

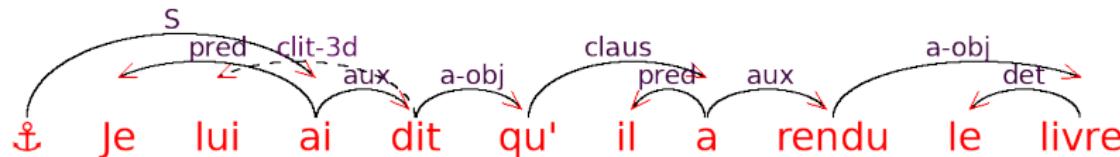
Categorial Dependency Grammars extended with typed barriers

Denis Béchet, Nantes University, France
Annie Foret, University of Rennes and IRISA, France

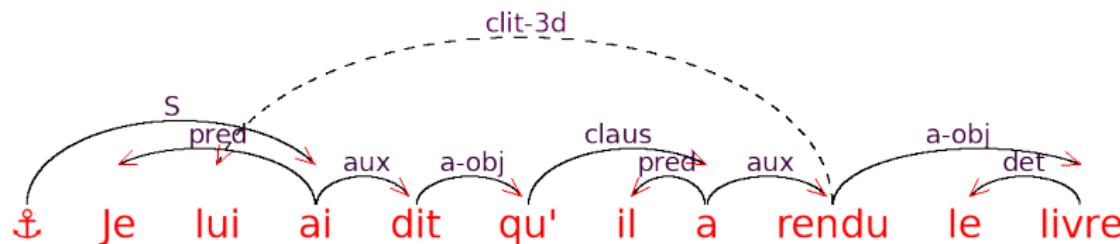
MCLP 2025, September 15–18, 2025, Orsay, France

CDG Problem 1: Overgeneration with non-projective dependencies

The CDG analyses of “Je lui ai dit qu'il a rendu le livre”
“I have told him that he has returned the book” – clitic “lui” (him)



The normal analysis



A wrong analysis

⇒ Our solution: CDG_{tb} (typed barriers)

No known construction for the Kleene plus (CDG):

Let G be a CDG. $L(G)$ is the formal language generated by G

$\exists G'$, a CDG such that $L(G') = L(G)^+$?

⇒ Our solutions: CDG_{tb} (typed barriers) or CDG_b (barriers)

- 1 CDG Languages
- 2 Product of CDG Languages
- 3 Product and Kleene Plus of CDG_{tb} Languages (with typed barriers)
- 4 CDG_{tb} for Natural Languages
- 5 Conclusion

- 1 CDG Languages
- 2 Product of CDG Languages
- 3 Product and Kleene Plus of CDG_{tb} Languages (with typed barriers)
- 4 CDG_{tb} for Natural Languages
- 5 Conclusion

Basics of Dependency Syntax

Surface Dependency Structures (DS) are graphs of surface syntactic relations between the words in a sentence.

A Dependency Structure



Dependencies are determined by valencies of words

brought has +valency **pred** of a **left adjacent** word

deal has -valency **pred** of a **right adjacent** word

Saturation of valency **pred** determines projective dependency

deal **pred** ← *brought* (**Governor:** *brought*, **Subordinate:** *deal*)

Basics of Dependency Syntax

Surface Dependency Structures (DS) are graphs of surface syntactic relations between the words in a sentence.

A Dependency Structure



Dependencies are determined by valencies of words

more has +valency **comp-conj** of a word *somewhere* on its right

than has -valency **comp-conj** of a word *somewhere* on its left

Saturation of **comp-conj** determines **non-projective dependency**

more **comp-conj** ---> *than* (**Governor:** *more*, **Subordinate:** *than*)

CDG Types express dependency valencies

PROJECTIVE DEPENDENCIES

Dependency: $\text{Gov} \xrightarrow{d} \text{Sub}$:

Governor Type: $\text{Gov} \mapsto [\dots / \dots / d / \dots]^P$

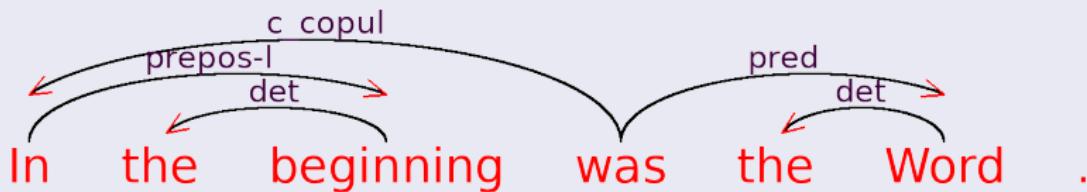
Subordinate Type: $\text{Sub} \mapsto [\dots \backslash d / \dots]^P$

