

# Categorical Dependency Grammars extended with typed barriers

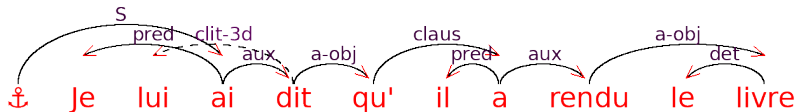
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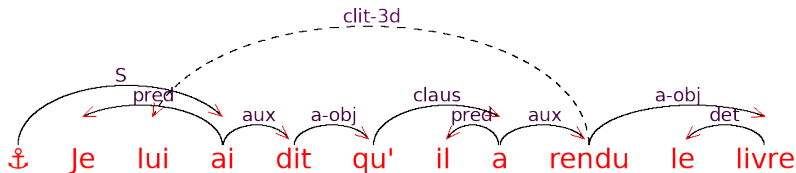
MCLP 2025, September 15–18, 2025, Orsay, France

# CDG Problem 1: Overgeneration with non-projective dependencies

The **CDG analyses** of “Je lui ai dit qu’il a rendu le livre”  
“I have told him that he has returned the book” – clitic “lui” (him)



The normal analysis



A wrong analysis

⇒ **Our solution:**  $CDG_{tb}$  (typed barriers)

# CDG Problem 2: No known construction for Kleene plus

No known construction for the Kleene plus (CDG):

Let  $G$  be a CDG.  $L(G)$  is the formal language generated by  $G$

$\exists G'$ , a CDG such that  $L(G') = L(G)^+$  ?

$\Rightarrow$  Our solutions:  $CDG_{tb}$  (typed barriers) or  $CDG_b$  (barriers)

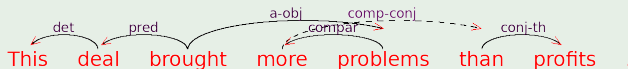
- 1 CDG Languages
- 2 Product of CDG Languages
- 3 Product and Kleene Plus of  $\text{CDG}_{tb}$  Languages (with typed barriers)
- 4  $\text{CDG}_{tb}$  for Natural Languages
- 5 Conclusion

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# Basics of Dependency Syntax

Surface Dependency Structures (DS) are graphs of surface syntactic relations between the words in a sentence.

## A Dependency Structure



Dependencies are determined by valencies of words

*brought* has +valency **pred** of a **left adjacent** word

*deal* has -valency **pred** of a **right adjacent** word

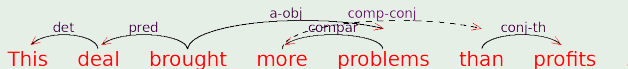
Saturation of valency **pred** determines **projective dependency**

*deal* <sup>**pred**</sup> ← *brought* (**Governor:** *brought*, **Subordinate:** *deal*)

# Basics of Dependency Syntax

Surface Dependency Structures (DS) are graphs of surface syntactic relations between the words in a sentence.

## A Dependency Structure



Dependencies are determined by valencies of words

*more* has +valency **comp-conj** of a word *somewhere* on its right

*than* has -valency **comp-conj** of a word *somewhere* on its left

Saturation of **comp-conj** determines *non-projective dependency*

*more* **comp-conj** *than* (**Governor:** *more*, **Subordinate:** *than*)

CDG Types express dependency valencies

## PROJECTIVE DEPENDENCIES

**Dependency:**  $Gov \xrightarrow{d} Sub$ :

**Governor Type:**  $Gov \mapsto [..\backslash..\backslash..\backslash d/..]^P$

**Subordinate Type:**  $Sub \mapsto [..\backslash d/..]^P$

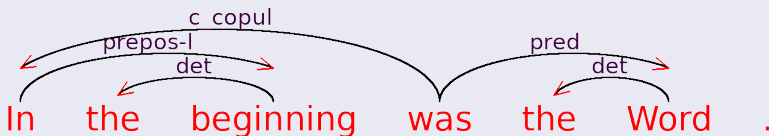
$[...]$  : Part of a type for projective relations (basic dependency type)

