Physics Informed Neural Networks: A Quantum Leap in Robust Control

MTHG103E

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Digging deep into intelligence & driving the future

Abstract

Modeling differential equation-governed physical systems is computationally intensive, especially without analytical solutions. We leverage Physics-Informed Neural Networks (**PINNs**) to efficiently model connected vehicle dynamics, achieving steady-state accuracy within < **2%** error (**20** m/s reference) and transient response with **1.2** s rise time (5 m/s changes) and < **0.3** m/s overshoot. Our physics-embedded model combines efficiency and accuracy, enabling robust control that maintains stability despite uncertainties, enhancing vehicle communication reliability, and advancing intelligent transportation systems.

Problem Definition

Road crashes cause **1.19** million annual deaths globally, primarily due to human error and limitations in current vehicle automation. While connected and automated vehicles (**CAVs**) promise significant safety improvements, existing systems struggle with real-world complexities unpredictable driver behavior, and imperfect sensors. The critical challenge lies in developing accident prevention systems that reliably handle these uncertainties while maintaining traffic flow efficiency and reducing emissions (potential **15%** fuel savings). This demands fundamentally new approaches to vehicle coordination and avoidance.

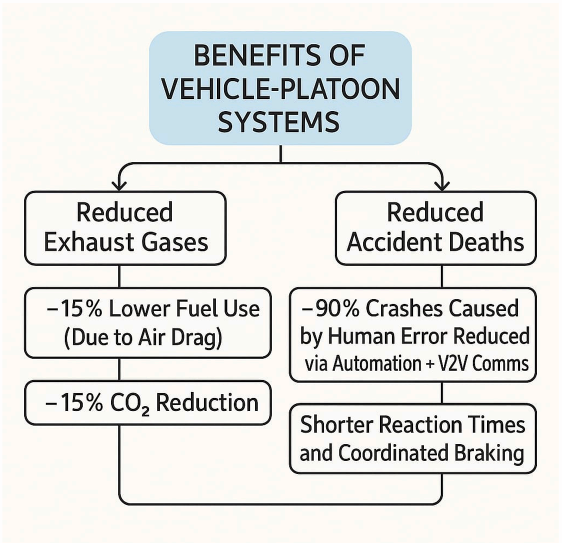

Figure 1

Literature Review

This review compares control methods for Connected and Automated Vehicles (**CAVs**). Model Predictive Control (**MPC**) predicts future states but struggles with uncertainties. Sliding Mode Control (**SMC**) counters disturbances but causes actuator wear. **H∞** control ensures robust stability, while Min-Max **MPC** optimizes for worst-case scenarios. Integrating these with physics-informed neural networks shows strong potential for managing complex vehicle dynamics, as shown in Figure 2.

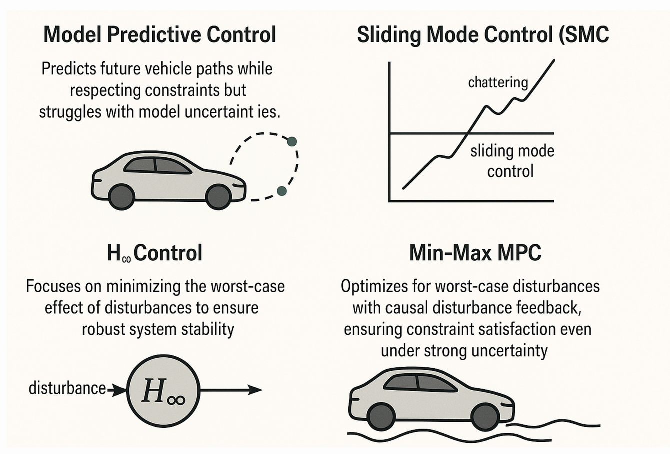

Figure 2

Mathematical Modeling

To model the motion of Connected and Autonomous Vehicles (**CAVs**), we use physics-based equations that describe how throttle, drag, and road resistance affect speed (longitudinal dynamics) and how steering influences yaw motion (lateral dynamics). These equations are derived from Newtonian mechanics and capture the essential behaviors needed for cruise control, platooning, and path tracking.

