2022 Endterm - Sommersemester

Computer Vision II: Multiple view geometry (Technische Universität München)

I9
Informatik
Technical University of Munich

Endterm

Computer Vision II: Multiple View Geometry

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13:45 – 15:45

Working instructions

- This exam consists of **16 pages** with a total of **6 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 120 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - a few **blank sheets** for working out the solutions before answering
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English ↔ native language
- Do not write with red or green colors nor use pencils.
- The online version of this exam uses the TUMexam platform which offers a student manual on their webpage[1]. Please also consult the information provided in Moodle.
- There is an additional 10 min submission (upload) buffer when the exam is submitted online.
- The boxes on the sides of the subproblems (those with numbers) are used for correction and ticking them is prohibited. A ticked box that is not part of a multiple choice question may result in zero points for the problem.
- In the multiple choice questions there is always **exactly one correct answer** (i.e., single choice). A correct answer (comprising of exactly one tick at the correct position) will give the indicated credits, whereas a wrong answer will result in 0 credits.

Mark correct answers with a cross	☒
To undo a cross, completely fill out the answer option	■
To re-mark an option, use a human-readable marking	×■

- **After you have uploaded this submission, check that all your annotations are still visible after you download it again.**
- **By submitting this online graded exercise to TUMexam you confirm that you did it completely alone without the help of anyone else.**

[1] https://www.tumexam.de/links.html

Problem 1 Linear Algebra (24 credits)

det $\neq 0$

a) Which of the following sets of vectors is linearly independent? **(6 credit(s))**

$a_1 \cdot v_1 + a_2 \cdot v_2 + a_3 \cdot v_3 = 0$

只有零解 $a_1 = a_2 = a_3 = 0$

☐ $B = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 10 \\ 1 \end{pmatrix} \right\}$ dependent

☒ $B = \left\{ \begin{pmatrix} 4 \\ -5 \\ 5 \end{pmatrix} \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix} \right\}$ $-8 - 20 - 25 - 10 - 40 + 30 = -123 \neq 0$

☐ $B = \left\{ \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} -1 \\ -7 \\ -2 \end{pmatrix} \right\}$ $-4 - 2 - 21 + 7 + 14 + 12 = 0$

☐ $B = \left\{ \begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix} \begin{pmatrix} 8 \\ 12 \\ 16 \end{pmatrix} \begin{pmatrix} -6 \\ -2 \\ 2 \end{pmatrix} \right\}$ $24 - 480 - 144 + 648 + 32 - 80 = 0$

b) Which of the following vectors in addition to the set of vectors $B = \left\{ \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix} \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \right\}$ spans $\mathbb{R}^3$?

(3 credit(s))

☒ $\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

☐ The given set of vectors spans $\mathbb{R}^3$ $-4 + 1 + 6 + 4 - 1 - 6 = 0$

☐ A set of 4 vectors in $\mathbb{R}^3$ cannot span $\mathbb{R}^3$

☐ $\begin{pmatrix} 15 \\ -25 \\ 15 \end{pmatrix}$

c) Which of the following statements is incorrect? **(4 credit(s))**

☒ In general, for a matrix $B \in GL(n+1)$, $\det(B) \neq 1$

☒ In general, for a matrix $B \in SE(n)$, $\det(B) \neq 1$

☐ $SO(3) \subset O(3)$

☐ $A(n) \subset \mathbb{R}^{n+1 \times n+1}$

☐ $SE(3) \subset A(3)$

无零特征值

d) Which of the following is **false** given a full rank symmetric matrix $B \in \mathbb{R}^{n \times n}$? **(4 credit(s))**

☒ The eigenvalues of B are always positive 可能有负值

☐ B is positive semidefinite if the minimum eigenvalue is 0

☐ If B is positive definite the minimum eigenvalue is greater than 0

$\sigma_i = \sqrt{\lambda_i^2}$ ☐ The singular values of B are always positive 对称实矩阵的奇异值就是其特征值的绝对值

☐ When B is positive definite, if the sigular vector decomposition of B is $\text{svd}(B) = U\Sigma V'$, then $U = V$

