Chair of Computer Vision & Artificial Intelligence
School of Computation, Information and Technology
Technical University of Munich

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Computer Vision II: Multiple View Geometry

Exam:	IN2228 / Retake	**Date:**	Tuesday 7th October, 2025
Examiner:	Prof. Dr. Daniel Cremers	**Time:**	08:00 – 10:00

	P 1	P 2	P 3	P 4	P 5	P 6
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Working instructions

- This exam consists of **12 pages** with a total of **6 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 91 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - one **analog dictionary** English ↔ native language
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.
- For all multiple choice questions, either one, or multiple answers can be correct. For each question, you'll receive full points (3p) if all boxes are answered correctly (i.e., correct answers are checked, wrong answers are not checked) and zero otherwise.

Mark correct answers with a cross ☒

To undo a cross, completely fill out the answer option ■

To re-mark an option, use a human-readable marking ×■

Left room from ________ to ________ / Early submission at ________

Problems

Problem 1 Mathematical Background (20 credits)

1.1 What is the dimension of the kernel of the matrix $C = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & -1 & 1 & 0 \end{bmatrix}$

- [ ] 0
- [ ] 1
- [x] 2
- [ ] 3

Since two rows are linearly independent, the rank is 2. Thus, the dimension of the kernel is $4 - 2 = 2$.

1.2 Which of the following sets represents all the possible values of determinants of a matrix in $SO(n)$?

- [ ] $\{0\}$
- [ ] $\mathbb{R} \setminus \{0\}$
- [ ] $\{-1, 1\}$
- [x] $\{1\}$

1.3 Which of the following is not a necessary condition for a set G with an operation $\circ : G \times G \to G$ to be a group?

- [ ] Associativity: $(g_1 \circ g_2) \circ g_3 = g_1 \circ (g_2 \circ g_3) \quad \forall g_1, g_2, g_3 \in G$.
- [x] Commutativity: $g_1 \circ g_2 = g_2 \circ g_1 \quad \forall g_1, g_2 \in G$.
- [ ] Closure: $g_1 \circ g_2 \in G \quad \forall g_1, g_2 \in G$.
- [ ] Neutral element: $\exists e \in G : e \circ g = g \circ e = g \quad \forall g \in G$.

1.4 Given a matrix $X = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \in \mathbb{R}^{2\times 2}$ and its vectorisation $X^s = [a, b, c, d]^\top \in \mathbb{R}^4$, **derive** and write down a matrix B such that the equation $BX^s = 0$ constrains the matrix X to be skew-symmetric.

0 1 2 3

Skew-symmetry means that $X^\top = -X \Leftrightarrow X^\top + X = 0$ (1). Hence, $a = 0$, $b + c = 0$, $d = 0$. (1)

Then, we can express these constraints in matrix form as $B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (1), where we have $BX^s = [a, b + c, d]^\top = 0 \Leftrightarrow X^\top = -X$.

NOTE: It is not sufficient to specify a matrix B which satisfies $BX^S = 0$ if X is skew-symmetric. In addition, $BX^S \neq 0$ must also hold if X is not skew-symmetric. Our B must satisfy $\ker(B) = \text{range}([0, -1, 1, 0]^T)$, where $[0, -1, 1, 0]^T$ is a basis for (vectorised) 2×2 skew-symmetric matrices.

-0.5p if wrong sign in B. 0.5p if only example of skew-symm amtrix given. 0.5p if b=-c but not a=d=0.
Not full points for B being a vector 4x1. +0.5p for $[f, 1, 1, g]$ with any values of f and g, 0p for $[0, 1, 1, 0]$.

1.5 Let $USV^\top$ be the singular value decomposition of $A \in \mathbb{R}^{n\times n}$ and $XLX^\top$ denote the eigenvalue decomposition of $AA^\top$. How can you compute X and L from U, S, V? Write down the solution with a **derivation**.

