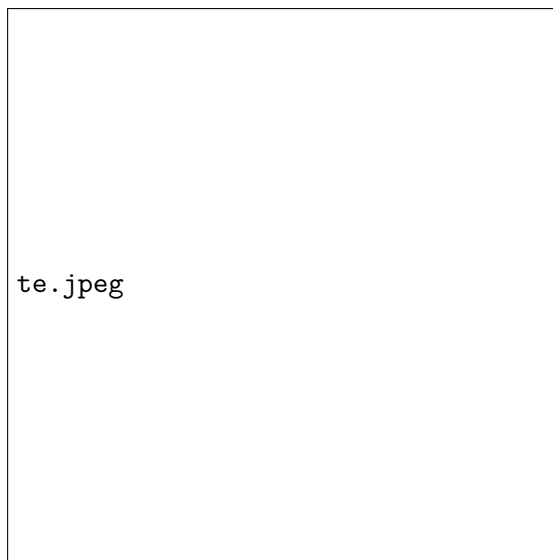


October Math Gems

PROBLEM OF THE WEEK 4

§1 problems

Problem 1.1. In the figure below ABC is a right-angled triangle and BD an angle bisector. If $AB = 3$, and the area of $ABD = 9$, what is the length of DC ?



Problem 1.2. If a, b, c are positive reals with $abc = 1$. what is the minimum value of

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(a+c)} + \frac{1}{c^3(a+b)}$$

?

Problem 1.3. How many ordered pairs of positive integers (M, N) satisfy the equation $\frac{M}{6} = \frac{6}{N}$?

Problem 1.4. Two median are drawn from acute angles a right angled triangle intersect at an angle $\frac{\pi}{6}$. If the length of the hypotenuse of the triangle = 3, then, find the area of triangle=

Problem 1.5. If the points $a(3, 4), b(7, 12), \text{ and } p(x, y)$ are such that $(pa^2 > (pb)^2 > (ab^2))$ Evaluate x where x is integral number.

Problem 1.6. Known that $a + b + c = \pi$, then

$$\frac{\sin 2a + \sin 2b + \sin 2c}{\cos a + \cos b + \cos c - 1} =$$

Problem 1.7. Given $g(x) = 9 \log_8(x - 3) - 5$, $g^{-1}(13) =$

Problem 1.8. let $x, y, z > 0$. Prove that

$$\frac{2}{x+y} + \frac{2}{y+z} + \frac{2}{z+x} \geq \frac{9}{x+y+z}$$

Problem 1.9. Determine the domain of the function

$$g(x) = \cot^{-1} \left(\frac{x}{\sqrt{x^2 - [x^2]}} \right)$$

Problem 1.10. Let α, β , and γ denote the angles of a triangle. Show that

$$\sin \alpha + \sin \beta + \sin \gamma = 4 \cos \frac{\alpha}{2} \cos \frac{\beta}{2} \cos \frac{\gamma}{2},$$

$$\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 4 \sin \alpha \sin \beta \sin \gamma$$

$$\sin 4\alpha + \sin 4\beta + \sin 4\gamma = -4 \sin 2\alpha \sin 2\beta \sin 2\gamma.$$

Problem 1.11. Prove that no number in the sequence

$$11, 111, 1111, 11111, \dots$$

is the square of an integer.

Problem 1.12. Solve the following system of three equations for the unknowns x , y and z :

$$5732x + 2134y + 2134z = 7866$$

$$2134x + 5732y + 2134z = 670$$

$$2134x + 2134y + 5732z = 11464$$

Problem 1.13. A pyramid is called "regular" if its base is a regular polygon and the foot of its altitude is the center of its base. A regular pyramid has a hexagonal base the area of which is one quarter of the total surface-area S of the pyramid. The altitude of the pyramid is h . Express S in terms of h .

Problem 1.14. Let a and b be positive real numbers satisfying

$$\frac{a}{b} \left(\frac{a}{b} + 2 \right) + \frac{b}{a} \left(\frac{b}{a} + 2 \right) = 2022.$$

Find the positive integer n such that

$$\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = \sqrt{n}$$

