

Lógica Clásica

Verdadera

1

Falsa

0

$$Veracidad \in \{0,1\}$$

Lógica Difusa

Parcialmente
verdadera

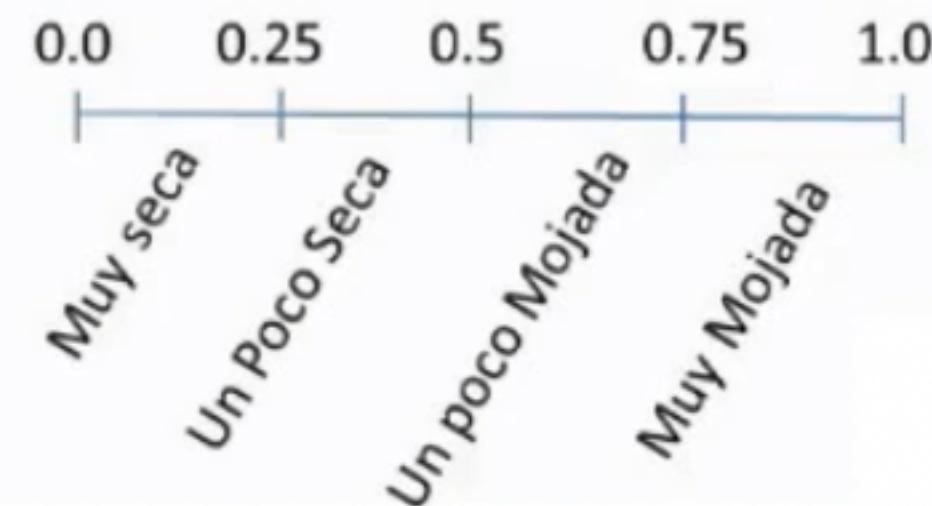
0.7

Parcialmente
Falsa

0.3

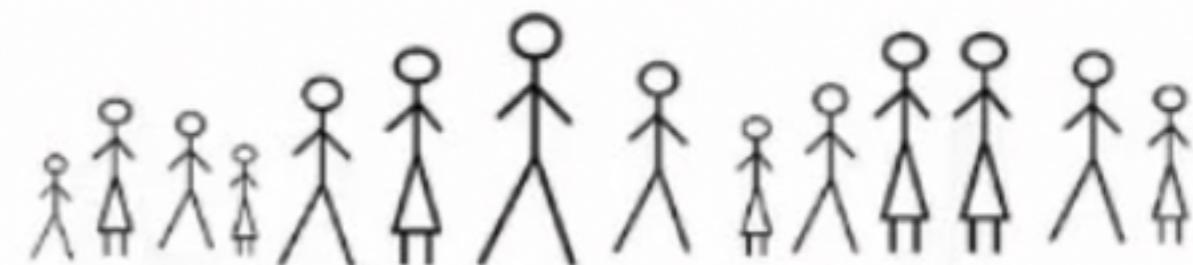
$$Veracidad \in [0.0,1.0]$$

Ejemplo: ¿Qué tan mojada está la ropa?



Ejemplo

Clasifique a las personas de acuerdo con su estatura en dos conjuntos

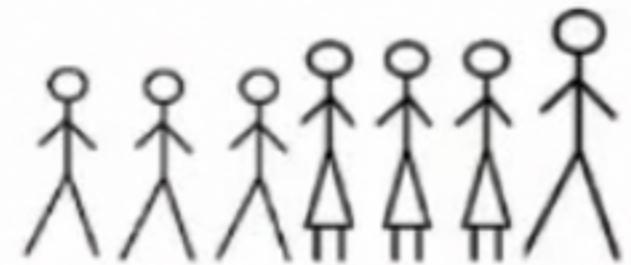


Conjuntos Clásicos (Certeros)

Los Bajos



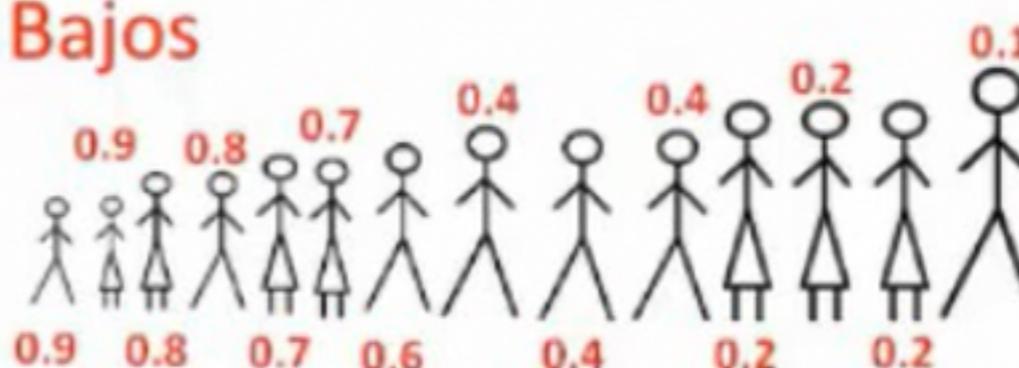
Los Altos



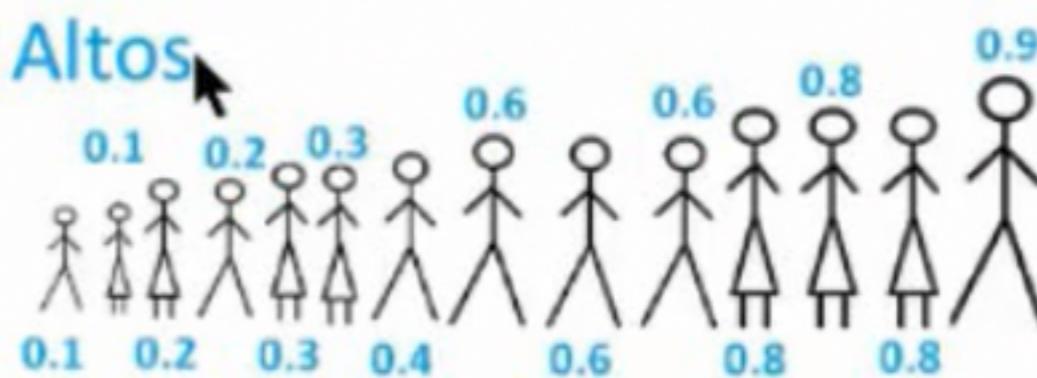
Un individuo **UNICAMENTE**
puede pertenecer a una sola
clase

