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CSC 8830 Computer Vision

Homework Assignment 1

Note: **I followed the videos camera calibration, Intrinsic and Extrinsic parameters under lecture 5: Camera Matrix and Calibration as reference**

Part A: Theory

Correspondences between 3D scene points and image points:
(x,y,z) are the coordinate system for the 3D to be images

$$3D: \; x_w = \begin{pmatrix} x_w \\ y_w \\ z_w \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

$$2D: \; u = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \\ \end{pmatrix}$$

Matrix form of point i in 3d and 2d:

$$\begin{pmatrix} u^{(i)} \\ v^{(i)} \\ 1 \end{pmatrix} = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{pmatrix} \begin{pmatrix} x_{w^{(i)}} \\ y_{w^{(i)}} \\ z_{w^{(i)}} \\ 1 \end{pmatrix}$$

Matrix in linear equation form:

$$u^{(i)} = \frac{p_{11}x_{w^{(i)}} + p_{12}y_{w^{(i)}} + p_{13}z_{w^{(i)}} + p_{14}}{p_{31}x_{w^{(i)}} + p_{32}y_{w^{(i)}} + p_{33}z_{w^{(i)}} + p_{34}}$$

$$v^{(i)} = \frac{p_{21}x_{w^{(i)}} + p_{22}y_{w^{(i)}} + p_{23}z_{w^{(i)}} + p_{24}}{p_{31}x_{w^{(i)}} + p_{32}y_{w^{(i)}} + p_{33}z_{w^{(i)}} + p_{34}}$$

World Coordinates

$$\mathbf{{}_o x_w} = \begin{pmatrix} x_w \\ y_w \\ z_w \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ 6 \\ 0 \end{pmatrix} \begin{pmatrix} 9 \\ 2 \\ 0 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix}$$

Image Coordinates

$$\mathrm{u} = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 618 \\ 686 \end{pmatrix} \begin{pmatrix} 274 \\ 288 \end{pmatrix} \begin{pmatrix} 908 \\ 914 \end{pmatrix} \begin{pmatrix} 310 \\ 308 \end{pmatrix} \begin{pmatrix} 432 \\ 336 \end{pmatrix}$$

$$\begin{pmatrix} 1074 \\ 416 \end{pmatrix} \begin{pmatrix} 596 \\ 270 \end{pmatrix} \begin{pmatrix} 918 \\ 442 \end{pmatrix}$$

Intrinsic and Extrinsic parameters

Linear perspective transforms as a matrix product

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{pmatrix} = \underbrace{\begin{pmatrix} f/s_x & 0 & 0_x & 0 \\ 0 & f/s_y & 0_y & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{M_{int}} \underbrace{\begin{pmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{M_{ext}}$$

M_{int} : from camera to image
M_{ext} : from world to camera frame

The matrix becomes 3 x 3 , therefore M_{int} only depends on the intrinsic parameters.

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} = \begin{pmatrix} f/s_x & 0 & 0_x \\ 0 & f/s_y & 0_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} = KR$$

K: upper right triangular matrix
R: is orthogonal: ($R^TR = RR^T = I$)

Take the last column of the image point p with the M_{int} parameters times the 3d translation of vector:

$$\begin{pmatrix} p_{14} \\ p_{24} \\ p_{34} \end{pmatrix} = \begin{pmatrix} f/s_x & 0 & 0_x \\ 0 & f/s_y & 0_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix} = KT$$

K: upper right triangular matrix
T: 3d translation vector T = $[x, y, z]^T$

Next step is to find the translation vector by doing the inverse:

$$T = K^{-1} \begin{pmatrix} p_{14} \\ p_{24} \\ p_{34} \end{pmatrix}$$

Intrinsic Matrix

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}$$

$$P = \begin{pmatrix} -0.9250 & 0.3800 & 0.0027 \\ -0.3800 & -0.9249 & -0.0094 \\ -0.0011 & -0.0097 & 1.0000 \end{pmatrix}$$

Extrinsic Matrix

$$P = \begin{pmatrix} 0.1695 & -0.0293 & 0.1304 & -0.9762 \\ 0 & 0.0042 & 0.0074 & -0.0198 \\ 0 & 0 & 5.5713e-05 & -1.3990e-05 \end{pmatrix}$$

