

Controlling computation in type theory, *locally*



Théo Winterhalter

INRIA Saclay

Computation in type theory ✨

Proofs by computation

```
refl : 2 + 2 = 4
```

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Proving equalities in a commutative ring done right in Coq

Grégoire, Mahboubi

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**Accelerating verified-compiler development
with a verified rewrite engine**

Gross, Erbsen, Philipoom, Poddar-Agrawal, Chlipala

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Observational equality, now!

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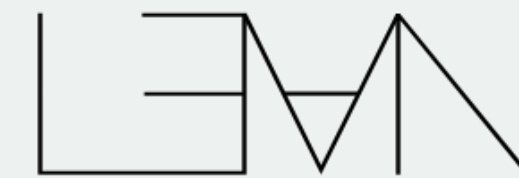
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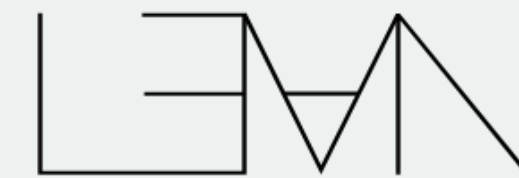
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Definitional proof irrelevance without K

Gilbert, Cockx, Tabareau

2019

 Agda

 ROCQ

and more!

Controlling and extending computation in ITPs

Coq modulo theory

Strub

2010

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Coq modulo theory

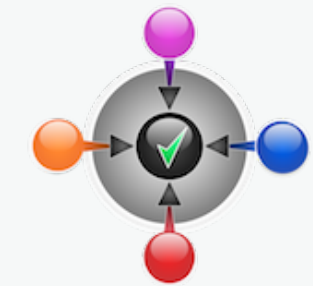
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Dedukti: a logical framework based on the $\lambda\Pi$ -calculus modulo theory

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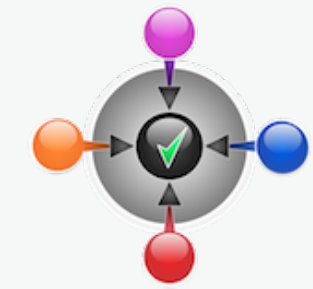
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Sprinkles of extensionality for your vanilla type theory

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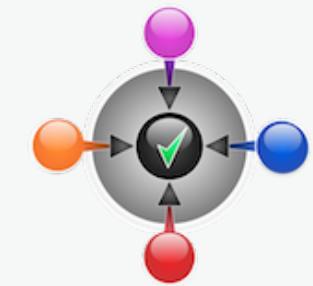
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Gratzer, Sterling, Angiuli, Coquand, Birkedal

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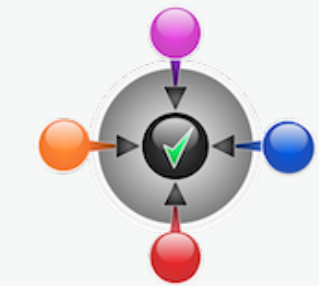
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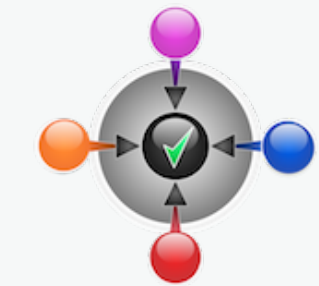
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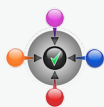



This one is **local!** 😎

Why locality matters

Example: exceptions

```
symb raise : ∀ {A}, A  
rule if raise then t else f ↦ raise  
defn nth_exn : list A → ℕ → A := ...
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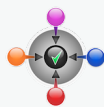

   **ROCQ**
Computation rules must be assumed **forever**

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● — No way to ensure the rules aren't used here
(unlike axioms, there is no **Print Assumptions**)

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Rules extend the **trusted computing base**

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Uncaught mistake without a model
(somewhat mitigated in UAgda)

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Overall, not very **modular**
(or type-theoretic)

The elephant in the room: why not use extensional type theory?

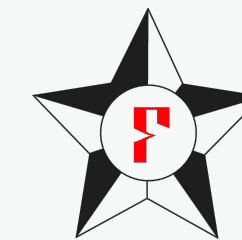
Equality reflection

$$\frac{\Gamma \vdash p : u =_A v}{\Gamma \vdash u \equiv v}$$

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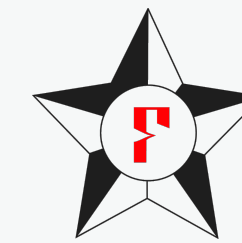


Undecidable type checking
need to rely on heuristics eg SMT solvers in F*
so no longer really computation

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Conservative over ITT + UIP + funext

Extensional concepts in intensional type theory

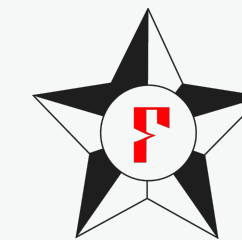
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Extensional concepts in intensional type theory

Hofmann

1995

Effective translation to ITT

Extensionality in the calculus of constructions

Oury

2005

Eliminating reflection from type theory

Winterhalter, Sozeau, Tabareau

2019

Terribly inefficient!

Prenex quantification over (directed) equations

```
interface Bool
  assumes
    bool : Type
    true, false : bool
    ifte :  $\forall (P : \text{bool} \rightarrow \text{Type}). P \text{ true} \rightarrow P \text{ false} \rightarrow \forall b. P b$ 
  where
    ifte P t f true  $\mapsto$  t
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interface Sum
  assumes
    sum : Type  $\rightarrow$  Type  $\rightarrow$  Type
    inl :  $\forall \{A B\}. A \rightarrow \text{sum } A B$ 
    inr :  $\forall \{A B\}. B \rightarrow \text{sum } A B$ 
    elim :
       $\forall \{A B\} (P : \text{sum } A B \rightarrow \text{Type}).$ 
         $(\forall a, P (\text{inl } a)) \rightarrow$ 
         $(\forall b, P (\text{inr } b)) \rightarrow$ 
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  where
    elim P l r (inl a)  $\mapsto$  l a
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```
instance Bool-as-sum( U : Unit, S : Sum ) : Bool
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  true := S.inl U.*
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  ifte P t f := S.elim P (U.elim _ t) (U.elim _ f)
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Equations are verified implicitly

My proposal

Prenex quantification over (directed) equations

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You can exploit the encoding without having to work directly with it!

Other examples include...