0 1 2 3 4

Given $A = USV^\top$, we have $AA^\top = USV^\top (USV^\top)^\top$ (1) $= USV^\top VS^\top U^\top = USS^\top U^\top$ (1).
Therefore, $X = U$ (1), $L = SS^\top$ (1) (since S is diagonal, $L = SS$ is also correct)
For $X = US$ and $L = V^T V$, give 2p if first derivation step is correct but missing $VV^T = I$ (Folgefehler).
For $X = U$ and $L = SVV^T S$, give 3.5p (not recognised that $VV^T = I$). If derivation missing but result correct, 2p. + 1p if equation present.

1.6 Derive the solution to the least squares problem $\min_{x \in \mathbb{R}^m} \|Ax - b\|_2^2$, with $A \in \mathbb{R}^{n\times m}$, $b \in \mathbb{R}^n$ and $\text{rank}(A) = m \leq n$?

0 1 2 3 4

Expand the objective: $\|Ax - b\|_2^2 = (Ax - b)^\top (Ax - b) = x^\top A^\top Ax - 2b^\top Ax + b^\top b$ (1)
Differentiate with respect to x: $\nabla f(x) = 2A^\top Ax - 2A^\top b$ (1)
Setting $\nabla f(x) = 0$ gives the *normal equations*: $A^\top Ax = A^\top b$ (1)
Since $\text{rank}(A) = m$, the $m \times m$ matrix $A^\top A$ is invertible. Thus, solution is $x^\star = (A^\top A)^{-1} A^\top b$ (1)
0.5p for only mentioning pseudo inverse without derivation

Problem 2 Representing a Moving Scene (16 credits)

2.1 Consider the Lie groups SE(n) (Special Euclidean Group) and SO(n) (Special Orthogonal Group) with $n \in \mathbb{N}$. The corresponding Lie algebras are $\mathfrak{se}(n)$ and $\mathfrak{so}(n)$. Which of the following statements are correct?

- [ ] $\dim(\mathfrak{se}(3)) = 9$
- [x] $\dim(\mathfrak{se}(n)) - \dim(\mathfrak{so}(n)) = n$
- [x] $\dim(\mathfrak{so}(2)) = 1$
- [x] $\dim(\mathfrak{so}(n)) = \frac{n(n-1)}{2}$

2.2 Let $\xi_1, \xi_2 \in \mathbb{R}^6$ be two vectors representing rigid-body motions, $\widehat{\cdot}\colon \mathbb{R}^6 \to \mathfrak{se}(3)$ the mapping to the rigid-body Lie-Algebra $\mathfrak{se}(3)$, and $\exp\colon \mathfrak{se}(3) \to \mathrm{SE}(3) \subset \mathbb{R}^{4\times 4}$ the exponential map. Which of the following statements hold?

- [ ] $\exp(\widehat{\xi_1}) = \exp(\widehat{\xi_2}) \Leftrightarrow \xi_1 = \xi_2$
- [ ] $\exp(\widehat{\xi_1}) \cdot \exp(\widehat{\xi_2}) = \exp(\widehat{\xi_1} + \widehat{\xi_2})$
- [x] $\exp(-\widehat{\xi_1}) = (\exp \widehat{\xi_1})^{-1}$
- [ ] $\exp(\widehat{\xi_1}) = \sum_{n=1}^{\infty} \frac{(\widehat{\xi_1})^n}{n!}$

0 1 2

2.3 Given a rotation matrix $R \in \mathrm{SO}(3)$ and a translation vector $T \in \mathbb{R}^3$, write down the corresponding rigid-body motion $g \in \mathrm{SE}(3)$ and its inverse g^{-1} in matrix form.

$g = \begin{pmatrix} R & T \\ 0 & 1 \end{pmatrix}$ (1) $\quad g^{-1} = \begin{pmatrix} R^\top & -R^\top T \\ 0 & 1 \end{pmatrix}$ (1)

-0.5p if R^{-1} instead of $R^\top$. -1p if one entry of g or g^{-1} is wrong

0 1 2 3 4 5

2.4 Consider a rotation $R \in \mathrm{SO}(3)$ being the results of first rotating by 180° around the x-axis (R_x), followed by a rotation of 180° around the y-axis (R_y). Write down the two rotation matrices R_x and R_y. Then, compute the combined rotation matrix R. What is the axis and angle of the rotation R?