[...] : Part of a type for projective relations (basic dependency type)

P : Part of a type for non-projective dependencies (potential)

Categorial Dependency Grammars

CDG Types express dependency valencies



in $\mapsto [c_copul / prepos-l]$

the $\mapsto [det]$

beginning $\mapsto [det \setminus prepos-l]$

was $\mapsto [c_copul \setminus S / pred]$

Word $\mapsto [det \setminus pred]$

CDG Types express dependency valencies

NON-PROJECTIVE DEPENDENCIES

Polarized valencies: $\nearrow d$, $\searrow d$, $\nwarrow d$, $\swarrow d$

Dependency: $Gov \xrightarrow{d} Sub$:

Governor Type Potential: $Gov \mapsto [...] \cdots \nearrow d \cdots$

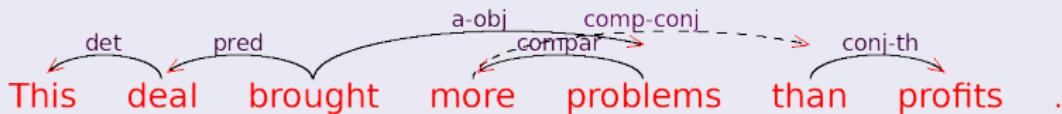
Subordinate Type Potential: $Sub \mapsto [...] \cdots \searrow d \cdots$

[...] : Part of a type for projective relations (**basic dependency type**)

$\cdots \nearrow d \cdots$: Part of a type for non-projective dependencies (**potential**)

Categorial Dependency Grammars

CDG Types express dependency valencies



this $\mapsto [det]$

deal $\mapsto [det \backslash pred]$

brought $\mapsto [pred \backslash S/a_obj]$

problems $\mapsto [compar \backslash a_obj]$

profits $\mapsto [conj_th]$

more $\mapsto [compar] \nearrow comp_conj$

than $\mapsto [/conj_th] \searrow comp_conj$

Left-oriented rules

 $\text{L}^1. \quad [C]^P[C\backslash\beta]^Q \vdash [\beta]^{PQ}$ $Gov \xrightarrow{C} Sub$

Left-oriented rules

 $\text{L}^!.$ $[C]^P[C \setminus \beta]^Q \vdash [\beta]^{PQ}$ $Gov \xrightarrow{C} Sub$ $\text{L}_\varepsilon^!.$ $[\]^P[\beta]^Q \vdash [\beta]^{PQ}$

(no new dependency)

Left-oriented rules

 $\text{L}^!. [C]^P[C \setminus \beta]^Q \vdash [\beta]^{PQ}$ $Gov \xrightarrow{C} Sub$ $\text{L}_\varepsilon^!. []^P[\beta]^Q \vdash [\beta]^{PQ}$

(no new dependency)

 $\text{I}!. [C]^P[C^* \setminus \beta]^Q \vdash [C^* \setminus \beta]^{PQ}$ $Gov \xrightarrow{C} Sub$ $\Omega^!. [C^* \setminus \beta]^P \vdash [\beta]^P$

(no new dependency)

Left-oriented rules

$$\text{L}^!. [C]^P[C \setminus \beta]^Q \vdash [\beta]^{PQ}$$

Gov \xrightarrow{C} *Sub*

$$\text{L}_\varepsilon^!. []^P[\beta]^Q \vdash [\beta]^{PQ}$$

(no new dependency)

$$\text{I}!. [C]^P[C^* \setminus \beta]^Q \vdash [C^* \setminus \beta]^{PQ}$$

Gov \xrightarrow{C} *Sub*

$$\Omega^!. [C^* \setminus \beta]^P \vdash [\beta]^P$$

(no new dependency)

$$\text{D}^!. \alpha^{P_1} (\swarrow C) P (\nwarrow C) P_2 \vdash \alpha^{P_1 P P_2}$$

Gov \dashrightarrow^C *Sub*

First-Available Rule

FA: in $(\swarrow C) P (\nwarrow C)$, the valency $\swarrow C$ is the **first available** for the dual valency $\nwarrow C$, i.e. P has no occurrences of $\swarrow C, \nwarrow C$

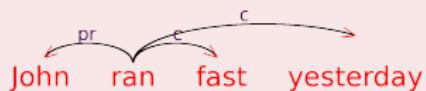
LEXICON:

Derivation

$$\begin{array}{c}
 \frac{\frac{[pr \setminus S / c^*] [c]}{[pr \setminus S / c^*]} \stackrel{\text{fast}}{r} \quad \frac{[pr \setminus S / c^*] [c]}{[pr \setminus S / c^*]} \stackrel{\text{yesterday}}{r}}{\frac{[pr \setminus S / c^*]}{[pr \setminus S]} \Omega^r} \\
 \frac{John \quad [pr]}{S} \stackrel{L}{r}
 \end{array}$$

John $\mapsto [pr]$
ran $\mapsto [pr \setminus S / c^*]$
fast, yesterday $\mapsto [c]$

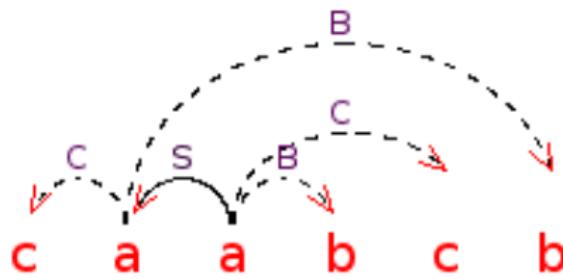
Dependency structure



$$\begin{array}{ll}
 L^l & [C]^P[C \setminus \beta]^Q \vdash [\beta]^{PQ} \\
 L_\varepsilon^l & [\]^P[\beta]^Q \vdash [\beta]^{PQ} \\
 I^l & [C]^P[C^* \setminus \beta]^Q \vdash [C^* \setminus \beta]^{PQ} \\
 \Omega^l & [C^* \setminus \beta]^P \vdash [\beta]^P \\
 D^l & \alpha^{P_1(\swarrow V)P(\nwarrow V)P_2} \vdash \alpha^{P_1PP_2}, \text{ if FA}
 \end{array}$$

$$\begin{array}{ll}
 L^r & [\beta/C]^P[C]^Q \vdash [\beta]^{PQ} \\
 L_\varepsilon^r & [\beta]^P[\]^Q \vdash [\beta]^{PQ} \\
 I^r & [\beta/C^*]^P[C]^Q \vdash [\beta/C^*]^{PQ} \\
 \Omega^r & [\beta/C^*]^P \vdash [\beta]^P \\
 D^r & \alpha^{P_1(\nearrow V)P(\searrow V)P_2} \vdash \alpha^{P_1PP_2}, \text{ if FA}
 \end{array}$$

- a
- $[S] \nwarrow B \searrow C$
 - $[S \backslash S] \nwarrow B \searrow C$
 - $[S] \nearrow B \nearrow C$
 - $[S \backslash S] \nearrow B \nearrow C$
 - $[S] \nwarrow B \nearrow C$
 - $[S \backslash S] \nwarrow B \nearrow C$
 - $[S] \nearrow C \nearrow B$
 - $[S \backslash S] \nearrow C \nearrow B$
- b
- $[] \swarrow B$
 - $[] \searrow B$
- c
- $[] \swarrow C$
 - $[] \searrow C$



A CDG for mix with a parse example

In the above grammar, some types have empty heads ; other grammars avoiding empty heads can be provided, but the above one is simpler.

- 1 CDG Languages
- 2 Product of CDG Languages
- 3 Product and Kleene Plus of CDG_{tb} Languages (with typed barriers)
- 4 CDG_{tb} for Natural Languages
- 5 Conclusion

CDG example: $a^n b^n c^n$

$$a \quad [A]^{\swarrow D} \\ [A \setminus A]^{\swarrow D}$$

$$b \quad [A \setminus S/C] \\ [B/C]$$

$$c \quad [C]^{\nwarrow D} \\ [B \setminus C]^{\nwarrow D}$$

$$\frac{[A]^{\swarrow D} [A \setminus A]^{\swarrow D}}{[A]^{\swarrow D \swarrow D}} \text{L}^l \quad [A \setminus S / C] \text{L}^l \quad \frac{[B / C] [C]^{\nwarrow D}}{[B]^{\nwarrow D}} \text{L}^r \quad [B \setminus C]^{\nwarrow D} \text{L}^l$$

$$\frac{[S / C]^{\swarrow D \swarrow D}}{[S]^{\swarrow D \nwarrow D \nwarrow D}} \text{L}^r$$

