

# Towards Practical and Rigorous Automated Grading in Functional Programming Courses

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Women in EuroProofNet 2023

## Warm-Up Exercise #1

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Write a function that deletes the duplicate elements from the stated list.

```
def uniq(lst: List[Int]): List[Int] = ???
```

# Warm-Up Exercise #1

Write a function that deletes the duplicate elements from the stated list.

```
def uniq(lst: List[Int]): List[Int] =  
  distinct(List(), lst)  
  
def distinct(a: List[Int], b: List[Int]): List[Int] =  
  b match  
    case Nil() => a  
    case Cons(x, xs) =>  
      if isin(x, a) then distinct(a, xs)  
      else distinct(a ++ List(x), xs)  
  
def isin(n: Int, lst: List[Int]): Boolean =  
  lst.foldRight(false){ (e, acc) =>  
    (e == n || acc)  
  }
```

Is this solution correct?

# Warm-Up Exercise #1

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```

Is this solution correct? 

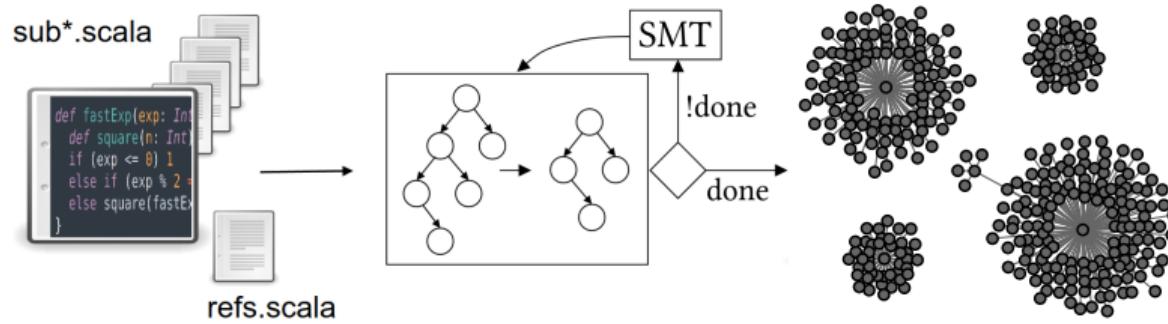
# How Does This Scale?



# How Does This Scale?

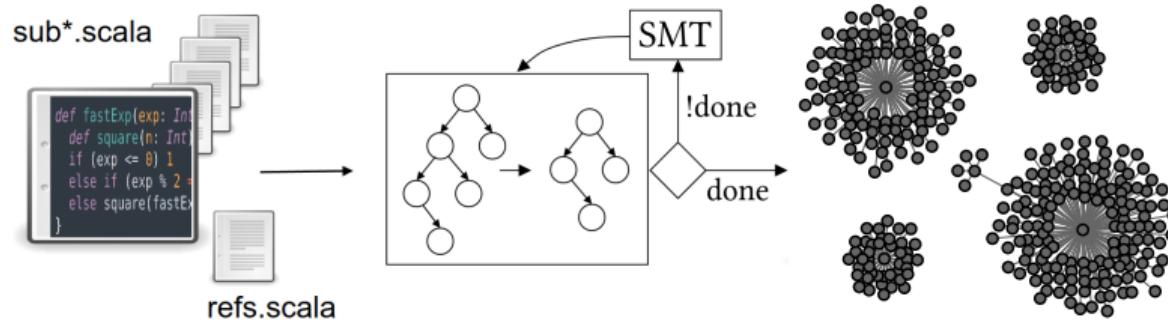


# Proving and Disproving Equivalence of Functional Programming Assignments<sup>1</sup>



<sup>1</sup>Dragana Milovancevic and Viktor Kunčak. 2023. Proving and Disproving Equivalence of Functional Programming Assignments. PLDI'23.

# Proving and Disproving Equivalence of Functional Programming Assignments<sup>1</sup>



- ▶ An interesting combination of program verification and clustering
- ▶ Program decomposition and function call matching (modular programs)
- ▶ A large variety of recursive problems, challenging for equivalence proofs

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<sup>1</sup>Dragana Milovancevic and Viktor Kunčak. 2023. Proving and Disproving Equivalence of Functional Programming Assignments. PLDI'23.