$P$  : Part of a type for non-projective dependencies (potential)



# Categorial Dependency Grammars

## CDG Types express dependency valencies



*in*  $\mapsto$   $[c\_copul / prepos - I]$

*the*  $\mapsto$   $[det]$

*beginning*  $\mapsto$   $[det \backslash prepos - I]$

*was*  $\mapsto$   $[c\_copul \backslash S / pred]$

*Word*  $\mapsto$   $[det \backslash pred]$

CDG Types express dependency valencies

## NON-PROJECTIVE DEPENDENCIES

Polarized valencies:  $\nearrow d$ ,  $\searrow d$ ,  $\nwarrow d$ ,  $\swarrow d$

Dependency:  $Gov \xrightarrow{d} Sub$ :

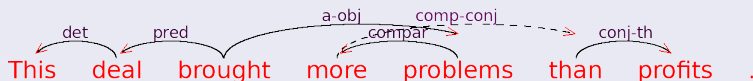
Governor Type Potential:  $Gov \mapsto [..] \nearrow d ..$

Subordinate Type Potential:  $Sub \mapsto [..] \searrow d ..$

$[..]$  : Part of a type for projective relations (basic dependency type)

$.. \nearrow d ..$  : Part of a type for non-projective dependencies (potential)

## CDG Types express dependency valencies



*this*  $\mapsto$  [*det*]

*deal*  $\mapsto$  [*det* \ *pred*]

*brought*  $\mapsto$  [*pred* \ *S* / *a-obj*]

*problems*  $\mapsto$  [*compar* \ *a-obj*]

*profits*  $\mapsto$  [*conj-th*]

*more*  $\mapsto$  [*compar*]  $\nearrow$  *comp-conj*

*than*  $\mapsto$  [ / *conj-th*]  $\searrow$  *comp-conj*

## Left-oriented rules

$$\mathbf{L}^!. \quad [C]^P [C \setminus \beta]^Q \vdash [\beta]^{PQ}$$

$$\text{Gov} \xrightarrow{C} \text{Sub}$$

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*Gov*  $\xrightarrow{\mathbf{C}}$  *Sub*

$$\mathbf{L}^!_{\epsilon} . \quad [ ]^P [\beta]^Q \vdash [\beta]^{PQ}$$

(no new dependency)

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*Gov*  $\xrightarrow{C}$  *Sub*

$$\mathbf{L}_\epsilon^!. \quad [ ]^P [\beta]^Q \vdash [\beta]^{PQ}$$

(no new dependency)

$$\mathbf{I}^!. \quad [C]^P [C^* \setminus \beta]^Q \vdash [C^* \setminus \beta]^{PQ}$$

*Gov*  $\xrightarrow{C}$  *Sub*

$$\mathbf{\Omega}^!. \quad [C^* \setminus \beta]^P \vdash [\beta]^P$$

(no new dependency)

## Left-oriented rules

- $L^!.$   $[C]^P [C \setminus \beta]^Q \vdash [\beta]^{PQ}$   $Gov \xrightarrow{C} Sub$   
 $L^!_{\epsilon}.$   $[ ]^P [\beta]^Q \vdash [\beta]^{PQ}$  (no new dependency)  
 $I^!.$   $[C]^P [C^* \setminus \beta]^Q \vdash [C^* \setminus \beta]^{PQ}$   $Gov \xrightarrow{C} Sub$   
 $\Omega^!.$   $[C^* \setminus \beta]^P \vdash [\beta]^P$  (no new dependency)  
 $D^!.$   $\alpha^{P_1(\swarrow C)P(\searrow C)P_2} \vdash \alpha^{P_1 P P_2}$   $Gov \dashrightarrow Sub$

## First-Available Rule

**FA:** in  $(\swarrow C)P(\nwarrow C)$ , the valency  $\swarrow C$  is the **first available** for the dual valency  $\nwarrow C$ , i.e.  $P$  has no occurrences of  $\swarrow C, \nwarrow C$

