Final Project – DCT Design

● 設計描述

設計一個 2-D 8-point Discrete Cosine Transform (DCT)，1-D DCT 猶如一個矩陣運算，下面即是設計描述：

$$Z(n) = \sqrt{\frac{2}{N}} c(n) \sum_{m=0}^{N-1} x(m) \times \cos\left(\frac{(2m+1)n\pi}{2N}\right)$$

where

$$C(n) = \begin{cases} 1/\sqrt{2} & \text{for } n = 0 \\ 1 & \text{for } others \end{cases}$$

利用矩陣展開

$$Z = \begin{bmatrix} Z_0 \\ Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \\ Z_7 \end{bmatrix} = \begin{bmatrix} \cos 4\theta & \cos 4\theta & \cos 4\theta & \cos 4\theta & \cos 4\theta & \cos 4\theta & \cos 4\theta & \cos 4\theta \\ \cos\theta & \cos 3\theta & \cos 5\theta & \cos 7\theta & -\cos 7\theta & -\cos 5\theta & -\cos 3\theta & -\cos\theta \\ \cos 2\theta & \cos 6\theta & -\cos 6\theta & -\cos 2\theta & -\cos 2\theta & -\cos 6\theta & \cos 6\theta & \cos 2\theta \\ \cos 3\theta & -\cos 7\theta & -\cos\theta & -\cos 5\theta & \cos 5\theta & \cos\theta & \cos 7\theta & -\cos 3\theta \\ \cos 4\theta & -\cos 4\theta & -\cos 4\theta & \cos 4\theta & \cos 4\theta & -\cos 4\theta & -\cos 4\theta & \cos 4\theta \\ \cos 5\theta & -\cos\theta & \cos 7\theta & \cos 3\theta & -\cos 3\theta & -\cos 7\theta & \cos\theta & -\cos 5\theta \\ \cos 6\theta & -\cos 2\theta & \cos 2\theta & -\cos 6\theta & -\cos 6\theta & \cos 2\theta & -\cos 2\theta & \cos 6\theta \\ \cos 7\theta & -\cos 5\theta & \cos 3\theta & -\cos\theta & \cos\theta & -\cos 3\theta & \cos 5\theta & -\cos 7\theta \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}$$

$where \quad \theta = \pi/16$

2-D DCT 運算為

$$Y(u,v) = \frac{1}{4} C(u) C(v) \sum_{i=0}^{7} \sum_{j=0}^{7} X(i,j) \times \cos\left(\frac{(2i+1)u\pi}{16}\right) \cos\left(\frac{(2j+1)v\pi}{16}\right)$$

Peak signal-to-noise ratio

PSNR is most easily defined via the mean squared error (*MSE*). Given a noise-free $m \times n$ monochrome image I and its noisy approximation K, *MSE* is defined as

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [I(i,j) - K(i,j)]^2.$$

The PSNR (in dB) is defined as

$$\begin{aligned} PSNR &= 10 \cdot \log_{10} \left(\frac{MAX_I^2}{MSE} \right) \\ &= 20 \cdot \log_{10} \left(\frac{MAX_I}{\sqrt{MSE}} \right) \\ &= 20 \cdot \log_{10}(MAX_I) - 10 \cdot \log_{10}(MSE). \end{aligned}$$

https://en.wikipedia.org/wiki/Peak_signal-to-noise_ratio

1. Requirement: 100MHz, PSNR=40dB, area -> as small as possible.
2. Input 8-bit (w/o signed bit); output 16-bit (w/ signed bit)
3. Please finish this DCT before 2023/1/5, and give a presentation on the class of 2022/1/5.
4. A technique report about this filter is also needed. Please upload the report to E-learning system before 2023/1/12.