Note: **I followed the videos camera calibration, Intrinsic and Extrinsic parameters under lecture 5: Camera Matrix and Calibration as reference and to get the rotation matrix I went on Matlab and used a script to get the matrices before the rotation matrix and the angles .**

7	5	0	1	0	0	0	0	-4326	-3090	0	-618
0	0	0	0	7	5	0	1	-1918	-1370	0	-274
1	3	4	1	0	0	0	0	-908	-2724	-3632	-908
0	0	0	0	1	3	4	1	-310	-930	-1240	-310
2	6	8	1	0	0	0	0	-864	-2592	-3456	-432
0	0	0	0	2	6	8	1	-672	-2016	-2688	-336
9	2	0	1	0	0	0	0	-9666	-2148	0	-1074
0	0	0	0	9	2	0	1	-3744	-832	0	-416
5	3	1	1	0	0	0	0	-2980	-1788	-596	-596
0	0	0	0	5	3	1	1	-1350	-810	-270	-270
0	4	3	1	0	0	0	0	0	-3672	-2754	-918
0	0	0	0	0	4	3	1	0	-1768	-1768	-442

_A^T **12 x 12 matrix**

A are the variables from the video and T is the transpose

7	0	1	0	2	0	9	0	5	0	0	0
5	0	3	0	6	0	2	0	3	0	4	0
0	0	4	0	8	0	0	0	1	0	3	0
1	0	1	0	1	0	1	0	1	0	1	0
0	7	0	1	0	2	0	9	0	5	0	0
0	5	0	3	0	6	0	2	0	3	0	4
0	0	0	4	0	8	0	0	0	1	0	3
0	1	0	1	0	1	0	1	0	1	0	1
-4326	-1918	-908	-310	-864	-672	-9666	-3744	-2980	-1350	0	0
-3090	-1370	-2724	-930	-2592	-2016.	-2148	-832	-1788	-810	-3672	-1768
0	0	-3632	-1240	-3456	-2688	0	0	-596	-270	-2754	-1768
-618	-274	-908	-310	-432	-336	-1074	-416	-596	-270	-918	-442

$\mathbf{A}^{\mathbf{T}} \times \mathbf{A}$ 12 x 12 Matrix

This is the matrix that shows the multiplication of both matrices above

8644375	12699900	12906335	4406340	12013965	9344160	49116242	19024512	18784779	8509860.	11913825	57362
2699900.	5630775	5722216	1953643	5326560	4142925	21776424	8434890	8328504	3773031	5282172	25
2906335	5722216	22260555	7599960	20789621	16169664.	15603088	6043648	10282211	4658040	20838625	116
406340	1953643	7599960	2594727	7097760	5520533	5327040	2063376	3510440	1590319	7114500	39
2013965	5326560	20789621	7097760	19595625	15240960	14383039	5571072	9526501	4315680	19432273	10
344160	4142925	16169664	5520533	15240960	11854185	11186784	4333087	7409472	3356677	15113952	846
9116242	21776524	15603088	5327040	14383039	11186784	99199022	38423424	33285460	15078960	8873397	42723
9024512	8434890	6043648	2063376	5571072	4333087	38423424	14882902	12892672	5840692	3436992	16
8784779	8328504	10282211	3510440	9526501	7409472	33285460	12892672	12787812	5793120	8754064	447
509860	3773031	4658040	1590319	4315680	3356677	15078960	5840692	5793120	2624436	3965760	20
1913825	5282172	20838625	7114500	19432273	15113952	8873397	3436992	8754064	3965760	21910850	1176
736276	2543289	11638744	3973605	10883808	8465233	4272372	1654857	4478344	2028796	11766924	6447

Rotation Matrix

r_A = Type equation here.

$$\begin{pmatrix} rx \\ ry \\ rz \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

$$= \begin{pmatrix} -0.9250 & 0.3800 & 0.0027 \\ -0.3800 & -0.9249 & -0.0094 \\ -0.0011 & -0.0097 & 1.0000 \end{pmatrix}$$

Angles

θ_x = atan2 (r_{32}, r_{33})

θ_x = atan2 (-0.0097,1.0000)

= 90.55 degrees

θ_y = atan2 ($-r_{31,}\sqrt{r^2{}_{32} + r^2{}_{33}}$)