$$\frac{[S]^{\swarrow D \nwarrow D}}{[S]} \text{D}^l$$

$$[S] \text{D}^l$$

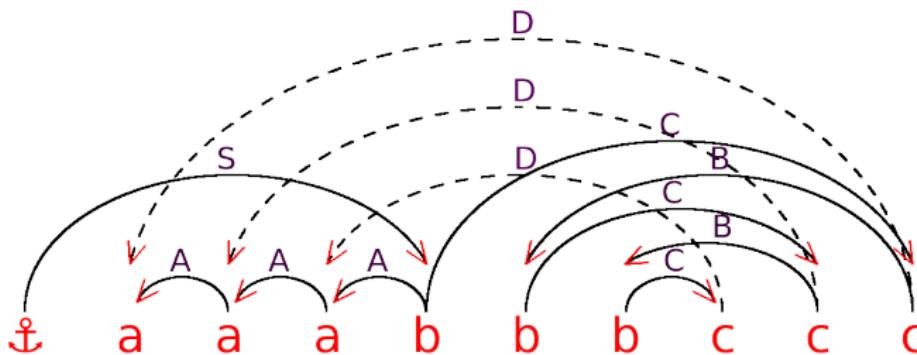
A CDG for $\{a^n b^n c^n, n \geq 1\}$ with a derivation for $aabbcc$ ($n = 2$)

CDG example: $a^n b^n c^n$

$$a \quad [A] \swarrow^D \\ [A \setminus A] \swarrow^D$$

$$b \quad [A \setminus S/C] \\ [B/C]$$

$$c \quad [C] \nwarrow^D \\ [B \setminus C] \nwarrow^D$$



The same CDG for $\{a^n b^n c^n, n \geq 1\}$ with the dependency structure for $aaabbbccc$ ($n = 3$)

Parsing time complexity : $\mathcal{O}(n^4)$

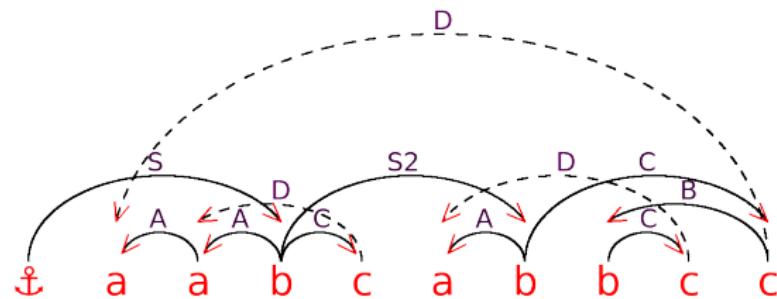
Is it possible to define a CDG that yields the product of $a^n b^n c^n$ with itself ?

$$\{a^p b^p c^p a^q b^q c^q, p \geq 1, q \geq 1\}$$

How can we find it from a CDG that yields $a^n b^n c^n$?

CDG example: An unsuccessful attempt for the product of $a^n b^n c^n$ with itself

a	$[A] \nwarrow^D$
	$[A \setminus A] \nwarrow^D$
b	$[A \setminus S / S2 / C]$
	$[A \setminus S2 / C]$
	$[B / C]$
c	$[C] \nwarrow^D$
	$[B \setminus C] \nwarrow^D$



The CDG is built from the initial CDG for $a^n b^n c^n$:

The initial type of b with S is duplicated and $S2$ is added

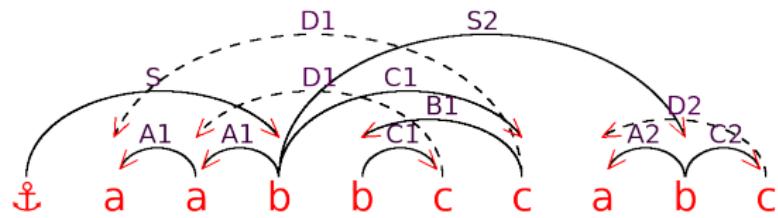
The CDG **doesn't yield** the product of $a^n b^n c^n$ with itself:

$aabcabbcc$ can be parsed but it isn't correct:

Non-projective dependencies between the parts are allowed

CDG example: A correct product of $a^n b^n c^n$ with itself

- a $[A_1] \swarrow^{D_1}$
- $[A_1 \setminus A_1] \swarrow^{D_1}$
- b $[A_1 \setminus S / S_2 / C_1]$
- $[B_1 / C_1]$
- c $[C_1] \nwarrow^{D_1}$
- $[B_1 \setminus C_1] \nwarrow^{D_1}$
- a $[A_2] \swarrow^{D_2}$
- $[A_2 \setminus A_2] \swarrow^{D_2}$
- b $[A_2 \setminus S_2 / C_2]$
- $[B_2 / C_2]$
- c $[C_2] \nwarrow^{D_2}$
- $[B_2 \setminus C_2] \nwarrow^{D_2}$