$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ (1) $\quad R_y = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ (1) $\quad R = R_y R_x = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (1)

The axis of R is $(0, 0, 1)^\top$ / the z-axis (1). The angle of R is 180° (1).
0.5p if $R = R_y R_x$ (order of rotations correct), Full points for matrices if sin/cos kept (e.g. cos(theta) or cos(pi))

0 1 2 3

2.5 Given any two rotation matrices $R_1, R_2 \in \mathrm{SO}(3)$, is the average matrix of them $R = \frac{1}{2}(R_1 + R_2)$ always a valid rotation matrix? Justify your answer.

No, the average matrix $R = \frac{1}{2}(R_1 + R_2)$ is not always a valid rotation matrix. (1)
A valid rotation matrix must be orthogonal ($R^\top R = I$) and have a determinant of +1 ($\det(R) = 1$). (1)
While the average of two rotation matrices will still be a square matrix, it may not satisfy these conditions. (A counter example also works.) (1)
1pt for the correct answer, 1p for the definition or that one of the constraints is not satisfied, 1p for a more or less correct proof (e.g., counterexample, rewriting the equation such that it is obvious). Saying that SO(3) is not closed under summation only gets 1p, but 0p for the proof and definition, because it doesn't discuss the example of 0.5 (R1 + R2)

Problem 3 Perspective Projection (10 credits)

A 3D point is given in the camera coordinate frame by $X = (-1, 2, 10)^\top$ (measured in meters). The camera intrinsic parameter matrix is given by:

$$K = \begin{pmatrix} 200 & 0 & 640 \\ 0 & 200 & 480 \\ 0 & 0 & 1 \end{pmatrix}$$

3.1 Compute the pixel-coordinate x of the point X with explicit steps. 0 1 2

$x = (\frac{-1\times 200}{10} + 640, \frac{2\times 200}{10} + 480)^\top$ (1) $= (640 - 20, 480 + 40)^\top = (620, 520)^\top$ (1)

3.2 Compute with explicit steps the new pixel-coordinate x' of the point X after the camera moves away from the point along the z axis by 10 meters. 0 1 2

The new position of the point X in the camera coordinate frame is $X' = (-1, 2, 10{+}10)^\top = (-1, 2, 20)^\top$ (1). Thus, the new pixel coordinate is $x' = (\frac{-1\times 200}{20} + 640, \frac{2\times 200}{20} + 480)^\top = (640 - 10, 480 + 20)^\top = (630, 500)^\top$ (1).

3.3 Knowing that the optical center of the camera is at the center of the image, what is the aspect ratio of the image? What will be the new intrinsic parameter matrix K' if the image is downsampled to the half resolution? 0 1 2

aspect ratio: $\frac{640}{480} = \frac{4}{3}$ (1) The new intrinsic parameter matrix: $K' = \begin{pmatrix} 100 & 0 & 320 \\ 0 & 100 & 240 \\ 0 & 0 & 1 \end{pmatrix}$ (1)

ratio should be expressed either with : or division or as a comma. Expressing as x is not a ratio. -0.5p for each of last column being wrong or first 2 columns

3.4 Knowing that the x-axis points to the right and the y-axis points downwards in the image coordinate frame, how should we modify the intrinsic parameter matrix K to rotate the image clockwise by 90 degrees? Write down the new intrinsic parameter matrix K'' after the rotation. 0 1 2

$$K'' = \begin{pmatrix} 0 & -200 & 480 \\ 200 & 0 & 640 \\ 0 & 0 & 1 \end{pmatrix} \quad (2)$$

-0.5p for each if sign swapped at the wrong position or for both positions or at wrong axis. Both the first 2 columns and the last column must be correct, otherwise further -0.5p

3.5 Is perspective projection a linear mapping in Euclidean 3D coordinates? Justify your answer. 0 1 2

No, perspective projection is not a linear mapping. (1) This is because it involves a division by the depth (z-coordinate) of the point, which introduces a non-linear relationship between the 3D coordinates and the 2D pixel coordinates. (1)

mentioning 4th dimension, homogenous coordinate or rotation is not relevant to the question about Euclidean space. Should explicitly mention about division by z which introduces non-linearity between 3D and 2D pixel coordinates