$\Lambda = \Sigma$

对称矩阵 SVD = 特征值分解

$B = Q\Lambda Q = U\Sigma V'$

$U = V$

e) Let $A \in \mathbb{R}^{n \times n}$ be a symmetric matrix with eigen values ordered as $\lambda_1 \geq \lambda_2 \geq \cdots > \lambda_{n-1} \geq \lambda_n$ with corresponding eigen vectors $v_1, v_2, \ldots v_{n-1}, v_n$. Write down the solution for the optimization problem

$$\min_{\|x\|=1} x^\top A x\,, \tag{1.1}$$

x 可以是正或负 v 但 $\min x^TAx$ 的值是 unique 的，符号相消

where $x \in \mathbb{R}^n$. Be as specific as possible. Further, when is the solution to this problem unique and what is the solution in this case? (writing the final equations is sufficient) **(4 credit(s))**

$A = V \Lambda V^T$ symmetric $\quad V^T V = I$ orthogonal

$x := Vy \qquad y = V^T x$

$\|y\|^2 = \|V^T x\|^2 = (V^T x)^T V^T x = x^T x = \|x\|^2 = 1 \Rightarrow \|y\| = 1$

$\min x^T A x = \min (Vy)^T V \Lambda V^T V y = y^T V^T V \Lambda V^T V y = \min y^T \Lambda y = \min \sum_{i=1}^{n} \lambda_i y_i^2 = \lambda_n$

λ_n is the smallest eigenvalue $\qquad x = \pm v_n$ the corresponding eigenvector

if λ_n is simple (multiplicity one), the solution is unique

f) A vector $v \in \mathbb{R}^3$ is translated by a vector $T_1 \in \mathbb{R}^3$, then rotated by a matrix $R \in SO(3)$, and finally translated by a vector $T_2 \in \mathbb{R}^3$. Let the transformed vector be $v' \in \mathbb{R}^3$. Write down the equation in homogeneous coordinate system to obtain v' from v. (writing the final equation is sufficient, no need to show intermediate steps) **(3 credit(s))**

$$\begin{bmatrix} v' \\ 1 \end{bmatrix} = \begin{bmatrix} R & T_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & T_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ 1 \end{bmatrix}$$

Problem 2 Image Formation (20 credits)

For this problem by a 'canonically oriented camera', we mean that the camera origin is at $[0,0,0]$, with respect to the world coordinate system, the camera's rotation is given by $R = \mathbb{I}_{3\times3}$, and the camera's optical axis points along positive direction of z-axis

For the next two subproblems, consider the following setting:

没有说是 normalized coordinates

A 3D point is given in the world coordinate system as $P = [\frac{1}{2}, \frac{3}{2}, 1]^\top$. The point is observed by a camera which is canonically oriented. The intrinsic parameter matrix K of the camera is given by

$$K = \begin{bmatrix} 200 & 0 & 500 \\ 0 & 200 & 500 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \\ 1 \end{pmatrix} = \begin{bmatrix} 100+500 \\ 300+500 \\ 1 \end{bmatrix}$$

a) Where is the point P observed on the image captured by the given camera? **(3 credit(s))**

- [ ] $[u, v] = [250, 400]$
- [ ] $[u, v] = [300, 400]$
- [x] $[u, v] = [600, 800]$
- [ ] The given point is not observed in the image
- [ ] $[u, v] = [500, 500]$

b) Where is the depth of the point P as observed by the given camera? **(2 credit(s))**

- [ ] 0
- [x] 1
- [ ] 2
- [ ] -1
- [ ] $\frac{1}{2}$

A 3D point is given in the world coordinate system as $P = [1, 2, -3]^\top$. The point is observed by a camera which is canonically oriented. The intrinsic parameter matrix K is given by

$$K = \begin{bmatrix} 125 & 0 & 250 \\ 0 & 125 & 250 \\ 0 & 0 & 1 \end{bmatrix}.$$

c) Where is the point P observed on the image captured by this camera? **(3 credit(s))**

- [ ] $[u, v] = [625, 500]$
- [x] The given point is not observed in the image
- [ ] $[u, v] = [125, 250]$
- [ ] $[u, v] = [250, 250]$
- [ ] $[u, v] = [250, 125]$

— The problem continues on the next page —

For the next four subproblems, consider the following setting.

