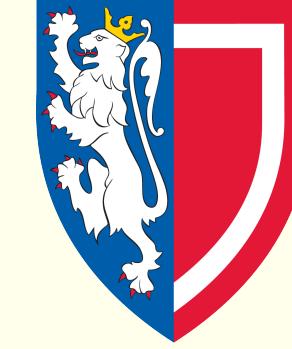




UNIVERSITY OF
OXFORD

DEPARTMENT OF
**COMPUTER
SCIENCE**



**BALLIOL
COLLEGE**
UNIVERSITY OF OXFORD

TUTORIAL ON PROVERIF

EuroProofNet 2024
Tutorial on Usable Formal Methods for Security of Systems

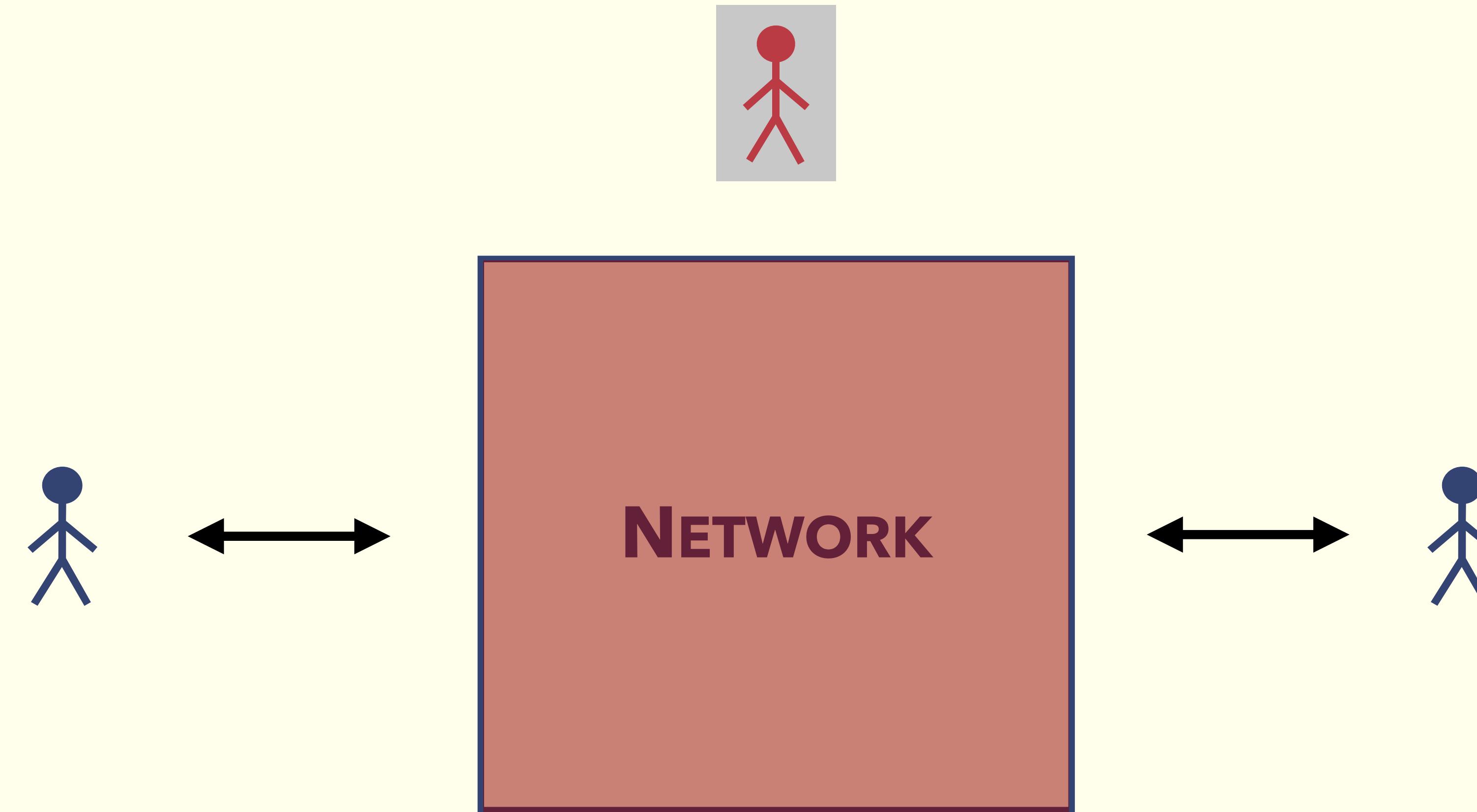
Vincent Cheval
University of Oxford

vincent.cheval@cs.ox.ac.uk

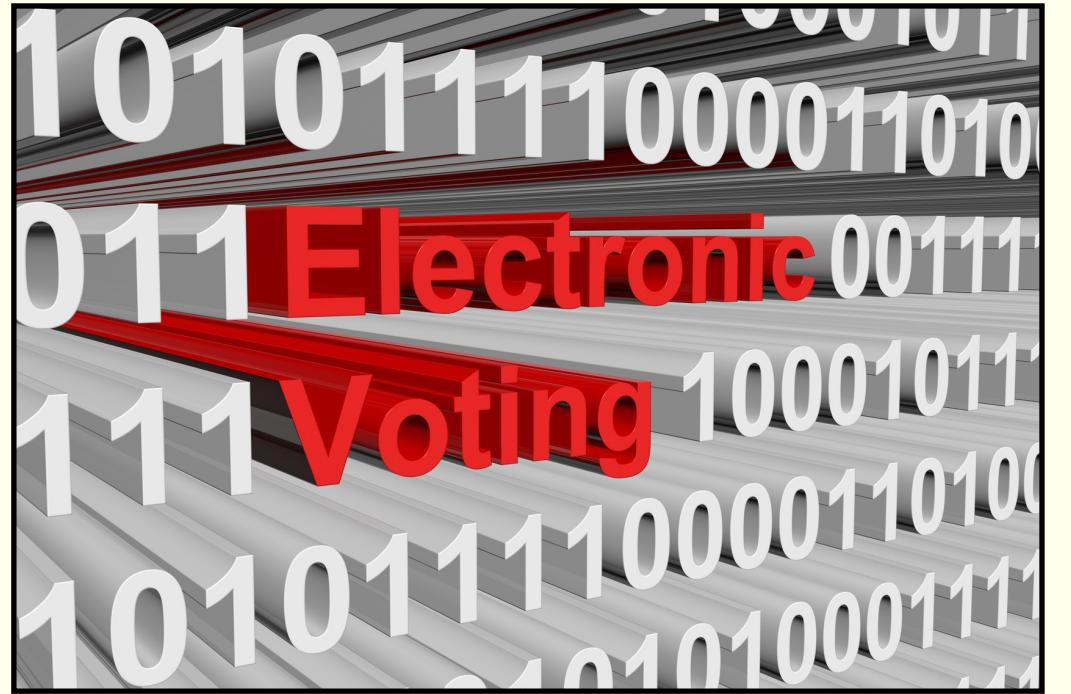
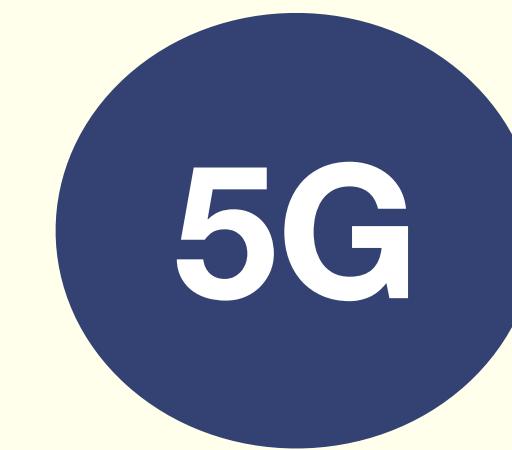
Dresden

27/03/2024

Communication and security over a network



Cryptographic protocols



- ▶ Concurrent programs designed to secure communications
- ▶ Rely on cryptographic primitives (encryption, digital signatures, ...)

Security properties

Each protocol have their own security goals



Transport Secure Layer

Authentication

Secrecy

Forward Secrecy



Electronic passport

Non-Malleability of coins

Balance property



Cryptocurrency

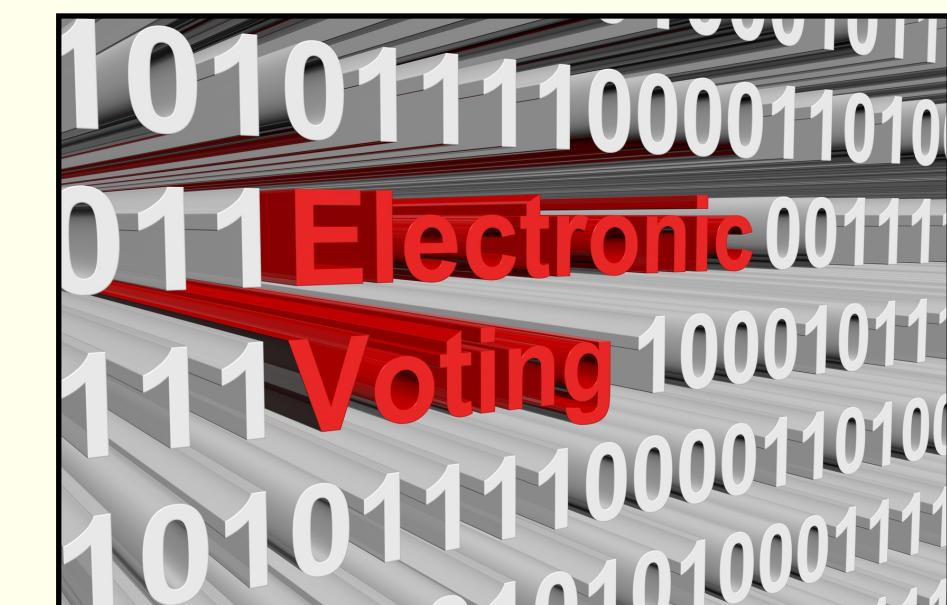
Unlinkability

Anonymity

Verifiability

Coercition resistance

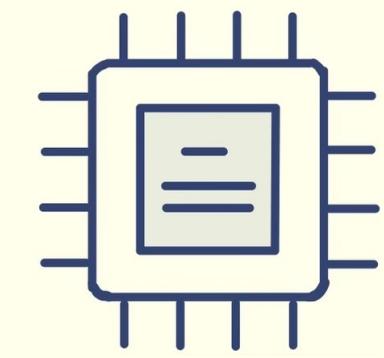
Vote privacy



Voting systems

Designing secure systems

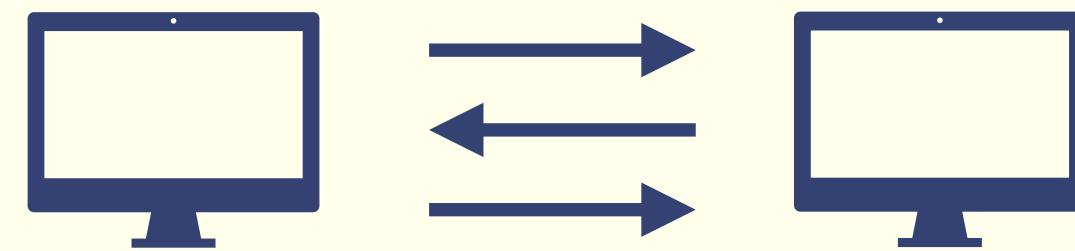
Multiple aspects to consider



Hardware



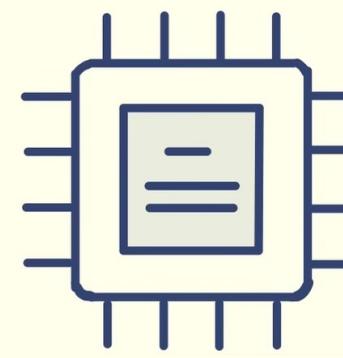
Primitives



Protocols

Designing secure systems is hard!

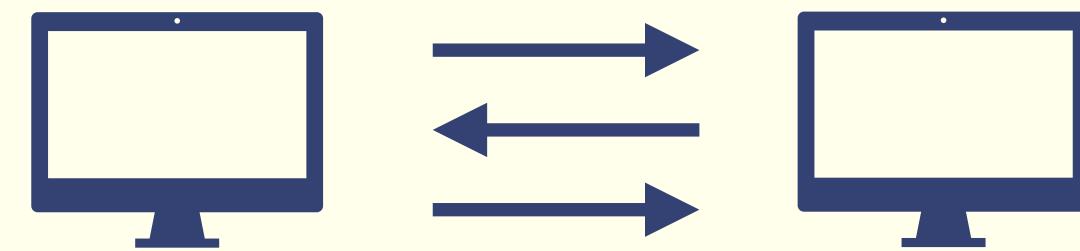
Multiple aspects to consider



Hardware



Primitives



Protocols

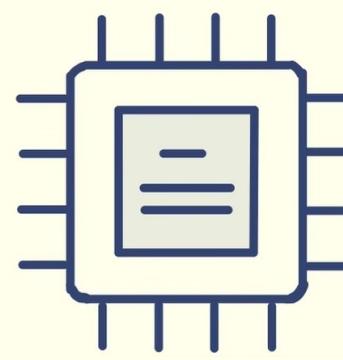
Attacks are common

- Google SSO (2008)
- Power fault attack on RSA (2010)
- BAC (2010)
- Helios (2011)
- Triple Handshake on TLS (2014)
- At least 15 on TLS
- Freak and Logjam attacks (2015)
- Spectre and Meltdown attacks (2017)
- WPA2 (2017)
- Practical collision in SHA-1 (2017)
- 5G Authentication (2018)
- PLATYPUS (2021)

...

Designing secure systems is hard!