Problem 4 Estimating Point Correspondences (9 credits)

The *Lucas-Kanade Optical Flow Algorithm* computes the best velocity vector v by minimizing the error

$$E(v) = \int_{W(x)} |\nabla I(x', t)^T v + I_t(x', t)|^2 dx',$$

where $W(x)$ is a neighborhood around x. Expanding the terms and setting the derivative to zero one can obtain

$$\frac{dE}{dv} = 2Mv + 2q = 0.$$

0 1 2

4.1 With the image gradient defined as $\nabla I = \begin{pmatrix} I_x \\ I_y \end{pmatrix}$, write down M and q.

$$M = \int_{W(x)} \nabla I(x', t) \nabla I(x', t)^T dx' = \int_{W(x)} \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix} dx' \; 1$$

$$q = \int_{W(x)} I_t(x', t) \nabla I(x', t) dx' = \int_{W(x)} I_t(x', t) \begin{pmatrix} I_x \\ I_y \end{pmatrix} dx' \; 1$$

-0.5p for missing the integral. -0.5p for every serious mistake / missing term

0 1 2

4.2 Ignoring the integration, show that $\nabla I^{\perp} \in Ker(M)$, where $\nabla I^{\perp}$ is an orthogonal vector of ∇I.

Ignoring the integration over the window $W(x)$, we have

$$M \nabla I^{\perp} = \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix} \begin{pmatrix} -I_y \\ I_x \end{pmatrix} \; 1 = \begin{pmatrix} I_x^2(-I_y) + I_x I_y I_x \\ I_x I_y(-I_y) + I_y^2 I_x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \; 1$$

Thus, $\nabla I^{\perp} \in Ker(M)$.

-0.5p for mistakes in M-matrix definition, -1p for stating $\nabla I^{\perp} * M = 0$

0 1 2

4.3 What does the aperture problem state? What is its relation to the matrix M?

The aperture problem states that the motion of a one-dimensional pattern (e.g. a line) is ambiguous when viewed through a small aperture. Only the component of motion perpendicular to the line can be determined, while the component parallel to the line remains unknown. 1

This is related to the matrix M because M is constructed from the image gradients, which encode the local structure of the image. If the image structure is not well-defined (as in the case of the aperture problem), then M may not have full rank, leading to a loss of information about the motion. 1

-0.5p for describing the no-texture case and missing the edge-case. -0.5p for explaining aperture problem without referring to motion estimation problem

4.4 What is / are the central assumption(s) in the *Lucas-Kanade Optical Flow Algorithm*?

- [x] The intensity of a given scene point stays constant (Brightness Constancy).
- [ ] The observed scene is planar (Planarity).
- [x] Neighbouring pixels have the same motion (Smoothness).
- [ ] The images show the same scene from different viewpoints (No moving objects).

Problem 5 Multi-view Reconstruction with Linear Algorithms (23 credits)

5.1 Let x_1, x_2 be the observed 2D points of a 3D point X with corresponding depths $\lambda_1, \lambda_2 \neq 0$. Derive the epipolar constraint given the following equations:

$$\begin{aligned} \lambda_1 x_1 &= X, \\ \lambda_2 x_2 &= RX + T. \end{aligned}$$

0 1 2 3

Substituting $X = \lambda_1 x_1$ into the second equation to eliminate the 3D point X from the equations:

$$\lambda_2 x_2 = R(\lambda_1 x_1) + T \quad (1)$$

Eliminateing T by taking the cross product of both sides with the translation vector T:

$$\widehat{T}(\lambda_2 x_2) = \widehat{T}(R(\lambda_1 x_1) + T) \Rightarrow \lambda_2 \widehat{T} x_2 = \lambda_1 \widehat{T} R x_1 \quad (1)$$

Projecting both sides onto x_2 and noting that $x_2^T \widehat{T} x_2 = 0$, we obtain:

$$0 = \lambda_1 x_2^T \widehat{T} R x_1$$

Since $\lambda_1 \neq 0$, we can divide both sides by λ_1 to get the epipolar constraint:

$$x_2^T \widehat{T} R x_1 = 0 \quad (1)$$

0.5p if tried a bit. -0.5p for each false claim. -0.5p if skipping over why premultiplying with x_2^T makes the LHS go to zero.