## LEXICON:

$John \mapsto [pr]$   
 $ran \mapsto [pr \setminus S / c^*]$   
 $fast, yesterday \mapsto [c]$

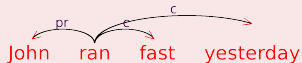
### Derivation

$$\begin{array}{c}
 \frac{[pr \setminus S / c^*] \quad [c]}{[pr \setminus S / c^*]} I^r \quad \text{yesterday} \\
 \frac{[pr \setminus S / c^*] \quad [c]}{[pr \setminus S / c^*]} I^r \\
 \frac{[pr \setminus S / c^*]}{[pr \setminus S]} \Omega^r \\
 \frac{John \quad [pr] \quad [pr \setminus S]}{S} L^I
 \end{array}$$

$L^I \quad [C]^P [C \setminus \beta]^Q \vdash [\beta]^{PQ}$   
 $L^I_\epsilon \quad [ ]^P [\beta]^Q \vdash [\beta]^{PQ}$   
 $I^I \quad [C]^P [C^* \setminus \beta]^Q \vdash [C^* \setminus \beta]^{PQ}$   
 $\Omega^I \quad [C^* \setminus \beta]^P \vdash [\beta]^P$   
 $D^I \quad \alpha^{P_1(\checkmark V)P(\checkmark V)P_2} \vdash \alpha^{P_1 P P_2}, \text{ if FA}$

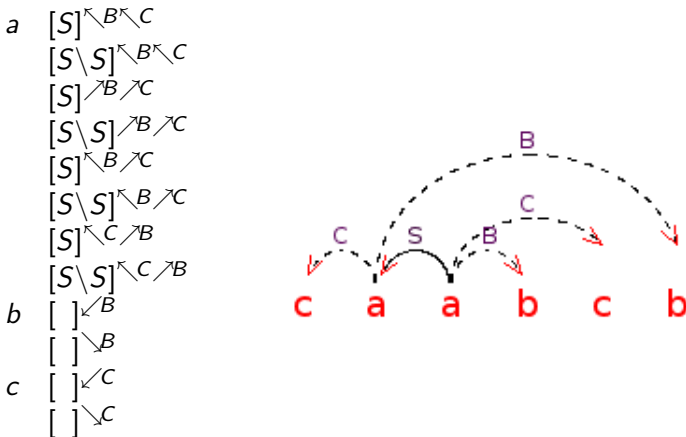
$L^r \quad [\beta/C]^P [C]^Q \vdash [\beta]^{PQ}$   
 $L^r_\epsilon \quad [\beta]^P [ ]^Q \vdash [\beta]^{PQ}$   
 $I^r \quad [\beta/C^*]^P [C]^Q \vdash [\beta/C^*]^{PQ}$   
 $\Omega^r \quad [\beta/C^*]^P \vdash [\beta]^P$   
 $D^r \quad \alpha^{P_1(\checkmark V)P(\checkmark V)P_2} \vdash \alpha^{P_1 P P_2}, \text{ if FA}$

### Dependency structure





# CDG formal example: mix of $n$ $a$ , $b$ and $c$



A CDG for mix with a parse example

In the above grammar, some types have empty heads ; other grammars avoiding empty heads can be provided, but the above one is simpler.

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# CDG example: $a^n b^n c^n$

$$a \quad \begin{array}{l} [A] \swarrow^D \\ [A \setminus A] \swarrow^D \end{array}$$

