CISC-235
Assignment 2
February 7, 2018

In this assignment you will explore some properties of Red-Black trees and compare them to "plain" Binary Search Trees.

Part 1:

Implement a **BinaryTreeVertex** class in which each object has at least these attributes:

- value: a value
- left (or left_child): a pointer to another BinaryTreeVertex object
- right or right_child): a pointer to another BinaryTreeVertex

and other attributes as needed for your implementation of Red-Black trees.

(Note: if you are completing your assignment in Python it is **not** acceptable to implement your vertices as Python dictionaries. One of the intended benefits of this assignment is to consolidate your programming skills with objects and explicit pointers.)

Implement a **BinarySearchTree** class in which each object has at least this attribute:

- root: a pointer to a BinaryTreeVertex object

and any other attributes that will help with the completion of this assignment.

Each BST object must also have an **Insert** method or function. You may assume that the values to be stored in the tree are unique integers.

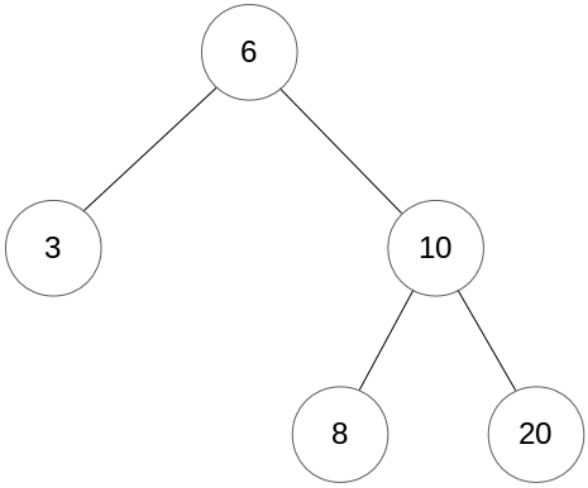
Your BST objects must also have a **SearchPath** method that returns a list of all the values visited on the search path. For example, on a particular tree T, T.SearchPath(12) might return

8, 20, 15, 14, 10, 12

8 would be the value stored in the root of the tree, 20 would be its right child, and so on down to 12

The purpose of this modified Search method is to allow checks to determine that the tree is properly structured.

Your BST objects must also have a **Total_Depth** method or function that computes the sum of the depths of all vertices in the tree, where the **depth** of a vertex is its level in the tree (the root has depth 1). For example, in this tree



the total depth is $1 + 2 + 2 + 3 + 3 = 11$

The Total Depth of a tree is important because if we divide it by n (the number of vertices) we get the average depth, which is the average number of vertices visited on a search of the tree (assuming all values in the tree are equally likely to be the target of a search). Thus by comparing the Total Depth of two trees that store the same set of values, we can determine which tree will have better average search performance.

Your Total_Depth method or function should run in O(n) time. Hint: the root is at depth 1. If a vertex knows its own depth, it can tell its children what their depth is. Each vertex can add its own depth to the total depth reported by its children, and return that sum. The value returned by the root will be the total depth of the tree.

Your BST objects must also have a **Max_Depth** method or function that computes the maximum of the depths of all vertices in the tree, where the **depth** of a vertex is its level in the tree (the root has depth 1). For example, in the tree shown above, the maximum depth is 3.

The Max Depth of a tree is important because it tells us the worst-case search time for the tree.

We can imagine a tree where the Total Depth is low but the Max Depth is high due to one long path. Part of this assignment will explore this possibility.

Your Max_Depth method or function should run in O(n) time.

All of your methods can be recursive or iterative.

Demonstrate the correctness of your methods by starting with an empty tree and inserting the values 6, 10, 20, 8 , 3 (which should produce the tree shown above), then search for each of the values and confirm that the search paths are correct (for example the search path for 8 should be 6, 10, 8)

Also run your Total_Depth and Max_Depth methods on this tree and confirm that they give the correct result.

Part 2:

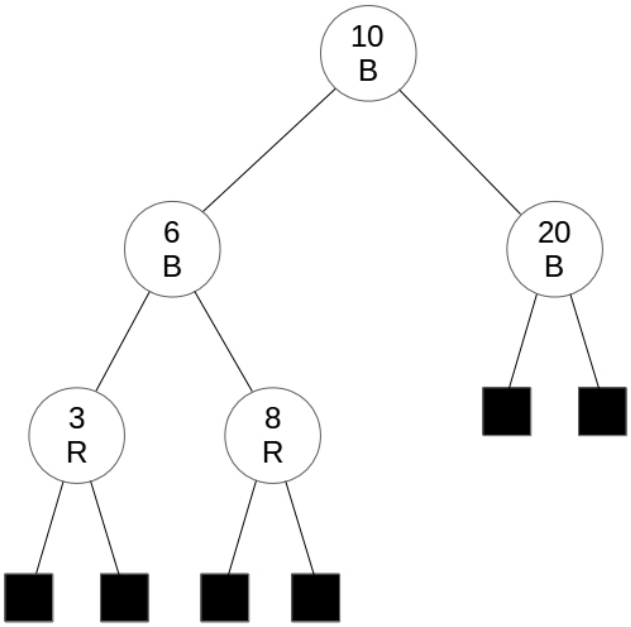
Repeat Part 1 with Red-Black Trees. Each vertex will need an additional field to store the vertex's colour (in real world applications this is usually done with a single bit, but you can use any data type you choose for this field. For those interested in the real nitty-gritty details of implementation, a popular method of storing the colour of a vertex is to use the low order bit of the left_child pointer. Obviously this requires some extra coding when properly interpreting the pointer.)

