October Math Gems

§1 problems

Problem 1.1. Prove that

$$\csc 6° + \ csc78° - \csc 42° - \csc 66° = 8$$

Problem 1.2. Find all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that $x, y \in \mathbb{R}^+$,

$$f\left(\frac{y}{f(x+1)}\right) + f\left(\frac{x+1}{xf(y)}\right) = f(y)$$

Problem 1.3. Find $f : \mathbb{R} \to \mathbb{R}$ which satisfied:

$$x^2 f(y) + y^2 f(x) = xy f(x+y)$$

Problem 1.4. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$\left(x\left(f(x) - \frac{f(y)+f(z)}{2}\right) + y\left(f(y) - \frac{f(z)+f(x)}{2}\right) + z\left(f(z) - \frac{f(x)+f(y)}{2}\right)\right) f(x+y+z) =$$
$$f(x^3) + f(y^3) + f(z^3) - 3f(xyz)$$

for all $x, y, z \in \mathbb{R}$.

Problem 1.5. In a triangle ABC, $\frac{cosA}{1+sinA} = \frac{sin2B}{1+cos2B}$. Find the minimum value of $\frac{a^2+b^2}{c^2}$.

Problem 1.6. Find all polynomials

$$(x-1)p(2x) = 4(x+1)p(x) + 5$$

Problem 1.7.
$$\log_{2x}^2\left(4x^3\right) - 2 = \log_{2x} 4x$$

Problem 1.8. Determine all the real numbers x for which

$$2\log_2(x-1) = 1 - \log_2(x+2)$$

Problem 1.9. Find the number of solution of $\log_4(x-1) = \log_2(x-3)$.

Problem 1.10. Given that :

$$log_4(log_{\sqrt{2}}x) + log_{\sqrt{2}}(log_4 x) = \frac{7}{2}$$

Find the value of:

$$log_2 2x + log_8 \frac{x}{2}$$

Problem 1.11. Solve over $\mathbb{R}$ the equation $4^{(sinx)^2} + 3^{(tanx)^2} = 4^{(cosx)^2} + 3^{(cotanx)^2}$.

Problem 1.12. Solve in $\mathbb{R}$ the equation $\frac{1}{2^x+x+1} + \frac{1}{3^x-4x-3} = \frac{1}{2^x-4x-2} + \frac{1}{x+3^x}$.

Problem 1.13. Solve in real numbers the following equation: $(\sqrt{6} - \sqrt{5})^x + (\sqrt{3} - \sqrt{2})^x + (\sqrt{3} + \sqrt{2})^x + (\sqrt{6} + \sqrt{5})^x = 32$.

Problem 1.14. Prove that $1 - \frac{sin^2\alpha}{1+cot\alpha} - \frac{cos^2\alpha}{1+tan\alpha} = cos\alpha.sin\alpha$

Problem 1.15. Prove that $\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} = \frac{1}{2}$

Problem 1.16. Determine all real numbers $x > 0$ for which

$$\log_4 x - \log_x 16 = \frac{7}{6} - \log_x 8$$

Problem 1.17. Solve the equation $x^{\log_{10} x} = 3y/2$

Problem 1.18. Find number of solutions to the equation $\log_{x+1}\left(x - \frac{1}{2}\right) = \log_{x-\frac{1}{2}}(x+1)$

Problem 1.19. A¡=b¡=c $a^2 + b^2 = c^2$ A,b,c are sides of a traingle $loga^2 + logb^2 = logc^2$ Find Max A

Problem 1.20. Solve for x

$$log_2 x = log_{5-x} 3$$