Hiding implementation details
while retaining computation

```
interface Shift
  assumes
    shift : list ℕ → list ℕ
  where
    shift (x :: l) ↦ suc x :: shift l
    shift [] ↦ []
```

```
instance ShiftasMap
  shift := map suc
```

This way, map never appears in goals out of nowhere
useful for automatically generated functions (eg. Equations in Rocq)
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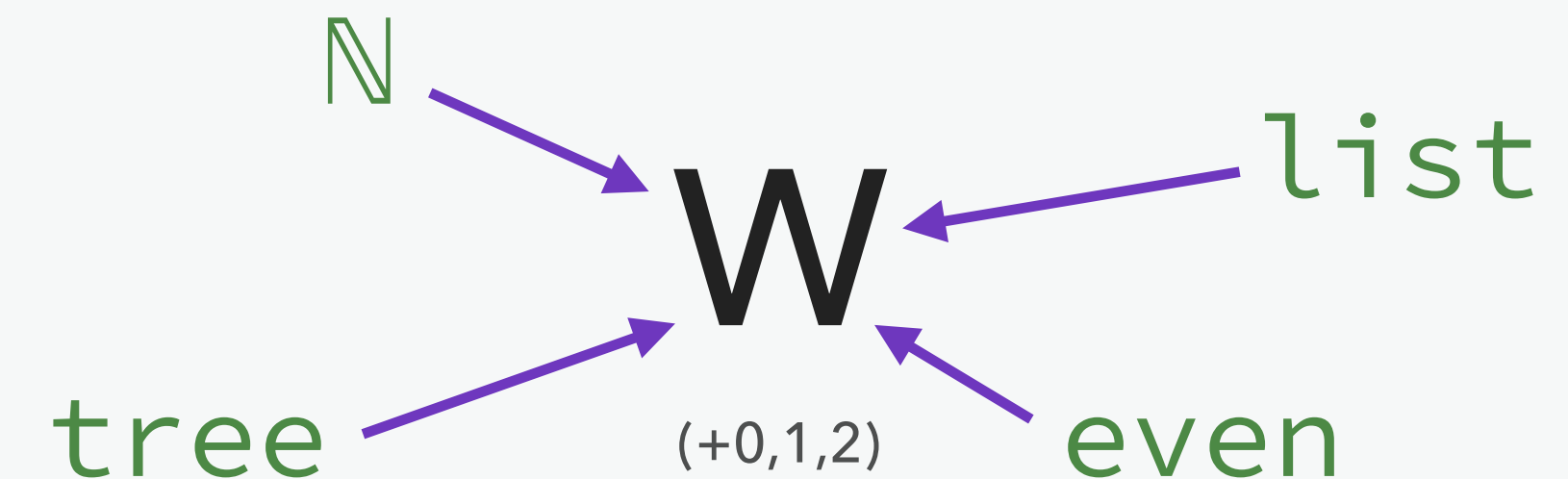
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Encode features using simpler ones



Why not W?

Hugunin

2021

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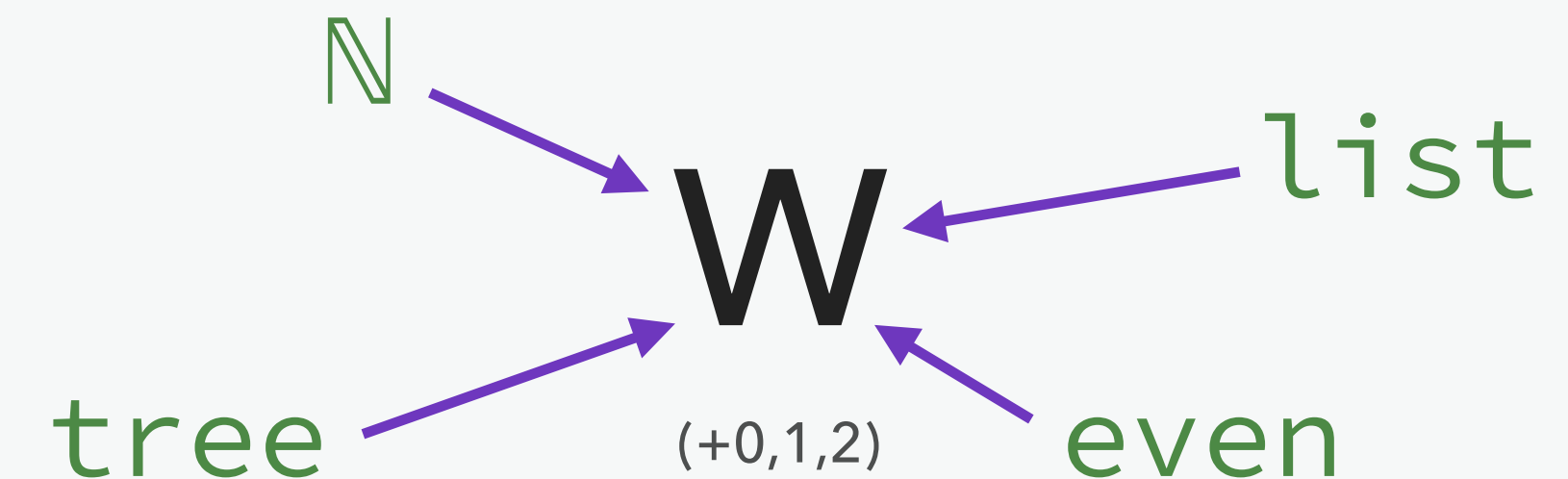
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Hugunin

2021

Use effects locally, eg. exceptions

The type theory

$\Sigma \mid \Xi \mid \Gamma \vdash t : A$

The type theory

```
interface E⟨  $\Xi'$  ⟩ assumes  $\Delta$  where R
```

```
def f⟨  $\Xi'$  ⟩ : A := t
```