All the types are duplicated from the initial CDG for $a^n b^n c^n$

- ⇒ Two non-projective dependency names: D_1 and D_2 rather than D
- ⇒ Higher parsing time complexity: $\mathcal{O}(n^5)$ rather than $\mathcal{O}(n^4)$

No known general construction for the Kleene plus of a CDG

- 1 CDG Languages
- 2 Product of CDG Languages
- 3 Product and Kleene Plus of CDG_{tb} Languages (with typed barriers)
- 4 CDG_{tb} for Natural Languages
- 5 Conclusion

Left-oriented rules

L!. $[C]^P[C \setminus \beta]^Q \vdash [\beta]^{PQ}$

$Gov \xrightarrow{C} Sub$

L_ε!. $[]^P[\beta]^Q \vdash [\beta]^{PQ}$

(no new dependency)

I!. $[C]^P[C^* \setminus \beta]^Q \vdash [C^* \setminus \beta]^{PQ}$

$Gov \xrightarrow{C} Sub$

Ω!. $[C^* \setminus \beta]^P \vdash [\beta]^P$

(no new dependency)

D!. $\alpha^{P_1(\swarrow C)P(\nwarrow C)P_2} \vdash \alpha^{P_1PP_2}$

$Gov \dashrightarrow \xrightarrow{C} Sub$

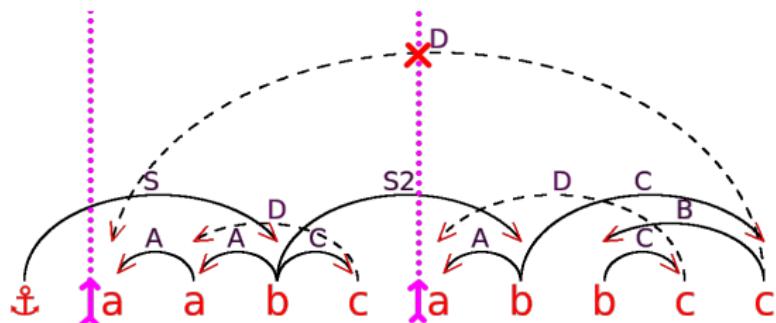
First-Available Rule (and no intermediate typed barrier)

FA_{tb}: in $(\swarrow C)P(\nwarrow C)$, the valency $\swarrow C$ is the **first available** for the dual valency $\nwarrow C$, i.e. P has no occurrences of $\swarrow C$, $\nwarrow C$ and $\uparrow C$

Potentials contain polarized valencies $\swarrow d$, $\nwarrow d$, $\searrow d$, $\nearrow d$ and typed barriers $\uparrow d$

CDG_{tb} with typed barriers: a simple product of $a^n b^n c^n$ with itself

a	$[A]$	$\uparrow D \swarrow D$
	$[A \backslash A]$	$\nwarrow D$
b	$[A \backslash [S/S2]/C]$	
	$[A \backslash S2/C]$	
	$[B/C]$	$\nwarrow D$
c	$[C]$	$\nwarrow D$
	$[B \backslash C]$	$\nwarrow D$



There is a typed barrier $\uparrow D$ on the rightmost a (for $aabcabbcc$)

⇒ The top non-projective dependency isn't allowed this time

The CDG_{tb} with typed barriers yields the product of $a^n b^n c^n$ with itself

Only one non-projective dependency name (D)

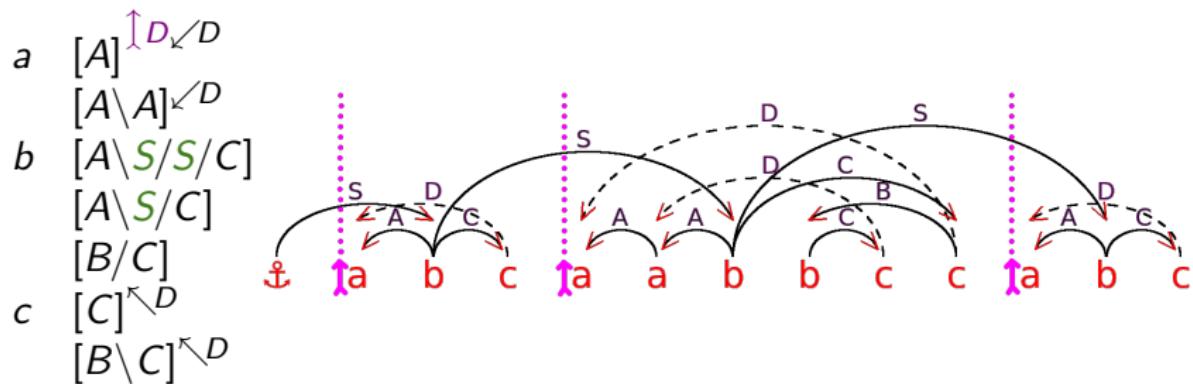
⇒ Same parsing time complexity as $a^n b^n c^n$

