Probability Theory

Lecture Note and some interesting problems

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组织：C • H 邮递公司

时间：花无凋零之日，意有传递之时

生活中最重要的问题，绝大部分其实只是概率问题。——拉普拉斯

目录

Syllabus

Teaching by *Lidan Wang*

Text 1 : A First Course in Probability (10th edition)

Text 2 : Elementary Probability Theory

We will cover Chapter 1-8 in the text

Combinatorial analysis: sets and counting;

Axioms of Probability;

Conditional probability, independence;

Random variables: discrete r.v.s, continuous r.v.s, multi-dim r.v.s;

Integration: mean, variance and transform;

Limit theorems: convergence of r.v.s, weak/strong law of large numbers,central limit theorem.

Grading Scheme

Homework(10%)

Quizzes twice(30%)

Final(60%)

第 1 章 Combinatorial Analysis

内容提要

- ❑ Counting
- ❑ Combinations
- ❑ Permutations
- ❑ Multinomials

1.1 Counting

1.1.1 Basic Counting Principles

定理 1.1

♡

第 1 章 练习

1.
2.
3.

第 2 章 Sets, Axioms of Probability

内容提要

- ❏
- ❏
- ❏
- ❏
- ❏

2.1 Sets

2.1.1 Subset/Superset

第 2 章练习

1.
2.
3.

第 3 章 Conditional Probability and Independence

内容提要

❏
❏
❏
❏
❏

3.1 Conditional Probabilities

3.1.1 Conditional probability

第 3 章 练习

1.
2.
3.

第 4 章 Discrete Random Variables

内容提要

- ❑
- ❑
- ❑
- ❑
- ❑

4.1 Counting

4.1.1 积分的定义

第 4 章 练习

1.
2.
3.

第 5 章 Combinatorial Analysis

内容提要

- ❑
- ❑
- ❑
- ❑
- ❑

5.1 Counting

5.1.1 积分的定义

第 5 章练习

1.
2.
3.

第 6 章 Quizz

6.1 Quizz 1 [2023]

6.1.1 Question 1

(15 points) Suppose there are two coins. Coin 1 lands on heads with probability $\dfrac{1}{2}$, coin 2 lands on heads with probability $dfrac13$. We randomly select one coin and repeatedly flip it. If the first flip results in heads, what is the conditional probability that there are exactly 3 heads in the first 5 flips? (Express your answer in the fraction form)

解 Let H_1 be the event that the first flip results in head. Let C_1, C_2 be the events that coin 1, 2 are selected, respectively. Let E be the event that there are exactly 3 heads in the first 5 flips. By law of total probability for conditional probability,

$$\begin{aligned}P(E|H_1) &= P(E|H_1C_1)P(C_1|H_1) + P(E|H_1C_2)P(C_2|H_1)\\ &= \binom{4}{2}(\frac{1}{2})^2(\frac{1}{2})^2P(C_1|H_1) + \binom{4}{2}(\frac{1}{3})^2(\frac{2}{3})^2P(C_2|H_1)\end{aligned}$$

By Bayes's formula,

$$P(C_1|H_1) = \frac{P(H_1|C_1)P(C_1)}{P(H_1|C_1)P(C_1) + P(H_1|C_2)P(C_2)} = \frac{3}{5}$$

and $P(C_2|H_1) = 1 - P(C_1|H_1) = \dfrac{2}{5}$.Altogether,

$$P(E|H_1) = \frac{3}{8}\cdot\frac{3}{5} + \frac{24}{81}\cdot\frac{2}{5}$$

6.1.2 Question 2

Answer the following questions:

(a) (5 points) How many terms are there in the expansion of $(2x + y + 3z)^5$? What is the coefficient of x^2y^2z?

(b) (5 points) 3 married couples are seated in a row, in how many ways can they sit if no person is next to their partner?

(c) (5points) E, F are mutually exclusive events with P(E) = 0.3, P(F) = 0.4. Compute $P(E|E \cup F)$.

(d) (5 points) A, B, C are three events. Show that $P(A \cup B \cup C) \geq P(A) + P(B) + P(C)2$.

(e) (5 points) Prove or give a counterexample: if E and F are independent, then E and F are conditionally independent.

解 (a)There are $\binom{5+3-1}{3-1} = 21$ terms and the coefficient is $\binom{5}{2,2,1}2^2 \cdot 3 = 360$.

(b)

(c)E, F are mutually exclusive, then $P(E \cup F) = P(E) + P(F) = 0.7$ and $P(E|E \cup F) = \dfrac{P(E)}{P(E \cup F)} = \dfrac{3}{7}$.

(d)

(e)

6.1.3 Question 3

6.1.4 Question 4

6.1.5 Question 5

6.1.6 Question 6

6.2 Quizz 2

6.2.1 Question 1

6.2.2 Question 2

6.2.3 Question 3

6.2.4 Question 4

6.2.5 Question 5

6.2.6 Question 6