The 3D points $X_1 = [1, 2, 5]^\top$, and $X_2 = [-1, 1, 5]^\top$ in the world coordinate system, appear in the image at pixel coordinates $x_1 = [315, 360]^\top$ and $x_2 = [185, 280]^\top$, respectively. The image is captured by a camera which is canonically oriented. Assuming a perspective pinhole camera with no skew, the intrinsic matrix is given by

$$K = \begin{bmatrix} f_x & 0 & o_x \\ 0 & f_y & o_y \\ 0 & 0 & 1 \end{bmatrix}.$$

$\frac{f_x \cdot X}{Z} + O_x = 315$

$\frac{f_x \cdot 1}{5} + O_x = 315$ $\quad f_x + 5O_x = 1575$

$-f_x + 5O_x = 925$

$\frac{-f_x}{5} + O_x = 185$

$10 O_x = 2500$

$O_x = 250$

$-f_x + 1250 = 925$

$f_x = 325$

What are the values of the intrinsic camera parameters?

d) What is the value of f_x? **(3 credit(s))**

- [x] $f_x = 325$
- [ ] $f_x = 250$
- [ ] $f_x = 125$
- [ ] $f_x = 225$
- [ ] $f_x = 180$

$\frac{2f_y}{5} + O_y = 360$

$\frac{f_y}{5} + O_y = 280$

$2f_y + 5 \cdot O_y = 1800$

$f_y + 5 \cdot O_y = 1400$

$f_y = 400$

$O_y = 200$

e) What is the value of f_y? **(3 credit(s))**

- [x] $f_y = 400$
- [ ] $f_y = 160$
- [ ] $f_y = 100$
- [ ] $f_y = 200$
- [ ] $f_y = 500$

f) What is the value of o_x? **(3 credit(s))**

- [ ] $o_x = 300$
- [x] $o_x = 250$
- [ ] $o_x = 200$
- [ ] $o_x = 400$
- [ ] $o_x = 150$

g) What is the value of o_y? **(3 credit(s))**

- [ ] $o_y = 400$
- [ ] $o_y = 150$
- [ ] $o_y = 250$
- [x] $o_y = 200$
- [ ] $o_y = 300$

Problem 3 Perspective projection (8 credits)

a) Which of the following statements is wrong? **(3 credit(s))**

- ☐ The coefficients in the intrinsic parameters matrix model linear distortions in the transformation to pixel coordinates
- ☐ Image, preimage and coimage uniquely determine oneanother
- ☐ The coimage of a line is the subspace in R^3 that is the orthogonal complement of its preimage
- ☐ A preimage of a line in the image plane is the largest set of 3D points that give an image equal to the given line
- ☒ There exists a unique representation for the two-dimensional projective space P^2

$\frac{\|x_d\|}{s\log(1+\frac{r}{k})}$ $\|x\| = \|x\|$

b) Consider the radial distortion model

$\frac{x_d}{g(r)}\|x\| = x$

$\frac{r_d}{s\log(1+\frac{r}{k})} = 1$

$$x_d = g(r)\frac{x}{\|x\|}, \tag{3.1}$$

$\frac{x_d}{s\log(1+\frac{r}{k})}\|x\| = x$

$r_d = s\log(1+\frac{r}{k})$

which maps coordinates $x \in \mathbb{R}^2$ in the normalized image plane to distorted coordinates $x_d \in \mathbb{R}^2$. The factor $g(r)$ depends on the radius $r = \|x\|$ and is given as

$\frac{r_d}{s} = \log(1+\frac{r}{k})$

$$g(r) = s\,\log(1+\frac{r}{k}), \tag{3.2}$$

where, $s \in \mathbb{R}^+$, and $k \in \mathbb{R}^+$ are model parameters. Choose the function f such that the inverse distortion can be written as

$\exp(\frac{r_d}{s}) = 1+\frac{r}{k}$

$$x = f(r_d)\frac{x_d}{\|x_d\|}, \tag{3.3}$$

where, $r_d = \|x_d\|$. **(5 credit(s))**

$k(\exp(\frac{r_d}{s})-1) = r$

- ☐ $f(r_d) = \frac{\exp(\frac{r_d}{s})-1}{k}$
- ☐ There is no such function f
- ☐ $f(r_d) = \frac{1-\exp(\frac{r_d}{s})}{k}$
- ☐ $f(r_d) = \frac{\exp(\frac{r_d}{s})-1}{k}$
- ☒ $f(r_d) = k(\exp(\frac{r_d}{s}) - 1)$