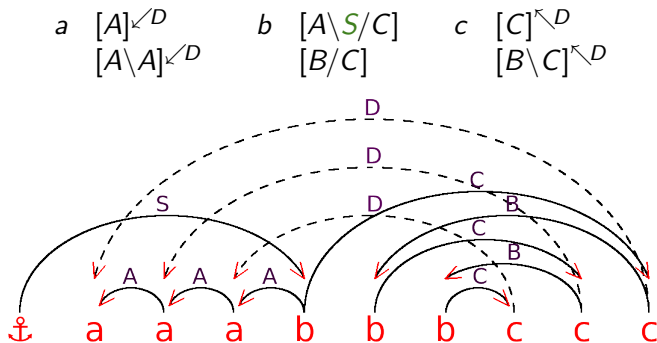
$$b \quad \begin{array}{l} [A \setminus S / C] \\ [B / C] \end{array}$$

$$c \quad \begin{array}{l} [C] \nwarrow^D \\ [B \setminus C] \nwarrow^D \end{array}$$

$$\begin{array}{c}
 \begin{array}{c} \begin{array}{c} \begin{array}{c} [A] \swarrow^D \quad [A \setminus A] \swarrow^D \\ \hline [A] \swarrow^D \swarrow^D \end{array} \quad \begin{array}{c} [A \setminus S / C] \\ \hline [S / C] \swarrow^D \swarrow^D \end{array} \\ \hline [S] \swarrow^D \swarrow^D \swarrow^D \swarrow^D \end{array} \quad \begin{array}{c} \begin{array}{c} \begin{array}{c} [B / C] \quad [C] \nwarrow^D \\ \hline [B] \nwarrow^D \end{array} \quad \begin{array}{c} [B \setminus C] \nwarrow^D \\ \hline [C] \nwarrow^D \nwarrow^D \end{array} \\ \hline [C] \nwarrow^D \nwarrow^D \nwarrow^D \nwarrow^D \end{array} \\
 \hline [S] \swarrow^D \swarrow^D \swarrow^D \nwarrow^D \nwarrow^D \\
 \hline [S] \swarrow^D \nwarrow^D \\
 \hline [S]
 \end{array}$$

A CDG for  $\{a^n b^n c^n, n \geq 1\}$  with a derivation for  $aabbcc$  ( $n = 2$ )

# CDG example: $a^n b^n c^n$



The same CDG for  $\{a^n b^n c^n, n \geq 1\}$  with the dependency structure for  $aaabbbccc$  ( $n = 3$ )

Parsing time complexity :  $\mathcal{O}(n^4)$

# CDG example: The product of $a^n b^n c^n$ with itself

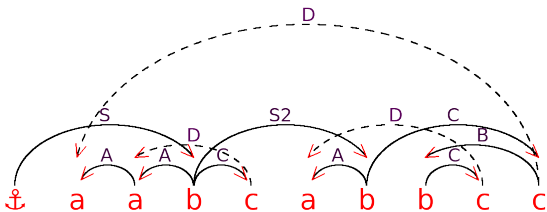
Is it possible to define a CDG that yields the product of  $a^n b^n c^n$  with itself ?

$$\{a^p b^p c^p a^q b^q c^q, p \geq 1, q \geq 1\}$$

How can we find it from a CDG that yields  $a^n b^n c^n$  ?

# CDG example: An unsuccessful attempt for the product of $a^n b^n c^n$ with itself

a  $[A] \swarrow^D$   
     $[A \setminus A] \swarrow^D$   
b  $[A \setminus \textcolor{green}{S} / \textcolor{blue}{S2} / C]$   
     $[A \setminus \textcolor{blue}{S2} / C]$   
     $[B / C]$   
c  $[C] \swarrow^D$   
     $[B \setminus C] \swarrow^D$



The CDG is built from the initial CDG for  $a^n b^n c^n$  :

The initial type of  $b$  with  $\textcolor{green}{S}$  is duplicated and  $\textcolor{blue}{S2}$  is added

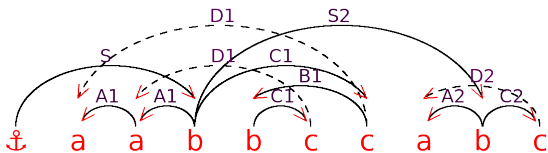
The CDG **doesn't yield** the product of  $a^n b^n c^n$  with itself:

$aabcabbcc$  can be parsed but it isn't correct:

**Non-projective dependencies between the parts are allowed**

# CDG example: A correct product of $a^n b^n c^n$ with itself

$a \quad [A_1] \swarrow^{D_1}$   
 $\quad [A_1 \setminus A_1] \swarrow^{D_1}$   
 $b \quad [A_1 \setminus \textcolor{green}{S}/\textcolor{blue}{S}_2/C_1]$   
 $\quad [B_1/C_1]$   
 $c \quad [C_1] \nwarrow^{D_1}$   
 $\quad [B_1 \setminus C_1] \nwarrow^{D_1}$   
  
 $a \quad [A_2] \swarrow^{D_2}$   
 $\quad [A_2 \setminus A_2] \swarrow^{D_2}$   
 $b \quad [A_2 \setminus \textcolor{blue}{S}_2/C_2]$   
 $\quad [B_2/C_2]$   
 $c \quad [C_2] \nwarrow^{D_2}$   
 $\quad [B_2 \setminus C_2] \nwarrow^{D_2}$



All the types are duplicated from the initial CDG for  $a^n b^n c^n$

⇒ Two non-projective dependency names:  $D_1$  and  $D_2$  rather than  $D$

⇒ Higher parsing time complexity:  $\mathcal{O}(n^5)$  rather than  $\mathcal{O}(n^4)$

No known general construction for the Kleene plus of a CDG

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# Our proposal : $CDG_{tb}$ calculus with **typed barriers**

## Left-oriented rules

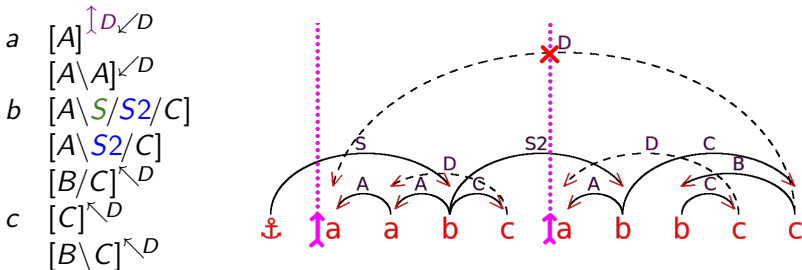
- $L^!$ .  $[C]^P [C \setminus \beta]^Q \vdash [\beta]^{PQ}$   $Gov \xrightarrow{C} Sub$
- $L^!_{\epsilon}$ .  $[ ]^P [\beta]^Q \vdash [\beta]^{PQ}$  (no new dependency)
- $I^!$ .  $[C]^P [C^* \setminus \beta]^Q \vdash [C^* \setminus \beta]^{PQ}$   $Gov \xrightarrow{C} Sub$
- $\Omega^!$ .  $[C^* \setminus \beta]^P \vdash [\beta]^P$  (no new dependency)
- $D^!$ .  $\alpha^{P_1(\swarrow C)P(\nwarrow C)P_2} \vdash \alpha^{P_1 P P_2}$   $Gov \dashrightarrow Sub$

## First-Available Rule (and no intermediate typed barrier)

$FA_{tb}$ : in  $(\swarrow C)P(\nwarrow C)$ , the valency  $\swarrow C$  is the **first available** for the dual valency  $\nwarrow C$ , i.e.  $P$  has no occurrences of  $\swarrow C$ ,  $\nwarrow C$  and  $\uparrow C$

Potentials contain polarized valencies  $\swarrow d$ ,  $\nwarrow d$ ,  $\searrow d$ ,  $\nearrow d$  and **typed barriers**  $\uparrow d$

# CDG<sub>tb</sub> with typed barriers: a simple product of $a^n b^n c^n$ with itself



There is a typed barrier  $\uparrow^D$  on the rightmost  $a$  (for  $aabcabbcc$ )

$\Rightarrow$  The top non-projective dependency isn't allowed this time

The CDG<sub>tb</sub> with typed barriers yields the product of  $a^n b^n c^n$  with itself