Global environment

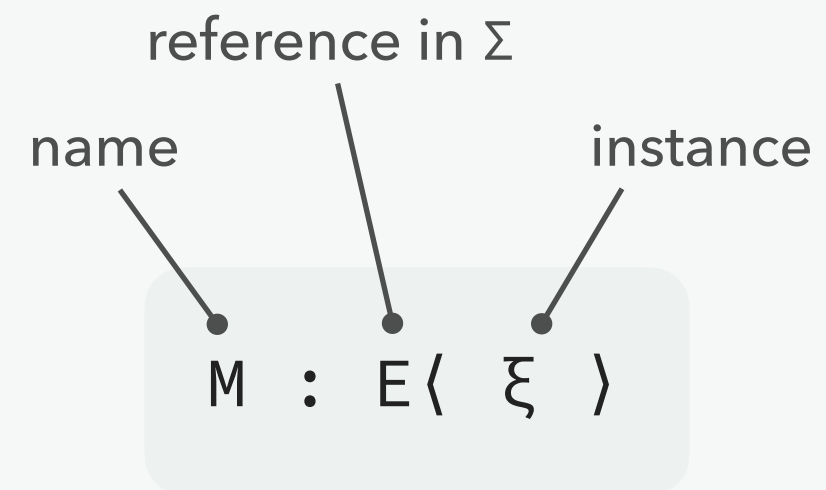


$\Sigma \mid \Xi \mid \Gamma \vdash t : A$

The type theory

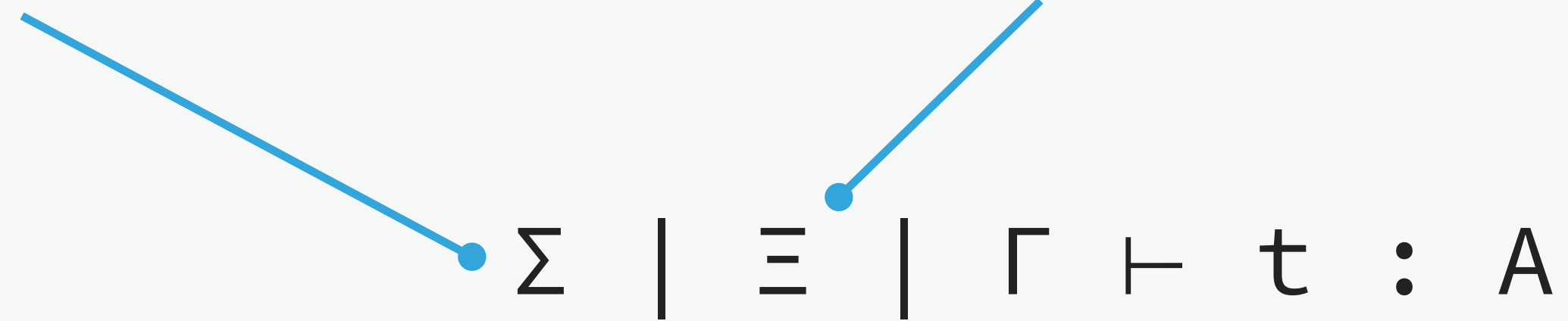
interface $E\langle \Xi' \rangle$ assumes Δ where R

def $f\langle \Xi' \rangle : A := t$



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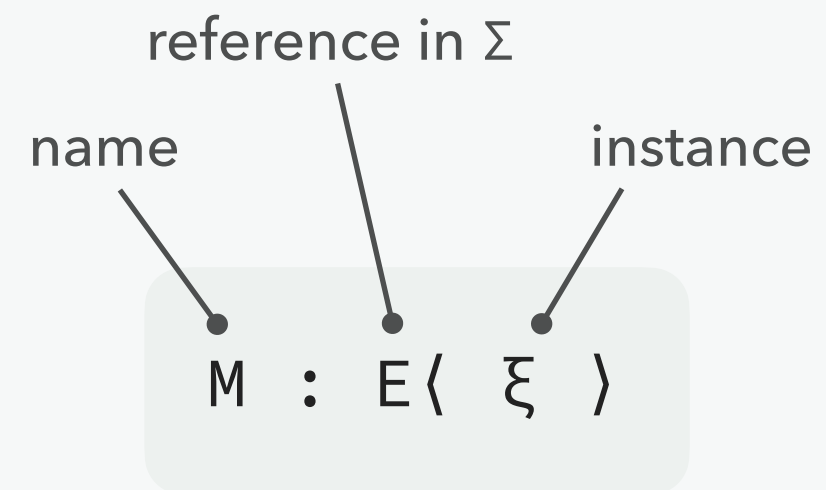
Extension environment



The type theory

`interface` $E\langle \Xi' \rangle$ `assumes` Δ `where` R

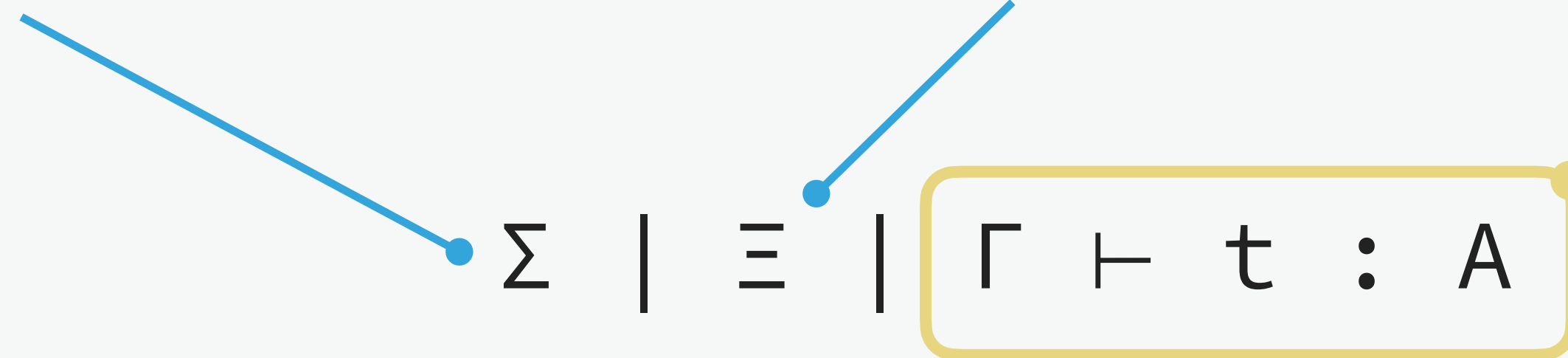
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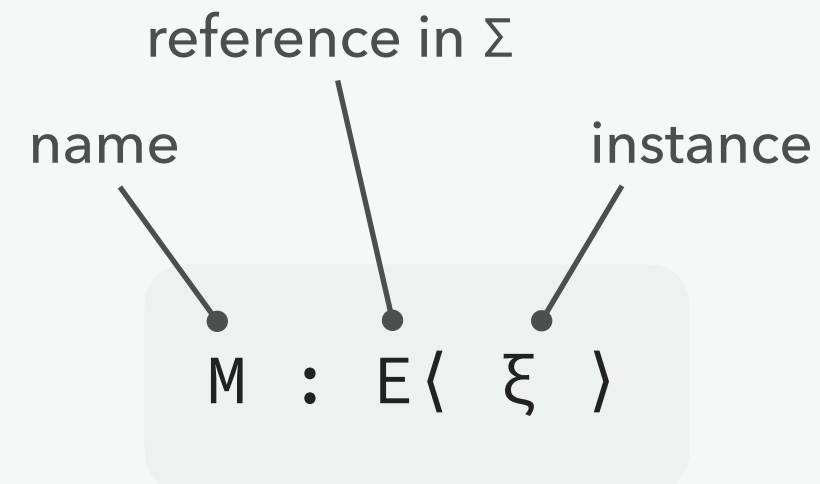
Basically regular MLTT



