Project A: An optical ray tracer

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Abstract

An optical ray tracer was designed and used to study different optical systems quantitatively. Rays were propagated through a spherical refracting surface and two different orientations of a plano-convex lens. Ray bundles, initial ray spot diagrams and focused output spot diagrams were plotted. The RMS values for the focused output spot diagrams and the diffraction scales of the plano-convex lens were calculated and compared. It was found that some RMS values were less than their corresponding diffraction scales in both orientations of the plano-convex lens. Therefore, both orientations are diffraction limited below a certain value of beam diameter.

Introduction

The wave theory of light, also known as Huygens' principle was discovered by Christiaan Huygens in the 1700s [1]. The Huygens' principle showed that light propagates in the form of wave and this is the basis of many properties of light, such as reflection, refraction, diffraction and interference [2]. The aim of this project is to design an optical ray tracer and use it to investigate optical systems quantitatively. Snell's law is also known as the law of refraction. It describes the change in direction of the light ray when it travels from a medium of refractive index, n_1 to another medium of refractive index, n_2. Snell's law in scalar form is given by

$$n_1 \cdot \sin\theta_1 = n_2 \cdot \sin\theta_2 \,, \tag{1}$$

where n_1 is the refractive index of the medium where the ray is coming from, n_2 is the refractive index of the medium where the ray is going into, θ_1 is the angle between the incident ray and the normal, and θ_2 is the angle between the refracted ray and the normal.

When a ray goes from one medium into another medium that has a larger refractive index, n_2, compared to n_1, the angle, θ_2 becomes smaller and the ray moves at a slower wave speed. So, the ray bends towards the normal. When the medium that the ray goes into has a smaller refractive index, n_1, compared to n_2, the ray moves at a faster wave speed and the ray bends away from the normal. Lenses usually have two spherical surfaces. When rays parallel to the optical axis are propagated towards a convex lens, the rays refract and converge to the focal point behind the lens. A real image is formed at the focal point. For a concave lens, the parallel rays diverge after refraction. The distance between the lens and the focal point is the focal length, f [3].

The refracted ray, t is split up into two components, the tangential component, $t_{\parallel}$ and the normal component $t_{\perp}$, and $t = t_{\parallel} + t_{\perp}$. The tangential component can be written as:

$$t_{\parallel} = \frac{n_1}{n_2}[\,\hat{k} + \cos\theta_1\hat{n}\,]\,, \tag{2}$$

where $\hat{k}$ is the incident direction, $\hat{n}$ is the surface normal, and $\cos\theta_1$ is $(-\hat{n} \cdot \hat{k}\,)$.The normal component is given by:

$$t_{\perp} = -\sqrt{1 - t_{\parallel}^2}\,\hat{n}\,. \tag{3}$$

The vector form of the Snell's law is given by:

$$t = \frac{n_1}{n_2}\hat{k} + \left(\frac{n_1}{n_2}\left(-\hat{n}\cdot\hat{k}\right) - \sqrt{1 - \frac{{n_1}^2}{n_2}\left(1 - \left(-\hat{n}\cdot\hat{k}\right)^2\right)}\right)\hat{n}\,, \tag{4}$$

where t is the refracted ray [4]. Critical angle, θ_c is the angle of the incident ray when the refracted ray exits the medium at $\theta_2 = 90°$. The refracted ray will be tangent to the surface. Total internal reflection occurs when $\theta_1 > \theta_c$. So, when $\sin\theta_1 > \frac{n_2}{n_1}$, total internal reflection occurs [3].

Experimental Method

A class Ray with a constructor and 4 different methods was created. It takes the parameters p, which is the starting point of the ray and k, which is the direction of the ray. This class generates a ray and updates the p value as well as storing them in a list.

A base class, OpticalElement was created, with two derived classes, SphericalRefraction class and OutputPlane class. The ray was made to intersect with a lens and underwent refraction in the SphericalRefraction class. It was then output onto a plane in the OutputPlane class. There were two main methods in the SphericalRefraction class, which were intercept and refraction. This method was created to produce an intersection point between the ray and the surface of the sphere. 'If' statements were used to account for 1, 2 or no intersection points. Equation (4) was used in the refraction method. This method was made to refract the ray after its intersection. An aperture radius condition was added so that refraction doesn't occur if the ray missed the lens.

A separate PlanoConvexLens class was created to allow a spherical surface refraction and a plane refraction. A Planerefraction class was created to allow plane refraction as the surface normal of a spherical surface is different to the surface normal of a plane surface. The output plane was placed at the paraxial focus, where the paraxial ray was taken to be at 0.1mm from the optical axis.

The code from the genpolar tutorial task was imported into this module. A starting point for the ray bundle was defined in a function. A for loop was used to plot ray bundles, initial spot diagrams and focused output spot diagrams for the 2 configurations of a plano-convex lens. The RMS values at the paraxial focus and the diffraction scales were calculated for different focused output spot diagrams.

Results and Discussion

A model of a spherical refracting surface was successfully created. The ray bundle propagation, initial ray spot diagram and focused output spot diagram were plotted.