# Why would students in an introductory course care about proofs?

*Automated techniques make it easy to perform **flawed** assessment at scale<sup>1</sup>*

- ▶ Because testing is not enough!
- ▶ Formal techniques as a guarantee that programs are never wrongly classified as correct, or incorrect

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<sup>1</sup>John Wrenn, Shriram Krishnamurthi, and Kathi Fisler. 2018. Who Tests the Testers?. ICER'18.

# Why equivalence checking?

- ▶ Because formal verification is hard!
- ▶ Case studies show ratios such as 9 lines of specifications per executable line<sup>1</sup>
- ▶ Impractical for automated grading

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<sup>1</sup>Mario Bucev and Viktor Kunčak. Formally verified quite ok image format. In 2022 Formal Methods in Computer-Aided Design (FMCAD'22).

## Overview of Our Approach

3. Clustering algorithm that finds the subset of correct solutions
2. Function call matching based on type- and test-directed search
1. Pairwise equivalence checking based on functional induction

# Pairwise Equivalence Checking



# Pairwise Equivalence Checking

A candidate program F is equivalent to a reference program M if:

- ▶ M and F have the same signature
- ▶ M and F terminate
- ▶ M and F return the same output for all inputs



# Pairwise Equivalence Checking: Recursion and Functional Induction

```
def isinM(lst:List[Int], n:Int): Boolean = def isin(lst:List[Int], n:Int): Boolean =
  if lst.isEmpty then false
  else if lst.head == n then true
  else isinM(lst.tail, n)
                                         lst.foldRight(false){ (e, acc) =>
                                         (e == n || acc)
                                         }
```

# Pairwise Equivalence Checking: Recursion and Functional Induction

```
def isinM(lst:List[Int], n:Int): Boolean = {
    if lst.isEmpty then false
    else if lst.head == n then true
    else isinM(lst.tail, n)
}

} ensuring(result => result == isin(lst, n))
```

# Pairwise Equivalence Checking: Recursion and Functional Induction

```
def isinM(lst:List[Int], n:Int): Boolean = {
  if lst.isEmpty then false
  else if lst.head == n then true
  else
    val tail = lst.tail
    val res =
      if tail.isEmpty then false
      else if tail.head == n then true
      else isinM(tail.tail, n)
    assume(res == isin(tail, n))
    res
} ensuring(result => result == isin(lst, n))
```

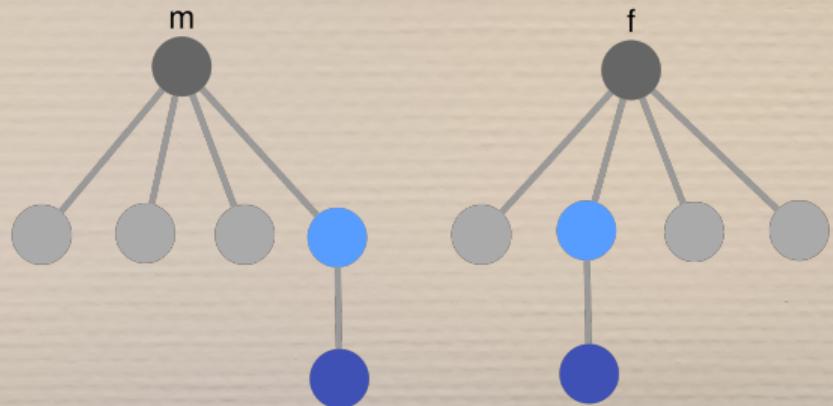
# Pairwise Equivalence Checking: Recursion and Functional Induction

```
def isinM(lst>List[Int], n:Int): Boolean = {  
    if lst.isEmpty then false  
    else if lst.head == n then true  
    else isinM(lst.tail, n)  
    val tail = lst.tail  
    val res =  
        if tail.isEmpty then false  
        else if tail.head == n then true  
        else isinM(tail.tail, n)  
    assume(res == isin(tail, n))  
    res  
} ensuring(result => result == isin(lst, n))
```

# Function Call Matching



# Function Call Matching



# Function Call Matching: An Example

```
def uniqM(lst: List[Int]): List[Int] =
  distinctM(lst, Nil())

def distinctM(l: List[Int], r: List[Int]): List[Int] =
  l match
    case Nil() => r
    case Cons(x, xs) =>
      if !isinM(r, x) then distinctM(xs, r ++ List(x))
      else distinctM(xs, r)

def isinM(lst: List[Int], n: Int): Boolean =
  if lst.isEmpty then false
  else if lst.head == n then true
  else isinM(lst.tail, n)
```