Is it possible to define a CDG that yields Kleene plus of $a^n b^n c^n$?
 $\{a^{p_1} b^{p_1} c^{p_1} a^{p_2} b^{p_2} c^{p_2} \dots a^{p_n} b^{p_n} c^{p_n}, n \geq 1, p_1 \geq 1, \dots, p_n \geq 1\}$

How can we find it from a CDG that yields $a^n b^n c^n$?

CDG_{tb} with typed barriers: Kleene plus of $a^n b^n c^n$

No known general construction for the Kleene plus of a CDG
Always possible with a CDG_{tb} (our proposal)



A typed barrier on the leftmost a of each part of the Kleene plus
⇒ Non-projective dependencies between parts aren't allowed
⇒ The CDG_{tb} yields the Kleene plus of $a^n b^n c^n$

Only one non-projective dependency name (D)
⇒ Same parsing time complexity as $a^n b^n c^n$

Kleene plus: The general construction for a CDG_{tb} language

Starting with G , a CDG_{tb} with typed barriers

- ① Transform G if G has types with empty heads in the lexicon
- ② Transform G if the axiom S is used as an argument of a type
- ③ Transform G such that the types in the lexicon are divided in two parts :
 - The types only used on the rightmost token of any derivation
 - The types never used on the rightmost token of any derivation
- ④ Add (typed) barriers in the potential of the types that can only be used on the rightmost token of any derivation
- ⑤ For each type with the axiom S as head type, duplicate the same type but where S is replaced with S/S

The final CDG_{tb} corresponds to the Kleene plus of the initial CDG_{tb}

Example: Kleene plus of $a^n b^n c^n$

$$\begin{array}{lll} a & [A]^{\swarrow D} & b & [A \setminus \textcolor{green}{S}/C] \\ & [A \setminus A]^{\swarrow D} & & [B/C] \\ & & c & [C]^{\nwarrow D} \\ & & & [B \setminus C]^{\nwarrow D} \end{array}$$

- ① Transform G if G has types with empty heads in the lexicon
⇒ Ok (no empty head)
- ② Transform G if the axiom S is used as an argument of a type
⇒ Ok (S only used as head type))

Example: Kleene plus of $a^n b^n c^n$

$$a \quad [A] \swarrow^D
[A \setminus A] \swarrow^D$$

$$b \quad [A \setminus \text{S/C}]
[B/C]$$

$$c \quad [C] \nwarrow^D
[B \setminus C] \nwarrow^D$$

- ③ Transform G such that the types in the lexicon are divided in two parts :

- The types only used on the rightmost token of any derivation
- The types never used on the rightmost token of any derivation

Types on the rightmost token: Types of c ($[C] \nwarrow^D$ and $[B \setminus C] \nwarrow^D$)

Types on other tokens: All the types

Not ok (the types of c)

⇒ We need to transform the grammar (axiom S_r):

$$a \quad [A_r] \swarrow^D$$

$$[A_o] \swarrow^D$$

$$[A_o \setminus A_r] \swarrow^D$$

$$[A_o \setminus A_o] \swarrow^D$$

$$b \quad [A_o \setminus S_r / C_r]$$

$$[A_o \setminus S_o / C_o]$$

$$[B_r / C_r]$$

$$[B_o / C_o]$$

$$c \quad [C_r] \nwarrow^D$$

$$[C_o] \nwarrow^D$$

$$[B_o \setminus C_r] \nwarrow^D$$

$$[B_o \setminus C_o] \nwarrow^D$$

Remark: The grammar can be simplified (useless types)

Example: Kleene plus of $a^n b^n c^n$

$$\begin{array}{lll} a & [A_o] \swarrow^D & \\ & [A_o \setminus A_o] \swarrow^D & \\ b & [A_o \setminus S_r / C_r] & \\ & [B_o / C_o] & \\ c & [C_r] \nwarrow^D & \\ & [C_o] \nwarrow^D & \\ & [B_o \setminus C_r] \nwarrow^D & \\ & [B_o \setminus C_o] \nwarrow^D & \end{array}$$