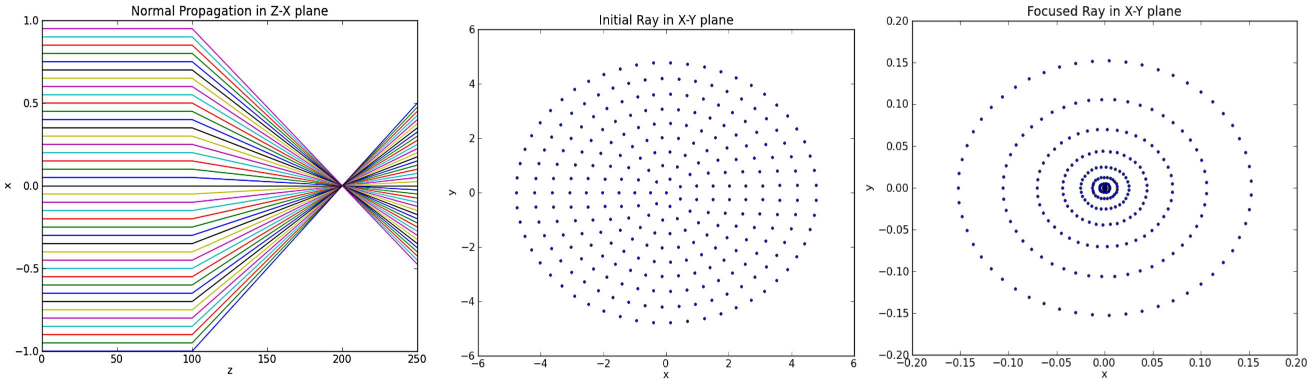

Figure 1: Plots for ray propagation in the z-x plane, initial ray spot diagram and focused output spot diagram in the x-y plane for a spherical refracting surface.

The root mean square (RMS) value for the focused output spot diagram was calculated to be 0.054. The rays were also propagated from an angle and the RMS value for the focused output spot diagram was calculated to be 0.105. Spherical aberration was observed in the ray propagation plots. This occurred because rays coming into the spherical refracting surface do not converge at the same point [5].

A plano-convex lens model was also created successfully. The refraction of a plano-convex lens was investigated by propagating the ray through the plane surface after the spherical surface, and vice versa. The ray bundle propagation, initial ray spot diagram and focused output spot diagram for both orientations were plotted.

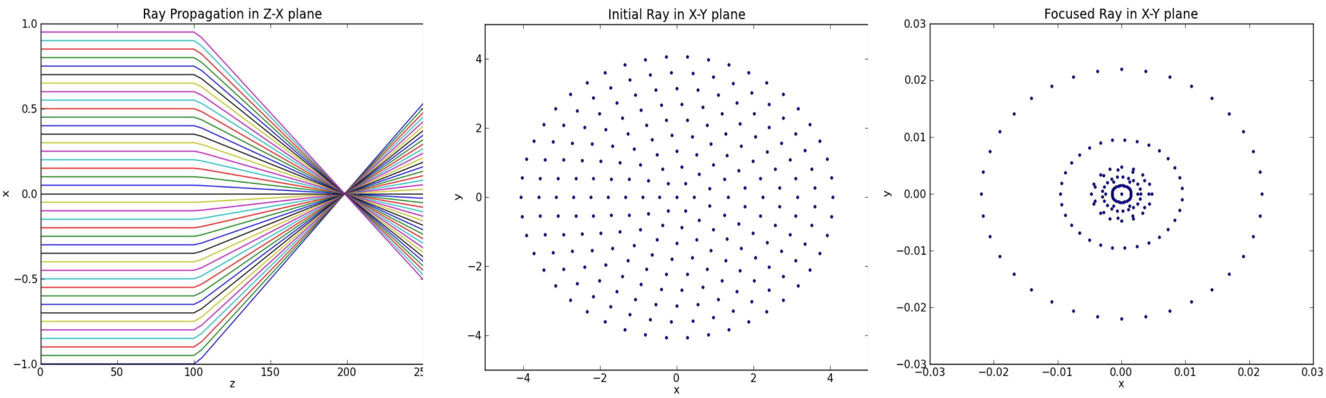

Figure 2: Plots for ray propagation in z-x plane, initial ray spot diagram and focused output spot diagram in the x-y plane for a curve-plane plano-convex lens.

The rays in both orientations converge. However, the refraction through the plane-curve orientation is similar to that of a normal spherical surface refraction. The paraxial focus of the plane-curve configuration was observed to be further than the paraxial focus of the curve-plane configuration of the plano-convex lens. Spherical aberrations were observed in all of the ray bundles plotted.

The diffraction scale was calculated using a formula, $\frac{\lambda f}{D}$, where λ is the wavelength ($\lambda = 588nm$), f is the focal length and D is the aperture diameter, but in this case, it is the beam diameter. The diffraction scale was calculated for both orientations of the plano-convex lens. A table of beam diameter, focal length, diffraction scale and RMS value was made for each orientation. It was found that there is a quadratic relationship between the beam diameter and the RMS value. A comparison was made between the diffraction scales and the RMS values for each case. When the RMS values are less than the values for diffraction scale, the lens is said to be diffraction limited. Both orientations of the lens are diffraction limited below a certain value of beam diameter [5].

Conclusion

The aim of this project is to use an optical ray tracer to investigate different optical systems quantitatively. A spherical refracting surface and a plano-convex lens were investigated by propagating a bundle of rays through them. Comparisons were made in between the two different optical systems after the RMS values and the diffraction scales were calculated.

Spherical aberrations occur in lenses made with spherical surfaces. There are two types of spherical aberrations; a transverse spherical aberration and a longitudinal spherical aberration. A lens can be optimized to reduce the effects of spherical aberration by changing its shape. A biconvex lens is the optimum shape to use in an optical system as it reduces most of the spherical aberration effects [6].

The diffraction limit is where diffraction starts to affect an image formation. An Airy disk is formed around a focused spot of light due to diffraction. The larger the aperture diameter, the smaller the Airy disk, the greater the resolution. However, there is a limit to how large the aperture diameter can be increased for a greater resolution. After that limit, the resolution remains the same regardless of any increase in the aperture diameter. Therefore, an optical system where the resolution is only limited by diffraction is a diffraction-limited system [3][7]. It is concluded that both configurations of a plano-convex lens are diffraction limited below a certain beam diameter.

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