# Function Call Matching: Type- and Test-Directed Search

```
def uniqM(lst: List[Int]): List[Int]
def distinctM(l: List[Int], r: List[Int]): List[Int]
def isinM(lst: List[Int], n: Int): Boolean

def uniq(lst: List[Int]): List[Int]
def distinct(a: List[Int], b: List[Int]): List[Int]
def isin(n: Int, lst: List[Int]): Boolean
```

# Function Call Matching: Type- and Test-Directed Search

```
def uniqM(lst: List[Int]): List[Int]
def distinctM(l: List[Int], r: List[Int]): List[Int]
def isinM(lst: List[Int], n: Int): Boolean

✗ def uniq(lst: List[Int]): List[Int]
✗ def distinct(a: List[Int], b: List[Int]): List[Int]
✓ def isin(n: Int, lst: List[Int]): Boolean
```

▶ Type-directed search

Test-directed search

# Function Call Matching: Type- and Test-Directed Search

```
def uniqM(lst: List[Int]): List[Int]
def distinctM(l: List[Int], r: List[Int]): List[Int]
def isinM( lst: List[Int] , n: Int ): Boolean
```

```
def uniq(lst: List[Int]): List[Int]
def distinct(a: List[Int], b: List[Int]): List[Int]
def isin( n: Int , lst: List[Int] ): Boolean
```

- ▶ Type-directed search

Test-directed search

# Function Call Matching: Type- and Test-Directed Search

```
def uniqM(lst: List[Int]): List[Int]
def distinctM( l: List[Int] , r: List[Int] ): List[Int]
def isinM(lst: List[Int], n: Int): Boolean

def uniq(lst: List[Int]): List[Int]
def distinct( a: List[Int] , b: List[Int] ): List[Int]
def isin(n: Int, lst: List[Int]): Boolean
```

Type-directed search

- ▶ Test-directed search

# Clustering Algorithm

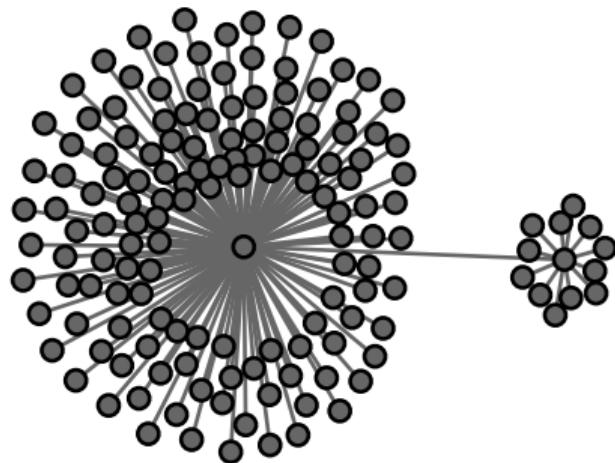


# Clustering Algorithm

- ▶ Classifies candidate programs using pairwise equivalence as a subroutine
- ▶ Prioritization of reference programs, including candidate programs proven correct
- ▶ Reuse of generated counterexamples



# Clustering Algorithm: Discovery of Intermediate Reference Solutions



- ▶ The single reference solution is in the center of the larger cluster
- ▶ Our system automatically finds the intermediate reference solution, from the set of student submissions
- ▶ Prioritization of reference programs to avoid the quadratic behavior

# Evaluation

1. Results on equivalence checking examples
2. Results on functional programming assignments

# Results on Equivalence Checking Examples

|                   | ackermann <sup>1</sup> | mccarthy91 <sup>1</sup> | limit1 <sup>1</sup> | limit2 <sup>1</sup> | limit3 <sup>1</sup> | add-horn <sup>1</sup> | triangular <sup>1</sup> | Inlining <sup>1</sup> | sum <sup>2</sup> | fibonacci-f <sup>2</sup> | pascal <sup>2</sup> | fibonacci-m <sup>2</sup> | fibonacci-t <sup>2</sup> | fibonacci-h <sup>2</sup> | fact4 <sup>3</sup> | fact13 <sup>3</sup> | fact14 <sup>3</sup> |
|-------------------|------------------------|-------------------------|---------------------|---------------------|---------------------|-----------------------|-------------------------|-----------------------|------------------|--------------------------|---------------------|--------------------------|--------------------------|--------------------------|--------------------|---------------------|---------------------|
| REVE <sup>1</sup> | ✓                      | ✓                       | ✗                   | ✓                   | ✓                   | ✓                     | ✓                       | ✓                     | ✓                | ✗                        | ✗                   | ✗                        | ✗                        | ✗                        | ✗                  | ✗                   | ✗                   |
| RVT <sup>23</sup> | ✓                      | ✓                       | ✓                   | ✗                   | ✗                   | ✓                     | ✗                       | ✗                     | ✓                | ✓                        | ✗                   | ✗                        | ✓                        | ✓                        | ✓                  | ✗                   | ✗                   |
| Our System        | ✗                      | ✗                       | ✓                   | ✓                   | ✓                   | ✓                     | ✗                       | ✓                     | ✓                | ✓                        | ✓                   | ✓                        | ✓                        | ✓                        | ✓                  | ✓                   | ✓                   |