- **Longitudinal Dynamics Equation:**

$$m\dot{v} = u - \frac{1}{2}\rho C_d A v^2 - F_{rr}$$

- **Lateral Dynamics (Yaw Motion):**

$$I_z\dot{r} = l_f C_{\alpha f}\left(\delta - \frac{r l_f}{v}\right) - l_r C_{\alpha r}\left(-\frac{r l_r}{v}\right)$$

- **MPC's Cost Function:**

$$J = \sum_{j=N_1}^{N_2} \|\mathbf{y}[k+j] - \mathbf{y}^{\text{ref}}[k+j]\|_{\mathbf{Q}}^2 + \sum_{i=0}^{N_u-1} \|\Delta\mathbf{u}[k+i]\|_{\mathbf{R}}^2$$

which is subject to:

$$\mathbf{y}[k+j+1] = \widehat{\mathbf{f}}_{\mathbf{w}}(\mathbf{y}[k+j], \mathbf{u}[k+j])$$

$$\mathbf{u}[k+j] = \mathbf{u}[k-1] + \sum_{i=0}^{j} \Delta\mathbf{u}[k+i],$$

- **Our Contribution: Neural Surrogate Modeling**

1. f_w

A neural surrogate model trained using physics-informed loss. It maps the time horizon , initial state, and control inputs to the predicted vehicle states.

2. $y[k]$

Discrete-time prediction used in closed-loop control, where the network recursively computes the next state from the previous prediction and current control input.

$$y[k] = f_w(T, y[k-1], u[k])$$

Results

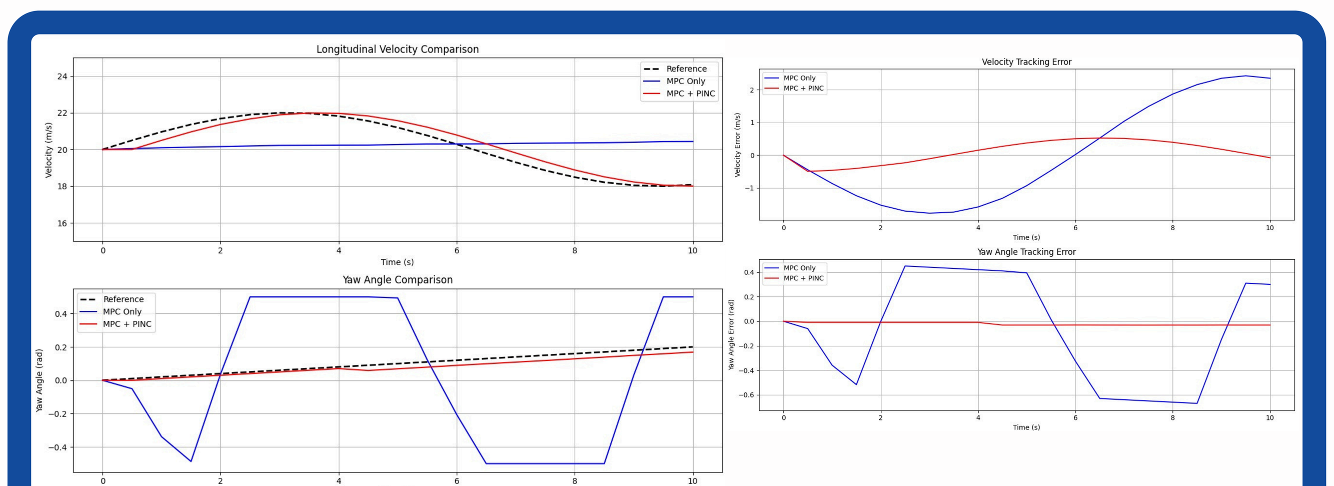

Figure 3

Tracking Performance:

Longitudinal Velocity:

- For the usual **MPC**, as shown in Figure 3, the velocity deviates by approximately **2** m/s from the reference by **10** seconds.
- The proposed combination of **MPC** and **PINC** keeps the deviation within **1** m/s, effectively cutting the error in half.

Yaw Angle:

- Traditional **MPC** results in large oscillations, with peak errors reaching around **0.4** radians.
- The **PINC-inspired MPC** accurately reproduces the reference yaw angle with a slight deviation at the **4** second mark.

