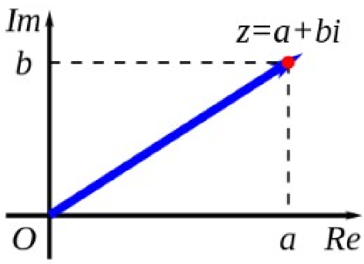
"~" - means i'm not sure if it is the correct name, or information is correct (only for that sentence)

The Mandelbrot Set (Numberphile)

Context:

- we have a complex number, that has real and ~imaginary part: a + bi
  - a and b are real numbers, i is $i=\sqrt{-1}$



- We can Graphically represent complex number in on complex plane:
  - real part (a) is on x axis, the imaginary (b) part is on y
    - Complex plane is a very geometric and intuitive way to visualse complex number and it's addition, subtraction e.g
- Magnitude (size, absolute value) of complex number: it is a distance from 0 to ~the point of a and b
  - $|a+bi|$ if $z=|x+yi|$ then $|z|=\sqrt{[\mathrm{Re}(z)]^2+[\mathrm{Im}(z)]^2}=\sqrt{x^2+y^2},$

~Functions for Mandelbrot set

- Let's say that our complex number is called c (can be c =1, can be c=1+2i)
- Let's say we have function $f_c(z)=z^2+c$
  - where c is our complex number, so $\mathbf{c=1}$ would be $f_1(z)$
- Let's see behavior of this function, from z = 0 (initial value is 0)
  - let's say we have c=1, so it's $f_1(z)$
  - Let's calculate some iterations:
    - $f_1(0)=0^2+1$ we insert starting number z = 0, and c =1, we've got **answer 1**
    - **Interation 2:** $f_1(\ f_1(0)\ )\ =\ f_1(1)=1^2+1$ we've got answer: 2
    - **Interation 3:** $f_1(2)=2^2+1=5$ then gets to 26, then bigger ....
  - **We can see how each iteration of c=1 , gives bigger and bigger number**

Overall with certain c values, iteration of fuction $f_c(z)=z^2+c$ may:

- give the <u>magnitude</u>($|a+bi|$) for each iteration, bigger and bigger
  - **it blows up, gets as large as you want it to be**
  - it is said that: *The interates go to infinity*
- Or <u>magnitude</u>($|a+bi|$) gets bound to certain range, value it remains small
  - For example, each iteration **never** gives number **larger than 2** (so it's quite close to 0)

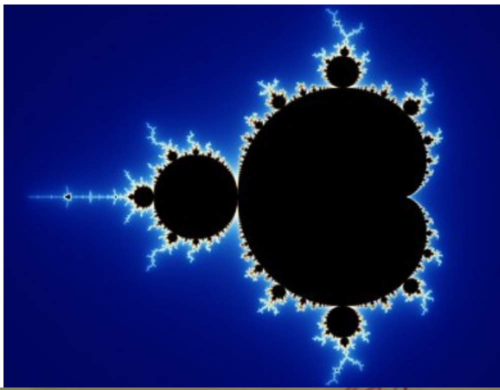
<u>The Mandelbrot Set:</u>

- It is a set of complex numbers c (the set usually called M)
  - **for which magnitude (size) never gets bigger than 2** (doesen't go towards infinity)

More examples:

- when c = -1:
  - $f_{-1}(0)=0^2-1 \quad \rightarrow -1$
  - $f_{-1}(-1)=-1^2-1 \quad \rightarrow -0$
  - and again we plug 0, then we get -1, and again 0 and so on ... it alternates between 0 and -1.

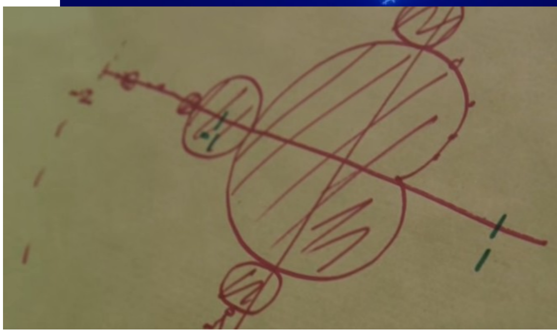
- when c = 1/8 ..... never gets to 2
  - $0^2+1/8=1/8$
  - $1/8^2+1/8= \quad 1/64+1/8=9/64 \;=\; 0{,}14 \;\; 063$
  - $9/64^2+1/8= \; 9/4096+1/8 \;=\; 0{,}14 \;\; 478$



The colors:

- If it takes long time to get big – we give one color, if it gets big very quickly – we give differnt color
  - once iterate is larger than 2 – **blue color**
  - iterate never gets larger than 2 – The **black color**

  - The **yellow'ish color** – it is hard to predict behavior
    - e.g 1/4, on the right side edge, if you take a little bit bigger value:
      - if we would take 1/3,99999, then after about 100000 iterations we would get answer of ∞
    - having smaller value, it stays under 2



That means $-2< \; |c| \; <2$

- code wise Re(c)^2 + Im(c)^2 < 4

- $|2.1|=2.1>2$, $|2.1i|=2.1>2$, $|1.5+1.5i|=2.12>2$ so anything outside the circle of r = 2, automatically gets the infinity color

The Filled Julia Set

(numberphile – Filled julia set)

let's fix c value:
Very similar to madelbrot - but we have fixed polynomial

It is a set of complex numbers c - while under iteration, the values don't "blow up" to infinity

if $c=0$ $\quad f_0(z)=z^2$

- we insert different z values: let's say we have 0.9
  - 0.9^2 = ^2 = eventually gets infinitely close to 0
  - 1.1^2 = ^2 = eventually goes to infinity

  anything bigger than 1 squared – gets bigger

rabbit c = -0.12 - 0.75i

Buntch of points, alot of seperate disconnected pieces c = 1

Another way to define Mandelbrot set, with julia set

- If with that c, julia set gives connected pieces – it is the converging point,
- if we get buntch of dots , disconnected pieces – it diverges, goes to infinity

Or one could say that Mandelbrot set classifies, what Julia sets we would get

https://www.youtube.com/watch?annotation_id=annotation_1578326063&feature=iv&src_vid=NGMRB4O922I&v=oCkQ7WK7vuY