Conjuntos Difusos

Los Bajos

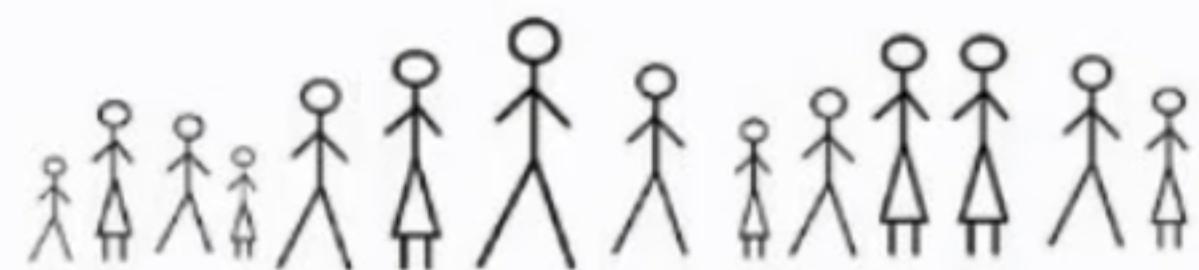


Los Altos



Un individuo puede pertenecer a
más de una clase

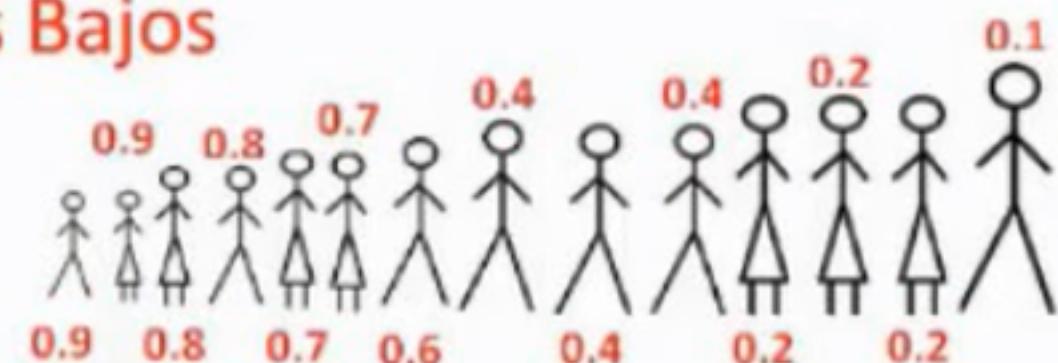




Universo de Discurso -> La totalidad

Por ejemplo, $X \in [1.0, 2.5] \text{ mts}$

Los Bajos

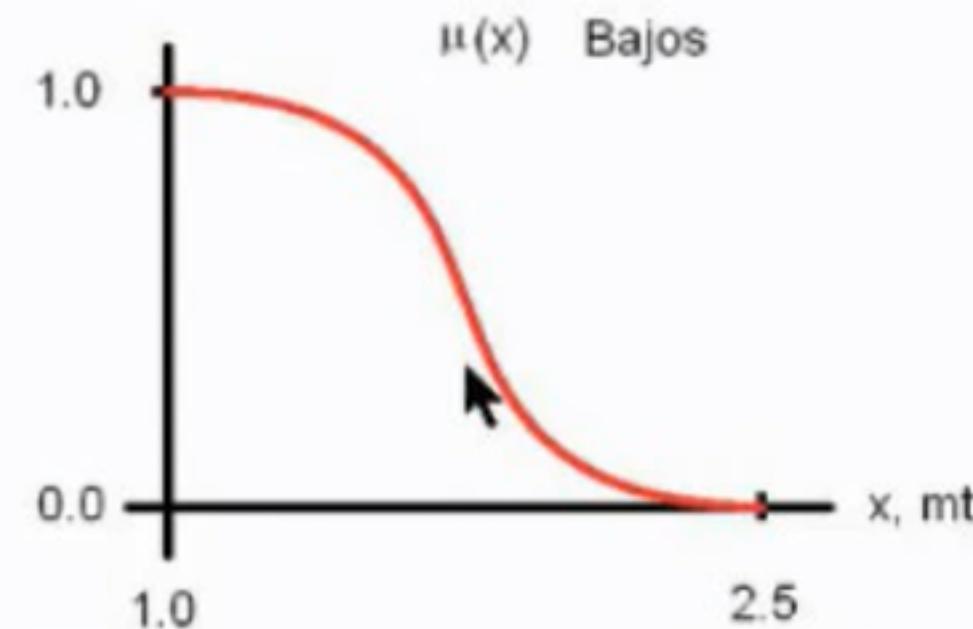
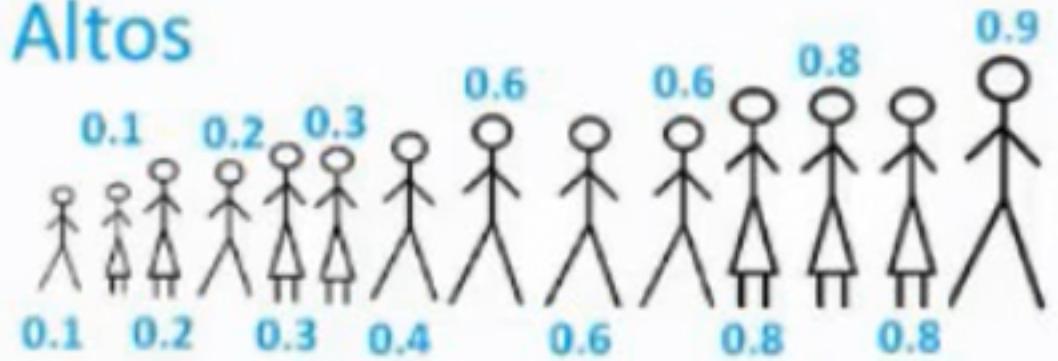


Función de Membresía $\mu(x)$

Dominio = Universo de Discurso, $x \in X$

Imagen -> $\mu \in [0,1]$

Los Altos

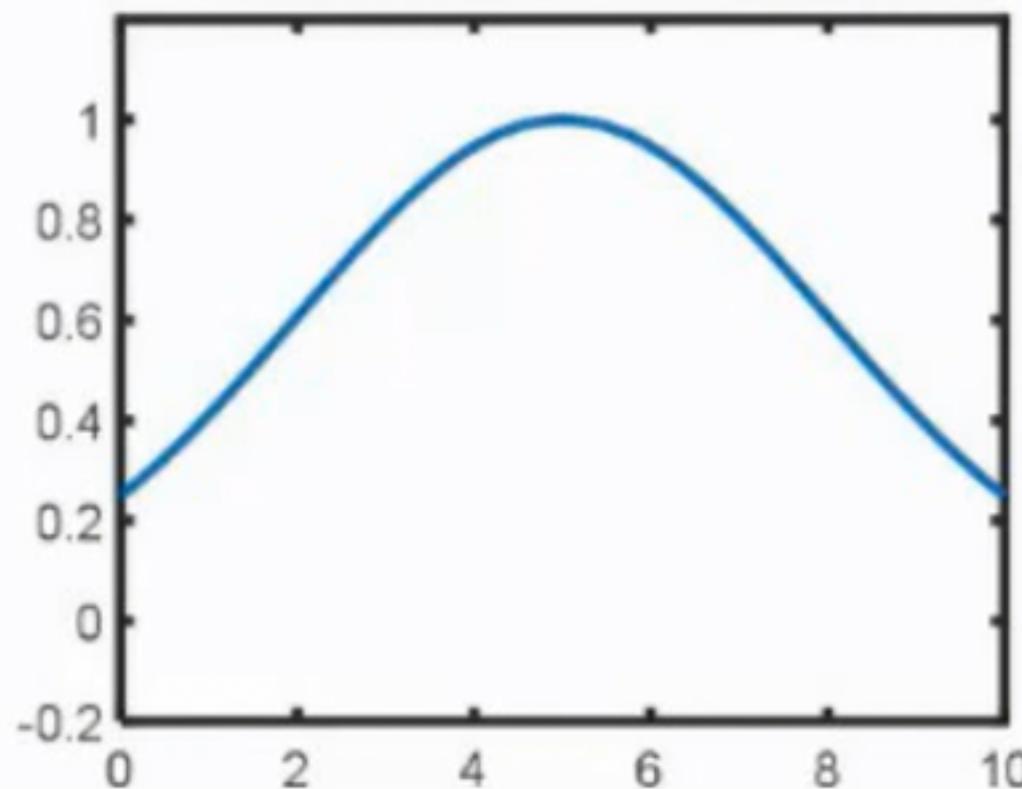


Conjuntos difusos

Continuos

$$A = \left\{ \int \frac{\mu(x)}{x} \right\}$$

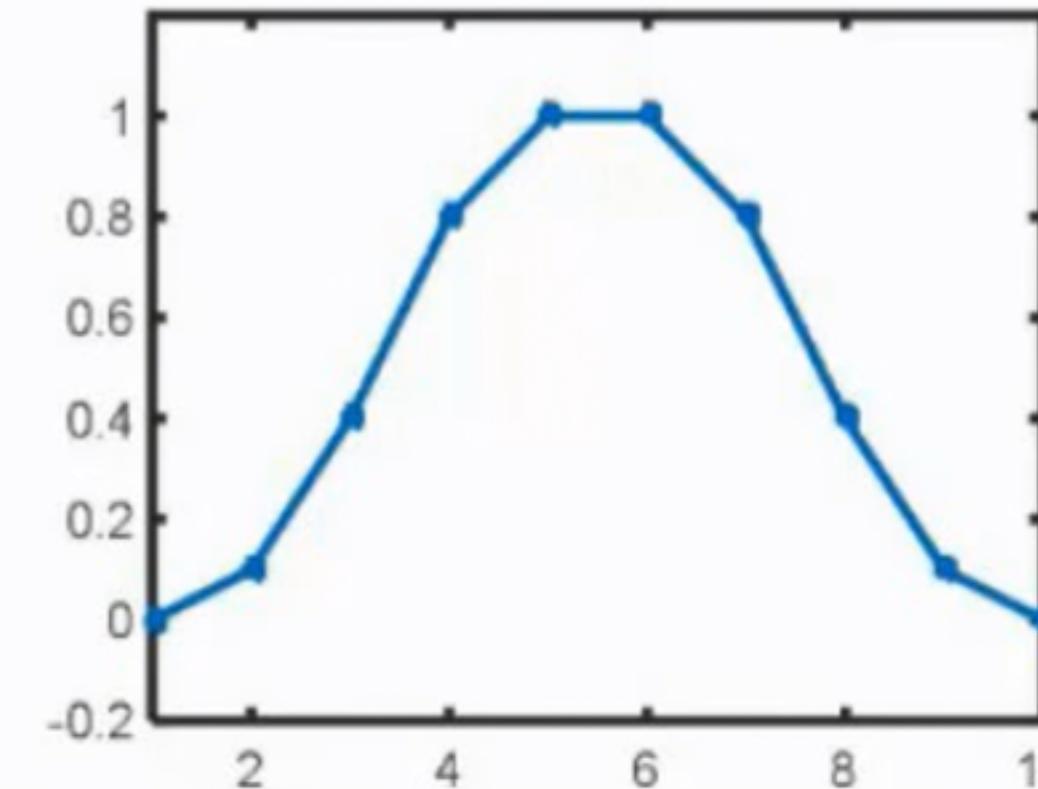
$$A = \left\{ \int_0^{10} \exp \left[-\frac{1}{2} \left(\frac{x-5}{3} \right)^2 \right] \right\}$$