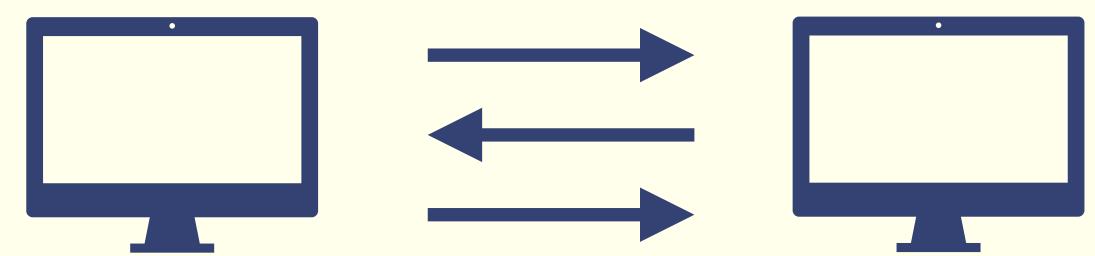
Multiple aspects to consider



Hardware



Primitives



Protocols

Automated verification to guarantee
the absence of logical attacks

14)

15)

17)

WPA2 (2017)

Formal methods to prevent large classes of attacks

17)

Attacks are common

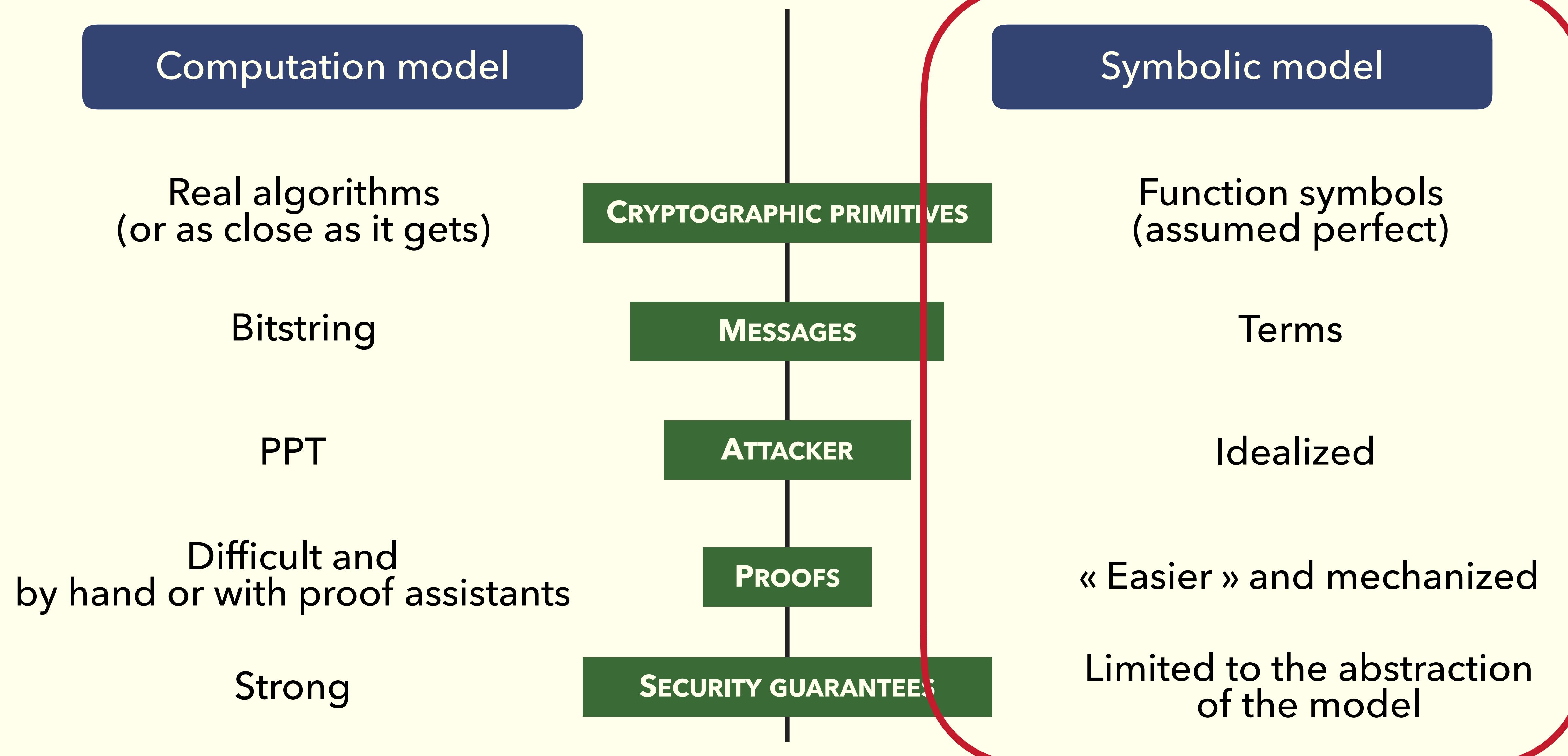
Google SSO (2008)

Power fault attack on RSA (2010)

BAC (2010)

Helios (2011)

Existing models



Symbolic (Dolev-Yao) models

The attacker can...



Read / Write



Intercept

But they cannot...



Break cryptography



Use side channels

Created in the 80' but we have come a long way!

Success stories (not exhaustif)



TLS 1.3 with Encrypted Client Hello



CHVote



Swiss Post



Wireguard



5G-AKA



Signal



ZCash



Certificate Transparency



Belenios



Noise Framework

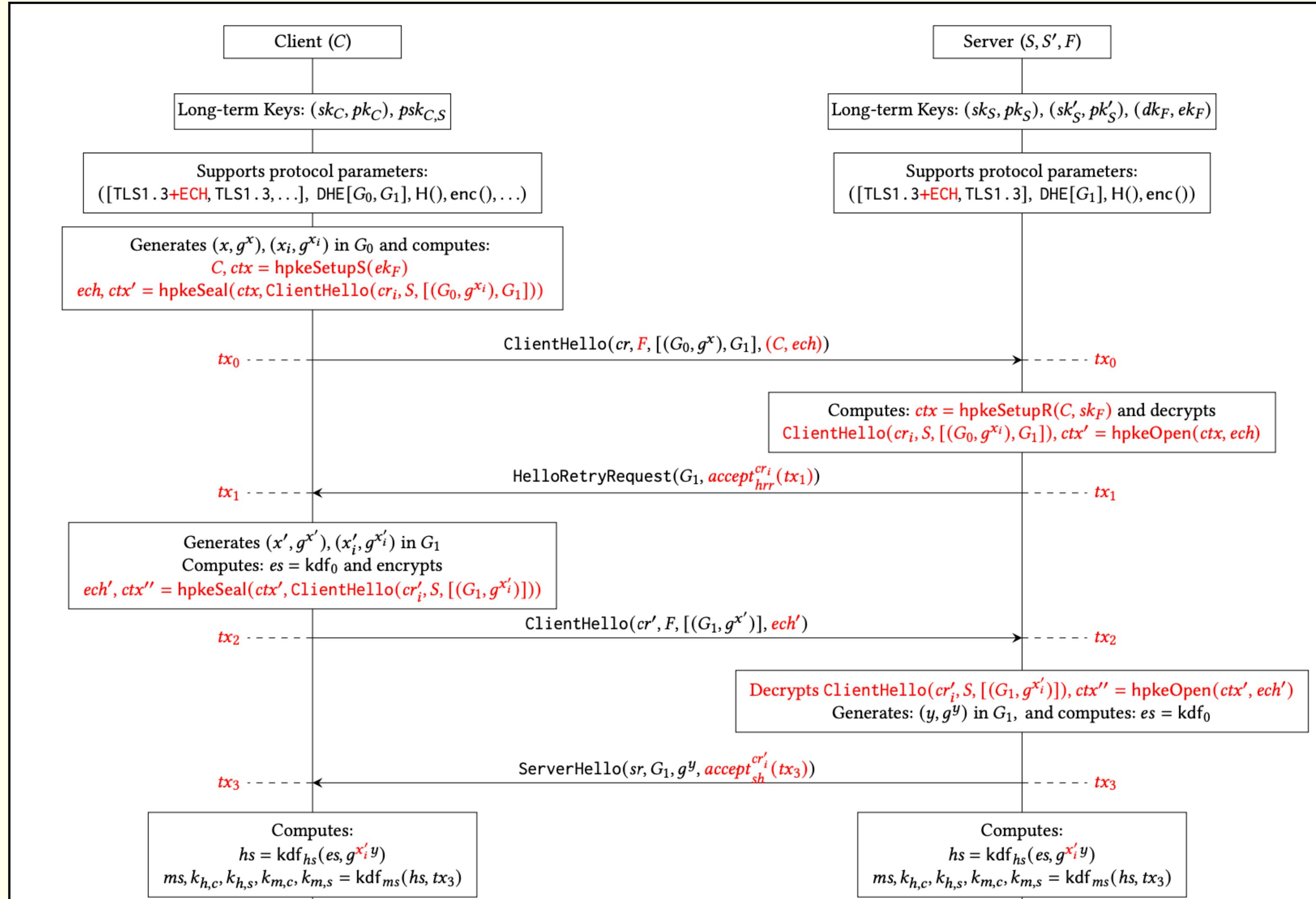


EMV

MODELLING A PROTOCOL AND ITS SECURITY PROPERTIES

A glimpse in the symbolic models

How do we translate an Alice-Bob description into something that we can analyse?



Symbolic terms

Nonces: a, b, c, \dots

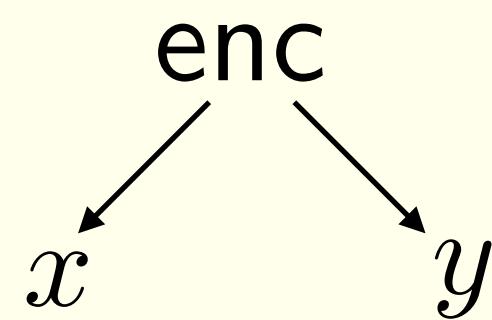
Variables: x, y, z, \dots

atomic elements (keys, random numbers, ...)

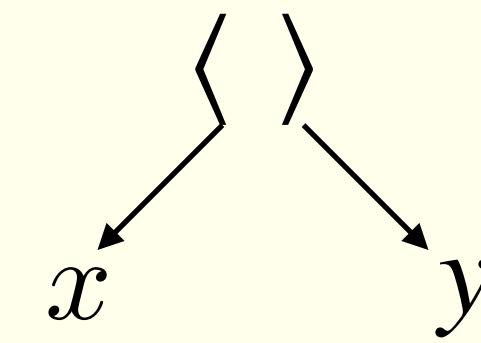
Functions symbols with their arity: enc/2, dec/2, $\oplus/2$, $\langle \rangle/2$, proj₁/1, proj₂/1, ...

Abstract functions

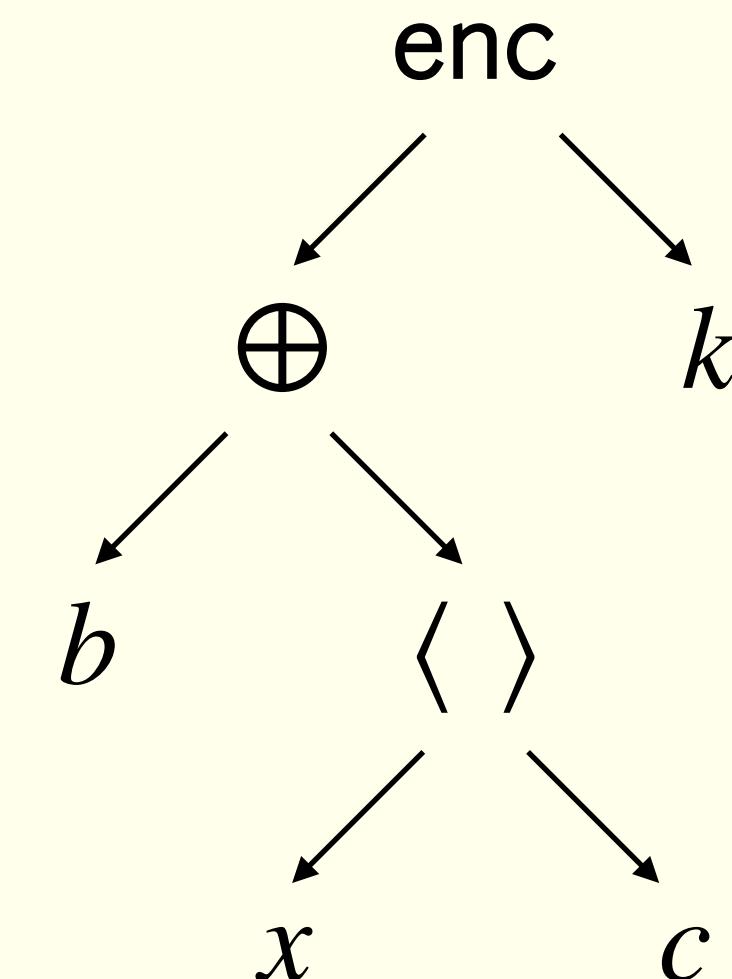
enc(x, y)



$\langle x, y \rangle$



enc($b \oplus \langle x, c \rangle, k$)





TYPES AND FUNCTIONS

Symbolic terms

If functions are kept abstracted and messages are not computed, what tests can we perform on messages?

Equality between terms

Syntactic equality: same term / tree

$$a \neq b$$

Ok two different represents two large random numbers

$$\text{dec}(\text{enc}(m, k), k) \neq k$$

Not Ok... Decryption of a cipher with the correct key should be equal to the plain text

Algebraic properties of cryptographic primitives

Algebraic properties of the cryptographic primitives must be modelled.

