Definition: *Request* $\rightarrow R$, *Traversal Path* $\rightarrow TP$, Region = {upper region, middle region, lower region}, *Target Key* $\rightarrow K$, *Target Internal Node* $\rightarrow N$, *Parent of Target Leaf Node* $\rightarrow X$, X'*s right sibling* $\rightarrow Y$, *Target Leaf Node* $\rightarrow \mathrm{N}_x$.
Theorem: The algorithm keeps the consistency.
Assume: R is in the region && TP is modified.
Prove: consistency of a traversal path is guaranteed.
⟨1⟩1.**CASE:** R is in upper region && TP is the in upper region.
PROOF: ⟨2⟩1.the modification will incur a conflict.
⟨2⟩2.R is protected by the upper HTM region.
⟨2⟩3.R will retry from the root.
⟨2⟩4.Q.E.D.

⟨1⟩2.**CASE:** R isn't in the upper region && TP is in the lower region.
PROOF: ⟨2⟩1.**CASE:** R is in the lower region.
PROOF: ⟨3⟩1.cause a HTM conflict in the lower region, which is similar to ⟨2⟩1., ⟨2⟩2.
⟨3⟩2.R will retry from the target leaf node.
⟨3⟩3.Q.E.D.

⟨2⟩2.**CASE:** R is in the middle region.
PROOF: ⟨3⟩1.**CASE:** N is consistent.
PROOF: R entering the lower region, it is similar to ⟨2⟩1.

⟨3⟩2.**CASE:** N is modified, N_x isn't valid.
PROOF: ⟨4⟩1. there must exist a new key $\mathrm{X}^{'}$ in node N s.t. $\mathrm{X} < \mathrm{X}^{'} \leq \mathrm{K} < \mathrm{Y}$.
⟨4⟩2. retry from the root.
⟨4⟩3. find a new leaf node $\mathrm{N}_{x'}$.
⟨4⟩4. Q.E.D.

⟨3⟩3.**CASE:** N's ancestor is modified.
PROOF: ⟨4⟩1. N_x is not valid(a new key is inserted into parent node only when the descendant node splits).
⟨4⟩2. retry from the root.
⟨4⟩3. Q.E.D.

⟨3⟩4. Q.E.D.

⟨2⟩3. Q.E.D.

⟨1⟩3. Q.E.D.