Analysis 2 Übungsblatt 5

Aufgabe 1

a) $\int_4^{16} \frac{x+\sqrt{x}}{x-\sqrt{x}}\,dx$ sei $t=\sqrt{x} \Rightarrow x=t^2$ $\frac{dx}{dt}=2t$

$$\int_{\sqrt{4}}^{\sqrt{16}} \frac{x+\sqrt{x}}{x-\sqrt{x}}\,dx = \int_{\sqrt{4}}^{\sqrt{16}} \frac{t^2+t}{t^2-t}\,2t\,dt$$

$$= 2\int_{\sqrt{4}}^{\sqrt{16}} (t+2)\,dt + 4\int_{\sqrt{4}}^{\sqrt{16}} \frac{dt}{t-1} = t^2+4t+4\ln|t-1|+c\,\Big|_{\sqrt{4}}^{\sqrt{16}}$$

$$= x+4\sqrt{x}+4\ln|\sqrt{x}-1|+c\,\Big|_4^{16}$$

$$= 16+16+4\ln 3-(4+8+4\ln 1) = 20+4\ln 3$$

b) $\int_{\pi/4}^{3\pi/4} \frac{dx}{\sin x}$, Sei $t=\tan\frac{x}{2}$ $\Rightarrow \frac{dt}{dx}=2$

$$\int_{\pi/4}^{3\pi/4} \frac{dx}{\sin x} = \int_{\pi/4}^{3\pi/4} \frac{t^2+1}{2t}\,\frac{2\,dt}{t^2+1} = \int_{\pi/4}^{3\pi/4} \frac{dt}{t} = \ln|t|+c\,\Big|_{\pi/4}^{3\pi/4}$$

$$= \ln|\tan\tfrac{x}{2}|+c\,\Big|_{\pi/4}^{3\pi/4} = \ln|2.4141|-\ln|0.4141| = 1.762$$

c) $\int_{5}^{8} \frac{x-\sqrt{x^2-5x+1}}{x^2}\,dx$ Sei $t=\sqrt{x^2-5x+1}-x$

$$= \int_{5t}^{8t} \frac{2t^3+10t^2+2t}{(1-t^2)^2}\,dt = \int_{5t}^{8t} \frac{2t^3+10t^2+2t}{(1+t)^2(1-t)^2}$$

Sei $A, B, C, D \in \mathbb{R}$.

$$\frac{2t^3+10t^2+2t}{(1+t)^2(1-t)^2} = \frac{A}{(1+t)} + \frac{B}{(1+t)^2} + \frac{C}{(1-t)} + \frac{D}{(1-t)^2}$$

$$2t^3+10t^2+2t = A(1+t)(1-t)^2 + B(1-t)^2 + C(1-t)(1+t)^2 + D(1+t)^2$$

$$= A(t^3-t^2-t+1) + B(1-2t+t^2) - C(t^3+t^2-t-1)$$

$$+ D(1+2t+t^2)$$

$$= (A-C)t^3 + (B+D-A-C)t^2 + (C+2D-A-2B)t$$

$$+ (A+B+C+D)$$

$$\Rightarrow \begin{cases} A-C=2 \\ B+D-A-C=10 \\ C+2D-A-2B=2 \\ A+B+C+D=0 \end{cases} \Rightarrow \begin{cases} A=-\frac{3}{2} \\ B=\frac{3}{2} \\ C=-\frac{7}{2} \\ D=\frac{7}{2} \end{cases}$$

$$\int \frac{2t^3+10t^2+2t}{(1-t^2)^2}\,dt = \int \frac{-\frac{3}{2}}{1+t} + \frac{\frac{3}{2}}{(1+t)^2} + \frac{-\frac{7}{2}}{(1-t)} + \frac{\frac{7}{2}}{(1-t)^2}$$

$$= -\frac{3}{2}\ln|1+t| + \frac{\frac{3}{2}}{(1+t)} + -\frac{7}{2}\ln|1-t| + \frac{\frac{7}{2}}{(1-t)} + C \Big|_{5t}^{8t}$$

$$= -\frac{3}{2}\ln|1+\sqrt{x^2-3x+1}-x| + \frac{\frac{3}{2}}{(1+\sqrt{x^2-5x+1}-x)} - \frac{7}{2}\ln|1-\sqrt{x^2-5x+1}+x| + \frac{\frac{7}{2}}{(1-\sqrt{x^2-5x+1}+x)}\Big|_5$$

$$= -\frac{3}{2}\ln|1+5-8| + \frac{\frac{3}{2}}{(1+5-8)} - \frac{7}{2}\ln|1-5+8| + \frac{\frac{7}{2}}{(1-5+8)}$$

$$-\left(-\frac{3}{2}\ln|1+1-5| + \frac{\frac{3}{2}}{(1+1-5)} - \frac{7}{2}\ln|1-1+5| + \frac{\frac{7}{2}}{(1-1+5)}\right)$$

$$= -\frac{3}{2}\ln 2 - \frac{3}{4} - \frac{7}{2}\ln 4 + \frac{7}{8} - \left(-\frac{3}{2}\ln 5 - \frac{3}{10} - \frac{7}{2}\ln 5 + \frac{7}{10}\right)$$

$$= \frac{3}{2}(\ln 5 - \ln 2) + \frac{7}{2}(\ln 5 - \ln 4) + \frac{1}{8} - \frac{4}{10}$$

$$= \frac{3}{2}(\ln 5 - \ln 2) + \frac{7}{2}(\ln 5 - \ln 4) - \frac{9}{40}$$

Aufgabe 2.

$\frac{1}{n!}\int_a^x (x-t)^n f^{(n+1)}(t)\, d(t)$ mit partielle Intergration

$$= \frac{1}{n!}\Big[(x-t)^n f^{(n)}(t) - n(x-t)^{n-1} f^{(n-1)}(t) - n\cdot(n-1)\,n^{(n-2)} f^{(n-2)}(t)$$

$$- n(n-1)(n-2)(x-t)^{n-3} f^{(n-3)}(t) - \cdots\cdots - n!\,f'(t) - f(t)\Big]\Big|_a^x$$

$$= 0 - \frac{1}{n!}\Big[(x-a)^n f^{n}(a) - n(x-a)^{n-1} f^{(n-1)}(a) - n(n-1)\,n^{(n-2)} f^{(n-2)}(a)$$

$$- n(n-1)(n-2)(x-a)^{n-3} f^{(n-3)}(a) - \cdots - n!\,f'(a) - f(a)$$