Equational theory: $\text{dec}(\text{enc}(x, y), y) = x$ $\text{proj}_1(\langle x, y \rangle) = x$ $\text{proj}_2(\langle x, y \rangle) = y$

$$x \oplus (y \oplus x) = (x \oplus y) \oplus z \quad x \oplus y = y \oplus x \quad x \oplus x = 0$$

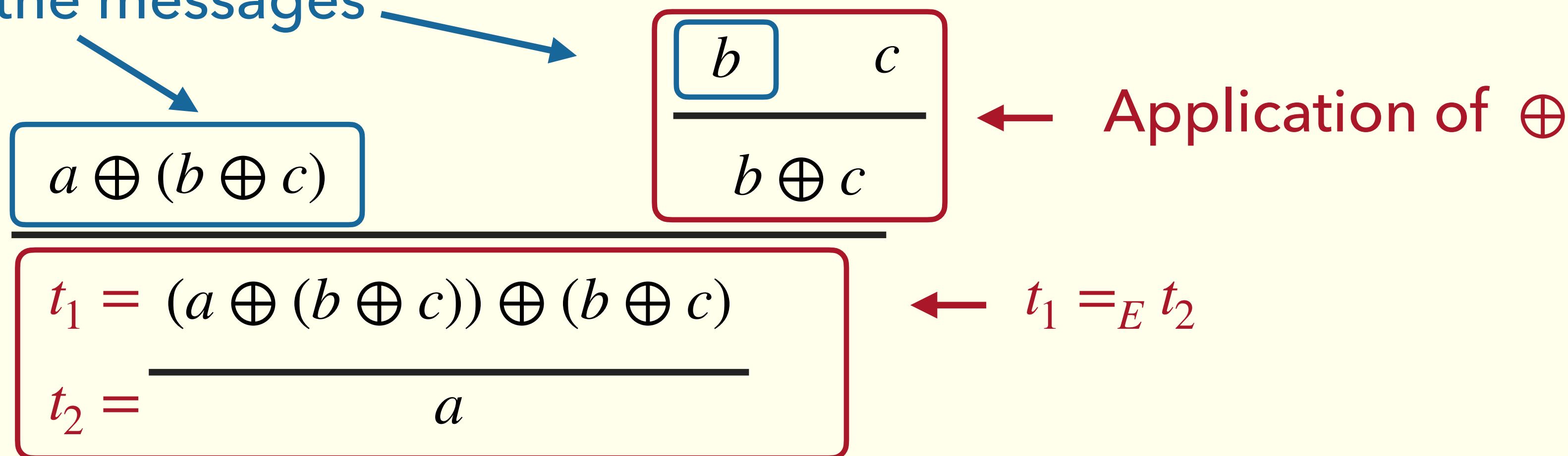
$$x \oplus 0 = x \quad (g^{\wedge} x)^{\wedge} y = (g^{\wedge} y)^{\wedge} x \quad (g^{\wedge} x) \times (g^{\wedge} y) = g^{\wedge}(x + y)$$

Deduction

Equational theory E : $x \oplus (y \oplus x) = (x \oplus y) \oplus z$ $x \oplus y = y \oplus x$ $x \oplus x = 0$ $x \oplus 0 = x$

Imagine an attacker intercepted the 3 messages $a \oplus (b \oplus c)$, b and c .
Can he deduce the name a ?

Leaves are the messages





EQUATIONS

Equational theory vs Rewrite rules

Strengths and weaknesses of rewrite rules

- + Verification efficient
- + Very expressive with **otherwise**

```
fun ifthenelse(bool,bitstring,bitstring):bitstring
reduc
  forall x,y:bitstring; ifthenelse(true,x,y) = x
  otherwise forall b:bool,x,y:bitstring; ifthenelse(b,x,y) = y.
```



the term
`ifthenelse(true,m,decrypt(a,k))`
fails

```
fun lazy_ite(bool,bitstring,bitstring):bitstring
reduc
  forall x:bitstring; y:bitstring or fail; lazy_ite(true,x,y) = x
  otherwise forall b:bool,x:bitstring or fail,y:bitstring; lazy_ite(b,x,y) = y.
```

Equational theory vs Rewrite rules

Strengths and weaknesses of rewrite rules

- + Verification efficient
- + Very expressive with **otherwise**
- Cannot call itself

Algebraic properties that cannot be modeled with rewrite rules in ProVerif

$$\text{dec}(\text{enc}(x, y), y) = x \quad \text{with} \quad \text{enc}(\text{dec}(x, y), y) = x$$

$$\text{exp}(\text{exp}(g, x), y) = \text{exp}(\text{exp}(g, y), x)$$

Diffie-Hellman

Equational theory vs Rewrite rules

Strengths and weaknesses of equational theory

- + Extremely expressive
- Makes the verification slow
- Not all equational theory can be handled
(may not terminate from the start)

```
fun enc(G, passwd): G.  
fun dec(G, passwd): G.  
equation forall x: G, y: passwd; dec(enc(x,y),y) = x.  
equation forall x: G, y: passwd; enc(dec(x,y),y) = x.
```

```
const g: G.  
fun exp(G, exponent): G.  
equation forall x: exponent, y: exponent; exp(exp(g, x), y) = exp(exp(g, y), x).
```



ROLES

Security properties

Type of security properties

Reachability

Bad event in one system



Authentication



Secrecy

Equivalence

Privacy as indistinguishability



Anonymity

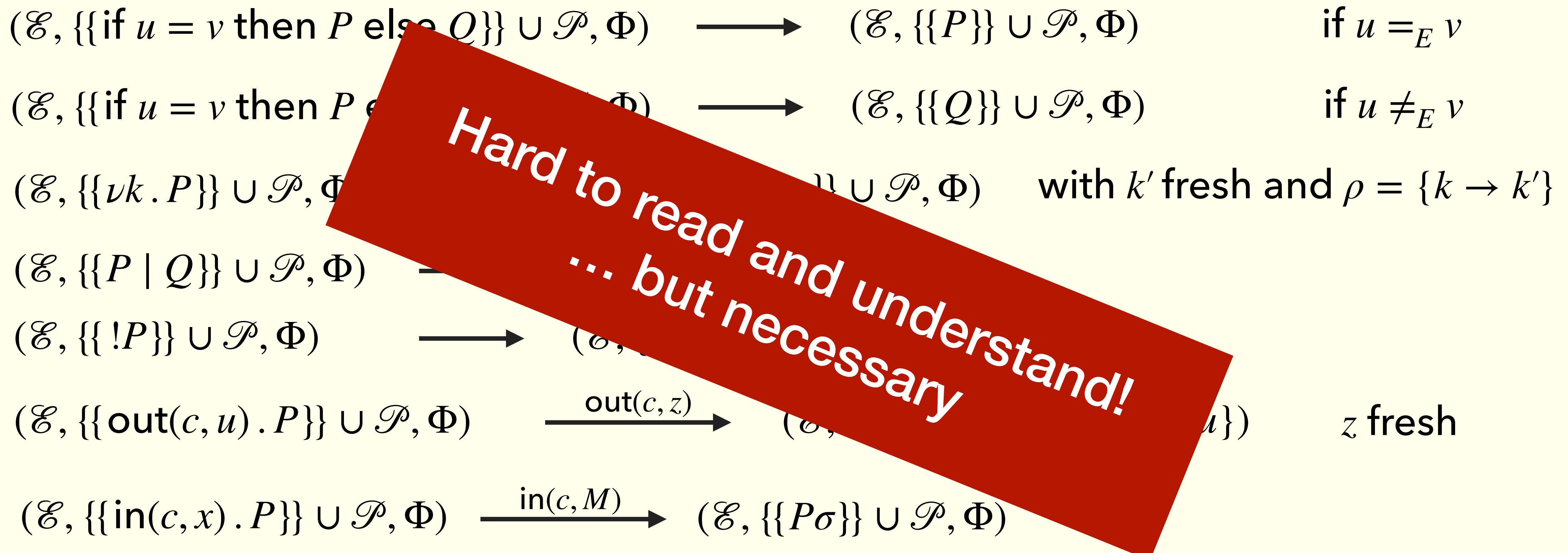


Vote privacy



Unlinkability

Semantics explains how the protocol can be executed in the presence of an attacker

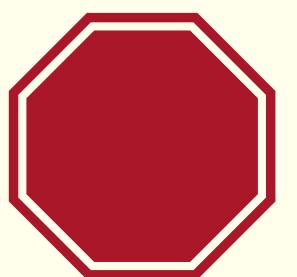


with $\sigma = \{x \mapsto t\}$ and $M\Phi =_E t$ and M does not contain names from \mathcal{E}
but M can contain variables from the domain of Φ

Expressing secrecy properties

Secrecy of k in P

For all transitions $P \longrightarrow \mathcal{C}_1 \xrightarrow{\dots\dots\dots} \mathcal{C}_{n-1} \longrightarrow \mathcal{C}_n$, the secret k is not
deducible from the attacker knowledge in \mathcal{C}_n