Only one non-projective dependency name ( $D$ )

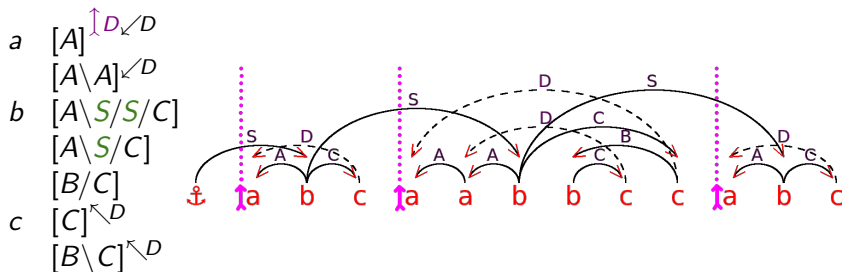
$\Rightarrow$  Same parsing time complexity as  $a^n b^n c^n$

Is it possible to define a CDG that yields Kleene plus of  $a^n b^n c^n$  ?  
 $\{a^{p_1} b^{p_1} c^{p_1} a^{p_2} b^{p_2} c^{p_2} \dots a^{p_n} b^{p_n} c^{p_n}, n \geq 1, p_1 \geq 1, \dots, p_n \geq 1\}$

How can we find it from a CDG that yields  $a^n b^n c^n$  ?

# CDG<sub>tb</sub> with typed barriers: Kleene plus of $a^n b^n c^n$

No known general construction for the Kleene plus of a CDG  
Always possible with a CDG<sub>tb</sub> (our proposal)



A typed barrier on the leftmost  $a$  of each part of the Kleene plus  
 $\Rightarrow$  Non-projective dependencies between parts aren't allowed  
 $\Rightarrow$  The CDG<sub>tb</sub> yields the Kleene plus of  $a^n b^n c^n$

Only one non-projective dependency name ( $D$ )  
 $\Rightarrow$  Same parsing time complexity as  $a^n b^n c^n$

# Kleene plus: The general construction for a $CDG_{tb}$ language

Starting with  $G$ , a  $CDG_{tb}$  with typed barriers

- ① Transform  $G$  if  $G$  has types with empty heads in the lexicon
- ② Transform  $G$  if the axiom  $S$  is used as an argument of a type
- ③ Transform  $G$  such that the types in the lexicon are divided in two parts :
  - The types only used on the rightmost token of any derivation
  - The types never used on the rightmost token of any derivation
- ④ Add (typed) barriers in the potential of the types that can only be used on the rightmost token of any derivation
- ⑤ For each type with the axiom  $S$  as head type, duplicate the same type but where  $S$  is replaced with  $S/S$

The final  $CDG_{tb}$  corresponds to the Kleene plus of the initial  $CDG_{tb}$

# Example: Kleene plus of $a^n b^n c^n$

$$\begin{array}{ccc} a & [A] \swarrow^D & b & [A \setminus \textcolor{green}{S} / C] \\ & [A \setminus A] \swarrow^D & & [B / C] \\ & & c & [C] \nwarrow^D \\ & & & [B \setminus C] \nwarrow^D \end{array}$$

- ① Transform  $G$  if  $G$  has types with empty heads in the lexicon  
 $\implies$  Ok (no empty head)
- ② Transform  $G$  if the axiom  $S$  is used as an argument of a type  
 $\implies$  Ok ( $S$  only used as head type))

# Example: Kleene plus of $a^n b^n c^n$

$$\begin{array}{lll} a & [A]^{\swarrow D} & b \quad [A \setminus S/C] \\ & [A \setminus A]^{\swarrow D} & [B/C] \end{array} \quad c \quad [C]^{\nwarrow D} \\ & & [B \setminus C]^{\nwarrow D}$$

- 3 Transform  $G$  such that the types in the lexicon are divided in two parts :
- The types only used on the rightmost token of any derivation
  - The types never used on the rightmost token of any derivation