Discretos

$$A = \left\{ \sum \frac{\mu(x)}{x} \right\}$$

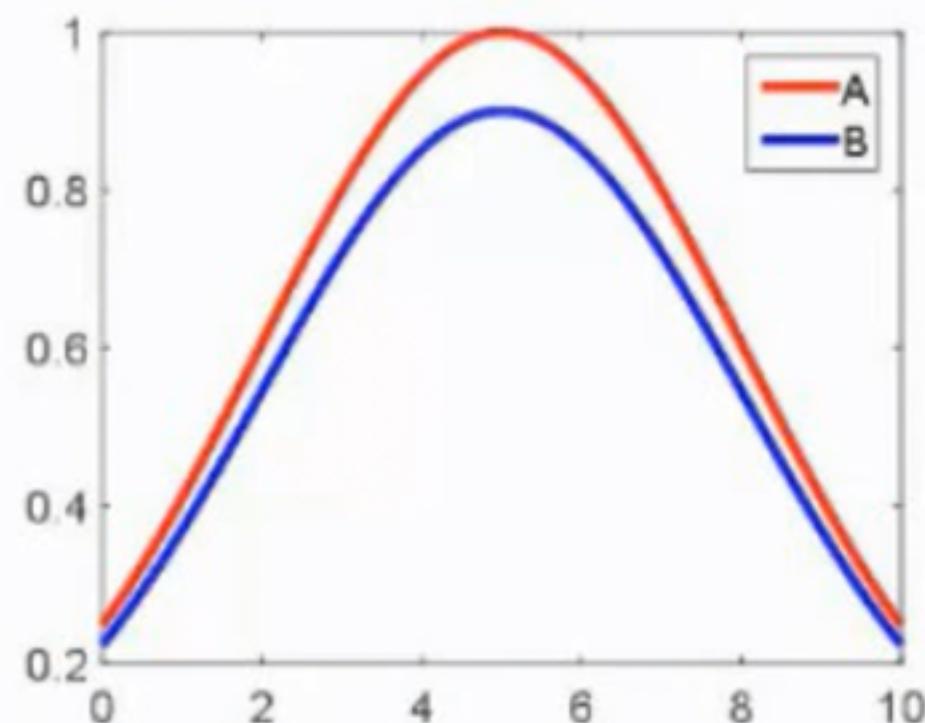
$$A = \left\{ \frac{0}{1} + \frac{0.1}{2} + \frac{0.4}{3} + \frac{0.8}{4} + \frac{1}{5} + \frac{1}{6} + \frac{0.8}{7} + \frac{0.4}{8} + \frac{0.1}{9} + \frac{0}{10} \right\}$$



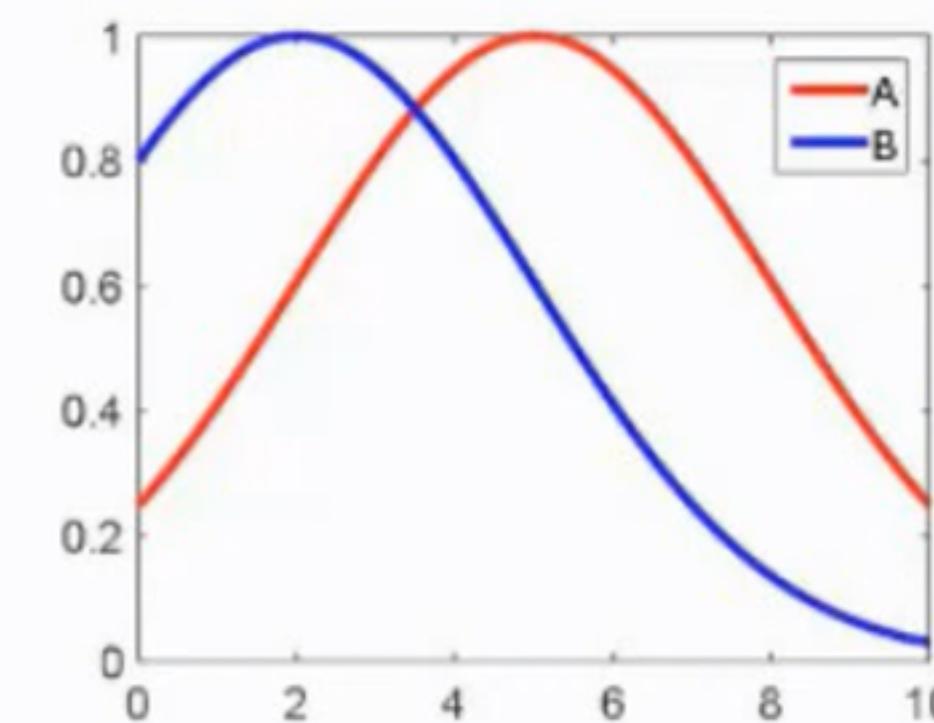
Subconjunto

Conjuntos Difusos

$$B \subseteq A \leftrightarrow \mu_B(x) \leq \mu_A(x) \text{ para } \forall x \in X$$