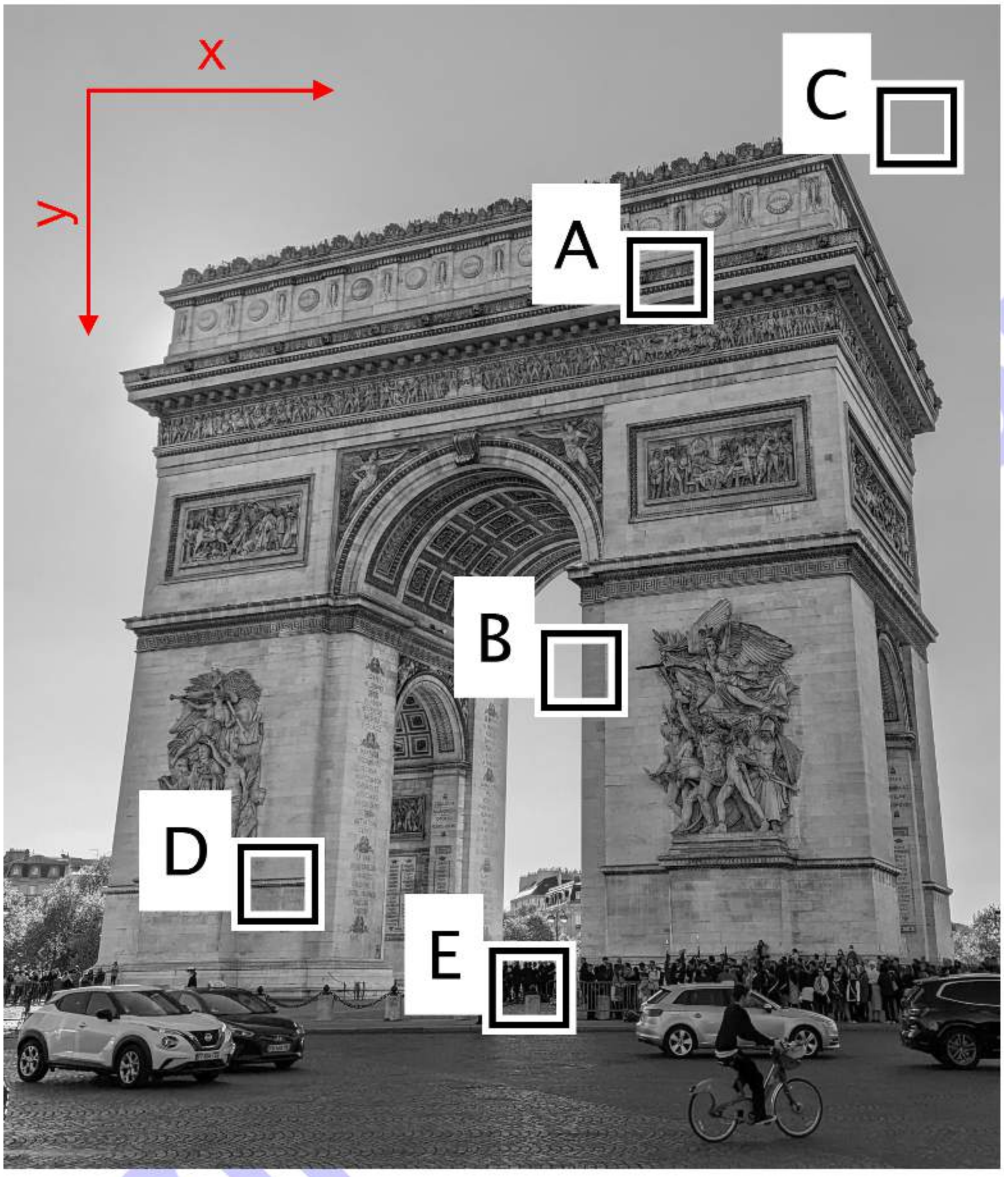
$x = \frac{x_d}{s\log(\exp(\frac{r_d}{s}))}\cdot\|x\|$

$x = \frac{x_d}{r_d}\cdot\|x\| = \frac{x_d}{\|x_d\|}\cdot r$

$= k(\exp(\frac{r_d}{s})-1)\frac{x_d}{\|x_d\|}$

Problem 4 Lucas-Kanade Algorithm: The Structure Tensor (17 credits)

The black and white boxes in the image below are annotations used within this exercise and were not present in the image used to calculate the structure tensors.