Types on the rightmost token: Types of  $c$  ( $[C]^{\nwarrow D}$  and  $[B \setminus C]^{\nwarrow D}$ )

Types on other tokens: All the types

Not ok (the types of  $c$ )

$\Rightarrow$  We need to transform the grammar (axiom  $S_r$ ):

$$\begin{array}{lll} a & [A_r]^{\swarrow D} & b \quad [A_o \setminus S_r / C_r] \\ & [A_o]^{\swarrow D} & [A_o \setminus S_o / C_o] \\ & [A_o \setminus A_r]^{\swarrow D} & [B_r / C_r] \\ & [A_o \setminus A_o]^{\swarrow D} & [B_o / C_o] \end{array} \quad c \quad [C_r]^{\nwarrow D} \\ & & [C_o]^{\nwarrow D} \\ & & [B_o \setminus C_r]^{\nwarrow D} \\ & & [B_o \setminus C_o]^{\nwarrow D}$$

Remark: The grammar can be simplified (useless types)

# Example: Kleene plus of $a^n b^n c^n$

$$a \quad \begin{array}{l} [A_o] \swarrow^D \\ [A_o \setminus A_o] \swarrow^D \end{array}$$

$$b \quad \begin{array}{l} [A_o \setminus S_r / C_r] \\ [B_o / C_o] \end{array}$$

$$c \quad \begin{array}{l} [C_r] \swarrow^D \\ [C_o] \swarrow^D \\ [B_o \setminus C_r] \swarrow^D \\ [B_o \setminus C_o] \swarrow^D \end{array}$$

- 4 Add typed barriers in the potential of the types that can only be used on the rightmost token of any derivation

$$a \quad \begin{array}{l} [A_o] \swarrow^D \\ [A_o \setminus A_o] \swarrow^D \end{array}$$

$$b \quad \begin{array}{l} [A_o \setminus S_r / C_r] \\ [B_o / C_o] \end{array}$$

$$c \quad \begin{array}{l} [C_r] \swarrow^D \uparrow^D \\ [C_o] \swarrow^D \\ [B_o \setminus C_r] \swarrow^D \uparrow^D \\ [B_o \setminus C_o] \swarrow^D \end{array}$$



# Example: Kleene plus of $a^n b^n c^n$

$$\begin{array}{lll}
 a & \begin{array}{l} [A_o] \swarrow^D \\ [A_o \setminus A_o] \swarrow^D \end{array} & b \quad \begin{array}{l} [A_o \setminus S_r / C_r] \\ [B_o / C_o] \end{array} \quad c \quad \begin{array}{l} [C_r] \swarrow^D \uparrow^D \\ [C_o] \swarrow^D \\ [B_o \setminus C_r] \swarrow^D \uparrow^D \\ [B_o \setminus C_o] \swarrow^D \end{array}
 \end{array}$$

- 5 For each type with the axiom  $S_r$  as head type, duplicate the same type but where  $S_r$  is replaced with  $S_r / S_r$

$$\begin{array}{lll}
 a & \begin{array}{l} [A_o] \swarrow^D \\ [A_o \setminus A_o] \swarrow^D \end{array} & b \quad \begin{array}{l} [A_o \setminus S_r / C_r] \\ [A_o \setminus S_r / S_r / C_r] \\ [B_o / C_o] \end{array} \quad c \quad \begin{array}{l} [C_r] \swarrow^D \uparrow^D \\ [C_o] \swarrow^D \\ [B_o \setminus C_r] \swarrow^D \uparrow^D \\ [B_o \setminus C_o] \swarrow^D \end{array}
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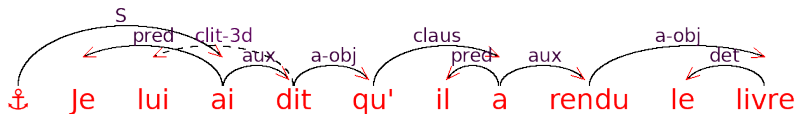
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# CDG and CDG<sub>tb</sub> for Natural Languages