<sup>1</sup>Dennis Felsing, Sarah Grebing, Vladimir Klebanov, Philipp Rümmer, and Mattias Ulbrich. 2014. Automating Regression Verification. ASE'14.

<sup>2</sup>Chaked R. J. Sayedoff and Ofer Strichman. 2022. Regression verification of unbalanced recursive functions with multiple calls (long version).

<sup>3</sup>Ofer Strichman and Maor Veitsman. 2016. Regression Verification for Unbalanced Recursive Functions. FM'16

# Results on FP Assignments

# Results on FP Assignments

| No | Name      | #S | #P  | #T  | #F  | LoC | Found/Term. |
|----|-----------|----|-----|-----|-----|-----|-------------|
| 1  | filter    | 1  | 210 | 210 | 1.1 | 9   | 99%         |
| 2  | max       | 1  | 216 | 216 | 1.5 | 15  | 92%         |
| 3  | mirror    | 1  | 97  | 96  | 1.1 | 19  | 100%        |
| 4  | mem       | 1  | 136 | 136 | 1.1 | 18  | 100%        |
| 5  | sigma     | 3  | 736 | 734 | 1.1 | 11  | 97%         |
| 6  | natadd    | 4  | 447 | 381 | 1.4 | 13  | 93%         |
| 7  | natmul    | 7  | 447 | 381 | 2.9 | 27  | 62%         |
| 8  | change    | 1  | 9   | 8   | 2.6 | 26  | 100%        |
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| 10 | uniq      | 4  | 157 | 147 | 2.8 | 23  | 81%         |
| 11 | iter      | 1  | 28  | 27  | 1.3 | 11  | 0%          |
| 12 | crazy2add | 1  | 240 | -   | 2.0 | 52  | NA          |
| 13 | formula   | 1  | 708 | 680 | 2.1 | 52  | 97%         |
| 14 | lambda    | 4  | 802 | 781 | 2.6 | 34  | 71%         |
| 15 | diff      | 1  | 409 | -   | 2.1 | 60  | NA          |

- ▶ Scala translations of public benchmarks from an OCaml FP course
- ▶ Thousands of programs
- ▶ Typical FP problems, include user-defined types and higher-order functions

## Example 1/3: Max

```
def maxM(lst: List[Int]): Int = lst match
  case Nil() => Int.MinValue
  case Cons(h, Nil()) => h
  case Cons(h, t) => if h > maxM(t) then h else maxM(t)

def maxT(lst: List[Int]): Int = lst match
  case Nil() => Int.MinValue
  case Cons(a, Nil()) => a
  case Cons(a, Cons(b, t)) =>
    if a > b then maxT(a :: t) else maxT(b :: t)

def maxF(lst: List[Int]): Int = lst match
  case Nil() => Int.MinValue
  case Cons(h, t) =>
    t.foldLeft(h)((a, b) => if a >= b then a else b)
```



## Example 2/3: Formula

```
def eval(p: Formula): Boolean =  
  p match  
    case True => true  
    case False => false  
    case Not(a) => !eval(a)  
    case AndAlso(a, b) => eval(a) && eval(b)  
    case OrElse(a, b) => eval(a) || eval(b)  
    case Imply(a, b) => eval(Not(a)) || eval(b)  
    case Equal(a, b) => if my_exp(a) == my_exp(b) then true else false  
  
def my_exp(p: Exp): Int =  
  p match  
    case Num(b) => b  
    case Plus(a, b) => my_exp(a) + my_exp(b)  
    case Minus(a, b) => my_exp(a) - my_exp(b)
```