In Equation 4.1 you are given the eigenvalues $\lambda_i^{(1)}, \dots, \lambda_i^{(5)}$ and eigenvectors $v_i^{(1)}, \dots, v_i^{(5)}$ for five structure tensors $M^{(1)}, \dots, M^{(5)}$. For each tensor you have to choose the corresponding image patch A – E. The neighborhood used to compute the structure tensor $W(x)$ is set to be the same as the inside of the corresponding annotation box. A uniform weight is used throughout the window. There is a one-to-one correspondence between tensors and patches. (To obtain readable numbers, the values below are scaled and rounded.)

$$
\begin{aligned}
&\text{E} \quad M^{(1)}: && \lambda_1^{(1)} = 4.84,\ \lambda_2^{(1)} = 3.22 && v_1^{(1)} = (-0.99, -0.16)^\top,\ v_2^{(1)} = (\ \ 0.16, -0.99)^\top \\
&\text{D} \quad M^{(2)}: && \lambda_1^{(2)} = 2.84,\ \lambda_2^{(2)} = 0.05 && v_1^{(2)} = (\ \ 0.05,\ \ 1.00)^\top,\ v_2^{(2)} = (-1.00,\ \ 0.05)^\top \\
&\text{C} \quad M^{(3)}: && \lambda_1^{(3)} = 0.00,\ \lambda_2^{(3)} = 0.00 && v_1^{(3)} = (-1.00,\ \ 0.06)^\top,\ v_2^{(3)} = (-0.06, -1.00)^\top \\
&\text{A} \quad M^{(4)}: && \lambda_1^{(4)} = 7.26,\ \lambda_2^{(4)} = 1.74 && v_1^{(4)} = (\ \ 0.27,\ \ 0.96)^\top,\ v_2^{(4)} = (-0.96,\ \ 0.27)^\top \\
&\text{B} \quad M^{(5)}: && \lambda_1^{(5)} = 0.90,\ \lambda_2^{(5)} = 0.07 && v_1^{(5)} = (-1.00, -0.00)^\top,\ v_2^{(5)} = (\ \ 0.00, -1.00)^\top
\end{aligned}
\tag{4.1}
$$

— The problem continues on the next page —

0 1 2 3 4

a) Choose the correct image patch for the structure tensor $M^{(1)}$ and explain your choice. **(4 credit(s))**

E, for both direction v_1, v_2 the eigenvalue λ_1, and λ_2 are relatively large, this means the patch has large intensity variation on v_1, v_2 direction

0 1 2 3 4

b) Choose the correct image patch for the structure tensor $M^{(2)}$ and explain your choice. **(4 credit(s))**

D, in direction v_1, it has large λ_1, means v_1 is dominant direction, edge is detected which is orthogonal to direction v_1.

c) Choose the correct image patch for the structure tensor $M^{(3)}$. **(3 credit(s))**

- [ ] D
- [ ] A
- [x] C
- [ ] B
- [ ] E

d) Choose the correct image patch for the structure tensor $M^{(4)}$. **(3 credit(s))**

- [ ] D
- [x] A
- [ ] B
- [ ] C
- [ ] E

e) Choose the correct image patch for the structure tensor $M^{(5)}$. **(3 credit(s))**

- [ ] A
- [ ] C
- [x] B
- [ ] E
- [ ] D

Problem 5 Essential and Fundamental Matrix (34 credits)

In this problem, given two vectors a and b in $\mathbb{R}^3$, we write the cross product $a \times b = \hat{a}b$.