$B \subseteq A$

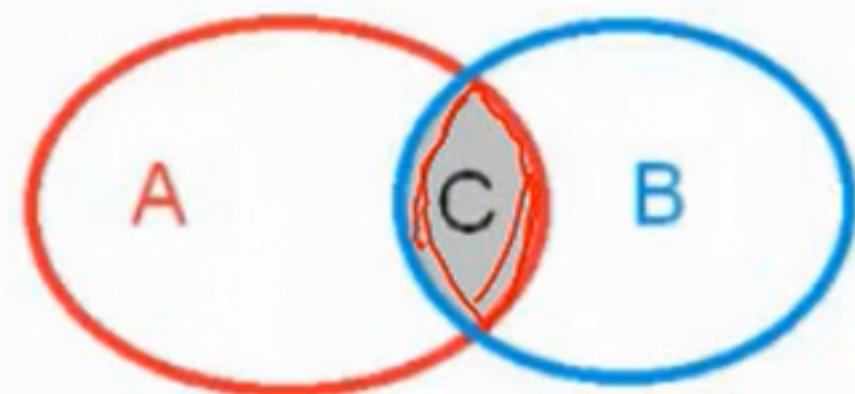


$B \not\subseteq A$

Intersección

Conjuntos Certeros

$$C = A \cap B$$

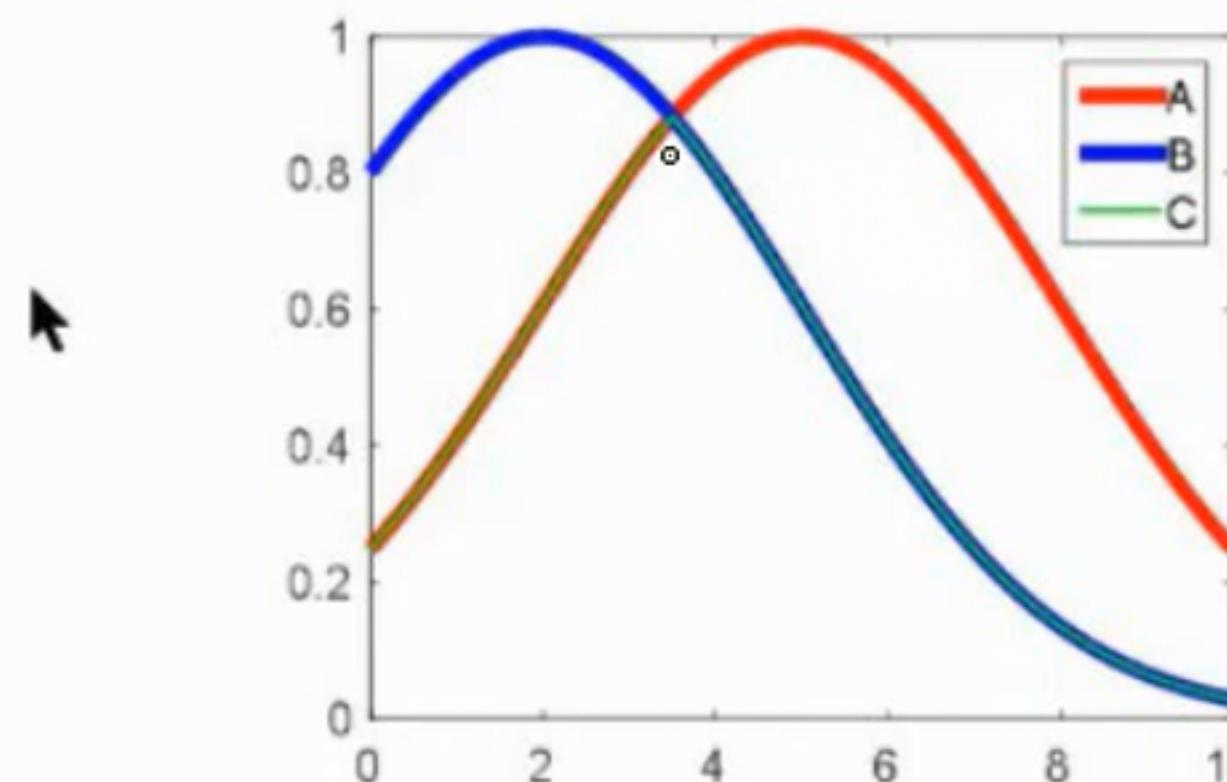


Todos los elementos que
estén en A y B.

Conjuntos Difusos

$$C = A \cap B \text{ si y solo si}$$

$$\mu_C(x) = \min(\mu_A(x), \mu_B(x)) = \mu_A(x) \wedge \mu_B(x) \text{ para } \forall x \in X$$



Propiedades de las operaciones: Conjuntos Clásicos

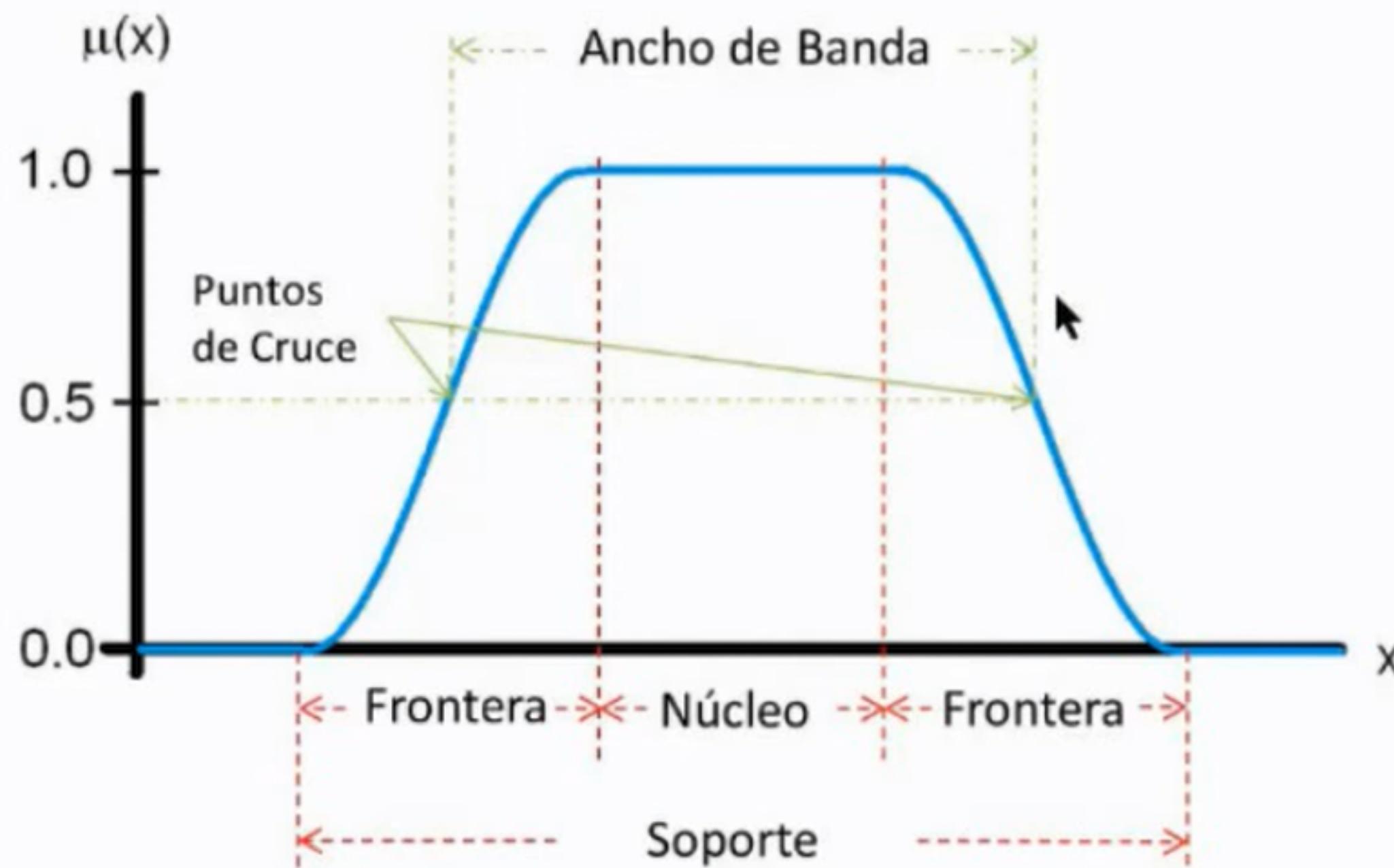
	Unión	Intersección
Commutativa	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Asociativa	$A \cup (B \cup C) = (A \cup B) \cup C$	$A \cap (B \cap C) = (A \cap B) \cap C$
Distributiva	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identidad	$A \cup \emptyset = A$ $A \cup X = X$	$A \cap \emptyset = \emptyset$ $A \cap X = A$
Transitiva	Si $A \subseteq B$ y $B \subseteq C$ entonces $A \subseteq C$	
Idempotencia	$A \cup A = A$	$A \cap A = A$
Complementariedad	$A \cup \bar{A} = X$	$A \cap \bar{A} = \emptyset$
Leyes de DeMorgan	$\overline{A \cup B} = \bar{B} \cap \bar{A}$	$\overline{A \cap B} = \bar{B} \cup \bar{A}$
Involutiva	$\bar{\bar{A}} = A$	

Todas las propiedades para conjuntos clásicos son validas para los Difusos, EXCEPTO la complementariedad