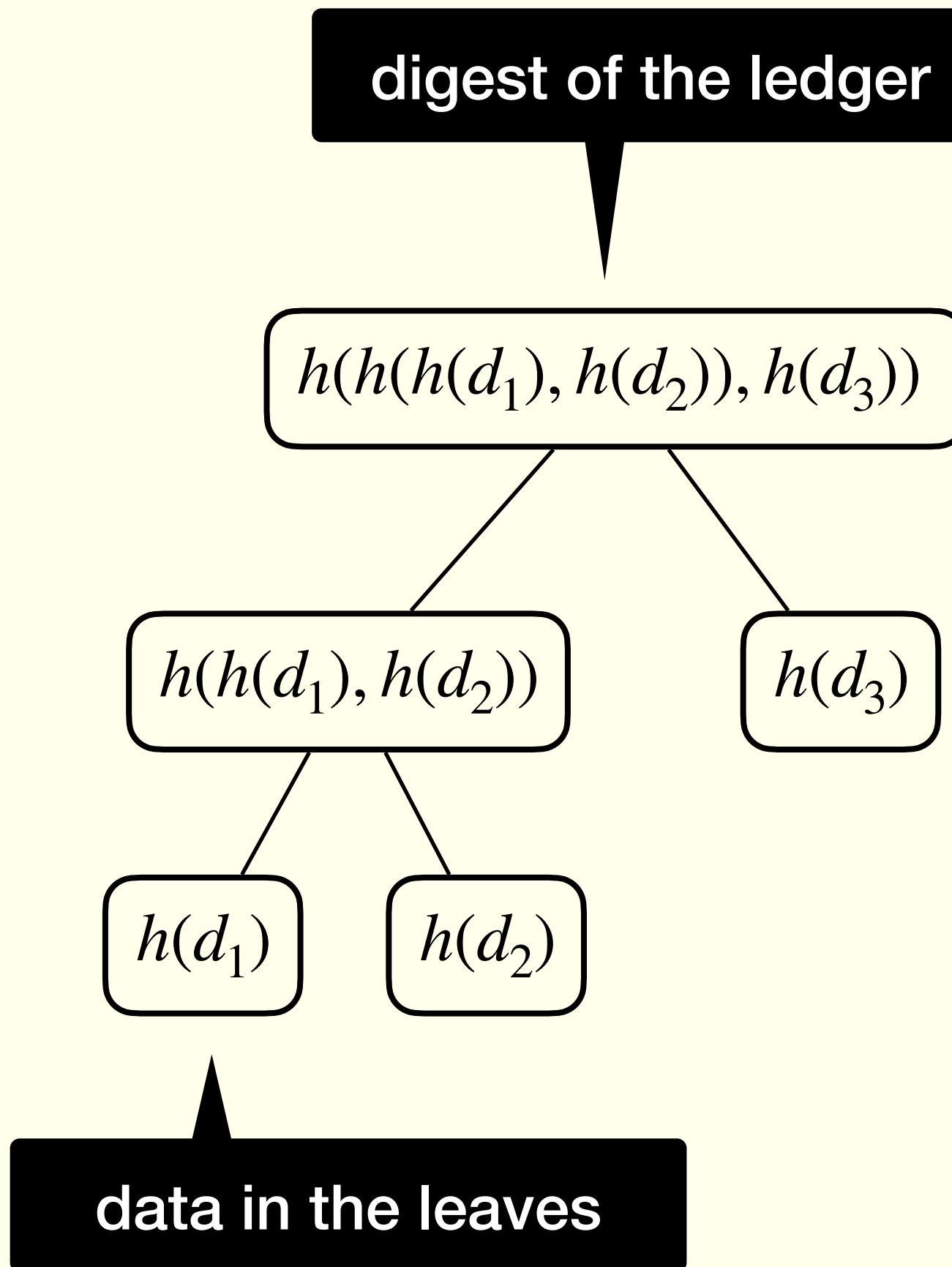


Secrecy problem undecidable for simple cryptographic primitives



SECURITY PROPERTIES

When equational theory fails? Example: Merkle Trees

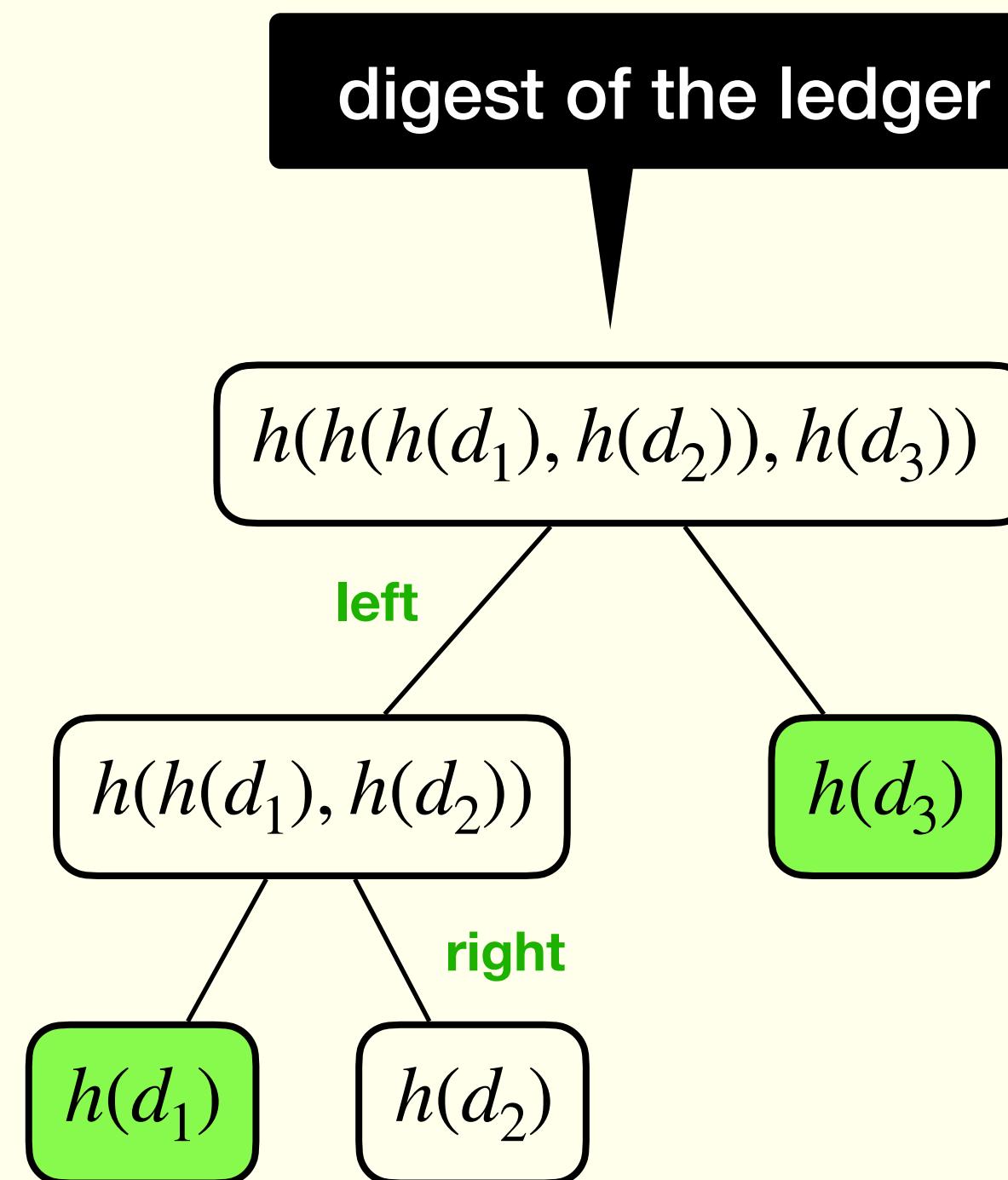


Append only structure

Proof of presence in $O(\log(n))$

Proof of extension in $O(\log(n))$

Proof of presence in a Merkle Tree



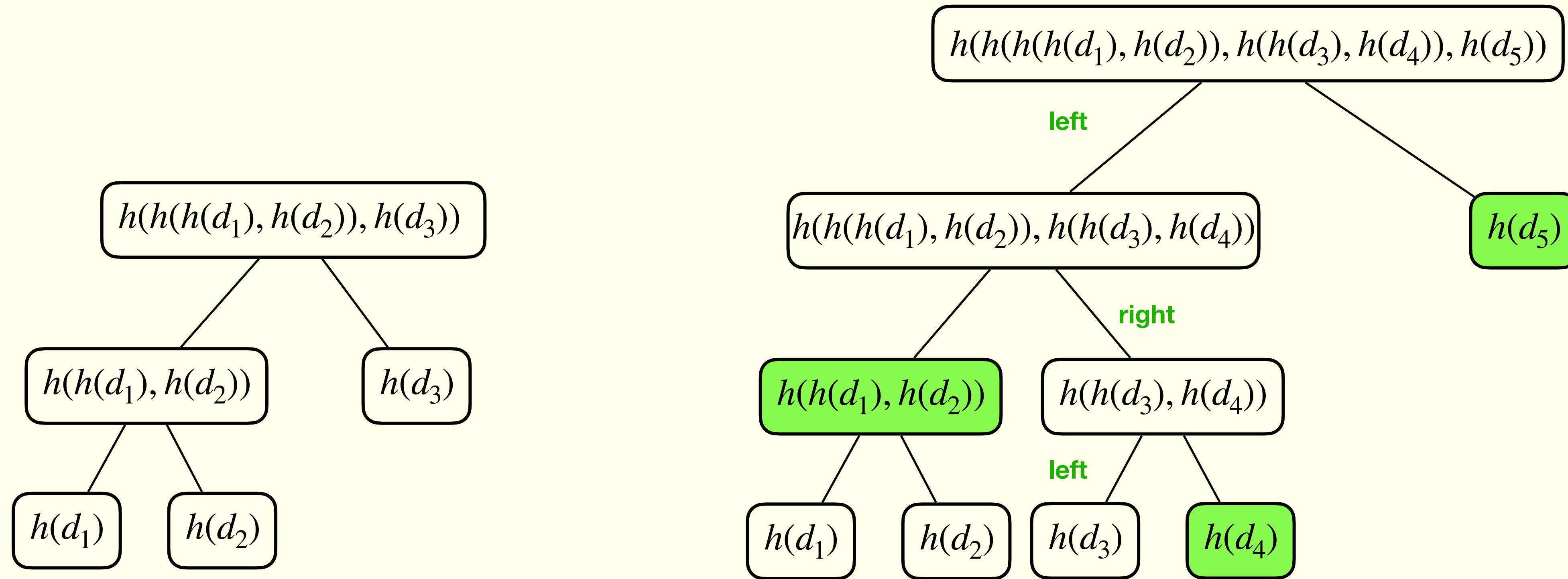
How to prove the presence of d_2
in digest $h(h(h(d_1), h(d_2)), h(d_3))$?

Proof contains:

- the data d_2
- the labels of siblings of the branch from the data to the root: $h(d_1)$ and $h(d_3)$
- The position of the data in the tree

To verify the proof, reconstruct the label of the root and compare with the digest of the ledger

Proof of extension in a Merkle Tree



In green, proof of extension between the two trees

Let's start with a simple list?

Digest has a list structure:

$$h(d_1, h(d_2, h(d_3, h(\dots, h(d_n, 0) \dots)))$$

How to prove the presence of d_3

Proof contains:

- the data
- the hash $h(d_4, h(\dots, h(d_n, 0) \dots))$
- The previous elements d_1, d_2



PREDICATES

Memory cell

ProVerif's calculus is stateless ... but we have private channels

A Ocaml like version

let x = ref 0

Initialisation

```
free cell:channel [private]
let init = out(cell,0).
```

!x

Reading

```
let P =
...
in(cell,x:nat); out(cell,x);
...
```

x := n

Writing

```
let Q =
...
in(cell,x:nat); out(cell,n);
...
```

Memory cell

Initialisation

```
free cell:channel [private]
let init = out(cell,0).
```

Reading

```
let P =
  ...
  in(cell,x:nat); out(cell,x);
  ...
```

Writing

```
let Q =
  ...
  in(cell,x:nat); out(cell,n);
  ...
```

The system

```
process
  init | P | Q | !in(cell,x:nat);out(cell,x)
```

Reading/Writing both consist of inputing the « current value » of the cell and outputting the « new value »

Communication are synchronous on private channels: always one single output available at all time.

Avoids « blocking » an agent

Locking memory cell

Initialisation

```
free cell:channel [private]
let init = out(cell,0).
```

Reading

```
let P =
...
in(cell,x:nat);
event B;
out(cell,x);
...
```

Writing

```
let Q =
...
in(cell,x:nat);
event A;
event C;
out(cell,n);
...
```

Communication are synchronous
on private channels:
If no output available, all processes
trying to input are « blocked »

The sequence of events A, B, C is not possible

Locking memory cell

Initialisation

```
free cell:channel [private]
let init = out(cell,0).
```

Lock and read

```
let P =
...
in(cell,x:nat);
event B;
out(cell,x);
...
```

Write and unlock

```
let Q =
...
in(cell,x:nat);
event A;
event C;
out(cell,n);
...
```

Communication are synchronous
on private channels:
If no output available, all processes
trying to input are « blocked »

The sequence of events A, B, C is not possible

Simplified Yubikey protocol

P only accepts increasing sequence of natural numbers.

Q emits sequentially all natural numbers encrypted with k

```
free k:key [private].
free cellP,cellQ:channel [private]

let P =
    in(c,x:bitstring);
    in(cellP,i:nat);
    let j = sdec(x,k) in
    if j > i
    then
        event Accept(j);
        out(cellP,j)
    else
        out(cellP,i).

let Q =
    in(cellQ,i:nat);
    out(c,senc(I,k));
    out(cellQ,i+1).

process out(cellP,0) | out(cellQ,0) | !P | !Q
```



SIGNAL: THE DOUBLE RATCHET ALGORITHM

Equivalence properties

Type of security properties

Reachability

Bad event in one system



Authentication



Secrecy

Equivalence

Privacy as indistinguishability



Anonymity



Vote privacy

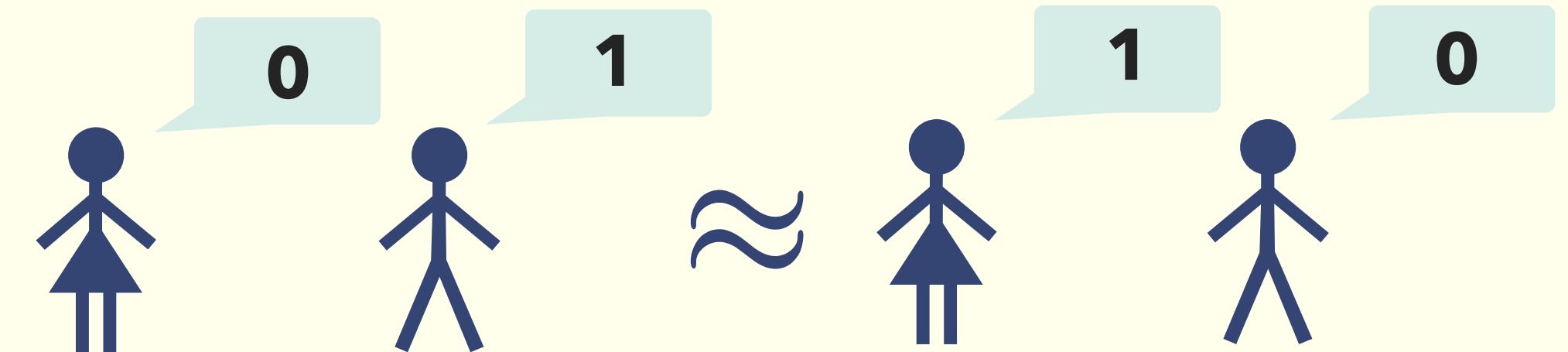
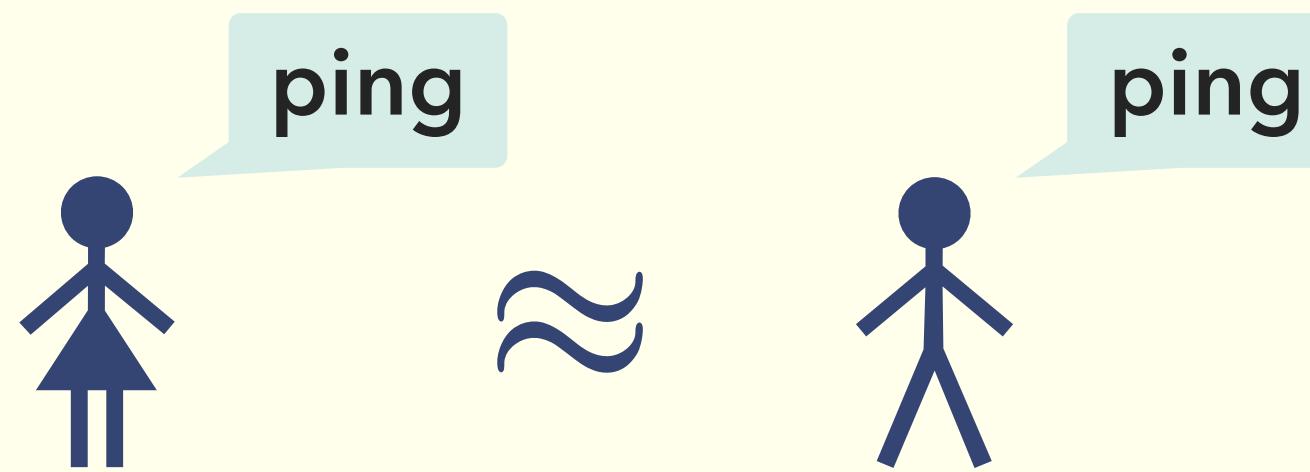