a) By denoting the projections of a point X onto two images S_1 and S_2 by x_1 and x_2, the camera rotation and translation that maps S_1 to S_2 by R and T respectively, state the epipolar constraint. **(2 credit(s))** 0 1 2

$$x_2^T \hat{T} R x_1 = 0$$

b) Give a geometrical interpretation of the epipolar constraint. **(2 credit(s))** 0 1 2

vector $\overrightarrow{O_1X}$, $\overrightarrow{O_1O_2}$ and $\overrightarrow{O_2X}$ are on the same plane, i.e. triple product of these vectors is zero. and volum of the parallelepiped

volum $= \underbrace{x_2^T}_{\text{proportional } \overrightarrow{O_2X}} (\underbrace{T}_{\overrightarrow{O_1O_2}} \times \underbrace{R x_1}_{\overrightarrow{O_1X}}) = 0$

c) State the formula for the fundamental matrix F using the essential matrix E and the intrinsic parameter matrix K. **(2 credit(s))** 0 1 2

$$F = K^{-T} E K^{-1}$$

d) State the epipolar constraint for uncalibrated cameras by using the fundamental matrix F and the image coordinates x_1', x_2' associated to the metric coordinates x_1, x_2 via the calibration matrix. **(2 credit(s))** 0 1 2

$$x_2'^T F x_1' = x_2'^T K^{-T} E K^{-1} x_1' = 0$$

x' 像素坐标 $(u, v, 1)^T$

x 相机三维坐标 $(x, y, z)^T$

$x' = K \cdot x$

$x = K^{-1} x'$ $\xrightarrow{\text{代入}}$ $x_2^T E x_1 = x_2' F x_1'$

e) Assume that we have a camera with motion constrained to the XZ-plane, i.e. the camera center can only be moved to locations of the form $O = [t_1, 0, t_2]$ for $t_1, t_2 \in \mathbb{R}$, and the rotation is constrained to a rotation around the Y-axis (both in world coordinates). For $a, b, c, d, e, f \in \mathbb{R}$, which form does the essential matrix E take for any two poses of this constrained camera? **(4 credit(s))**

$$T = \begin{pmatrix} t_1 \\ 0 \\ t_2 \end{pmatrix} \qquad R_y = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\hat{T} = \begin{pmatrix} 0 & -t_2 & 0 \\ t_2 & 0 & -t_1 \\ 0 & t_1 & 0 \end{pmatrix}$$

☐ $E = \begin{bmatrix} 0 & 0 & a \\ 0 & 0 & b \\ c & d & 0 \end{bmatrix}$

☐ $E = \begin{bmatrix} 0 & a & b \\ c & d & 0 \\ e & 0 & f \end{bmatrix}$

☐ E does not have a special structure, all entries can be non-zero

☒ $E = \begin{bmatrix} 0 & a & 0 \\ b & 0 & c \\ 0 & d & 0 \end{bmatrix}$

$$E = \begin{pmatrix} 0 & -t_2 & 0 \\ t_2\cos\theta + t_1\sin\theta & 0 & t_2\sin\theta - t_1\cos\theta \\ 0 & t_1 & 0 \end{pmatrix}$$

☐ $E = \begin{bmatrix} a & 0 & b \\ 0 & c & d \\ e & f & 0 \end{bmatrix}$

☐ $E = \begin{bmatrix} 0 & a & b \\ c & 0 & 0 \\ d & 0 & 0 \end{bmatrix}$

— The problem continues on the next page —

In the following questions, we denote a camera matrix $P = K[R|t]$ with K its intrinsic parameter matrix, $R \in SO(3)$, $t \in \mathbb{R}^3$. P maps point coordinates X to image coordinates x' via $PX \sim x'$. A given pair of camera matrices (P, P') such that $P = [I|0]$ and $P' = [A|a]$ (with $K = K' = 1$), is said to be canonical, with I the identity matrix. Questions f) and g) are linked together.

f) Let F be a non-zero matrix and (P, P') a camera pair. Show that if $P'^\top FP$ is skew-symmetric, then F is the fundamental matrix corresponding to the pair (P, P'). **(4 credit(s))**