$$A \cup \bar{A} = X$$

$$A \cap \bar{A} \neq \emptyset$$

Características de las funciones de membresía



$$\text{núcleo}(A) = \{x | \mu_A(x) = 1\}$$

$$\text{fronteras}(A) = \{x | 0 < \mu_A(x) < 1\}$$

$$\text{soporte}(A) = \{x | \mu_A(x) > 0\}$$

$$\text{cruce}(A) = \{x | \mu_A(x) = 0.5\}$$

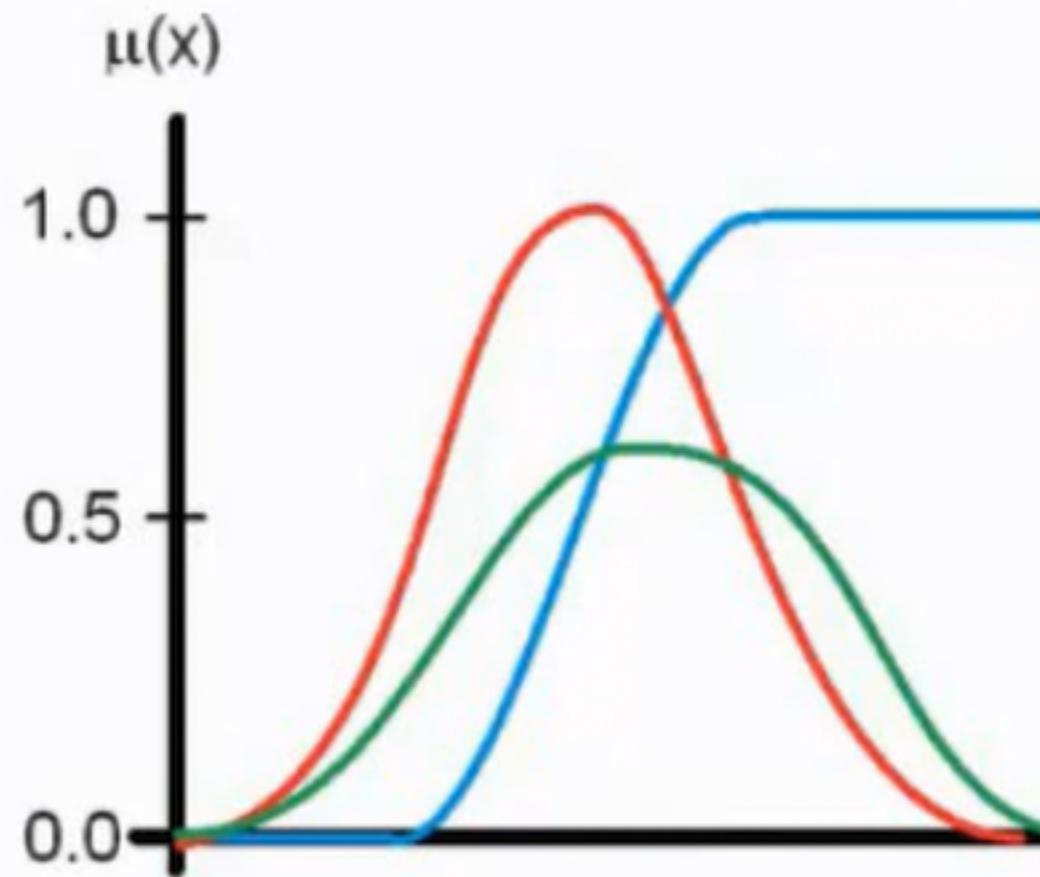
$$\text{ancho}(A) = |x_2 - x_1|$$

Tipos de Funciones de Membresía

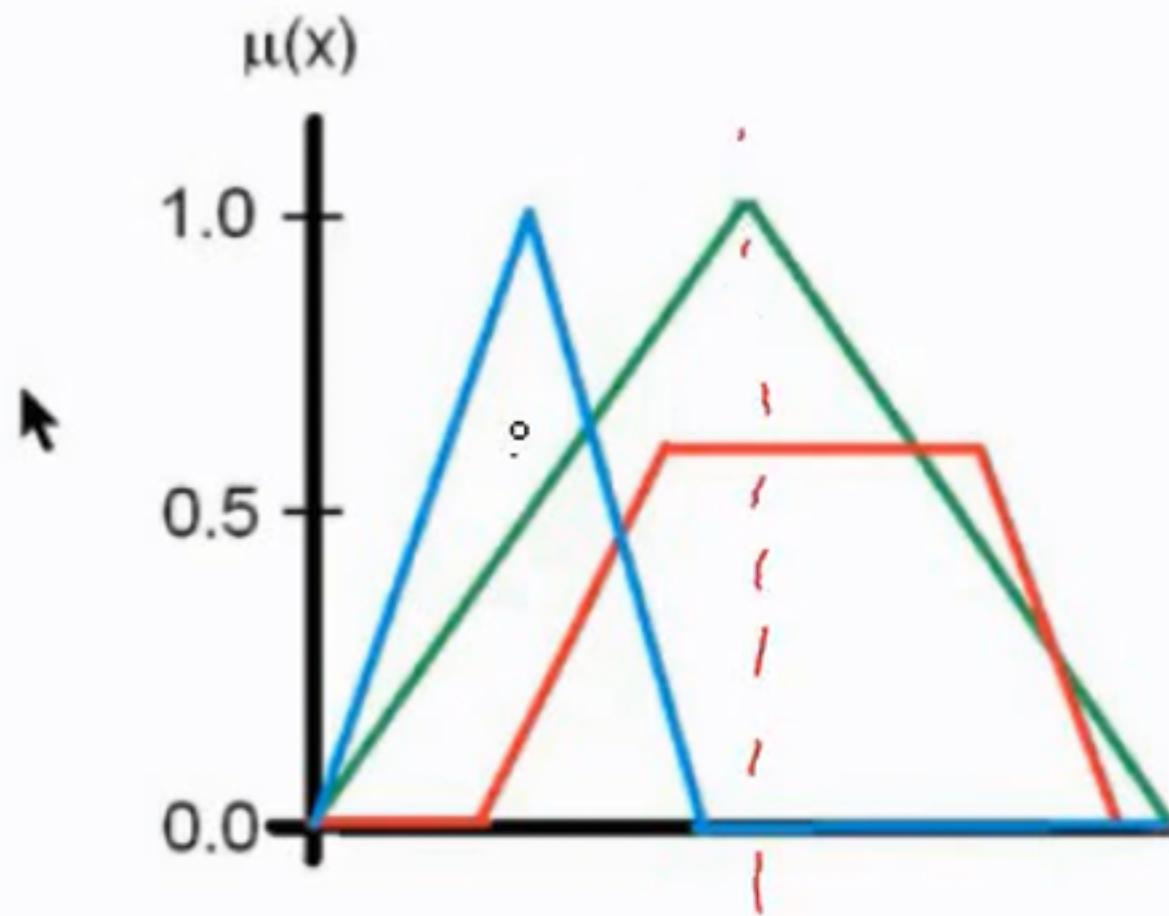
Normal vs Subnormal

(Con núcleo)

(Sin núcleo)