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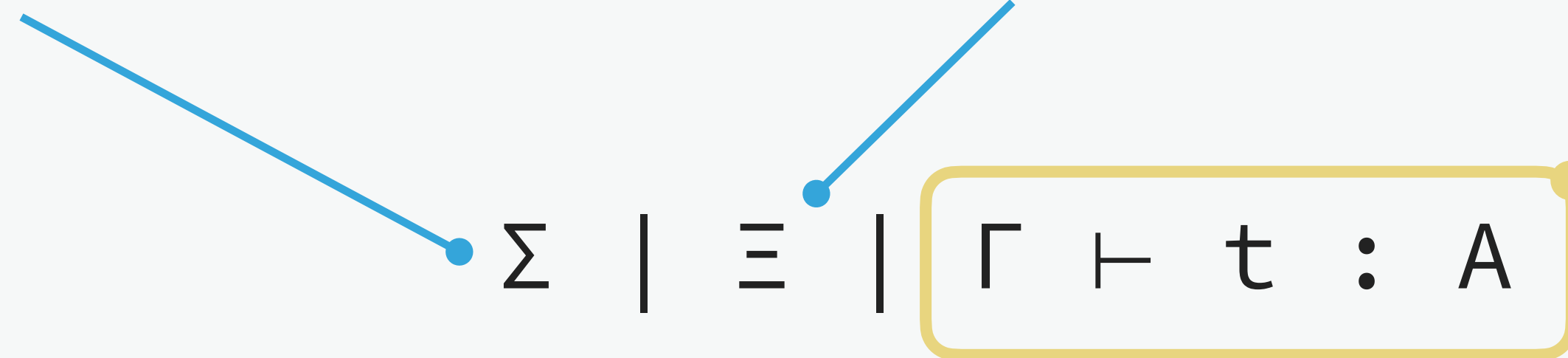
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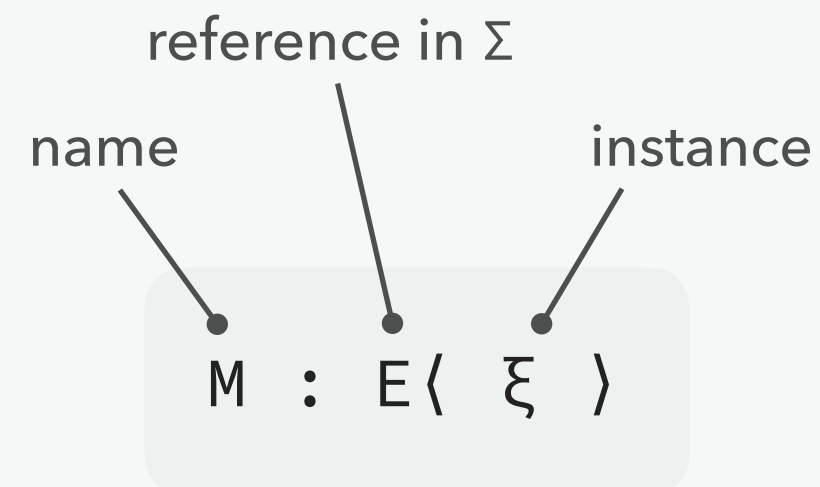
Computation rule (simplified)

$$\frac{\begin{array}{l} (\text{interface } E\langle \Xi' \rangle \text{ assumes } \Delta \text{ where } R) \in \Sigma \\ (M : E\langle \xi \rangle) \in \Xi \end{array} \quad (\lambda \mapsto r) \in R}{\Sigma \mid \Xi \mid \Gamma \vdash \lambda \xi \sigma \equiv r \xi \sigma}$$

The type theory

`interface` $E\langle \Xi' \rangle$ `assumes` Δ `where` R

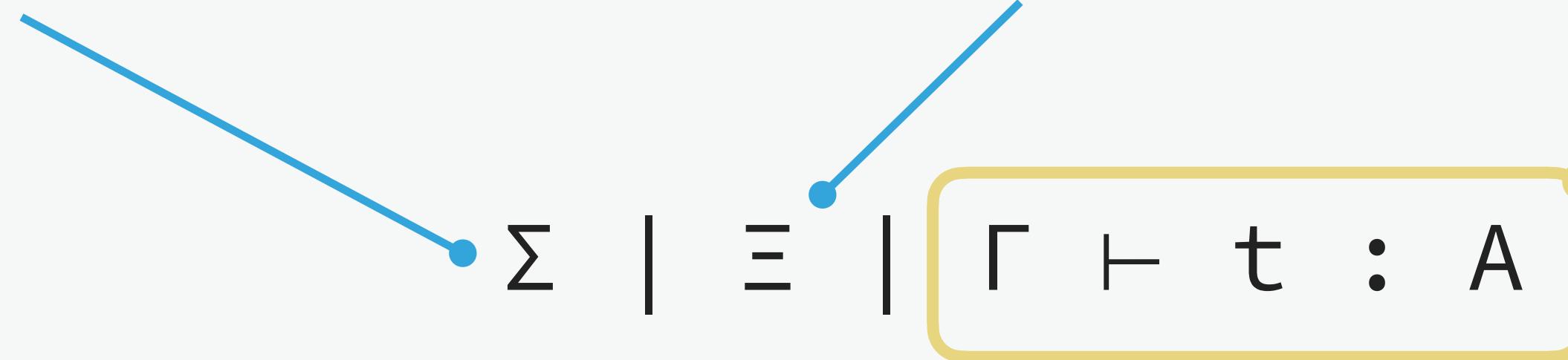
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Unfolding rule

$$\frac{(\text{def } f\langle \Xi' \rangle : A := t) \in \Sigma \quad \Sigma \mid \Xi \mid \Gamma \vdash \xi : \Xi'}{\Sigma \mid \Xi \mid \Gamma \vdash f\langle \xi \rangle \equiv t \xi}$$