- ④ Add typed barriers in the potential of the types that can only be used on the rightmost token of any derivation

$$\begin{array}{lll} a & [A_o] \swarrow^D & \\ & [A_o \setminus A_o] \swarrow^D & \\ b & [A_o \setminus S_r / C_r] & \\ & [B_o / C_o] & \\ c & [C_r] \nwarrow^D \uparrow D & \\ & [C_o] \nwarrow^D & \\ & [B_o \setminus C_r] \nwarrow^D \uparrow D & \\ & [B_o \setminus C_o] \nwarrow^D & \end{array}$$

Example: Kleene plus of $a^n b^n c^n$

$$a \quad [A_o] \swarrow^D \\ [A_o \setminus A_o] \swarrow^D$$

$$b \quad [A_o \setminus S_r / C_r] \\ [B_o / C_o]$$

$$c \quad [C_r] \stackrel{D}{\nwarrow} \\ [C_o] \stackrel{D}{\nwarrow} \\ [B_o \setminus C_r] \stackrel{D}{\nwarrow} \\ [B_o \setminus C_o] \stackrel{D}{\nwarrow}$$

- ⑤ For each type with the axiom S_r as head type, duplicate the same type but where S_r is replaced with S_r / S_r

$$a \quad [A_o] \swarrow^D \\ [A_o \setminus A_o] \swarrow^D$$

$$b \quad [A_o \setminus S_r / C_r] \\ [A_o \setminus S_r / S_r / C_r] \\ [B_o / C_o]$$

$$c \quad [C_r] \stackrel{D}{\nwarrow} \\ [C_o] \stackrel{D}{\nwarrow} \\ [B_o \setminus C_r] \stackrel{D}{\nwarrow} \\ [B_o \setminus C_o] \stackrel{D}{\nwarrow}$$

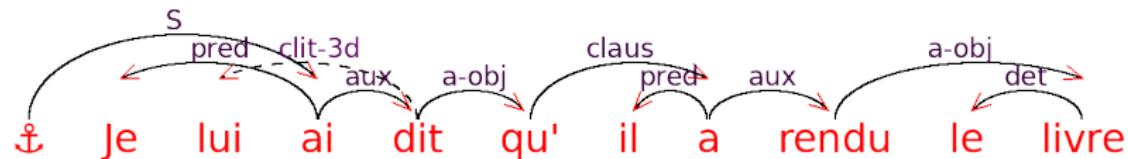
- 1 CDG Languages
- 2 Product of CDG Languages
- 3 Product and Kleene Plus of CDG_{tb} Languages (with typed barriers)
- 4 CDG_{tb} for Natural Languages
- 5 Conclusion

CDG and CDG_{tb} for Natural Languages

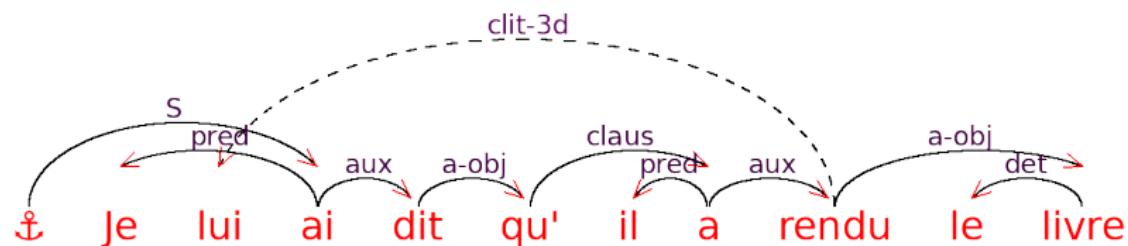
The CDG analyses of “Je lui ai dit qu'il a rendu le livre”

“I have told him that he has returned the book” clitic “lui” (him)

$il \mapsto [] \xleftarrow{\text{clit-3d}}$ $dit, rendu \mapsto [\text{aux/a-obj}], [\text{aux/a-obj}] \xleftarrow{\text{clit-3d}}$



The normal analysis: $dit \mapsto [\text{aux/a-obj}] \xleftarrow{\text{clit-3d}}$ $rendu \mapsto [\text{aux/a-obj}]$



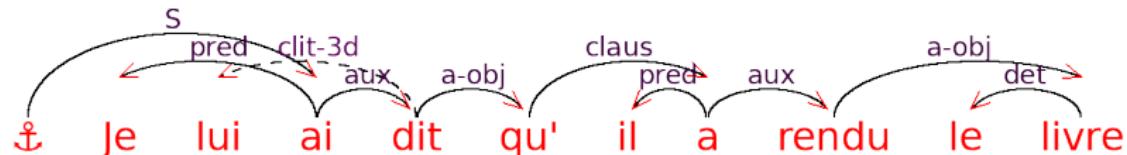
A wrong analysis: $dit \mapsto [\text{aux/a-obj}]$ $rendu \mapsto [\text{aux/a-obj}] \xleftarrow{\text{clit-3d}}$