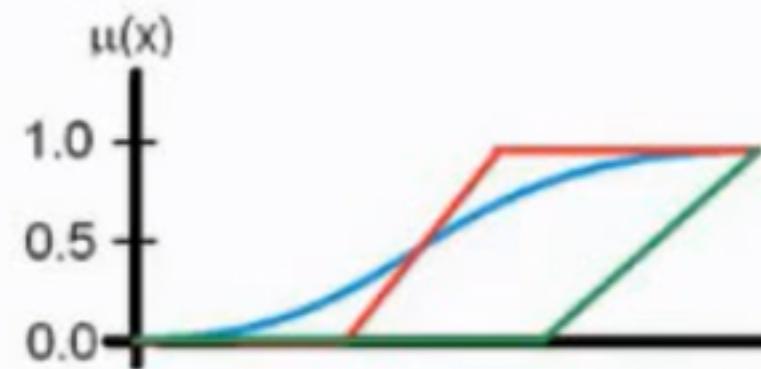


Simétricas vs No simétricas

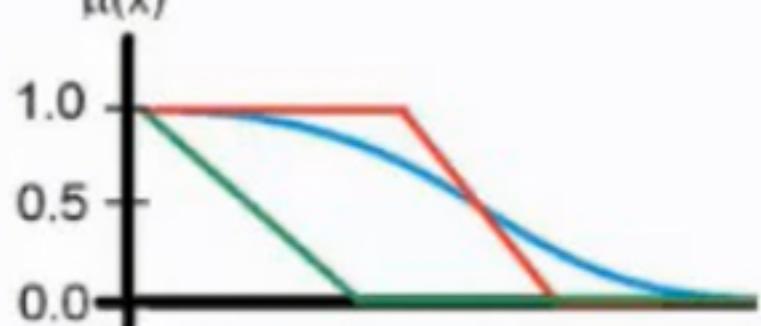


Tipos de Funciones de Membresía

Abiertas vs Cerradas



Abiertos por la derecha

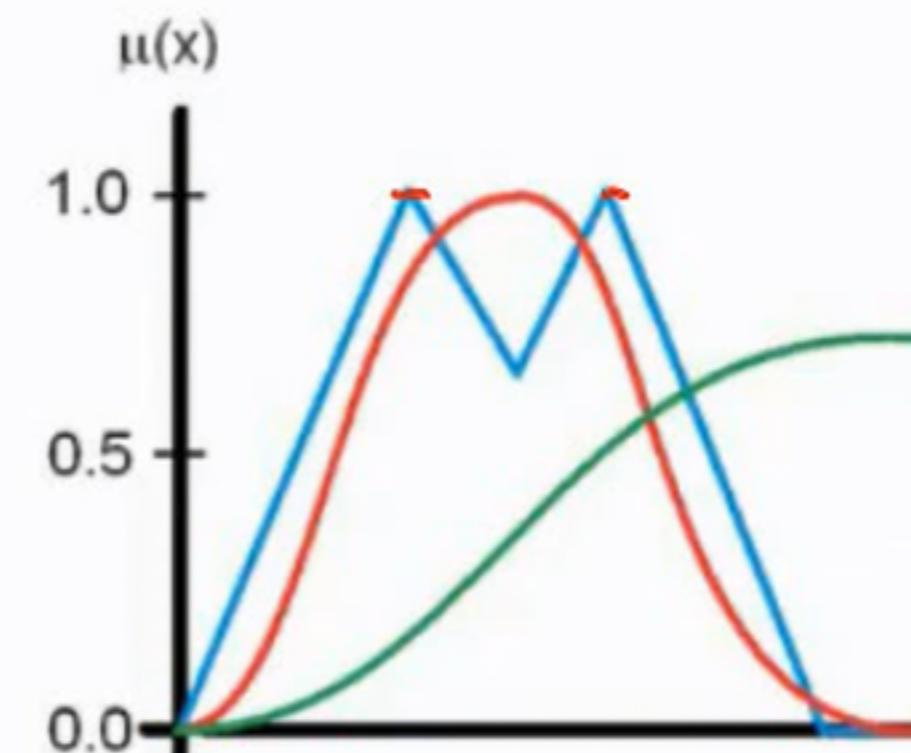


Abiertos por la izquierda



Cerrados

Convexas vs No convexas

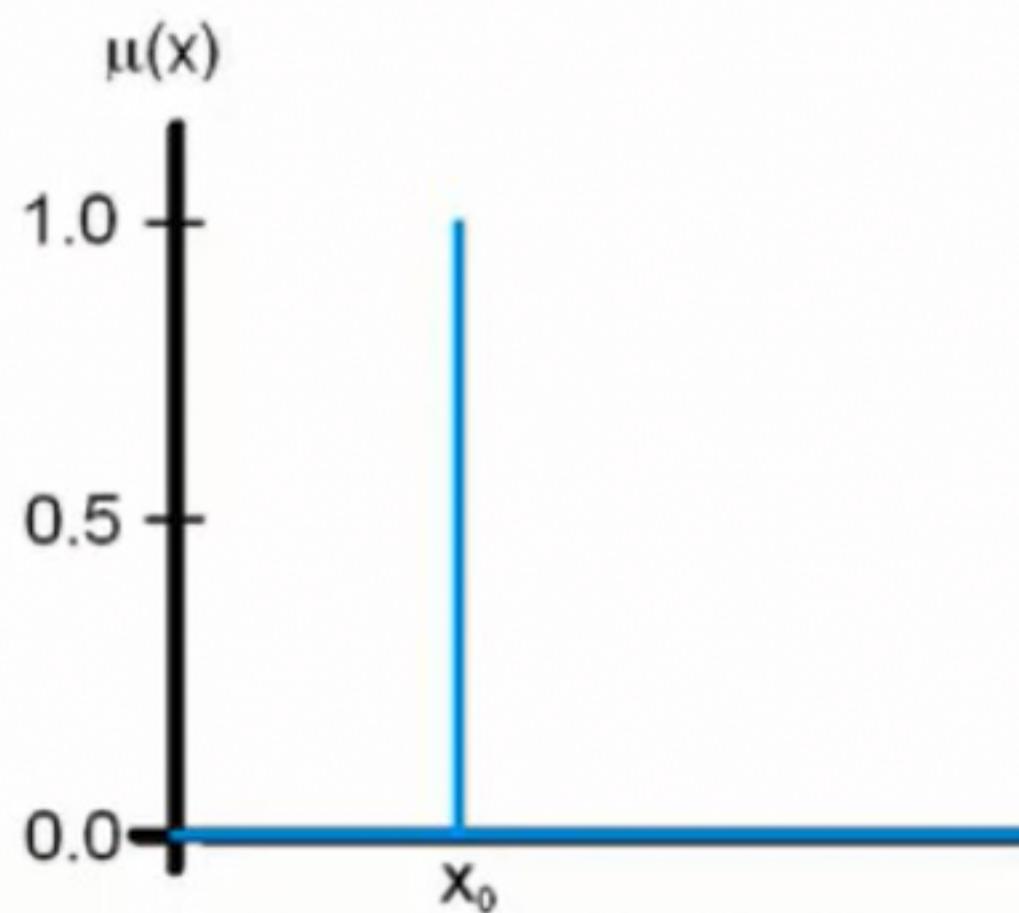


$$\mu_A(x_1\lambda + (1 - \lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$$
$$\forall x_1, x_2 \in X, \forall \lambda \in [0, 1]$$

Formas Especiales de F. de Membresía

Singleton

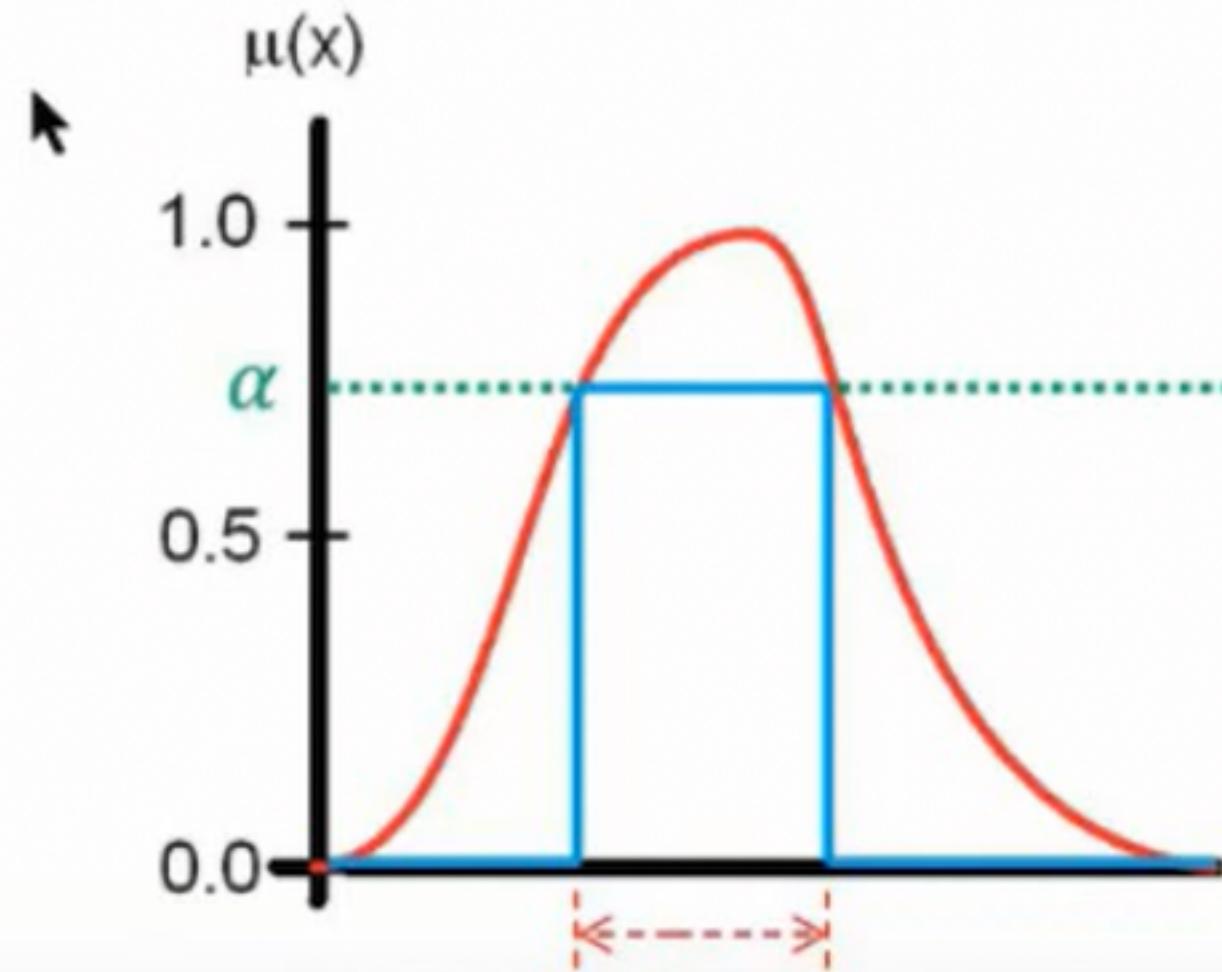
$$\mu_A(x_0) = \begin{cases} 1 & x = x_0 \\ 0 & x \neq x_0 \end{cases}$$



