Computing Method - Chapter2&5

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在(2.33)式中，令 $n=2$，求 $f'(x_0), f'(x_1), f'(x_2)$.

解: 令 $x=x_0+th, x_k=x_0+kh(k=0,1,2)$，

$$\varphi_2(x)=f_0+t\Delta f_0+\frac{t(t-1)}{2}\Delta^2 f_0=f_0+\frac{x-x_0}{h}(f_1-f_0)+\frac{(x-x_0)(x-x_0-h)}{2h^2}(f_2-2f_1+f_0)$$

所以

$$\varphi_{2'}(x)=\frac{f_1-f_0}{h}+\frac{f_2-2f_1+f_0}{2h^2}(2x-2x_0-h)$$

代入 x_0, x_1, x_2 得

$$\begin{cases} f'(x_0)=\frac{f_1-f_0}{h}-\frac{f_2-2f_1+f_0}{2h}=\frac{4f_1-f_2-3f_0}{2h} \\ f'(x_1)=\frac{f_1-f_0}{h}+\frac{f_2-2f_1+f_0}{2h}=\frac{f_2-f_0}{2h} \\ f'(x_2)=\frac{f_1-f_0}{h}+\frac{3(f_2-2f_1+f_0)}{2h}=\frac{-4f_1+f_0+3f_2}{2h} \end{cases}$$

□

补充题

求 x_1, x_2, A_1, A_2，满足

$$\int_0^1 e^x f(x)\mathrm{d}x=A_1 f(x)+A_2 f(x_2)$$

具有最高代数精度（精确到 0.0001）

解:

$$\begin{cases} \int_0^1 e^x \mathrm{d}x=e-1=A_1+A_2 \\ \int_0^1 xe^x \mathrm{d}x=1=A_1x_1+A_2x_2 \\ \int_0^1 x^2e^x \mathrm{d}x=e-2=A_1x_1^2+A_2x_2^2 \\ \int_0^1 x^3e^x \mathrm{d}x=6-2e=A_1x_1^3+A_2x_2^3 \end{cases}$$

求解的 python 代码如下：

```python
from scipy.optimize import fsolve
import numpy as np

def equations(vars):
    A1, A2, x1, x2 = vars

    e_minus_1 = np.exp(1) - 1
    integral_x_ex = 1
```

```
    integral_x2_ex = np.exp(1) - 2
    integral_x3_ex = 6 - 2 * np.exp(1)

    eq1 = A1 + A2 - e_minus_1
    eq2 = A1 * x1 + A2 * x2 - integral_x_ex
    eq3 = A1 * x1**2 + A2 * x2**2 - integral_x2_ex
    eq4 = A1 * x1**3 + A2 * x2**3 - integral_x3_ex

    return np.array([eq1, eq2, eq3, eq4])

initial_guesses = [0.5, 0.5, 0.25, 0.75]

result = fsolve(equations, initial_guesses)
print(f"Results: A1 = {result[0]}, A2 = {result[1]}, x1 = {result[2]}, x2 = {result[3]}")

# Results: A1 = 0.7131482910221459, A2 = 1.005133537436899, x1 = 0.24760397941963827, x2
= 0.8192161683583419
```

所以，

$$\begin{cases} A_1 = 0.7131 \\ A_2 = 1.0051 \\ x_1 = 0.2476 \\ x_2 = 0.8192 \end{cases}$$

$\square$