Unlinkability

Equivalence properties

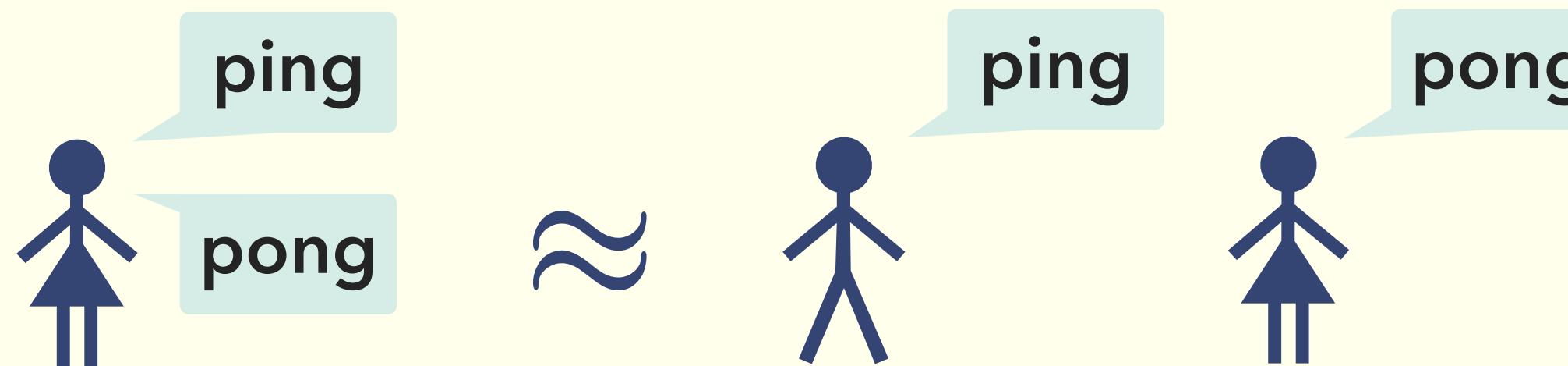
Indistinguishability

of two situations where the private attribute differs



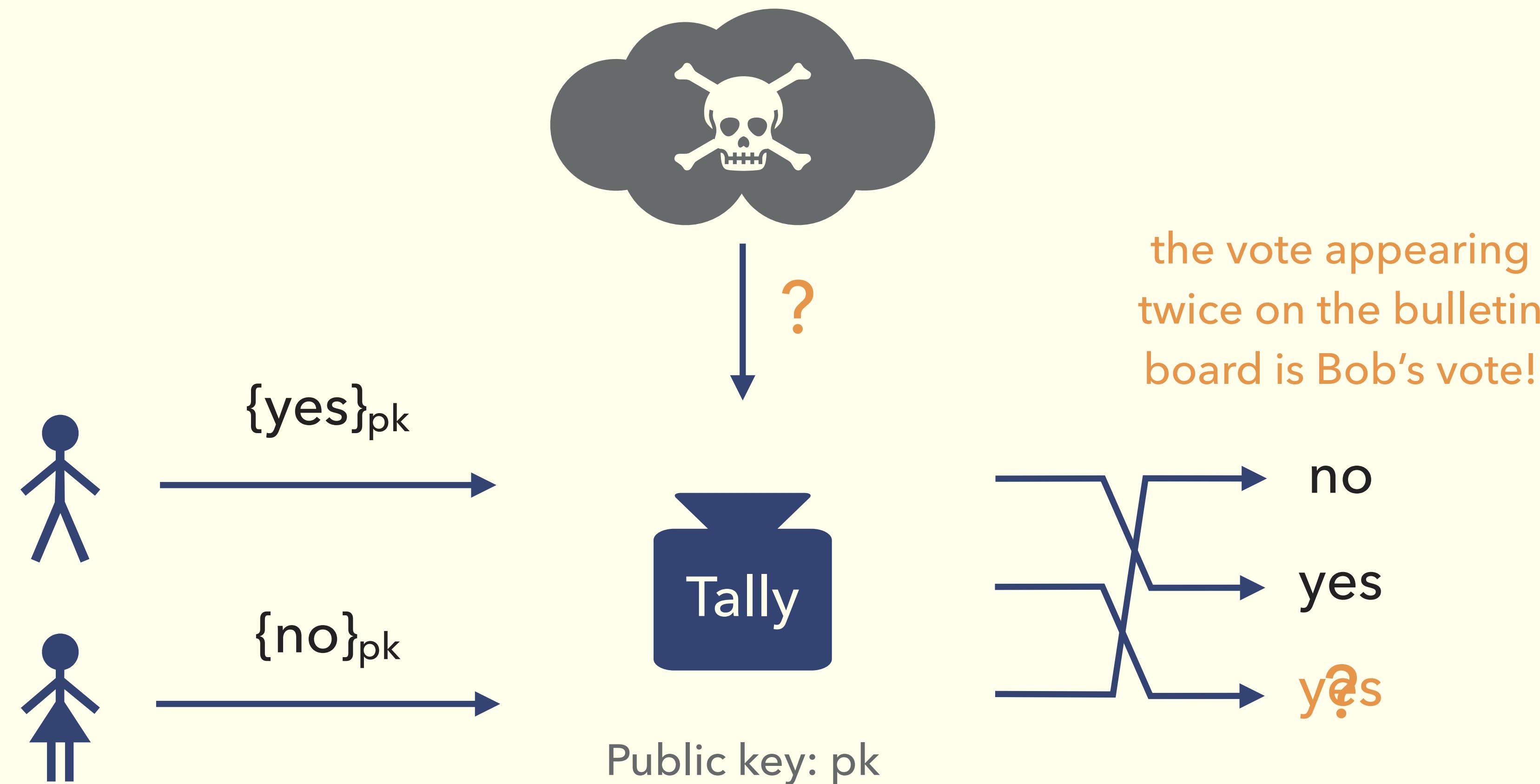
Anonymity

Vote privacy

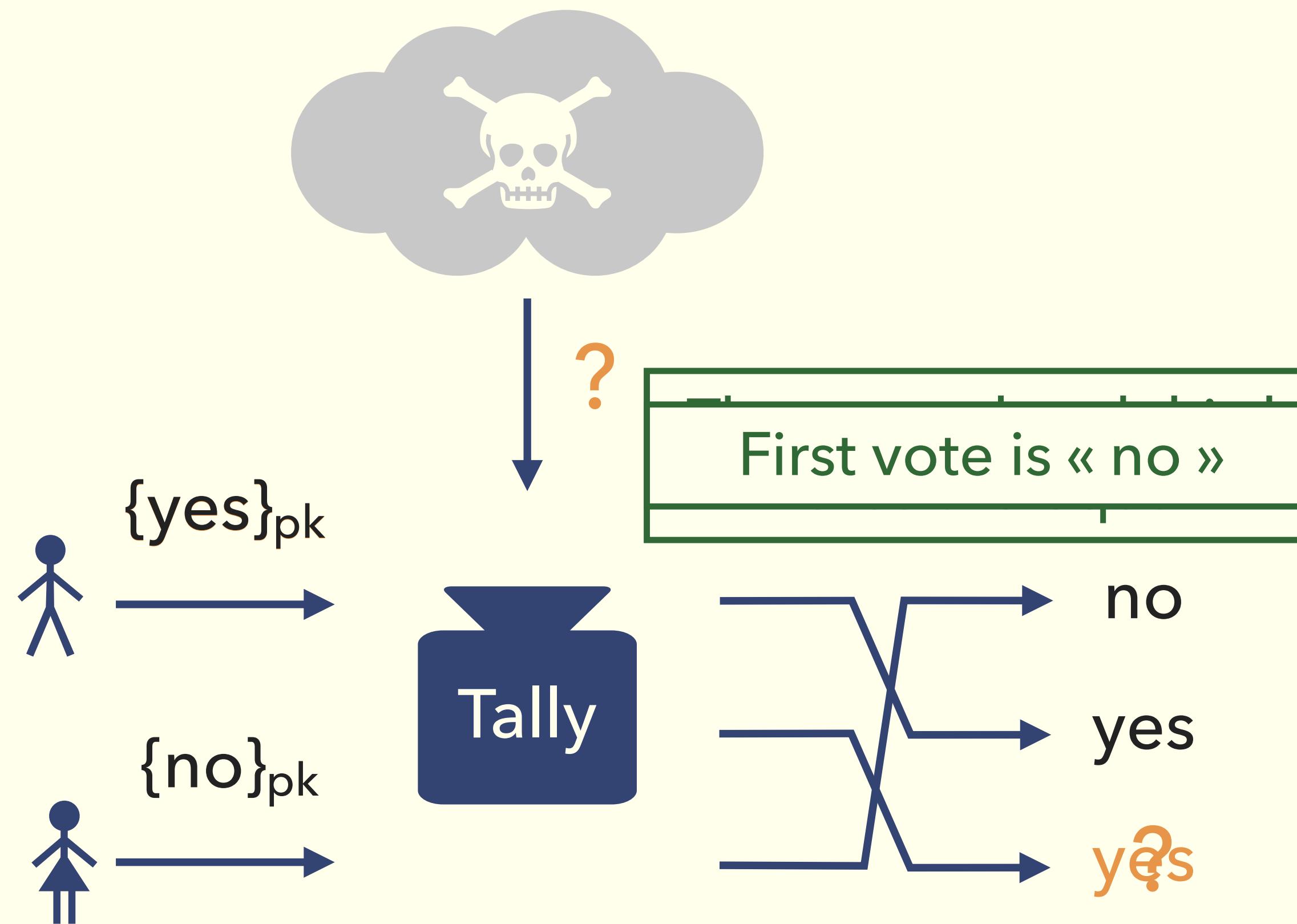


Unlinkability

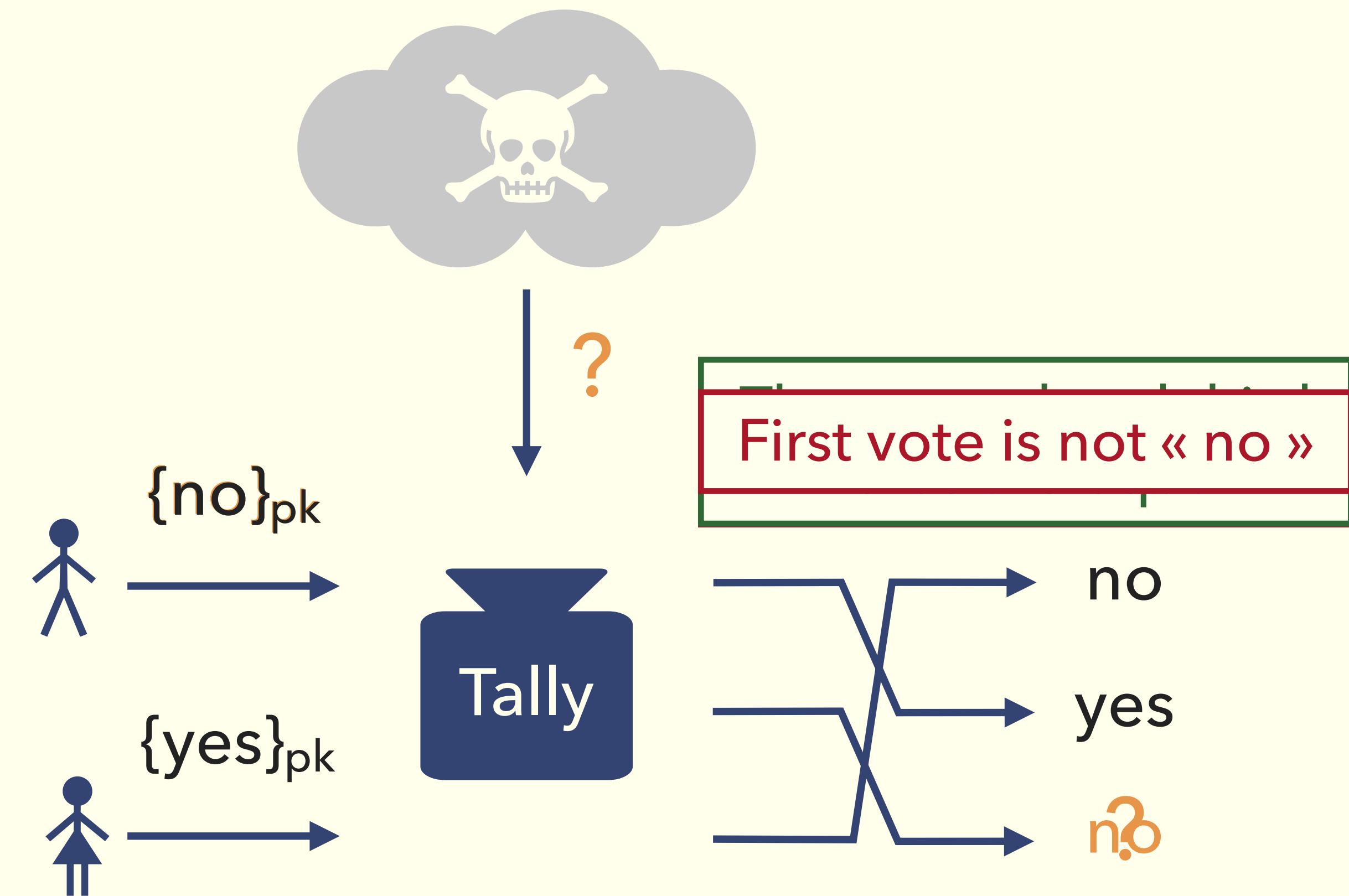
A simple e-voting protocol



A simple e-voting protocol



the vote appearing twice on the bulletin board is Bob's vote!



the vote appearing twice on the bulletin board is Bob's vote!