Conjunto Cortado

$$A_\alpha = \{x | \mu_A(x) \geq \alpha\}$$

$$A'_\alpha = \{x | \mu_A(x) > \alpha\} \quad (\text{Fuertemente})$$



Este es el conjunto cortado

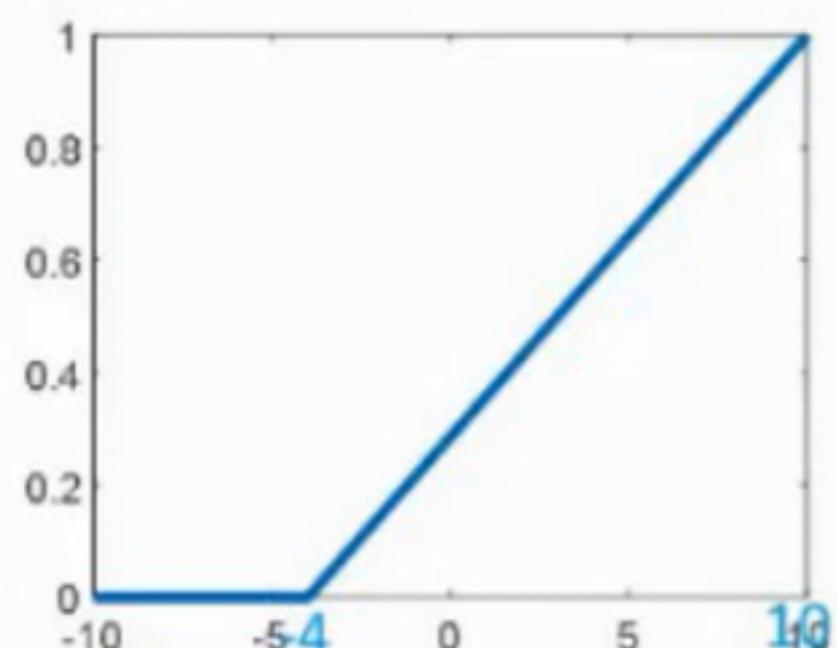
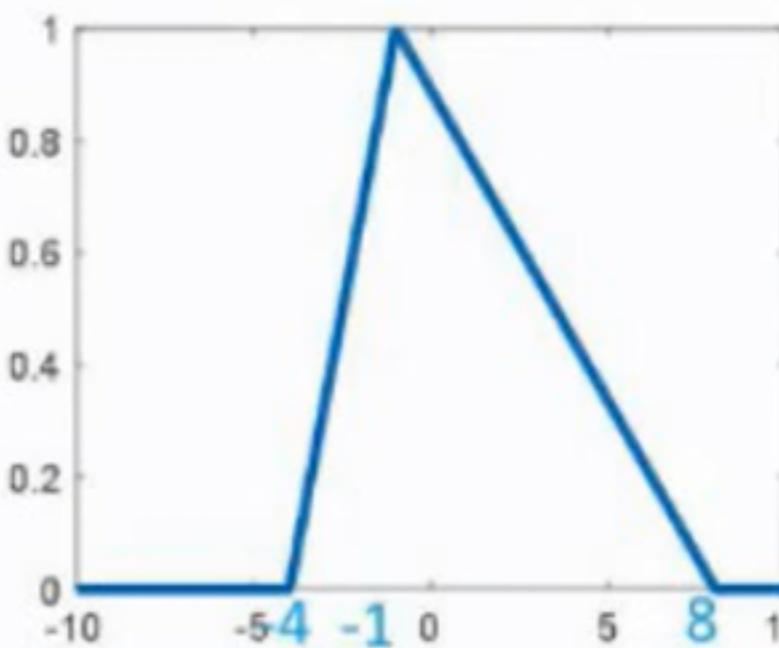
Funciones de membresía típicas

Con derivadas discontinuas

Triangular

$$f(x; a, b, c) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{c-x}{c-b} & b \leq x \leq c \\ 0 & x \geq c \end{cases}$$

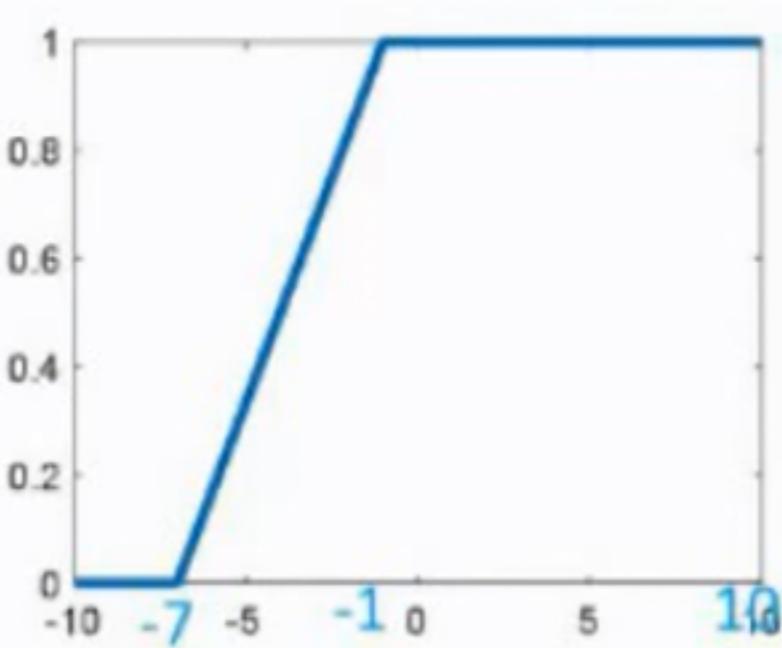
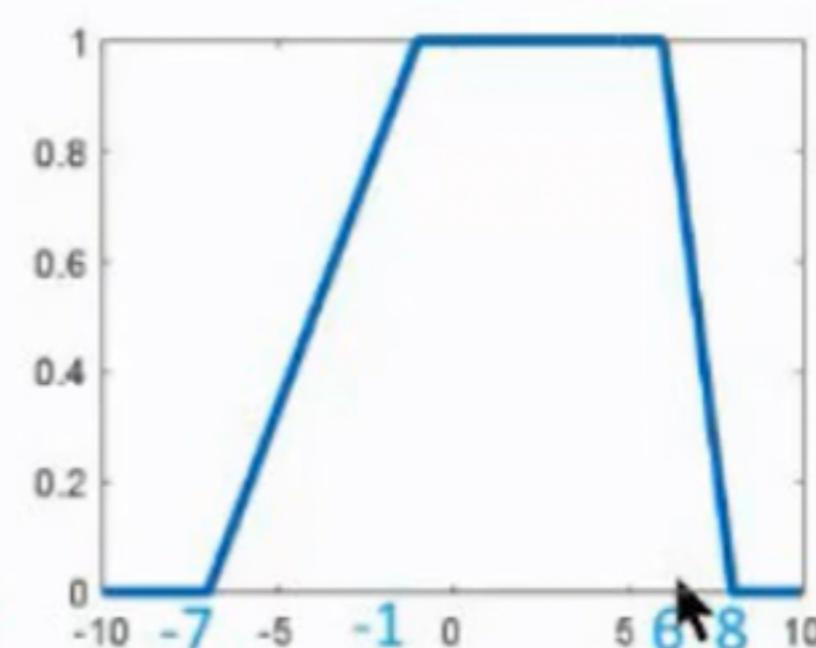
`y = trimf(x,[a b c])`



Trapezoidal

$$f(x; a, b, c, d) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{d-x}{d-c} & c \leq x \leq d \\ 0 & x \geq d \end{cases}$$