Fundamental Matrix with epipolar constrain $x_2'^T F x_1' = 0$

$x_1' = PX_1 \quad x_2' = P'X_2$

$(P'X_2)^T F P X_1 = X_2^T P'^T F P X_1$

if $P'^T F P$ is skew symmetric

$\Rightarrow P'^T F P = -(P'^T F P)^T$

$X_2^T M X_1 = (X_2^T M X_1)^T$: skalar

$M := P'^T F P$

$= X_1^T M^T X_2 = -X_2^T M X_1$

$\Rightarrow X_2^T P'^T F P X_1 = 0 \Rightarrow F$ is Fundamental matrix

g) Let F be any matrix in $\mathbb{R}^{3\times3}$ and S any skew-symmetric matrix in $\mathbb{R}^{3\times3}$. Let (P,P') be the pair of camera matrices that satisfies $P = [I|0]$ and $P' = [SF|e']$ where e' is the epipole such that $e'^\top F = 0$. We assume that P' is valid (has rank 3). Prove that F is the fundamental matrix corresponding to the pair (P, P'). **(4 credit(s))**

$x_2'^T F x_1' \overset{!}{=} 0 \quad x_2' = P'X_2 \quad x_1' = PX_1 \quad \Rightarrow X_2^T P'^T F P X_1 \overset{!}{=} 0$ Prove

$x_2' = [SF | e'] X_2 = SFX_2 + e'$

$x_1' = X_1$

$= (SFX_2 + e')^T F X_1$

$= \underbrace{X_2^T F^T S^T F X_1}_{=0} + \underbrace{e'^T F}_{0} X_1$

$= 0$

$X_2^T F^T S^T F X_1 = (X_2^T F^T S^T F X_1)^T \quad -S^T = S$

$= X_1^T F^T S F X_2$

$= -X_2^T F^T S^T F X_1 = 0$

— The problem continues on the next page —

Questions h) to k) are linked together. The goal is to prove the following theorem:

Theorem 1 *Let F be a matrix and let (P_1, P_1'),(P_2, P_2') be two canonical pairs of camera matrices such that F is the fundamental matrix corresponding to both pairs, $P_1 = P_2 = [I|0]$, $P_1' = [A_1|a_1]$ and $P_2' = [A_2|a_2]$. Then there exists a non-singular 4x4 matrix H such that $P_2 \sim P_1 H$ and $P_2' \sim P_1' H$.*

$\underbrace{P = K[R|t]}_{3\times 4}$ $F = \hat{e}' P_2 P_1^+$ $\hat{e}'$ epipolar in second image
P_1^+: Pseudo inverse of P_1

0 1 2 h) Show that $\widehat{a_1} A_1 = \widehat{a_2} A_2$. **(2 credit(s))**

$P_1 = P_2 = [I|0]$ $P_1' = [A_1|a_1]$ $P_2' = [A_2|a_2]$

for (P_1, P_1'), $F_1 = \hat{a}_1 A_1$ for (P_2, P_2'), $F_2 = \hat{a}_2 A_2$

$F_1 = F_2$

$\hat{a}_1 A_1 = \hat{a}_2 A_2$

$F = K^{-T} E K^{-1} = K^{-T} \hat{T} R K$

for (P_1, P_1') $P = K[R|t]$

$K_1 = 1$ $R = A_1$ $\hat{T} = \hat{a}_1$

0 1 2 3 4 i) Show that $a_2 = ua_1$ for some non-zero constant scalar u. **(4 credit(s))**

$\hat{a}_1 A_1 = \hat{a}_2 A_2$

$\hat{a}_1 A_1 \cdot x = \hat{a}_2 A_2 \cdot x$

$a_1 \times (A_1 x) = a_2 \times (A_2 x)$

$a_1 = u a_2$

$[A_2|a_2] \sim [A_1|a_1] \cdot H$

$\sim [A_1|a_1] \cdot \begin{bmatrix} M & v \\ w^T & u \end{bmatrix}$

$\sim [A_1 M + a_1 w^T \quad A_1 \cdot v + u a_1]$

$a_2 \sim A_1 \cdot v + u a_1 = \lambda a_1 + M a_1 = u a_1$

0 1 2 3 j) Show that $A_2 = u^{-1}(A_1 + a_1 v^\top)$ for some non-zero 3-vector v. **(3 credit(s))**

$A_2 = A_1 M + a_1 w^T$

0 1 2 3 4 5 k) Conclude. Hint: Use the matrix $H = \begin{pmatrix} u^{-1} I & 0 \\ u^{-1} v^\top & u \end{pmatrix}$. **(5 credit(s))**

