

Report on the outcomes of a Short-Term Scientific Mission¹

Action number: CA20111

Grantee name: Melanie Taprogge

Details of the STSM

Title: Verification of Higher-Order Logic Automated Reasoning within the Dedukti Framework

Start and end date: 18/05/25 to 29/05/25

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

This report discusses Melanie Taprogge's Short-Term Scientific Mission to visit Alexander Steen at the University of Greifswald.

During the visit, we continued our joint work on enabling the Leo-III higher-order logic prover to output proofs verifiable in the Dedukti (Lambdapi) format. The mission closely followed the initial working plan, with minor adjustments prompted by discussions and findings during the visit. Furthermore, Geoff Sutcliffe was present for part of the visit, which provided an opportunity for collaboration. He intends to use the version of Leo-III that produces Lambdapi proofs in his general-purpose TSTP checker, GDV-LP, once the extension has been completed. His extensive expertise in proof checking and output standards in automated reasoning was invaluable.

Firstly, we discussed the current status of the implementation, with a particular focus on the encoding of the classification steps, which was only recently included. We then systematically reviewed Leo-III's inference rules and considered aspects of the implementation that complicate verification. One notable issue identified was the unpredictable ordering of terms in the generated proofs, which is introduced by specific inference rules. While this ordering is inconsequential for TSTP outputs, it significantly complicates verification, necessitating additional intermediate steps. We discussed solutions for adapting Leo-III to produce consistently ordered terms and identified the necessary modifications to the core prover. Similarly, other implementation-related challenges were identified, and changes to Leo-III's reasoning process were agreed upon. Additionally, an error in the current output generation process was

¹ This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

discovered, which occasionally causes definitions given in the input problems and used by certain inference rules to be omitted from the proof certificate. We also examined the verification and encoding of Skolemisation steps and how they are represented using choice. We decided to include explicit definitions of Skolem terms in both the standard and Lambdapi outputs. Further discussions covered the impact of the developments planned by Alexander Steen on the base prover, such as translating external proofs from provers like E, which are integrated into the Leo-III reasoning workflow. We discussed how to implement this automated translation to allow verification using Lambdapi seamlessly.

In collaboration with Geoff Sutcliffe, we held extensive discussions about the optimal level of detail for proof outputs. This collaboration was instrumental in the decision to produce Leo-III proofs in greater detail, which will significantly simplify future verification efforts and enhance compatibility with external proof-checkers. We also discussed verifying Skolemisation steps in the GDV tool.

Overall, the STSM successfully identified several improvements that were necessary for Leo-III, providing clarity and structure for subsequent integration steps.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

The STSM successfully achieved its primary objective of advancing the integration of Leo-III with the Dedukti framework by encoding proofs in Lambdapi, while also coordinating this development with the ongoing prover work. Through in-depth discussions and joint analysis, several concrete achievements were made that contribute directly to the goals of the COST Action, particularly Research Coordination Objectives 1 and 2, as well as deliverables D9 and D11.

One of the most significant outcomes was the identification of technical obstacles in Leo-III's current proof output that hinder formal verification. We proposed solutions to produce consistent and verifiable outputs, including potential changes to the prover's core implementation. Furthermore, we identified an issue resulting in missing definitions in certain proof outputs, which will be addressed as part of the ongoing work. This contributes to Objective 1: expressing a new proof system in the Dedukti framework.

A key decision made during the STSM, in collaboration with Geoff Sutcliffe, was to enhance the granularity of Leo-III's proof output. Finer granularity will simplify verification efforts for both our Lambdapi encoding and other tools, such as GDV. Consequently, Leo-III will become a more robust and accessible source of verifiable higher-order logic proofs. Additionally, the treatment of Skolemisation in Leo-III was carefully reviewed. It was agreed that Skolem terms and their definitions should be made explicit in the output, thereby improving the clarity and completeness of the encoding. Both directly support Objective 2, which is to promote detailed, checkable proofs.

Although no publications resulted immediately from the STSM, the insights gained and decisions made will form the basis of ongoing work expected to lead to future publications. These publications will document the encoding strategies, the practical challenges of implementation, the changes made to Leo-III as a result, and the verification of higher-order proofs in Dedukti.

Planned follow-up activities include continuing to implement the Lambdapi output extension and coordinating with the upcoming changes to Leo-III. We will also continue to work with Geoff Sutcliffe to integrate the extended Leo-III into the GDV tool, which will enable higher-order TSTP proofs to be verified using Lambdapi (contributing to D9).

In summary, the STSM achieved its intended goals, overcoming several technical and conceptual challenges and laying the groundwork for future collaborative work that will directly contribute to the COST Action's objectives and deliverables.



Prof. Dr. Alexander Steen, 11.06.2025