`y = trapmf(x,[a b c d])`



Funciones de membresía típicas

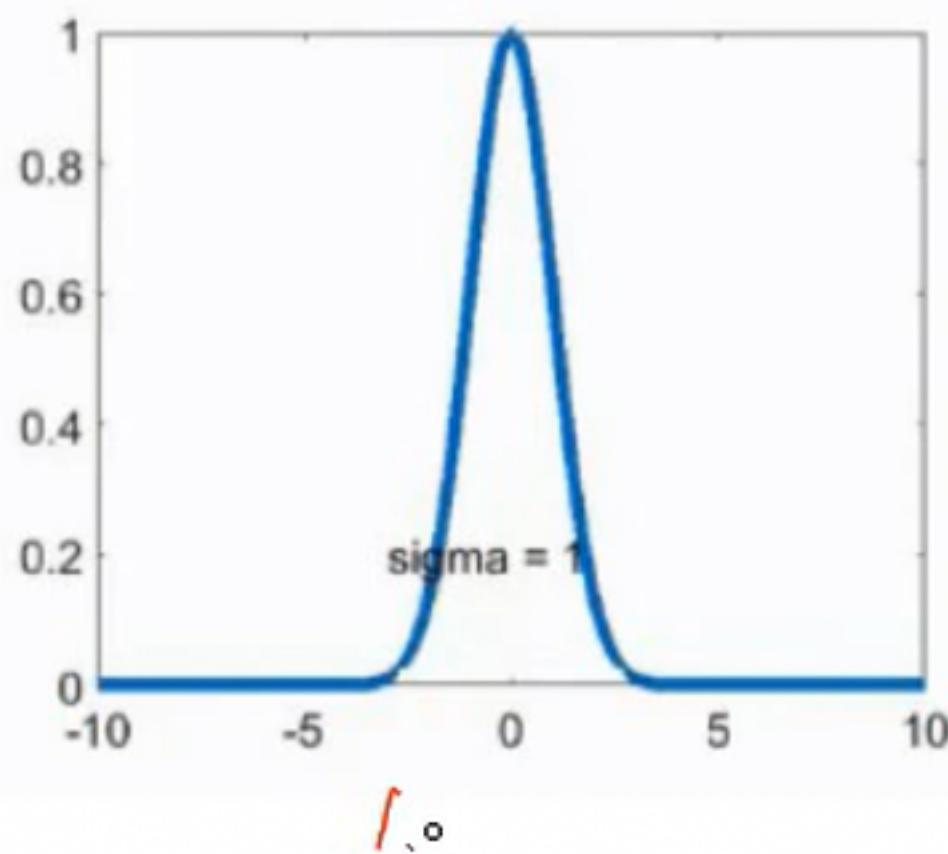
Con derivadas continuas y son cerradas-simétricas

Gaussiana

$$f(x; \sigma, x_0) = e^{-\frac{1}{2} \left(\frac{x-x_0}{\sigma} \right)^2}$$

σ determina el ancho
 x_0 fija el centro

`y = gaussmf(x,[sig x0])`

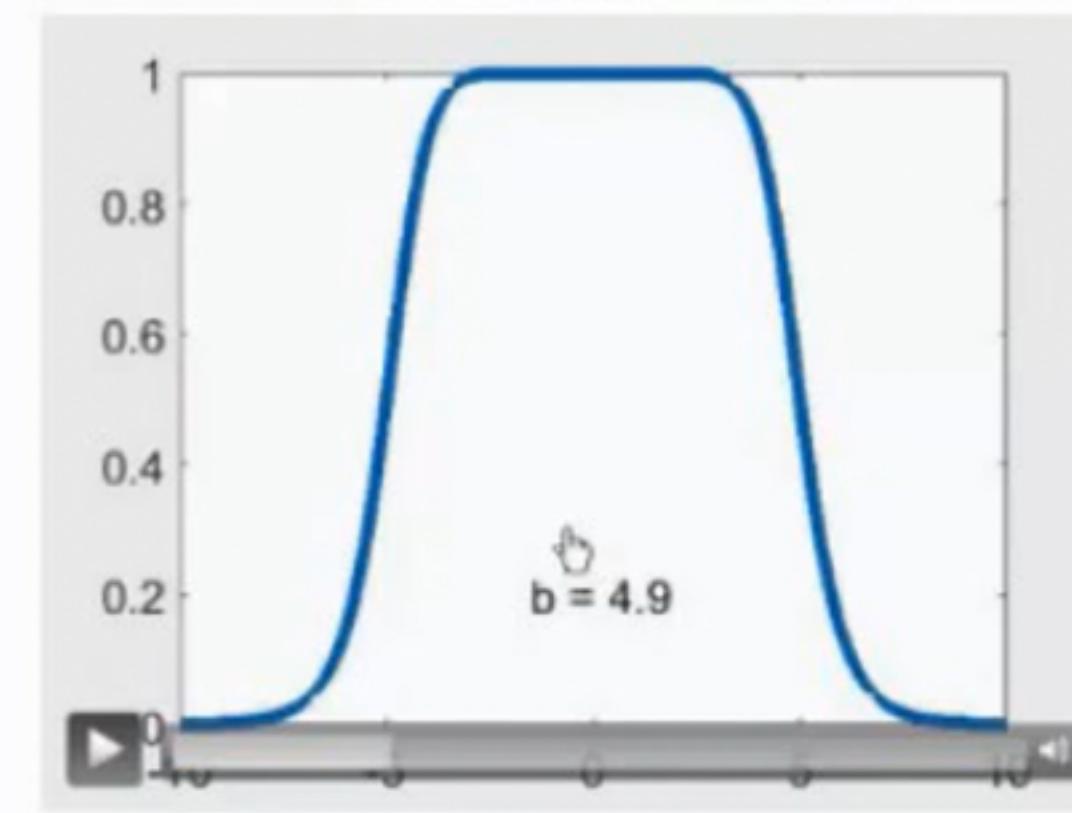


Campana Generalizada

$$f(x; a, b, x_0) = \frac{1}{1 + \left| \frac{x-x_0}{a} \right|^{2b}}$$

a determina el ancho
 b determina la pendiente
 x_0 fija el centro

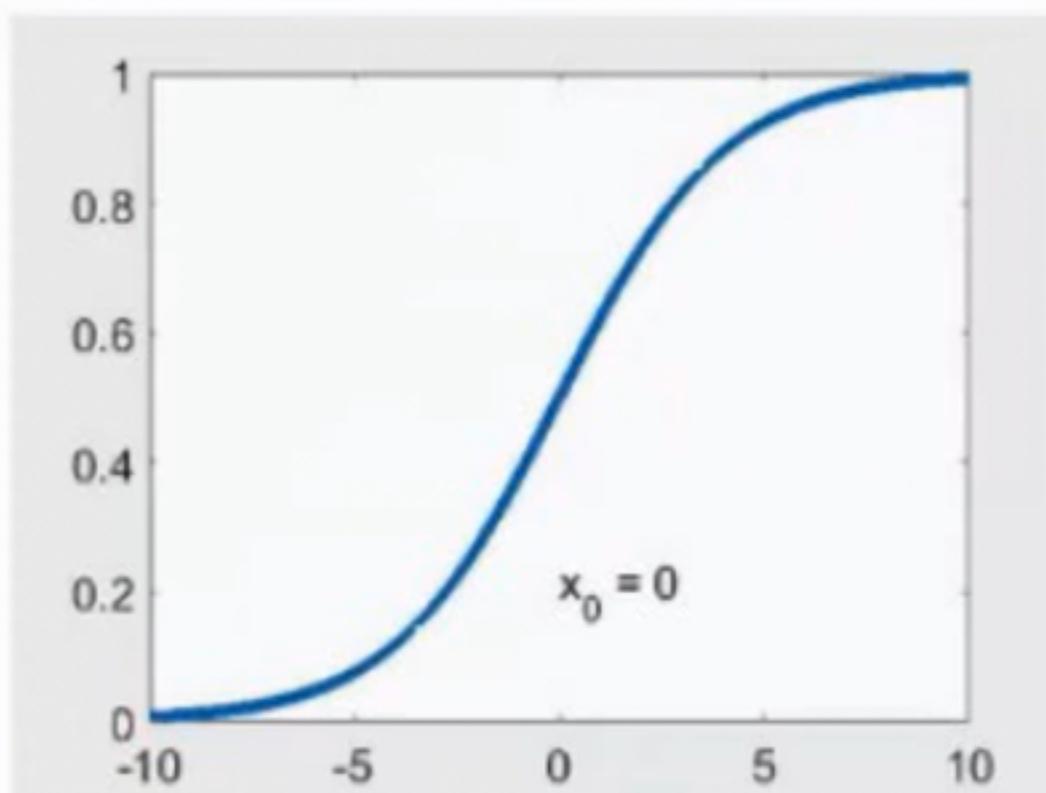
`y = gbellmf(x,[a b x0])`



Sigmoidal

$$f(x; a, x_0) = \frac{1}{1 + e^{-a(x-x_0)}}$$

`y = sigmf(x,[a x0])`



a determina la pendiente
 x_0 fija el punto de cruce

Si $a > 0$, abre a la derecha
Si $a < 0$, abre a la izquierda