θ_y = atan2 ($-0.0011, \sqrt{r^2{}_{-0.0097} + r^2{}_{1.0000}}$

θ_y = atan2 ($-0.0011, \sqrt{-0.00009409 + 1}$

θ_y = atan2 ($-0.0011, 0.9999$)

= 90.06 degrees

θ_x = atan2 (r_{21}, r_{11})

θ_x = atan2 (-0.3800, -0.9250)

= -112.33 degrees

<u>The Perspective projection matrix</u>

The projection matrix correlates with the eigenvectors

$$E = \begin{pmatrix} e_{11} & e_{12} & e_{13} & e_{14} \\ e_{21} & e_{22} & e_{23} & e_{24} \\ e_{31} & e_{32} & e_{33} & e_{34} \end{pmatrix}$$

$$= \begin{pmatrix} -0.1568 & 0.0287 & -0.1179 & 0.8954 \\ -0.0644 & 0.0073 & -0.0564 & 0.3893 \\ -1.8051e\text{-}04 & -9.5421e\text{-}06 & -1.5453e\text{-}04 & -0.0012 \end{pmatrix}$$

$$E' = \begin{pmatrix} 0.1568 & -0.0287 & 0.1179 & -0.8954 \\ 0.0644 & -0.0073 & 0.0564 & -0.3893 \\ 1.8051e\text{-}04 & 9.5421e\text{-}06 & 1.5453e\text{-}04 & 0.0012 \end{pmatrix}$$

2. **<u>Homography Matrix</u>**

Projection matrix acts on homogenous coordinates , the x and y coordinates are the same for both images

$$\begin{pmatrix} -9.9651e\text{-}04 & 0.0016 & 0.3442 \\ -2.2680e\text{-}04 & -9.872e\text{-}04 & -0.9385 \\ 2.8724e\text{-}04 & -2.8691e\text{-}04 & 0.0288 \end{pmatrix}$$

$$\begin{pmatrix} -x_s{}^{(1)} & y_s{}^{(1)} & 1 & 0 & 0 & 0 & -x_d{}^1 x_s{}^1 & -x_d{}^1 \\ 0 & 0 & 0. & x_s{}^{(1)} & y_s{}^{(1)} & 1 & -y_d{}^1 x_s{}^1 & -y_d{}^1 \\ x_s{}^{(2)} & y_s{}^{(2)} & 1. & 0 & 0 & 0 & -x_d{}^2 x_s{}^2 & -x_d{}^2 \\ 0 & 0 & 0 & x_s{}^{(2)} & y_s{}^{(2)} & 1 & -y_d{}^2 x_s{}^2 & -y_d{}^2 \\ x_s{}^{(3)} & y_s{}^{(3)} & 1 & 0 & 0 & 0 & -x_d{}^3 x_s{}^3 & -x_d{}^3 \\ 0 & 0 & 0 & x_s{}^{(3)} & y_s{}^{(3)} & 1. & -y_d{}^3 x_s{}^3 & -y_d{}^3 \\ x_s{}^{(4)} & y_s{}^{(4)} & 1 & 0 & 0 & 0 & -x_d{}^4 x_s{}^4 & -x_d{}^4 \\ 0 & 0 & 0 & x_s{}^{(4)} & y_s{}^{(4)} & 1 & -y_d{}^4 x_s{}^4 & -y_d{}^4 \\ x_s{}^{(5)} & y_s{}^{(5)} & 1 & 0 & 0 & 0 & -x_d{}^5 x_s{}^5 & -x_d{}^5 \\ 0 & 0 & 0 & x_s{}^{(5)} & y_s{}^{(5)} & 1 & -y_d{}^5 x_s{}^5 & -y_d{}^5 \\ x_s{}^{(6)} & y_s{}^{(6)} & 1 & 0 & 0 & 0 & -x_d{}^6 x_s{}^6 & -x_d{}^6 \\ 0 & 0 & 0 & x_s{}^{(6)} & y_s{}^{(6)} & 1 & -y_d{}^6 x_s{}^6 & -y_d{}^6 \end{pmatrix}$$

Part B: MATLAB Prototype

File submitted : partbmatlabprototy.m

Part C : Application development

5. Setup your application to show a RGB stream from the mono camera and a depth map stream from the stereo camera simultaneously. Is it feasible? What is the maximum frame rate and resolution achievable?

6. Run the camera calibration tutorial. Compare the output with answers from Part A and Matlab calibration exercise.