Meta-theory

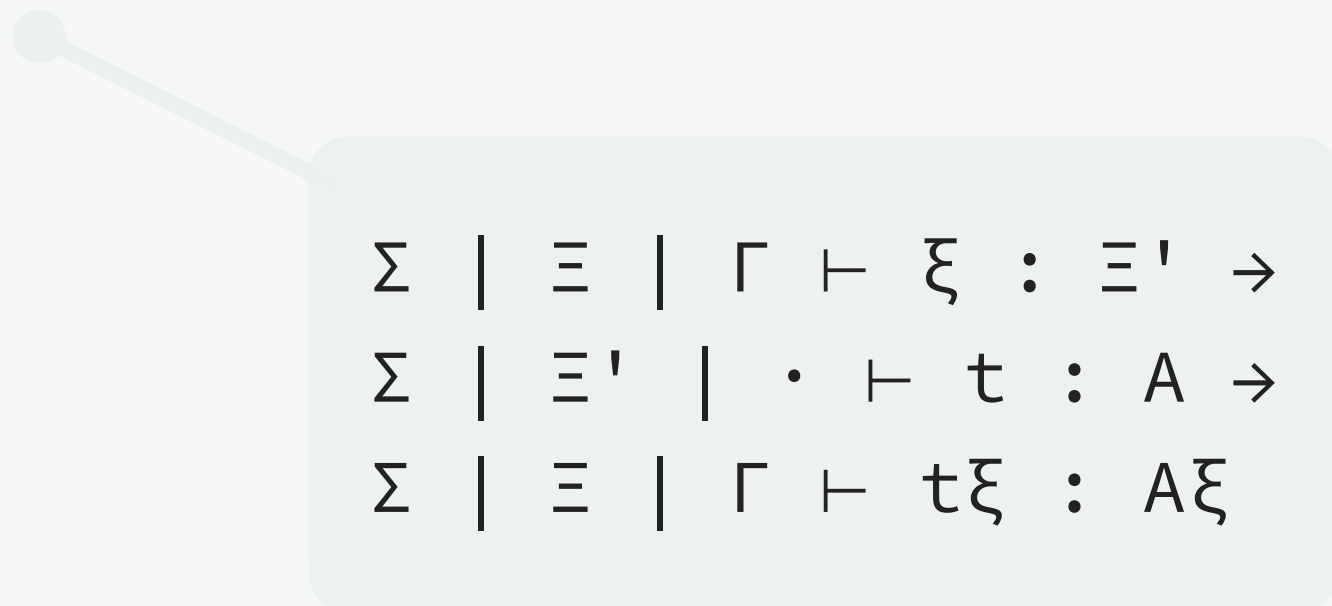
mostly usual

Environment weakening (Σ , Ξ , Γ), substitution, instantiation, validity

Meta-theory

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Environment weakening (Σ, Ξ, Γ) , substitution, instantiation, validity


$$\begin{array}{l} \Sigma \mid \Xi \mid \Gamma \vdash \xi : \Xi' \rightarrow \\ \Sigma \mid \Xi' \mid \cdot \vdash t : A \rightarrow \\ \Sigma \mid \Xi \mid \Gamma \vdash t\xi : A\xi \end{array}$$

Meta-theory

mostly usual

Environment weakening (Σ, Ξ, Γ) , **substitution, instantiation, validity**

Consistency

A given by embedding into ETT 

$\Sigma \mid \Xi \mid \Gamma \vdash \xi : \Xi' \rightarrow$
 $\Sigma \mid \Xi' \mid \cdot \vdash t : A \rightarrow$
 $\Sigma \mid \Xi \mid \Gamma \vdash t\xi : A\xi$

Meta-theory

mostly usual

Environment weakening (Σ, Ξ, Γ) , substitution, instantiation, validity

Consistency

A given by embedding into ETT 

$\Sigma \mid \Xi \mid \Gamma \vdash \xi : \Xi' \rightarrow$
 $\Sigma \mid \Xi' \mid \cdot \vdash t : A \rightarrow$
 $\Sigma \mid \Xi \mid \Gamma \vdash t\xi : A\xi$

more interesting:

Conservativity over MLTT

$\cdot \mid \cdot \mid \cdot \vdash A : \text{Type} \rightarrow$
 $\Sigma \mid \cdot \mid \cdot \vdash t : A \rightarrow$
 $\exists t'. \cdot \mid \cdot \mid \cdot \vdash t' : A$

Obtained by **inlining** definitions

Inlining

if $\Sigma \mid \Xi \mid \Gamma \vdash t : A$

then $\llbracket \Sigma \rrbracket \mid \llbracket \Xi \rrbracket \langle \kappa \rangle \mid \llbracket \Gamma \rrbracket \langle \kappa \rangle \vdash \llbracket t \rrbracket \langle \kappa \rangle : \llbracket A \rrbracket \langle \kappa \rangle$

where κ interprets the definitions of Σ

Inlining

if $\Sigma \mid \Xi \mid \Gamma \vdash t : A$

then $[\Sigma] \mid [\Xi]\langle \kappa \rangle \mid [\Gamma]\langle \kappa \rangle \vdash [t]\langle \kappa \rangle : [A]\langle \kappa \rangle$

where κ interprets the definitions of Σ

removes all definitions
and unfolds them in extensions

Inlining

if $\Sigma \mid \Xi \mid \Gamma \vdash t : A$

then $\llbracket \Sigma \rrbracket \mid \llbracket \Xi \rrbracket \langle \kappa \rangle \mid \llbracket \Gamma \rrbracket \langle \kappa \rangle \vdash \llbracket t \rrbracket \langle \kappa \rangle : \llbracket A \rrbracket \langle \kappa \rangle$

where κ interprets the definitions of Σ

removes all definitions
and unfolds them in extensions

with κ fixed (and abstract):

$\llbracket x \rrbracket := x$

$\llbracket \lambda (x : A). t \rrbracket := \lambda (x : \llbracket A \rrbracket). \llbracket t \rrbracket$

$\llbracket u v \rrbracket := \llbracket u \rrbracket \llbracket v \rrbracket$

$\llbracket M.x \rrbracket := M.x$

$\llbracket f \langle \xi \rangle \rrbracket := (\kappa f) \llbracket \xi \rrbracket$

Inlining

if $\Sigma \mid \Xi \mid \Gamma \vdash t : A$

then $\llbracket \Sigma \rrbracket \mid \llbracket \Xi \rrbracket \langle \kappa \rangle \mid \llbracket \Gamma \rrbracket \langle \kappa \rangle \vdash \llbracket t \rrbracket \langle \kappa \rangle : \llbracket A \rrbracket \langle \kappa \rangle$

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κ is then defined by induction on $\vdash \Sigma$ such that

when $(\text{def } f \langle \Xi' \rangle : A := t) \in \Sigma$ we have $\kappa f := \llbracket t \rrbracket \langle \kappa_{\text{rec}} \rangle$

