October Math Gems

PROBLEM OF THE WEEK 14

§1 Problems

Problem 1.1. The sum of integers from 1 to 100 that are divisible by 2 or 5 is

Problem 1.2. If $f(x) = \cos(\log x)$, then $f(\frac{1}{x})f(\frac{1}{y}) - \frac{1}{2}(f(\frac{x}{y}) + f(xy)) =$

Problem 1.3. If $f(x) = x^2 - 1, g(x) = 2x + 3$ then $fogof(1) =$

Problem 1.4. $\log x + \log x^3 + \log x^5 .... + \log x^{2n-1} =$

Problem 1.5. If $\log 2 + \frac{1}{2}\log a + \frac{1}{2}\log b = \log(a+b)$ then find the value of a, b

Problem 1.6. If $f(x+1) = x^2 - 3x + 2$ then $f(x) =$

Problem 1.7. If $f(x) = \frac{3x+2}{5x-3}$, then $f^{-1}(x) =$

Problem 1.8. Domain of $f(x) = \log |\log x| is$

Problem 1.9. Given that $f(x) = \log \frac{1+x}{1-x}$ and $g(x) = \frac{3x+x^3}{1+3x^2}$, then $f(g(x)) =$ (with respct to $f(x)$)

Problem 1.10. The domain of the function $f(x) = \sqrt{\cos^{-1}(\frac{1-|x|}{2})}$ is

Problem 1.11. The value of $\frac{1+\cos 2\theta + \sin 2\theta}{1-\cos 2\theta + \sin 2\theta}$ is

Problem 1.12. If $\cot(\alpha + \beta) = 0$ (where $\alpha, \beta \in 1^{st}$ quadrant), then $\sin(\alpha + 2\beta) =$

Problem 1.13. If $\frac{\tan \alpha + \tan \beta}{\cot \alpha + \cot \beta} + \cos(\alpha - \beta)\sec(\alpha + \beta) + 1^{-1} = 1$ then $\tan \alpha \tan \beta$

Problem 1.14. The least value of $2\log_{10} x - \log_x 0.01$ for $x > 1$ is

Problem 1.15. If $e^{\ln x + \log_{\sqrt{e}} x + \log_{\sqrt[3]{e}} x + ... \log_{\sqrt[10]{e}} x} = x^{11n}$

Problem 1.16. The identity $\log_b n \log_a n + \log_b n \log_c n + \log_c n \log_a n$ equals

Problem 1.17. If $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$ then the value of $x^a y^b z^c$ is

Problem 1.18. If $1 < a \le x$, then the minimum value of $\log_a x + \log_x a$ is

Problem 1.19. Find the values of θ for which the function $f(\theta) = \frac{\sin \theta}{-\sin \theta}$ is not defined

Problem 1.20. If $\cos p\theta = \cos q\theta$, p not equal q, then $\theta =$