Equivalence of processes in ProVerif

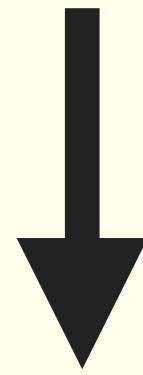
```
let system1 = setup | voter(skA,v1) | voter(skB,v2).  
let system2 = setup | voter(skA,v2) | voter(skB,v1).  
equivalence system1 system2
```

Equivalence between two
processes

```
let system(vA,vB) = setup | voter(skA,vA) | voter(skB,vB).  
process system(choice[v1,v2],choice[v2,v1])
```

Internally

Equivalence as a biprocess



Equivalence of processes in ProVerif

Equivalence between two processes

- + Easier to model,
- + No need to know « how to match » the processes
- Can be slow
- Difficult to « fix » when not working

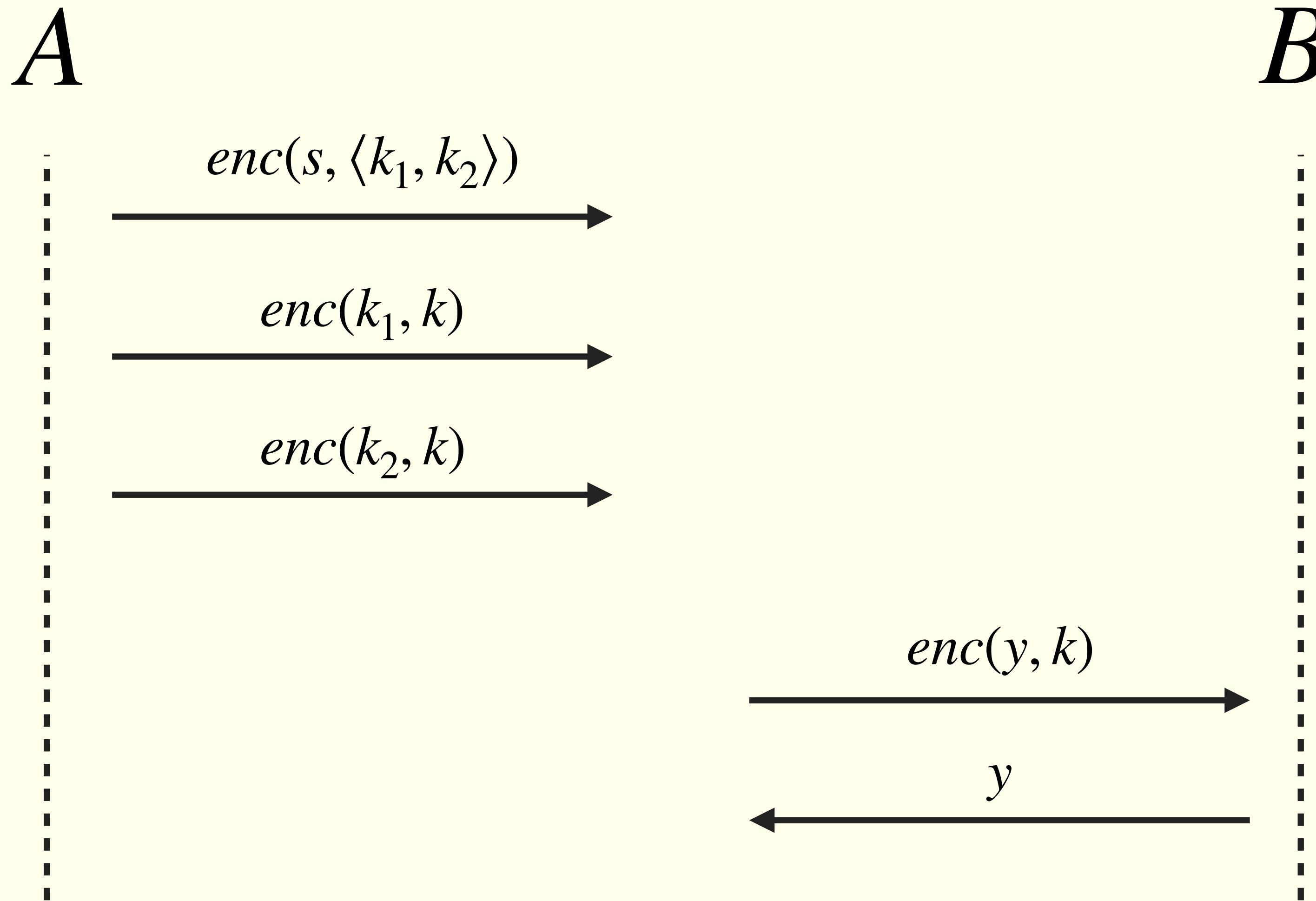
Equivalence as a biprocess

- + Also easy to model,
- + Works better with other features (e.g. lemmas, axioms)
- + More efficient
- Need to have a good idea why processes are equivalent

DEALING WITH “CANNOT BE PROVED”

Toy-example

*B acts as an oracle for decryption with the key k
but only one time !*



How does ProVerif work (high level) ?

```
free s,k1,k2,k:bitstring [private].  
  
let A =  
  out(c,enc(s,(k1,k2)));  
  out(c,enc(k1,k));  
  out(c,enc(k2,k)).  
  
...  
  
query attacker(s) ==> false.
```

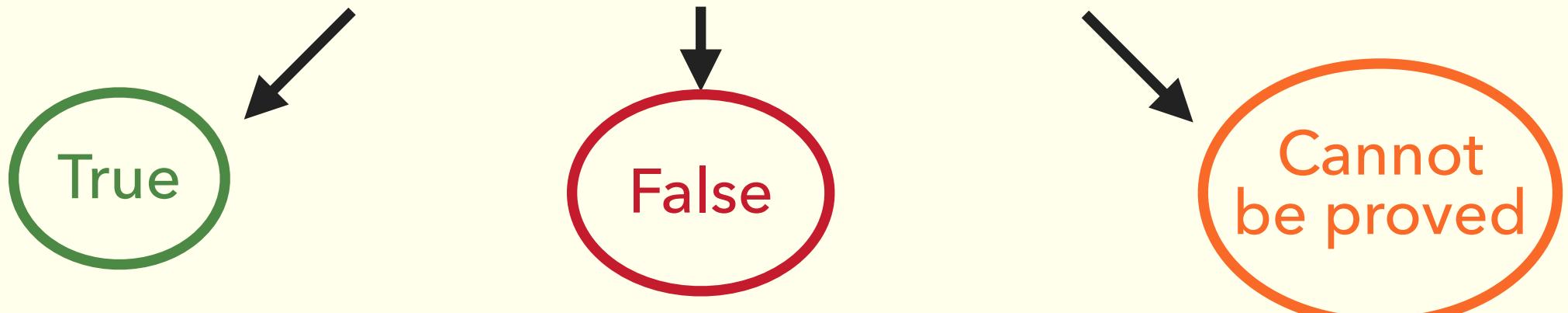
Abstraction in Horn clauses

→ $\text{att}(\text{enc}(s, \langle k_1, k_2 \rangle))$
→ $\text{att}(\text{enc}(k_1, k))$
→ $\text{att}(\text{enc}(k_2, k))$
 $\text{att}(\text{enc}(y, k)) \rightarrow \text{att}(y)$

...

Verification of the query on all saturated clauses

Saturation of the set of Horn clauses



Why does it fail ?

Transform process
in Horn clauses

Horn clauses for
the attacker

```
free s,k1,k2,k:bitstring [private].  
  
let A =  
    out(c,senc(s,(k1,k2)));  
    out(c,senc(k1,k));  
    out(c,senc(k2,k)).  
  
let B =  
    in(c,x);  
    out(c,dec(x,k)).  
  
process A | B
```

$$\begin{aligned} att(x) \wedge att(y) &\rightarrow att(\text{enc}(x, y)) \wedge att(y) \\ att(\text{enc}(x, y)) \wedge att(y) &\rightarrow att(x) \wedge att(y) \rightarrow att(\langle x, y \rangle) \end{aligned}$$

$\rightarrow att(\text{enc}(s, \langle k_1, k_2 \rangle))$

$\rightarrow att(\text{enc}(k_1, k))$

$\rightarrow att(\text{enc}(k_2, k))$

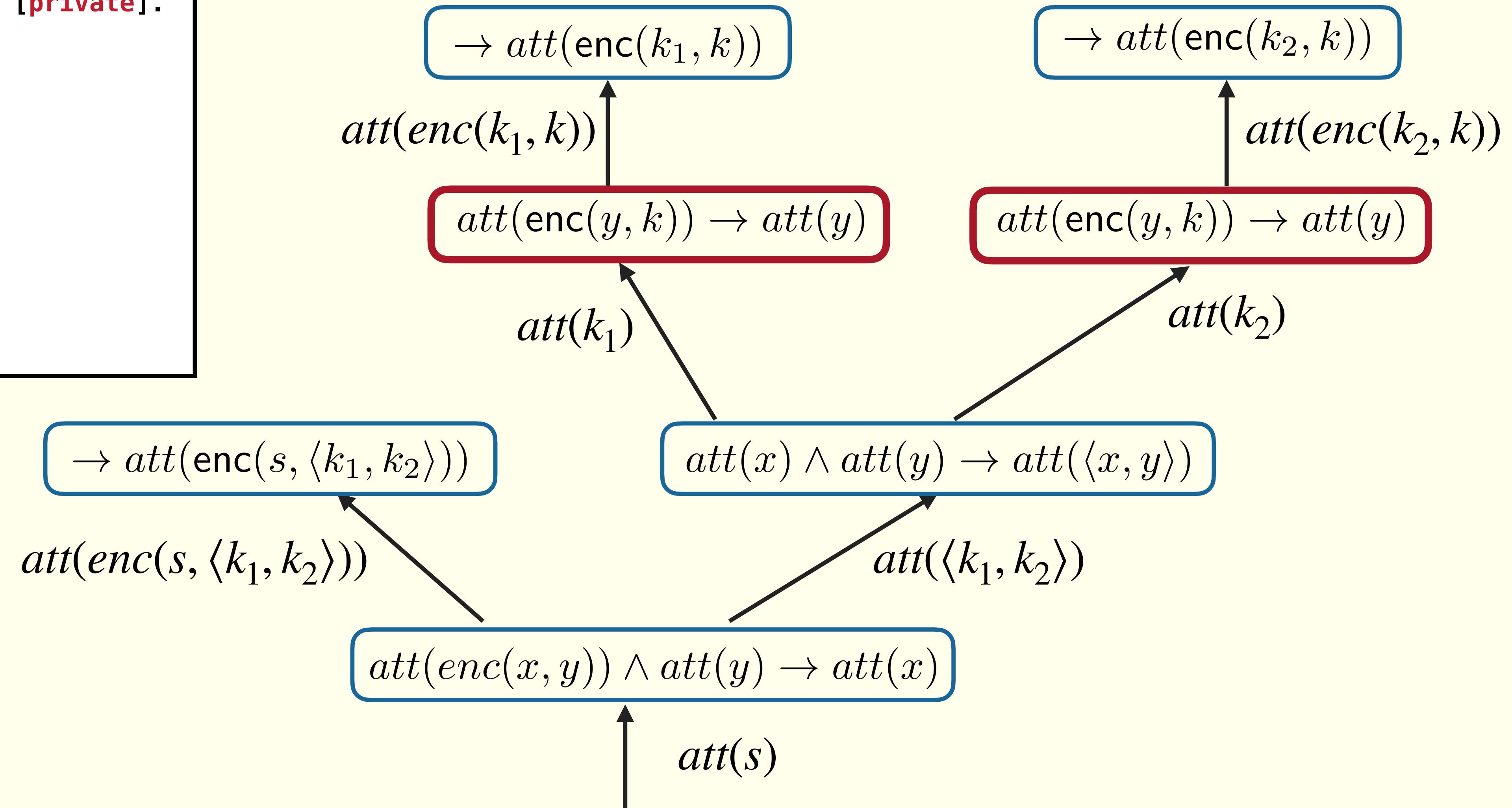
$att(\text{enc}(y, k)) \rightarrow att(y)$

Horn clauses can be applied an
arbitrary number of times for
arbitrary instantiations

Secrecy of s is preserved if $att(s)$ is not logically
deducible from the set of Horn clauses

Why does it fail ?

```
free s,k1,k2,k:bitstring [private].  
  
let A =  
  out(c,senc(s,(k1,k2)));  
  out(c,senc(k1,k));  
  out(c,senc(k2,k)).  
  
let B =  
  in(c,x);  
  out(c,dec(x,k)).  
  
process A | B
```



What to do ?

```
free s,k1,k2,k:bitstring [private].  
  
let A =  
  out(c,senc(s,(k1,k2)));  
  out(c,senc(k1,k));  
  out(c,senc(k2,k)).  
  
let B =  
  in(c,x) [precise];  
  out(c,dec(x,k)).  
  
process A | B
```

Add a [precise] option to the problematic input !

```
-- Query not attacker([]) in process 0.  
Translating the process into Horn clauses...  
Completing...  
Starting query not attacker([])  
  
RESULT not attacker([]) is true.  
  
-----  
Verification summary:  
  
Query not attacker([]) is true.  
-----
```

Global setting

```
set preciseActions = true.
```



Adding [precise] options may increase the verification time or lead to non-termination

How to know where to put precise ?

Going through
the derivation !

Find two different
messages received
by the same input {n}

Check on your
process if it should
be possible

Derivation:

1. The message $\text{enc}(k2[], k[])$ may be sent to the attacker at output {5}.
 $\text{attacker}(\text{enc}(k2[], k[]))$.
2. The message $\text{enc}(k2[], k[])$ that the attacker may have by 1 may be received at input {7}.
So the message $k2[]$ may be sent to the attacker at output {8}.
 $\text{attacker}(k2[])$.
3. The message $\text{enc}(k1[], k[])$ may be sent to the attacker at output {4}.
 $\text{attacker}(\text{enc}(k1[], k[]))$.
4. The message $\text{enc}(k1[], k[])$ that the attacker may have by 3 may be received at input {7}.
So the message $k1[]$ may be sent to the attacker at output {8}.
 $\text{attacker}(k1[])$.
5. By 4, the attacker may know $k1[]$.
By 2, the attacker may know $k2[]$.
Using the function 2-tuple the attacker may obtain $(k1[], k2[])$.
 $\text{attacker}((k1[], k2[]))$.
6. The message $\text{enc}(s[], (k1[], k2[]))$ may be sent to the attacker at output {6}.
 $\text{attacker}(\text{enc}(s[], (k1[], k2[])))$.
7. By 6, the attacker may know $\text{enc}(s[], (k1[], k2[]))$.
By 5, the attacker may know $(k1[], k2[])$.
Using the function dec the attacker may obtain $s[]$.
 $\text{attacker}(s[])$.
8. By 7, $\text{attacker}(s[])$.
The goal is reached, represented in the following fact:
 $\text{attacker}(s[])$.