Inlining

if $\Sigma \mid \Xi \mid \Gamma \vdash t : A$

then $\llbracket \Sigma \rrbracket \mid \llbracket \Xi \rrbracket \langle \kappa \rangle \mid \llbracket \Gamma \rrbracket \langle \kappa \rangle \vdash \llbracket t \rrbracket \langle \kappa \rangle : \llbracket A \rrbracket \langle \kappa \rangle$

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recursive call ok because t lives
in an environment *smaller* than Σ

Inlining

Compared to conservativity,
we need full generality here

if $\Sigma \mid \Xi \mid \Gamma \vdash t : A$

then $\llbracket \Sigma \rrbracket \mid \llbracket \Xi \rrbracket \langle \kappa \rangle \mid \llbracket \Gamma \rrbracket \langle \kappa \rangle \vdash \llbracket t \rrbracket \langle \kappa \rangle : \llbracket A \rrbracket \langle \kappa \rangle$

where κ interprets the definitions of Σ

removes all definitions
and unfolds them in extensions

with κ fixed (and abstract):

$\llbracket x \rrbracket := x$

$\llbracket \lambda (x : A). t \rrbracket := \lambda (x : \llbracket A \rrbracket). \llbracket t \rrbracket$

$\llbracket u v \rrbracket := \llbracket u \rrbracket \llbracket v \rrbracket$

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$\llbracket f \langle \xi \rangle \rrbracket := (\kappa f) \llbracket \xi \rrbracket$

κ is then defined by induction on $\vdash \Sigma$ such that

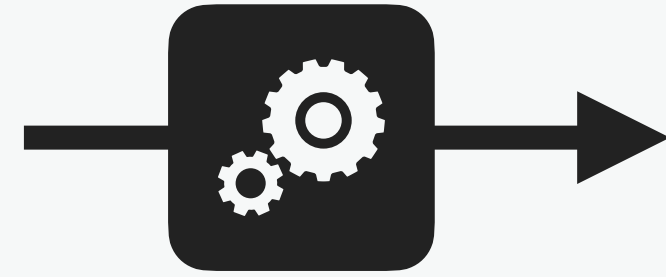
when $(\text{def } f \langle \Xi' \rangle : A := t) \in \Sigma$ we have $\kappa f := \llbracket t \rrbracket \langle \kappa_{\text{rec}} \rangle$

recursive call ok because t lives
in an environment *smaller* than Σ

Conclusion

Conservative extension of MLTT with **local computation**

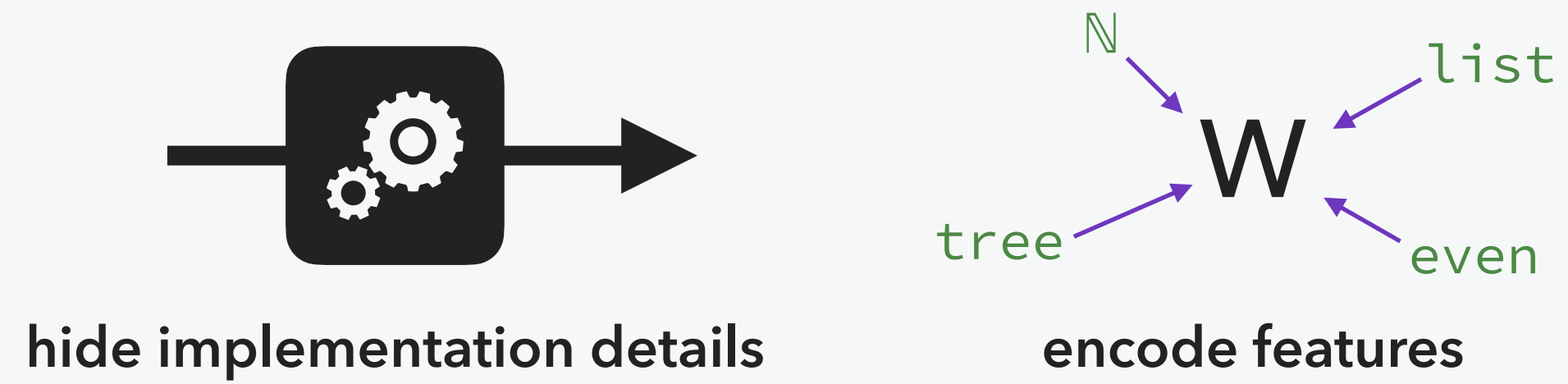
Conclusion



hide implementation details

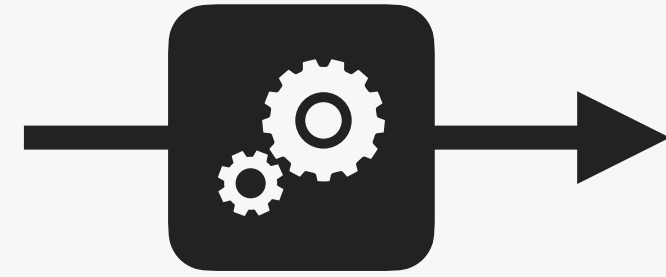
Conservative extension of MLTT with **local computation**