## Example 3/3 Change

```
def change(coins: List[BigInt], amount: BigInt): BigInt = {
    require(amount >= 0)
    def coin_reduce(lst: List[BigInt], v: BigInt): BigInt = {
        require(v >= 0)
        lst match
            case Nil() => 0
            case Cons(hd, tl) =>
                if hd <= 0 then 999
                else if v - hd == 0 then 1
                else if v - hd < 0 then 0
                else coin_reduce(tl, v) + coin_reduce(lst, v-hd)
    }
    if amount == 0 then if coins == Nil() then 0 else 1
    else if amount < 0 then 0
    else coin_reduce(coins, amount)
}
```



coins: List(2, 0, 1), amount: 1

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- ▶ Overall 86% success rate (Found/Term.)
- ▶ Overall 414/420 of counterexamples discovered (99%)

## Results on FP Assignments

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- ▶ Large numbers of submissions (over 300 per benchmark on avg.)
- ▶ Variety of recursive problems (15 in total, with 28 LoC on avg.)
- ▶ Mostly correct submissions (~90%)

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- ▶ Overall 96% success rate on single-function benchmarks
- ▶ One reference solution is typically sufficient

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## Resources: Implementation, Benchmarks



- ▶ Implementation on top of the Stainless verifier for Scala
- ▶ Open source: [github.com/epfl-lara/stainless](https://github.com/epfl-lara/stainless)
- ▶ Accompanying artifact with all our benchmarks: [zenodo.org/record/7810840](https://zenodo.org/record/7810840)
- ▶ Additional examples: [stainless/frontends/benchmarks/equivalence](https://stainless.epfl.ch/frontends/benchmarks/equivalence)

# Example Run

```
milovanc@thinkpadx1e:~/equivalence/benchmarks/fastexp$ ls
A14.scala  A2.scala  B11.scala  B16.scala  C18.scala  C6.scala
A1.scala   A5.scala  B14.scala  C15.scala  C1.scala   refs.scala
milovanc@thinkpadx1e:~/equivalence/benchmarks/fastexp$ cd ../../stainless
milovanc@thinkpadx1e:~/equivalence/stainless$ ./stainless ../../benchmarks/fastexp/*.scala --batched=true --comparefun=fastExp --models=fastExpM --timeout=0.5

[ Info ] Printing equivalence checking results:
[ Info ] List of functions that are equivalent to model Model.fastExpM: B14.fastExp, A14.fastExp, C15.fastExp, A1.fastExp
[ Info ] List of erroneous functions: C1.fastExp
[ Info ] List of timed-out functions: A5.fastExp
[ Info ] List of wrong functions:
[ Info ] Printing the final state:
[ Info ] Path for the function B16.fastExp: Model.fastExpM
[ Info ] Path for the function B11.fastExp: Model.fastExpM
[ Info ] Path for the function C13.fastExp: Model.fastExpM
[ Info ] Path for the function C6.fastExp: Model.fastExpM
[ Info ] Path for the function A5.fastExp:
[ Info ] Path for the function A2.fastExp: Model.fastExpM
[ Info ] Path for the function A1.fastExp: Model.fastExpM
[ Info ] Path for the function C1.fastExp: Model.fastExpM
[ Info ] Path for the function C15.fastExp: Model.fastExpM
[ Info ] Path for the function A14.fastExp: Model.fastExpM
[ Info ] Path for the function B14.fastExp: Model.fastExpM
[ Info ] Counterexample for the function C1.fastExp: Map(base -> BigInt("0"), exp -> BigInt("1"))
```

## Resources: Related Work

- ▶ Automated grading
  - ▶ Clustering: OverCode, Clara, CoderAssist, LEGeNt, ZEUS...
  - ▶ LearnML: counterexample + feedback generation (FixML, TestML, CAFE)
- ▶ Equivalence checking
  - ▶ Regression verification: REVE, RVT
  - ▶ Translation validation
- ▶ Automated proofs by induction
  - ▶ Recursion induction in Isabelle, functional induction in Coq
  - ▶ Functional induction is the default induction heuristic in ACL2

## Warm-Up Exercise #2

## Warm-Up Exercise #2

Given the following lemmas:

$$(\text{MAPNIL}) \text{ Nil.map}(f) == \text{Nil}$$

$$(\text{MAPCONS}) \text{ (x :: xs).map}(f) == f(x) :: xs.\text{map}(f)$$

$$(\text{MAPTRNIL}) \text{ Nil.mapTr}(f, ys) == ys$$

$$(\text{MAPTRCONS}) \text{ (x :: xs).mapTr}(f, ys) == xs.\text{mapTr}(f, ys ++ (f(x) :: \text{Nil}))$$

$$(\text{NILAPPEND}) \text{ Nil} ++ xs == xs$$

$$(\text{CONSAPPEND}) \text{ (x :: xs)} ++ ys == x :: (xs ++ ys)$$

Let us prove the following lemma for the base case: I is Nil.