Problem 6 Bundle Adjustment and Optimization (17 credits)

a) Consider the minimization problem

☐ 0
☐ 1
☐ 2
☐ 3

$$\Delta x^* = \arg\min_{\Delta x} \|r(x+\Delta x)\|^2 + \lambda \Delta x^\top D^\top D \Delta x\,. \qquad (6.1)$$

By using the Taylor approximation at first-order and by denoting the Jacobian of the residual r by J, express Δx^*. (writing the final equation is sufficient) **(3 credit(s))**

$\|r(x)+J\Delta x\|^2 = (r(x)+J\Delta x)^T(r(x)+J\Delta x) = r(x)^T r(x) + \Delta x^T J^T r(x) + r(x)^T J\Delta x + \Delta x^T J^T J \Delta x$

$\frac{\partial}{\partial \Delta x} = 2r(x)^T J + 2J^T J\Delta x + 2\lambda D^T D \Delta x \overset{!}{=} 0$

$(J^T J + \lambda D^T D)\Delta x = -r(x)^T J$

$\Delta x = -(J^T J + \lambda D^T D)^{-1} r(x)^T J$

b) Let $A \in \mathbb{R}^{nxn}$ be a symmetric square matrix and $x \in \mathbb{R}^n$. The eigenvalues of A are ordered as $0.8 = \lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n = 0.4$. What is the value of a^* in the following optimization problem?

$$a^* = \min_{\|x\|=1} \langle 2x, -Ax + x\rangle \qquad (6.2)$$

$= \min 2x^T \cdot (-Ax + x)$ **(5 credit(s))**

$= \underbrace{-2x^T A x}_{-1.6} + \underbrace{2x^T x}_{2} = 0.4$

- ☐ 7.6
- ☐ 9.4
- ☐ 2.4
- ☒ 0.4
- ☐ None of the other values

c) Which of the following statements is wrong? **(2 credit(s))**

- ☐ Bundle adjustment type cost functions are typically minimized by nonlinear least squares algorithms ✓
- ☐ Direct methods tend to be more robust to noise ✓
- ☐ Bundle adjustment jointly estimates the pose and the landmark updates ✓
- ☒ Direct methods provide a sparse (semi-dense) geometric reconstruction of the scene
- ☐ Bundle adjustment problem needs to be initialized ✓

— The problem continues on the next page —

normalized : $\begin{pmatrix} \frac{X}{Z} \\ \frac{Y}{Z} \\ 1 \end{pmatrix}$ homogeneous : $\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$

Consider the bundle adjustment energy for two views

$$E(R, T, X_1, \dots X_N) = \sum_{j=1}^{N} \|\tilde{x}_1^j - \pi(X_j)\|^2 + \|\tilde{x}_2^j - \pi(R, T, X_j)\|^2. \tag{6.3}$$

We denote by $\tilde{x}_i^j \in \mathbb{R}^2$ the observed position of point j in image i. Note that these points are not in homogeneous coordinates. For this exercise we assume that we are working in normalised image coordinates.

0 1 2

d) Does equation 6.3 define a direct approach? Why / Why not? **(2 credit(s))**

no, because equation 6.3 want to minimize the reprojection error between a 3D Point and its 2D projected point. It's based on the extrated feature points $\tilde{X}_i^j \in \mathbb{R}^2$, so its a feature based methode not direct.

0 1 2 3

e) What does the term $\pi(R, T, X_j)$ represent? Describe all the steps involved in evaluating it. **(3 credit(s))**

$\pi(R,T,X_j)$ represent the perspective projection of X_j after rotation and transition

$X_c = RX_j + T$

$\pi(R,T,X_j) = \pi(RX_j+T) = \begin{pmatrix} \frac{X_c}{Z_c} \\ \frac{Y_c}{Z_c} \end{pmatrix}$ $X_c = \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix}$

0 1 2

f) Why is $\pi(X_j)$ in the first term in the summation of equation 6.3 not depending on the pose parameters? **(2 credit(s))**

cause in the first term the camera didn't move, $P = [I \mid 0]$ in canonical form

no Rotation and Transition $\pi(X_j) = \begin{pmatrix} \frac{X_j^{(1)}}{X_j^{(3)}} \\ \frac{X_j^{(2)}}{X_j^{(3)}} \end{pmatrix}$

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.