Conclusion

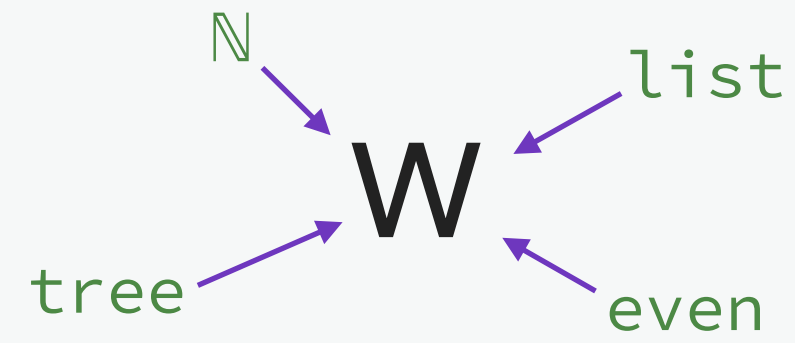


Conservative extension of MLTT with **local computation**

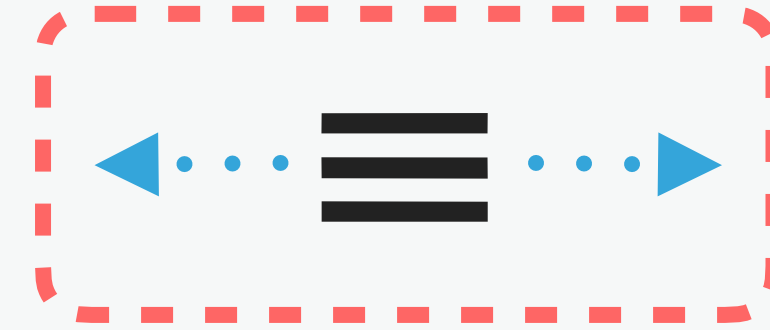
Conclusion



hide implementation details



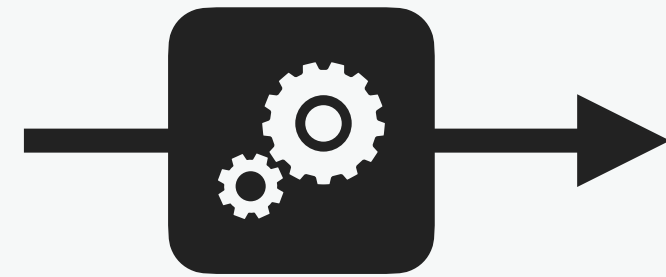
encode features



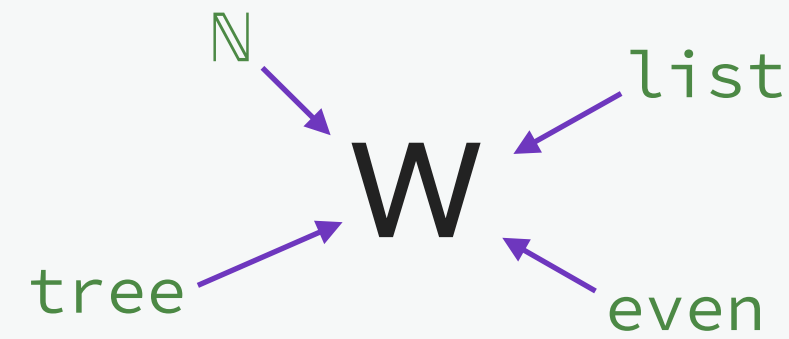
contained extensions (safer)

Conservative extension of MLTT with **local computation**

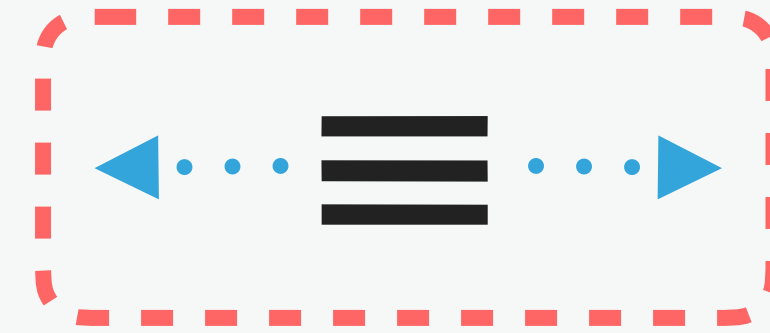
Conclusion



hide implementation details



encode features



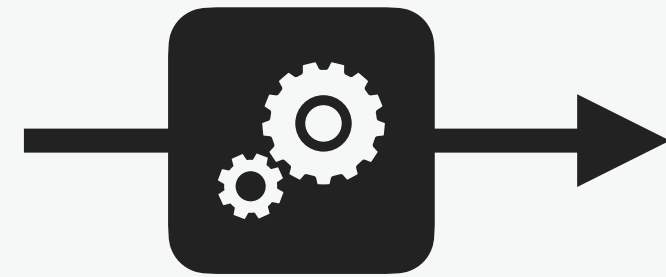
contained extensions (safer)

Conservative extension of MLTT with **local computation**

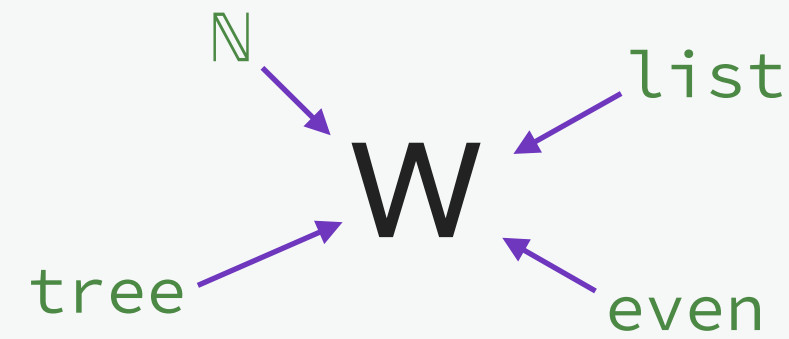


 /TheoWinterhalter/[local-comp](https://github.com/TheoWinterhalter/local-comp)

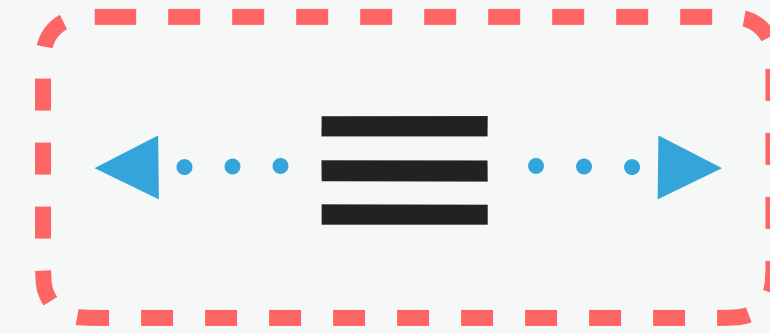
Conclusion



hide implementation details



encode features



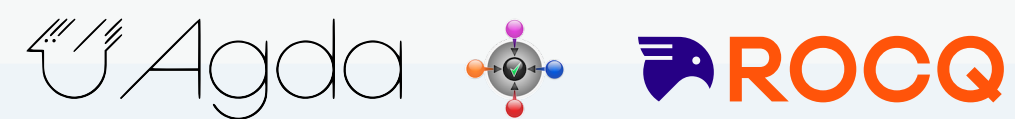
contained extensions (safer)

Conservative extension of MLTT with **local computation**



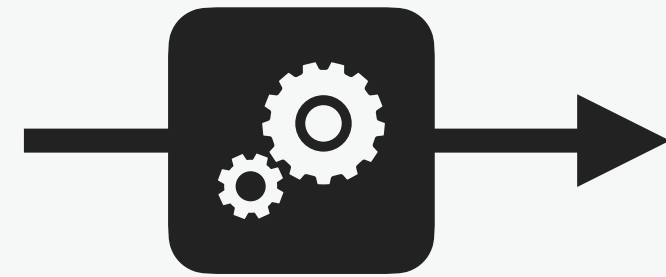
 /TheoWinterhalter/[local-comp](https://github.com/TheoWinterhalter/local-comp)

