vec{E} = -\nabla V = -\left(\frac{1}{\rho}\frac{\partial \rho D_\rho}{\partial \rho}, \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi}, \frac{\partial D_z}{\partial z}\right)$$

$$\vec{E} = -\left(0 + \frac{1}{\rho}\left(\frac{-60}{\pi}\right) + 0\right) \longrightarrow \vec{E} = \frac{60}{\pi r} a_\phi$$

$$w = \frac{1}{2}\int E \cdot D\, dv \qquad dv = d\rho \cdot \rho \cdot d\phi \cdot dz \qquad \rho = r$$

$$w = \frac{1}{2}\varepsilon_0 \int_0^1 \int_0^{\frac{\pi}{6}} \int_{0,1}^{0,6} \left(\frac{60}{\pi r}\right)^2 r\, dr\, d\phi\, dz$$

$$w = \frac{1}{2}\varepsilon_0 \cdot z\Big|_0^1 \cdot \phi\Big|_0^{\frac{\pi}{6}} \cdot \left(\frac{60}{\pi}\right)^2 \int_{0,1}^{0,6} \frac{1}{r}\, dr$$

$$w = \frac{1}{2}\varepsilon_0 \cdot \frac{\pi}{6} \cdot \left(\frac{60}{\pi}\right)^2 \cdot \ln\left(\frac{0{,}6}{0{,}1}\right) = \frac{\cancel{\pi}\, 60 \cdot 60}{2 \cdot 6 \cdot \pi \cancel{\pi}} \ln(6)$$

$$w = \boxed{1{,}51\, mJ}$$

(10) $r_1 = 0{,}01\,m$ $\quad E = \frac{10^5}{r} a_r \frac{V}{m}$

$r_2 = 0{,}05\,m$

$d = 0{,}5\,m$

$$w = \frac{1}{2}\int D\,E\,dv = \frac{1}{2}\int \varepsilon_0 E^2 dv = \frac{1}{2}\int \left(\frac{105}{r}\right)^2 r\,dr\,d\phi\,dz$$

$$w = \frac{\varepsilon_0}{2}\int_0^{0,5}\int_0^{2\pi}\int_{0,01}^{0,05} \frac{10^{2.5}}{r}\,dr\,d\phi\,dz = \frac{\varepsilon_0}{2} \cdot z\Big|_0^{0,5} \cdot \phi\Big|_0^{2\pi} \cdot 10^{10} \ln(r)\Big|_{0,01}^{0,05}$$

$$w = \frac{\varepsilon_0}{\cancel{2}} \cdot 0{,}5 \cdot \cancel{2}\pi \cdot 10^{10} \cdot \ln\left(\frac{0{,}05}{0{,}01}\right) = \varepsilon_0 \cdot 0{,}5\pi \cdot 100 \cdot 1{,}61$$

$w = \boxed{0{,}22\,J}$

(11) $\rho_l = 20\,pC/m$ $\quad (0,5,12) \rightarrow$ superfície

linha: $(x,0,0)$

em $(2,4,-4)$

$R_A = (x-2, 0-4, 0+4) \rightarrow x=2 \quad |R_A| = \sqrt{16+16} = 2\sqrt{2}$

$R_B = (x-0, 0-5, 0-12) \rightarrow x=0 \quad |R_B| = \sqrt{25+144} = 13$

$$V_{AB} = -\int_{r_B}^{r_A} \frac{\rho_l\,dr}{2\pi\varepsilon_0 r} = -\frac{\rho_l}{2\pi\varepsilon_0}\ln\left(\frac{r_A}{r_B}\right) = -\frac{\overset{10}{\cancel{20p}}}{\cancel{2}\pi \cdot 8{,}85 \cdot \cancel{10^{-12}}} \cdot \ln\left(\frac{2\sqrt{2}}{13}\right)$$

$V_{AB} = \boxed{0{,}54\,V}$