Your Total_Depth and Max_Depth methods will be unchanged from Part 1, but your Search method should be modified to also display the colour of each vertex encountered (this will give evidence that the tree is properly coloured). Naturally your Insert method will need to incorporate the Red-Black colouring and balancing operations. In addition, you will need some way to distinguish leaf vertices from internal vertices. Please refer to the text and the course notes for pseudocode versions of the Insert operations for Red-Black trees. It is important to note that the text uses Vertex objects that contain pointers to their parents. If you choose to follow the pseudocode given in the text you will need to add this attribute to your Vertex objects.

As in Part 1, you have the option of using recursion or iteration.

Test your methods by starting with an empty tree and inserting the values 6, 10, 20, 8 , 3

This should produce the tree



I have drawn the leaves of the tree as black squares. In your implementation you should

not include them in the Total_Depth and Max_Depth calculations.

Use your SearchPath method to confirm that the tree is correctly structured.

To test your single rotation, start with an empty tree and insert the values 6, 1, 10, 20, 30. I will let you work out what the tree should look like. Use your SearchPath method to confirm that the resulting Red-Black tree is correctly structured.

To test your double rotation, start with an empty tree and insert the values 6, 10, 8, 20, 3. Confirm that the resulting Red-Black tree is correctly structured.

Part 3:

Experiment:

The raison d'etre for Red-Black trees is that they guarantee O(log n) height. In this experiment you will compare Red-Black trees to simple Binary Search trees. The measurements for comparison will be the Total Depth and Max Depth as explained in the implementation section. It is generally accepted that "most" Binary Search Trees have "reasonable" Total Depth and Max Depth. Your task here is to collect data to evaluate this claim by comparing simple Binary Search Trees to Red-Black Trees.

To conduct your experiments, use the following procedure:

```
For  n in {1000, 2000, 4000, 8000, 16000}:
        # conduct 500 trials
        For i = 1 .. 500:
                Generate a random permutation of the set {1,2, ..., n}
                Build a Binary Search Tree by inserting the values in the permutation
                Build a Red-Black Tree by inserting the values in the permutation
                Compute the Total Depth and Max Depth of the two trees
```

This will generate a lot of data – you will need to summarize. For each value of n, you should determine the approximate distribution of the Total Depth and Max Depth differences. One way to do this is to compute the ratio $R_T = \frac{BST\ Total\ Depth}{RB\ Total\ Depth}$ for each trial. If R_T is less than 1 then the average search time of the BST is better than the RB tree, and if R_T is greater than 1then the average search time of the RB tree is better than the BST. On the other hand, if R_T is close to 1 then the two trees are similar in structure. Your summary for the Total Depth comparison might include a table such as this:

n	$R_T < 0.5$	$0.5 \leq R_T < 0.75$	$0.75 \leq R_T \leq 1.25$	$1.25 < R_T \leq 1.5$	$R_T > 1.5$
1000	2 %	8 %	15 %	50 %	25 %
2000	1 %	10 %	10 %	54%	25 %
...					

A similar table can be constructed for $R_M = \frac{BST\ Max\ Depth}{RB\ Max\ Depth}$

The numbers in the table shown here are **completely made up** – your results will not necessarily show the same pattern as these.

Discuss the results of your comparisons. Do they support the claim that Red-Black Trees are superior to simple Binary Search Trees? Does the difference (if any) become more or

less significant as n becomes larger?

For example, in the table shown above both values of n indicate that the BST Total Depth is at least 25% greater than the RB Total Depth in at least 75% of the trials. If that continues to be true for larger values of n, it is evidence that RB Trees usually have better average search time than simple Binary Search Trees.

You are not required to do sophisticated statistical analyses on your collected data. Also, if you find that your computer is not able to deal with trees with 16000 vertices in a reasonable amount of time, feel free to cap your trees at a smaller size.

Logistics:

You may complete the programming part of this assignment in Python, Java, C or C++.

Your implementation must include classes as specified. If you choose Python, you may not use the native Dictionary type to implement your vertex and tree objects.

You must submit your source code, properly documented according to standards established in CISC-121 and CISC-124, and taking into consideration any feedback you received on Assignment 1. You must also submit a PDF file summarizing the results of your experiments and containing your conclusions regarding the comparison of Red-Black Trees and simple Binary Search Trees. Both files must contain your name and student number, and must contain the following statement: “I confirm that this submission is my own work and is consistent with the Queen's regulations on Academic Integrity.”

You are required to work individually on this assignment. You may discuss the problem in general terms with others and brainstorm ideas, but you may not share code. This includes letting others read your code or your written conclusions. The course TAs will be available to advise and assist you regarding this assignment.