Perspectives

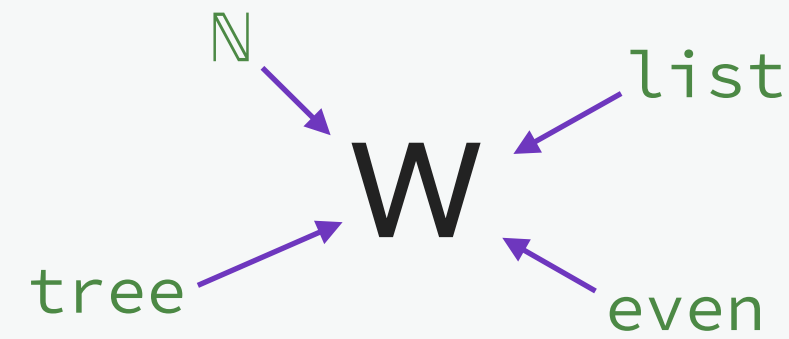


Concrete implementation

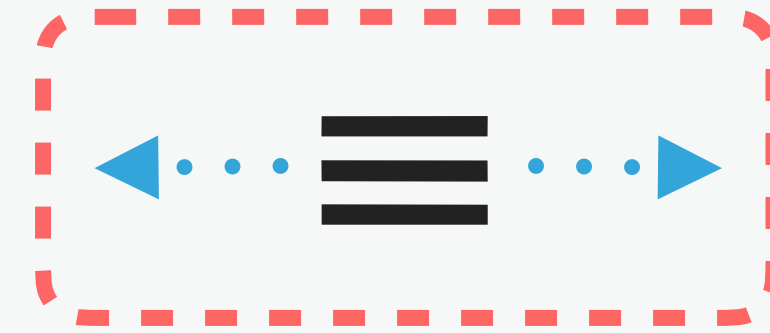
Conclusion



hide implementation details



encode features



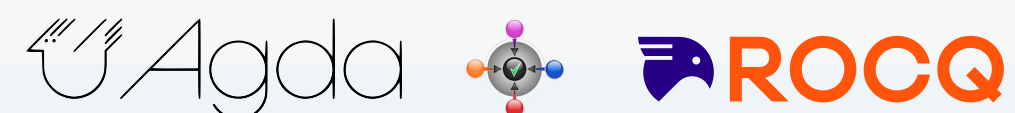
contained extensions (safer)

Conservative extension of MLTT with **local computation**



 /TheoWinterhalter/[local-comp](#)

Perspectives

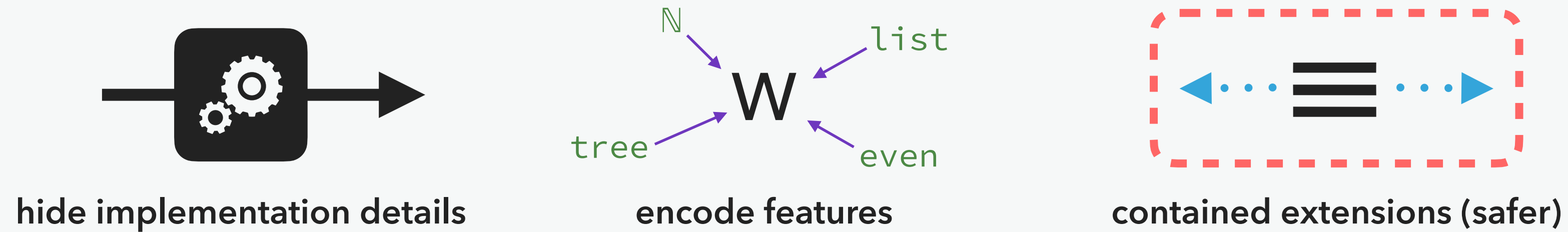


Concrete implementation



Decidability of type checking

Conclusion

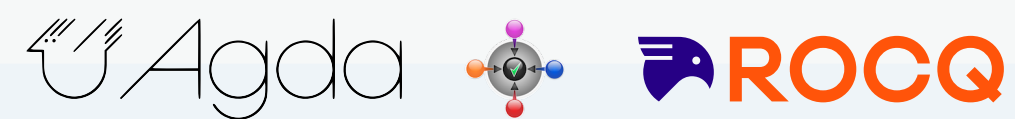


Conservative extension of MLTT with **local computation**



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Perspectives



Concrete implementation

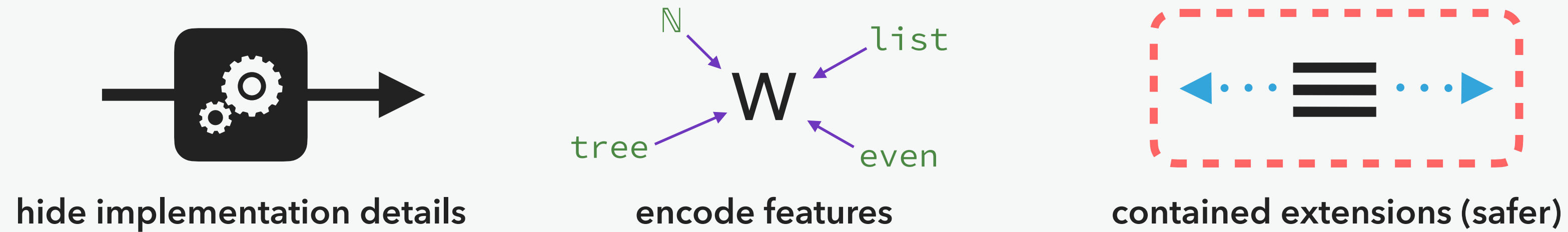


Decidability of type checking



Propositional instances

Conclusion

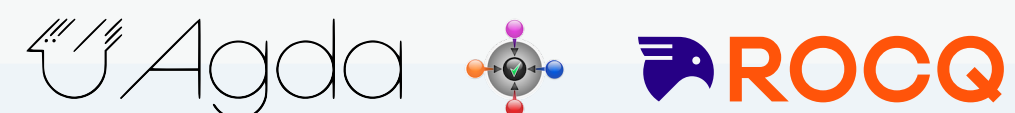


Conservative extension of MLTT with **local computation**



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Perspectives



Concrete implementation



Decidability of type checking



Propositional instances

Thank you!