Tracking Errors:

Longitudinal Velocity:

- Traditional **MPC** shows a peak error of **1.5** m/s and maintains an error between **1–2** m/s.
- **MPC** with **PINC** has the peak reduced to **0.5** m/s and quickly settle at that level, achieving a **66%** improvement.

Yaw Angle:

- Standalone MPC shows peak errors of **0.4** radians and suffers from instability.
- The combined approach reduces the peak to **0.1** radians, offering a **75%** improvement with more stable performance.

Experimental Setup

Approach:

- Use a hybrid **MPC-PINN** controller for real-time autonomous driving.
- Combine physics-based dynamics with neural network predictions in solving the two coupled problems.

Neural Network Architecture

- **7** input features: time, velocity, acceleration, yaw angle , yaw rate, throttle, steering, as shown in Figure 4.
- **256**-unit input layer (tanh activation, He init)
- **5** residual blocks: Dense + skip connection + LayerNorm
- **4** outputs: predicted velocity, acceleration, yaw angle, and yaw rate

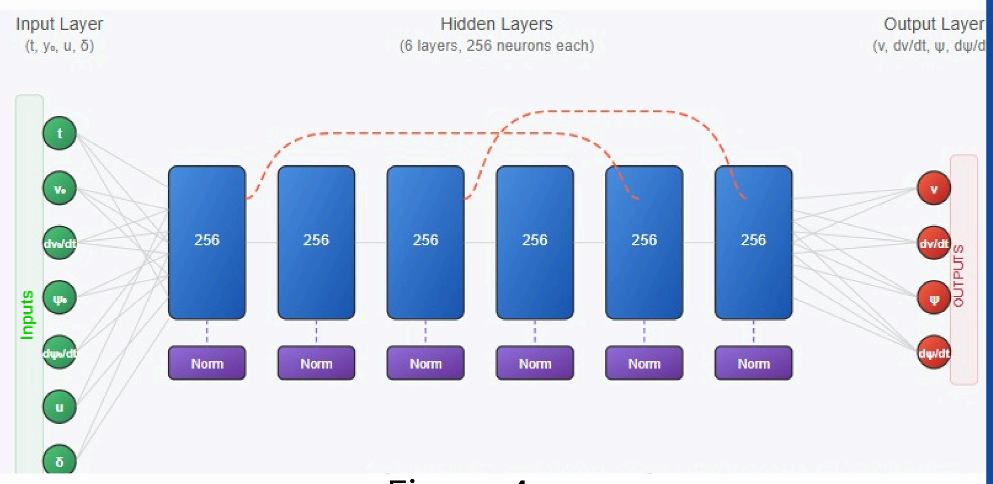

Figure 4

Training Protocol

- Loss: $\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{data}} + \lambda\mathcal{L}_{\text{phys}}$ where $\lambda = 0.1\frac{\mathcal{L}_{\text{data}}}{\mathcal{L}_{\text{phys}}}$
- Optimizers: **ADAM** (η = **1e-3**)
- Batch size: **256**
- Normalization:

Feature	Normalization
Velocity (v)	$v/30$
Acceleration ($\dot{v}$)	$\dot{v}/5$
Yaw angle (ψ)	$\psi/0.5$
Yaw rate (r)	$r/1$
Control input (u)	$u/3000$
Steering angle (δ)	$\delta/0.5$

- MPC formulation

$$\min_{\mathbf{u}} \sum_{k=0}^{19} \left(\underbrace{10(v_k - v_{ref})^2 + 50(\psi_k - \psi_{ref})^2}_{\text{tracking}} + \underbrace{0.001(u_k^2 + \delta_k^2)}_{\text{effort}} + \underbrace{1000\max(0, |r_k| - 0.5)^2}_{\text{constraints}} \right)$$

Conclusion

To further advance this research, the following directions are proposed:

1. Extended Testing: Validate the framework under more diverse and extreme driving scenarios (e.g., urban traffic, adverse weather) to assess scalability.
2. Energy Efficiency: Explore the framework's impact on fuel consumption and emissions reduction, particularly in heavy-duty vehicle platoons.
3. Multi-Agent Coordination: Expand the model to optimize cooperative control for large-scale **CAV** platoons, incorporating V2X (vehicle-to-everything) communication.

This work lays a foundation for next-generation autonomous systems, combining data-driven learning with physics-based constraints to achieve safer, more efficient transportation solutions.

References

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References

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