How to know where to put precise ?

Going through
the derivation !

Find two different
messages received
by the same input {n}

Check on your
process if it should
be possible

Derivation:

1. The message $\text{enc}(k2[], k[])$ may be sent to the attacker at output {5}.
 $\text{attacker}(\text{enc}(k2[], k[]))$.
- 2. The message $\text{enc}(k2[], k[])$ that the attacker may have by 1 may be received at input {7}.**
So the message $k2[]$ may be sent to the attacker at output {8}.
 $\text{attacker}(k2[])$.
3. The message $\text{enc}(k1[], k[])$ may be sent to the attacker at output {4}.
 $\text{attacker}(\text{enc}(k1[], k[]))$.
- 4. The message $\text{enc}(k1[], k[])$ that the attacker may have by 3 may be received at input {7}.**
So the message $k1[]$ may be sent to the attacker at output {8}.
 $\text{attacker}(k1[])$.
5. By 4, the attacker may know $k1[]$.
By 2, the attacker may know $k2[]$.
Using the function 2-tuple the attacker may obtain $(k1[], k2[])$.
 $\text{attacker}((k1[], k2[]))$.
6. The message $\text{enc}(s[], (k1[], k2[]))$ may be sent to the attacker at output {6}.
 $\text{attacker}(\text{enc}(s[], (k1[], k2[])))$.
7. By 6, the attacker may know $\text{enc}(s[], (k1[], k2[]))$.
By 5, the attacker may know $(k1[], k2[])$.
Using the function dec the attacker may obtain $s[]$.
 $\text{attacker}(s[])$.
8. By 7, $\text{attacker}(s[])$.
The goal is reached, represented in the following fact:
 $\text{attacker}(s[])$.

Two strange situations !

Simplified Yubikey

```
free k:key [private].  
free cellP,cellQ:channel [private]  
  
let P =  
  in(c,x:bitstring);  
  in(cellP,i:nat);  
  let j = sdec(x,k) in  
  if j > i  
  then  
    event Accept(j);  
    out(cellP,j)  
  else  
    out(cellP,i).  
  
let Q =  
  in(cellQ,i:nat);  
  out(c,senc(I,k));  
  out(cellQ,i+1).  
  
process out(cellP,0) | out(cellQ,0) | !P | !Q
```

Can't disprove the sanity check...

```
query i:nat; event(Accept(i)).
```

```
-- Query not event(Accept(i_2)) in process 0.  
Translating the process into Horn clauses...  
mess(cellQ[],i_2) -> mess(cellQ[],i_2 + 1)  
select mess(cellQ[],i_2)/-5000  
Completing...  
Starting query not event(Accept(i_2))  
  
goal reachable: i_2 ≥ 1 && mess(cellQ[],i_2) -> end(Accept(i_2))  
  
Derivation:  
  
1. We assume as hypothesis that  
mess(cellQ[],i_2).  
  
2. The message i_2 that may be sent on channel cellQ[] by 1 may be received  
at input {12}.  
So the message senc(i_2,k[]) may be sent to the attacker at output {13},  
attacker(senc(i_2,k[])).
```

...

Could not find a trace corresponding to this derivation.

Two strange situations !

```
-- Query not attacker(S(kAminus[!1 = v],x_1)) in process 1.  
select member(*x_1,y)/-5000  
select memberid(*x_1,y)/-5000  
Translating the process into Horn clauses...  
Completing...  
  
...  
  
A more detailed output of the traces is available with  
  set traceDisplay = long.  
  
new exponent: channel creating exponent_3 at {1}  
new honestC: channel creating honestC_3 at {8}  
new kAminus: skey creating kAminus_3 at {10} in copy a  
  
...  
  
event enddosi(Pk(kAminus_3),NI_3) at {37} in copy a, a_4, a_8  
event mess3(Pk(kAminus_3)...)) at {46} in copy a, a_4, a_8  
out(c, cons3(~M_9,...)) at {47} in copy a, a_4, a_8  
  
The attacker has the message 3-proj-3-tuple(D(H(...)).  
A trace has been found, assuming the following hypothesis:  
memberid(Pk(a_12[]),a_5[])  
Stopping attack reconstruction attempts. To try more traces,  
modify the setting reconstructTrace.  
RESULT not attacker(S(kAminus[!1 = v],x_1)) cannot be proved.
```

A trace is found... but ProVerif
assume that the attacker has
magically a term

A closer look

```
-- Query not attacker(S(kAminus[!1 = v],x_1)) in process 1.  
select member(*x_1,y)/-5000  
select memberid(*x_1,y)/-5000  
Translating the process into Horn clauses...  
Completing...  
...  
A more detailed output of the traces is available with  
set traceDisplay = long.  
  
new exponent: channel creating exponent_3 at {1}  
  
new honestC: channel creating honestC_3 at {8}  
  
new kAminus: skey creating kAminus_3 at {10} in copy a  
...  
...  
event enddosi(Pk(kAminus_3),NI_3) at {37} in copy a, a_4, a_8  
  
event mess3(Pk(kAminus_3)...)) at {46} in copy a, a_4, a_8  
  
out(c, cons3(~M_9,...)) at {47} in copy a, a_4, a_8  
  
The attacker has the message 3-proj-3-tuple(D(H(...)).  
A trace has been found, assuming the following hypothesis:  
memberid(Pk(a_12[],a_5[])  
Stopping attack reconstruction attempts. To try more traces,  
modify the setting reconstructTrace.  
RESULT not attacker(S(kAminus[!1 = v],x_1)) cannot be proved.
```

```
-- Query not event(Accept(i_2)) in process 0.  
Translating the process into Horn clauses...  
mess(cellQ[],i_2) -> mess(cellQ[],i_2 + 1)  
select mess(cellQ[],i_2)/-5000  
Completing...  
Starting query not event(Accept(i_2))  
  
goal reachable: i_2 ≥ 1 && mess(cellQ[],i_2) ->  
end(Accept(i_2))  
  
Derivation:  
  
1. We assume as hypothesis that  
mess(cellQ[],i_2).  
  
2. The message i_2 that may be sent on channel cellQ[] by 1 may  
be received at input {12}.  
So the message senc(i_2,k[]) may be sent to the attacker at  
output {13}.  
attacker(senc(i_2,k[])).  
...  
  
Could not find a trace corresponding to this derivation.
```

ProVerif decided to prevent
resolution on some facts

Why ProVerif prevent resolution ?

Simplified Yubikey

```
free k:key [private].  
free cellP,cellQ:channel [private]  
  
let P =  
  in(c,x:bitstring);  
  in(cellP,i:nat);  
  let j = sdec(x,k) in  
  if j > i  
  then  
    event Accept(j);  
    out(cellP,j)  
  else  
    out(cellP,i).  
  
let Q =  
  in(cellQ,i:nat);  
  out(c,senc(I,k));  
  out(cellQ,i+1).  
  
process out(cellP,0) | out(cellQ,0) | !P | !Q
```

```
select mess(cellQ[],i_2)/-5000
```

Clauses generated from the process Q

$$\text{mess}(\text{cellQ}, i) \rightarrow \text{mess}(\text{cellQ}, i + 1)$$

$$\rightarrow \text{mess}(\text{cellQ}, 0)$$

If $\text{mess}(\text{cellQ}, i)$ was selected then by resolution:

$$\rightarrow \text{mess}(\text{cellQ}, 1)$$

$$\rightarrow \text{mess}(\text{cellQ}, 2)$$

⋮
⋮
⋮

What to do to solve the problem ?

Use a new setting

```
set nounifIgnoreAFewTimes = auto.
```

When solving the query, ProVerif will ignore
a « few times » the prevention of resolution.

By default, only one time but it can be parametrized

```
set nounifIgnoreNtimes = 3.
```



The bigger the number, the slower the verification will be



Useful for proofs and finding attacks



Not always enough !

Proof of queries by induction

Simplified Yubikey

```
free k:key [private].
free cellP,cellQ:channel [private]

query i:nat; mess(cellQ,i) ==> is_nat(i).

let P =
  in(c,x:bitstring);
  in(cellP,i:nat);
  let j = sdec(x,k) in
  if j > i
  then
    event Accept(j);
    out(cellP,j)
  else
    out(cellP,i).

let Q =
  in(cellQ,i:nat);
  out(c,senc(I,k));
  out(cellQ,i+1).

process out(cellP,0) | out(cellQ,0) | !P | !Q
```

Even with

```
set nounifIgnoreAFewTimes = auto.
set nounifIgnoreNtimes = 10.
```

With obtain

```
goal reachable: is_not_nat(i_2 + 10) && mess(cellQ[],i_2) ->
mess(cellQ[],i_2 + 10)
```

...

...

```
Could not find a trace corresponding to this derivation.
RESULT mess(cellQ[],i_2) ==> is_nat(i_2) cannot be proved.
```



The attacker is untyped !

Proof of queries by induction

Simplified Yubikey

```
free k:key [private].
free cellP,cellQ:channel [private]

query i:nat; mess(cellQ,i) ==> is_nat(i).

let P =
  in(c,x:bitstring);
  in(cellP,i:nat);
  let j = sdec(x,k) in
  if j > i
  then
    event Accept(j);
    out(cellP,j)
  else
    out(cellP,i).

let Q =
  in(cellQ,i:nat);
  out(c,senc(I,k));
  out(cellQ,i+1).

process out(cellP,0) | out(cellQ,0) | !P | !Q
```

```
goal reachable: is_not_nat(i_2 + 10) && mess(cellQ[],i_2) ->
mess(cellQ[],i_2 + 10)
```

The fact `mess(cellQ[],i_2)` occurred strictly before `mess(cellQ[],i_2 + 10)` in the trace.

Induction on the size of the trace !

```
query i:nat; mess(cellQ,i) ==> is_nat(i) [induction].
```

Proof of queries by induction

It also works for a group of queries !

Proof by mutual induction

```
query i:nat, ...;
mess(cellQ,i) ==> is_nat(i);
mess(cellP,i) ==> is_nat(i);

query_3;

...
query_n [induction].
```



As usual it, it may slow down the verification
or lead to non-termination



Does not work as well for injective correspondence

Lemmas, axioms, restrictions

Restrictions « restrict » the traces considered in axioms, lemmas and queries.

```
restriction phi_1.  
...  
restriction phi_n.  
  
axiom aphi_1.  
...  
axiom aphi_m.  
  
lemma lphi_1.  
lemma lphi_k.  
  
query attacker(s).
```

query attacker(s). holds if no trace satisfying **phi_1, ..., phi_n** reveals **s**

1 Proverif assumes that the axioms **aphi_1, ..., aphi_n** hold.

2 Proverif tries to prove in order the lemmas **lphi_1, ..., lphi_k** reusing all axioms and previously proved lemmas

3 Proverif tries to prove the query **query attacker(s).** reusing all axioms and all lemmas.

The precise option under the hood

Option [precise] for inputs, table lookup and predicate testing
is coded as an axiom internally .

```
free s,k1,k2,k:bitstring [private].  
  
let A =  
  out(c,senc(s,(k1,k2)));  
  out(c,senc(k1,k));  
  out(c,senc(k2,k)).  
  
let B =  
  in(c,x) [precise];  
  out(c,dec(x,k)).  
  
process A | B
```

Encoded as

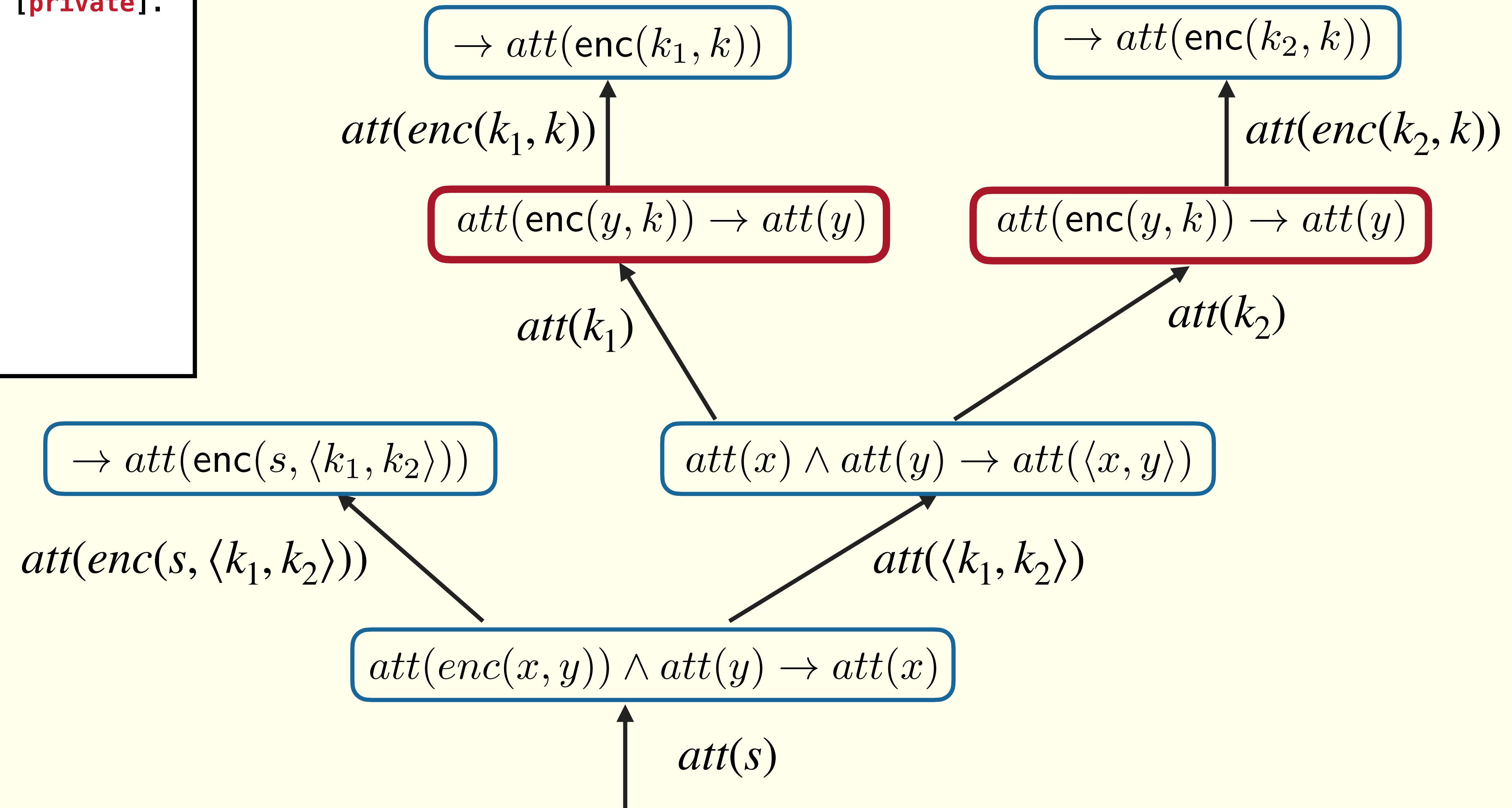


```
type occurrence.  
  
free s,k1,k2,k:bitstring [private].  
event Precise(occurrence,bitstring).  
  
axiom occ:occurrence,x1,x2:bitstring;  
  event(Precise(occ,x1)) && event(Precise(occ,x2)) ==> x1 = x2.  
  
let A =  
  out(c,senc(s,(k1,k2)));  
  out(c,senc(k1,k));  
  out(c,senc(k2,k)).  
  
let B =  
  in(c,x);  
  new occ[]:occurrence;  
  event Precise(occ,x);  
  out(c,dec(x,k)).  
  
process A | B
```