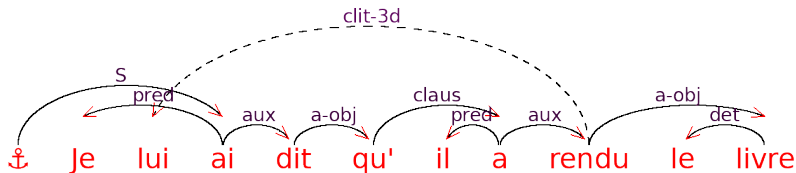
The CDG analyses of “Je lui ai dit qu’il a rendu le livre”

“I have told him that he has returned the book” clitic “lui” (him)

$il \mapsto [ ] \xleftarrow{\text{clit-3d}}$   $dit, rendu \mapsto [aux/a-obj], [aux/a-obj] \xleftarrow{\text{clit-3d}}$



The normal analysis:  $dit \mapsto [aux/a-obj] \xleftarrow{\text{clit-3d}}$   $rendu \mapsto [aux/a-obj]$

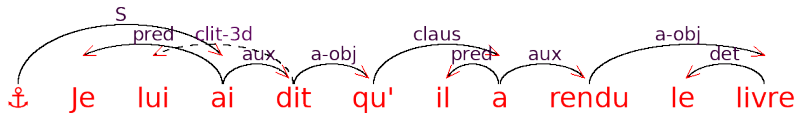


A wrong analysis:  $dit \mapsto [aux/a-obj] \xleftarrow{\text{clit-3d}}$   $rendu \mapsto [aux/a-obj] \xleftarrow{\text{clit-3d}}$

# CDG and CDG<sub>tb</sub> for Natural Languages

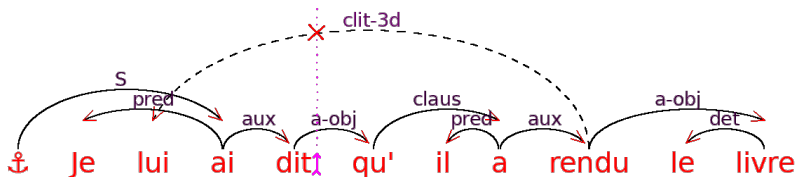
The CDG<sub>tb</sub> analyses of “Je lui ai dit qu’il a rendu le livre”  
 “I have told him that he has returned the book” clitic “lui” (him)

$il \mapsto [ ] \swarrow \text{clit-3d}$   $dit, rendu \mapsto [aux/a-obj] \nearrow \text{clit-3d}$ ,  $[aux/a-obj] \swarrow \text{clit-3d}$



The normal analysis:  $dit \mapsto [aux/a-obj] \swarrow \text{clit-3d}$

$rendu \mapsto [aux/a-obj] \nearrow \text{clit-3d}$



No analysis:  $dit \mapsto [aux/a-obj] \nearrow \text{clit-3d}$   $rendu \mapsto [aux/a-obj] \swarrow \text{clit-3d}$

⇒ Typed barriers can control the range of specific non-projective dependencies

# Conclusion and Open Questions

- The **product** and the **Kleene plus** of languages may be useful, for instance, to model the **conjunction of parts of speech** or the **list of complex complements** in a lot of **natural languages**.
- Our proposal **allows** such constructions for **CDG<sub>tb</sub> languages** (with typed barriers).
- There is **no parsing complexity penalty** for the **product** and the **Kleene plus**.
- **Categorial Dependency Grammars extended with typed barriers** define an **Abstract Family of Languages** (closed under union, product, Kleene plus,  $\varepsilon$ -free homomorphism, inverse homomorphism, intersection with regular sets).
- The same **questions** remain **opened** for **classical CDG**.
- For natural languages, **typed barriers** can control the **range** of **specific** non-projective dependencies.

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THANK YOU !



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