CDG and CDG_{tb} for Natural Languages

The CDG_{tb} analyses of “Je lui ai dit qu'il a rendu le livre”

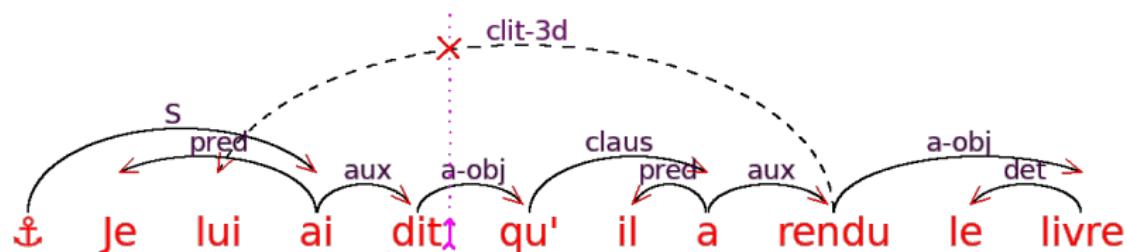
“I have told him that he has returned the book” clitic “lui” (him)

$il \mapsto [] \xleftarrow{\text{clit-3d}}$ $dit, rendu \mapsto [\text{aux/a-obj}] \xleftarrow{\text{clit-3d}}, [\text{aux/a-obj}] \xleftarrow{\text{clit-3d}}$



The normal analysis: $dit \mapsto [\text{aux/a-obj}] \xleftarrow{\text{clit-3d}}$

$rendu \mapsto [\text{aux/a-obj}] \xleftarrow{\text{clit-3d}}$



No analysis: $dit \mapsto [\text{aux/a-obj}] \xleftarrow{\text{clit-3d}}$ $rendu \mapsto [\text{aux/a-obj}] \xleftarrow{\text{clit-3d}}$

⇒ Typed barriers can control the range of specific non-projective dependencies

Conclusion and Open Questions

- The **product** and the **Kleene plus** of languages may be useful, for instance, to model the **conjunction of parts of speech** or the **list of complex complements** in a lot of **natural languages**.
- Our proposal **allows** such constructions for **CDG_{tb}** languages (**with typed barriers**).
- There is no parsing complexity penalty for the **product** and the **Kleene plus**.
- Categorial Dependency Grammars extended with **typed barriers** define an **Abstract Family of Languages** (closed under union, product, Kleene plus, ε -free homomorphism, inverse homomorphism, intersection with regular sets).
- The same **questions** remain **opened** for **classical CDG**.
- For natural languages, **typed barriers** can control the **range of specific non-projective dependencies**.

Conclusion and Open Questions

- The **product** and the **Kleene plus** of languages may be useful, for instance, to model the **conjunction of parts of speech** or the **list of complex complements** in a lot of **natural languages**.
- Our proposal **allows** such constructions for **CDG_{tb}** languages (**with typed barriers**).
- There is no parsing complexity penalty for the **product** and the **Kleene plus**.
- Categorial Dependency Grammars extended with **typed barriers** define an **Abstract Family of Languages** (closed under union, product, Kleene plus, ε -free homomorphism, inverse homomorphism, intersection with regular sets).
- The same **questions** remain **opened** for **classical CDG**.
- For natural languages, **typed barriers** can control the **range of specific non-projective dependencies**.

THANK YOU !

Bibliography

-  Michael Dekhtyar, Alexander Dikovsky, and Boris Karlov.
Categorial dependency grammars.
Theoretical Computer Science, 579:33–63, 2015.
-  Y. Bar-Hillel, H. Gaifman, and E. Shamir.
On categorial and phrase structure grammars.
Bull. Res. Council Israel, 9F:1–16, 1960.
-  I. Mel'čuk.
Dependency Syntax.
SUNY Press, Albany, NY, 1988.
-  Denis Béchet and Annie Foret.
Categorial dependency grammars extended with barriers
(CDGb) yield an abstract family of languages (AFL).
In *David C. Wyld and Dhinaharan Nagamalai, editors,*
*Proceedings of the 5th International Conference on Natural
Language Processing and Computational Linguistics (NLPCL
2024), Copenhagen, Denmark*, pages 53–66, September