- ▶  $\text{Nil.mapTr}(f, y :: ys) == y :: \text{Nil.mapTr}(f, ys)$
-

## Warm-Up Exercise #2

Given the following lemmas:

$$(\text{MAPNIL}) \text{ Nil.map}(f) == \text{Nil}$$

$$(\text{MAPCONS}) \text{ (x :: xs).map}(f) == f(x) :: xs.\text{map}(f)$$

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►  $\text{Nil.mapTr}(f, y :: ys) == y :: \text{Nil.mapTr}(f, ys)$

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$$\text{Nil.mapTr}(f, y :: ys)$$

$$== y :: ys \quad (\text{by MapTrNil}(f, y :: ys))$$

$$== y :: \text{Nil.mapTr}(f, ys) \quad (\text{by MapTrNil}(f, ys))$$

Is this solution correct?

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# How Does This Scale?



# How Does This Scale?



- MAPCONS, IH, NILAPPEND, MAPTRCONS, IH
- MAPCONS, IH, IH, NILAPPEND, MAPTRCONS
- MAPCONS, ACCOUT, IH, NILAPPEND, MAPTRCONS
- MAPTRCONS, ACCOUT, NILAPPEND, IH, MAPCONS
- MAPCONS, IH, NILAPPEND, MAPTRCONS, ACCOUT
- MAPCONS, NILAPPEND, ACCOUT, MAPTRCONS, ACCOUT
- MAPCONS, NILAPPEND, ACCOUT, MAPTRCONS, IH
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# How Does This Scale?



- MAPCONS, IH, NILAPPEND, MAPTRCONS, IH
- MAPCONS, IH, IH, NILAPPEND, MAPTRCONS
- MAPCONS, ACCOUT, IH, NILAPPEND, MAPTRCONS
- MAPTRCONS, ACCOUT, NILAPPEND, IH, MAPCONS
- MAPCONS, IH, NILAPPEND, MAPTRCONS, ACCOUT
- MAPCONS, NILAPPEND, ACCOUT, MAPTRCONS, ACCOUT
- MAPCONS, NILAPPEND, ACCOUT, MAPTRCONS, IH
- MAPTRCONS, IH, NILAPPEND, ACCOUT, MAPCONS
- MAPCONS, IH, ACCOUT, NILAPPEND, MAPTRCONS
- MAPCONS, NILAPPEND, IH, ACCOUT, MAPTRCONS
- MAPCONS, NILAPPEND, ACCOUT, ACCOUT, MAPTRCONS

# Proving Correctness of Proof Assignments – Future Directions

```
val AccOutNil = Theorem( Nil.mapTr(f, (x :: xs)) ≡ (x :: Nil.mapTr(f, xs)) ) {  
  have      ( Nil.mapTr(f, (x :: xs)) ≡ (x :: xs) )  
            by Apply(mapTr.NilCase of (acc → (x :: xs)))  
  thenHave( Nil.mapTr(f, (x :: xs)) ≡ (x :: Nil.mapTr(f, xs)) )  
            by Apply(mapTr.NilCase of (acc → xs)) }
```

---

<sup>1</sup>Simon Guilloud, Sankalp Gambhir, and Viktor Kuncak. 2023. LISA – A Modern Proof System. ITP'23.

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                by Apply(mapTr.NilCase of (acc → xs)) }
```

- ▶ Our candidate: LISA<sup>1</sup>, a new proof assistant based on set theory
- ▶ Why LISA?
  - ▶ Does not require deep knowledge of proof assistants
  - ▶ Provides an intuitive and programmer-friendly environment
  - ▶ Key features: DSL + high-level interface

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# Conclusion

- ▶ Practical and rigorous autograding of FP assignments
- ▶ Combination of simple techniques
- ▶ Fully automated
  - ▶ The only inputs are student submissions and reference solution
- ▶ Rigorous
  - ▶ Relies on program verifiers (or proof assistants)
- ▶ Effective in practice
  - ▶ Outperforms equivalence checking tools on their own benchmarks
  - ▶ 86% success rate on thousands of submissions from an actual FP course