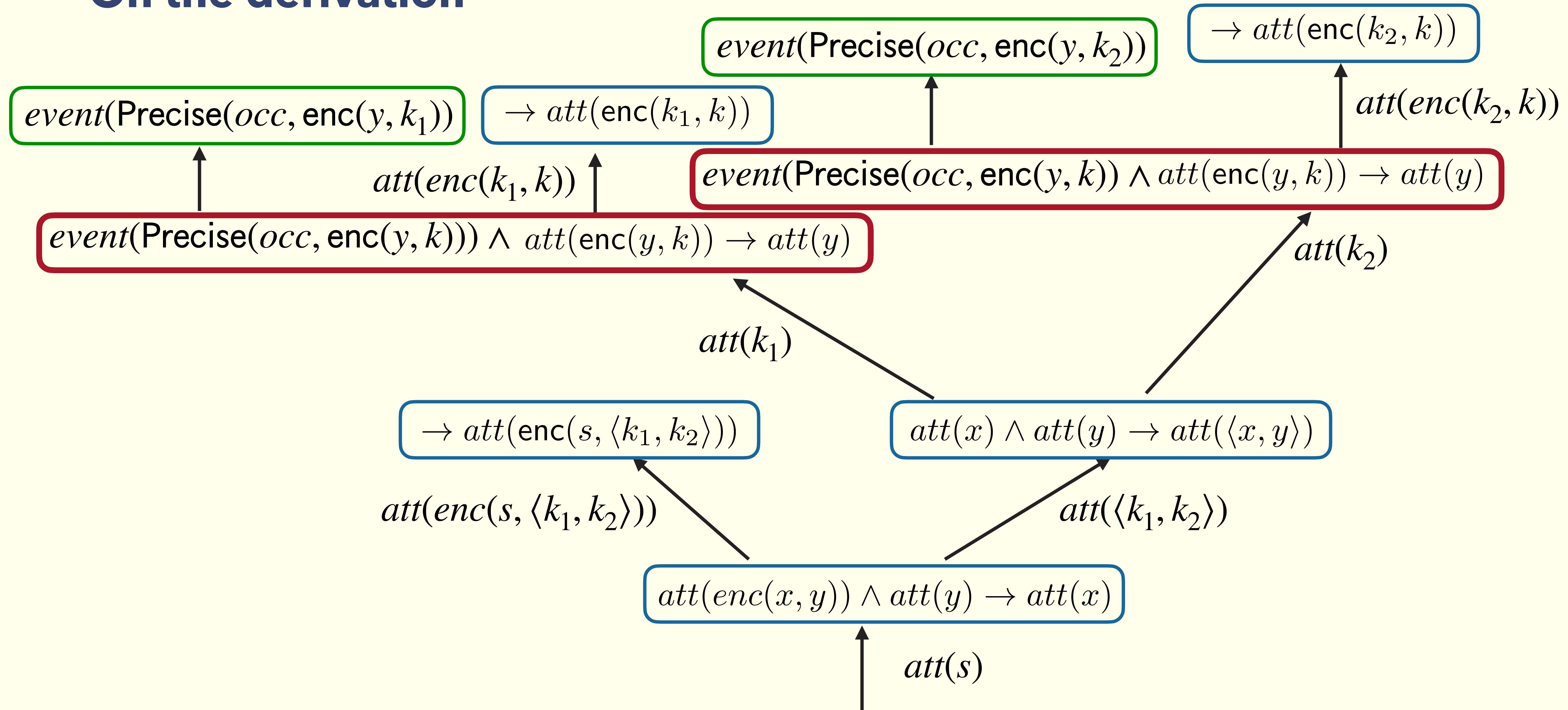
On the derivation

```

free s,k1,k2,k:bitstring [private].
let A =
  out(c,senc(s,(k1,k2)));
  out(c,senc(k1,k));
  out(c,senc(k2,k));
let B =
  in(c,x);
  out(c,dec(x,k));
process A | B
  
```



On the derivation



When to use Lemmas, Axioms and restrictions ?

Restriction

To avoid heavy encoding in the calculus

Ex : To model that a process does not accept twice the same message through multiple session

```
restriction
occ1,occ2:occurrence,x:bitstring;
event(Unique(occ1,x)) &&
event(Unique(occ2,x)) ==> occ1 = occ2.
```

```
let P =
in(c,x);
new occ[]:occurrence;
event Unique(occ,x);
...
```

Lemma

When you the property can help proving the main query.

Axiom

Ideally, always use lemma. Use axiom when you can prove by hand (or with another tool) that your property holds ... and ProVerif cannot.

DEALING WITH NON-TERMINATION

How to determine if ProVerif does not terminate ?

Translating the process into Horn clauses...

Completing...

```
200 rules inserted. Base: 200 rules (97 with conclusion selected). Queue: 679 rules.  
400 rules inserted. Base: 400 rules (133 with conclusion selected). Queue: 481 rules.  
600 rules inserted. Base: 600 rules (133 with conclusion selected). Queue: 291 rules.  
800 rules inserted. Base: 800 rules (133 with conclusion selected). Queue: 135 rules.  
1000 rules inserted. Base: 997 rules (157 with conclusion selected). Queue: 184 rules.  
1200 rules inserted. Base: 1093 rules (204 with conclusion selected). Queue: 134 rules.  
1400 rules inserted. Base: 1253 rules (293 with conclusion selected). Queue: 208 rules.  
1600 rules inserted. Base: 1420 rules (352 with conclusion selected). Queue: 281 rules.  
1800 rules inserted. Base: 1596 rules (382 with conclusion selected). Queue: 315 rules.  
2000 rules inserted. Base: 1790 rules (394 with conclusion selected). Queue: 369 rules.  
2200 rules inserted. Base: 1970 rules (400 with conclusion selected). Queue: 387 rules.  
2400 rules inserted. Base: 2166 rules (400 with conclusion selected). Queue: 393 rules.  
2600 rules inserted. Base: 2323 rules (402 with conclusion selected). Queue: 423 rules.  
2800 rules inserted. Base: 2507 rules (402 with conclusion selected). Queue: 447 rules.  
3000 rules inserted. Base: 2644 rules (416 with conclusion selected). Queue: 484 rules.  
3200 rules inserted. Base: 2790 rules (416 with conclusion selected). Queue: 500 rules.  
3400 rules inserted. Base: 2933 rules (443 with conclusion selected). Queue: 547 rules.  
3600 rules inserted. Base: 3068 rules (443 with conclusion selected). Queue: 571 rules.  
3800 rules inserted. Base: 3209 rules (464 with conclusion selected). Queue: 617 rules.  
4000 rules inserted. Base: 3320 rules (484 with conclusion selected). Queue: 715 rules.  
4200 rules inserted. Base: 3408 rules (484 with conclusion selected). Queue: 747 rules.  
4400 rules inserted. Base: 3529 rules (498 with conclusion selected). Queue: 756 rules.  
4600 rules inserted. Base: 3637 rules (530 with conclusion selected). Queue: 804 rules.  
4800 rules inserted. Base: 3705 rules (530 with conclusion selected). Queue: 882 rules.  
...  
...
```

Number of rules generated

Current size of the set of rules

Number of rules left to handle

The first clues

- Size of the queue seems to always increase
- Size of the queue seems to be cyclic
- No rule inserted for ages (can happen with lemmas)
- Termination warnings



Be patient 😊

On TLS 1.3, terminates with 200k rules inserted.

How to determine if ProVerif does not terminate ?

The real way to do it...

```
set verboseRules = true.
```

Display all the rules generated

```
Rule with hypothesis fact 0 selected: mess(cellQ[],i_2)
mess(cellQ[],i_2) -> mess(cellQ[],i_2)
The hypothesis occurs before the conclusion.
1 rules inserted. Base: 1 rules (0 with conclusion selected). Queue: 3 rules.
```

```
Rule with hypothesis fact 0 selected: mess(cellQ[],i_2)
is_nat(i_2) && mess(cellQ[],i_2) -> mess(cellQ[],i_2 + 1)
The hypothesis occurs strictly before the conclusion.
2 rules inserted. Base: 2 rules (0 with conclusion selected). Queue: 5 rules.
```

```
Rule with conclusion selected:
mess(cellQ[],0)
3 rules inserted. Base: 3 rules (1 with conclusion selected). Queue: 4 rules.
```

```
Rule with hypothesis fact 0 selected: attacker(cellQ[])
attacker(cellQ[]) && attacker(i_2) -> mess(cellQ[],i_2)
The 1st, 2nd hypotheses occur before the conclusion.
4 rules inserted. Base: 4 rules (1 with conclusion selected). Queue: 3 rules.
```

```
Rule with hypothesis fact 0 selected: mess(cellQ[],i_2)
is_nat(i_2) && mess(cellQ[],i_2) -> mess(cellQ[],i_2 + 2)
The hypothesis occurs strictly before the conclusion.
5 rules inserted. Base: 5 rules (1 with conclusion selected). Queue: 5 rules.
```

```
Rule with conclusion selected:
mess(cellQ[],1)
6 rules inserted. Base: 6 rules (2 with conclusion selected). Queue: 4 rules.
```

```
Rule with hypothesis fact 0 selected: attacker(cellQ[])
is_nat(i_2) && attacker(cellQ[]) && attacker(i_2) -> mess(cellQ[],i_2 + 1)
The 1st, 2nd hypotheses occur strictly before the conclusion.
7 rules inserted. Base: 7 rules (2 with conclusion selected). Queue: 3 rules.
```

Signs of a cycle

- Size of the term in the conclusion increases
- Number of hypotheses increases



Very long and painful to read



Best way to find the problem



Best way to understand how to solve it

Not attacker declaration and lemmas

Look if some facts should not be true

```
Rule with hypothesis fact 0 selected: attacker(cellQ[])
attacker(cellQ[]) && attacker(i_2) -> mess(cellQ[],i_2)
The 1st, 2nd hypotheses occur before the conclusion.
4 rules inserted. Base: 4 rules (1 with conclusion selected). Queue: 3 rules.
```

The cell should be private

```
Rule with hypothesis fact 1 selected: attacker(h(i))
is_not_nat(i_2) && event(Accept(i_2)) && attacker(h(i)) -> mess(cellQ[],i_2)
The 1st, 2nd hypotheses occur before the conclusion.
14 rules inserted. Base: 3 rules (2 with conclusion selected). Queue: 3 rules.
```

The content of the cell should be natural numbers

`not attacker(cellQ).`

Faster but semantically equivalent to

`Lemma attacker(cellQ).`

`lemma i:nat; mess(cellQ,i) ==> is_nat(i).`

Playing with the selection function

The fact that will be selected for resolution

```
Rule with hypothesis fact 1 selected: attacker(h(i))
is_not_nat(i_2) && event(Accept(i_2)) && attacker(h(i)) -> mess(cell0[],i_2)
The 1st, 2nd hypotheses occur before the conclusion.
14 rules inserted. Base: 3 rules (2 with conclusion selected). Queue: 3 rules.
```

The automatic detection of selections to avoid is not perfect

If you think this fact will lead to non-termination, you can tell it to ProVerif

```
noselect i:nat, attacker(h(i)).  
noselect i:nat, attacker(h(*i)).
```

*i means « any term »

```
select mess(cellQ[],i_2)/-5000
```

Clauses generated from the process Q

$$\text{mess}(\text{cell}Q, i) \rightarrow \text{mess}(\text{cell}Q, i + 1)$$
$$\text{mess}(\text{cell}Q, i_1) \rightarrow \text{mess}(\text{cell}Q, i_1 + 2)$$
$$\text{mess}(\text{cell}Q, i_2) \rightarrow \text{mess}(\text{cell}Q, i_2 + 3)$$

⋮

Try it!

<http://proverif.inria.fr>

Mailing list

<https://sympa.inria.fr/sympa/subscribe/proverif>

To ask questions: proverif@inria.fr

To report bug or ask for features: proverif-dev@inria.fr