5.2 Which part of the epipolar constraint is the essential matrix E? How many degrees of freedom does E have? Explain your answer.

0 1 2 3

The essential matrix is given by $E = \widehat{T}R$. (1)
It has 5 degrees of freedom (1): 3 from the rotation matrix R and 2 from the translation vector T (since the scale of T is not determined) (1).
0.5p if mentioning that scale is free or that $\widehat{T}$ has rank 2. -0.5p for each false claim.

5.3 Given $n >= 8$ point correspondences $\{x_1^i, y_1^i, x_2^i, y_2^i\}_{i=1}^n$ between two views, the eight-point algorithm can be used to solve for the essential matrix with the linear system $\chi E_s = 0$, where χ is a matrix constructed from the point correspondences, and $E_s = [e_{11}, e_{21}, e_{31}, e_{12}, e_{22}, e_{32}, e_{13}, e_{23}, e_{33}]^\top$ is the vectorization of the essential matrix E. Write down the shape of χ and the expression for one row of χ corresponding to the i-th point correspondence.

0 1 2

χ is an $n \times 9$ matrix. (1)
One row of χ corresponding to the i-th point correspondence $(x_1^i, y_1^i, x_2^i, y_2^i)$ is given by:

$$\begin{bmatrix} x_2^i x_1^i & x_2^i y_1^i & x_2^i & y_2^i x_1^i & y_2^i y_1^i & y_2^i & x_1^i & y_1^i & 1 \end{bmatrix} \quad (1)$$

5.4 Once you are given χ, how do you calculate E^s in practice?

0 1 2

E^s is calculated as the ninth column of V_χ (1) in the SVD $\chi = U_\chi \Sigma_\chi V_\chi^\top$ (1).

0 1 2 3

5.5 Is the non-trivial solution of the linear system $\chi E_s = 0$ guaranteed to be the essential matrix? Why?

No, the solution of the linear system $\chi E_s = 0$ is not necessarily the essential matrix. (1)
An essential matrix must have rank 2 (1) and two equal nonzero singular values (1).
0.5p if mentioning that solutions are not guaranteed to live in essential space but do not mention what characterizes the essential space. -0.5p if referring to eigenvalues instead of singular values. Only 1p if saying "no" but base the argumentation on numerical inaccuracies.

0 1 2 3 4

5.6 How many possible solutions for R and T should we get from the essential matrix E? What are they? How can you rule out the incorrect solutions?

There are four possible solutions for R and T from the essential matrix E. (1)
This is because there are two possible solutions for R (due to the ambiguity in the rotation matrix decomposition) (1) and two possible directions for T (since T is determined up to scale) (1).

To rule out the incorrect solutions, we can use the cheirality condition, which states that the reconstructed 3D points must be in front of both cameras. We can triangulate the 3D points for each of the four solutions and check which solution results in all points being in front of both cameras. (1)
-0.5p for imprecise answer to ruling out incorrect solutions, -0.5p if just "two for T"

0 1 2 3

5.7 The essential matrix E and hence the translation T are only defined up to an arbitrary scale $\gamma \in \mathbb{R}^+$, with $||E|| = ||T|| = 1$. After recovering R and T from the essential matrix, we therefore have the relation

$$\lambda_2^j \mathbf{x}_2^j = \lambda_1^j R \mathbf{x}_1^j + \gamma T \quad \forall j = 1, \dots, n,$$

with unknown scale parameters λ_i^j. To recover the depth of each point in the first camera coordinate system, we can solve the equation $M\vec{\lambda} = 0$ with $\vec{\lambda} = (\lambda_1^1, \lambda_1^2, \dots, \lambda_1^n, \gamma)^T \in \mathbb{R}^{n+1}$. Write down $M \in \mathbb{R}^{3n\times(n+1)}$ without justification.

$$M = \begin{pmatrix} \widehat{\mathbf{x}_2^1} R \mathbf{x}_1^1 & 0 & 0 & \dots & 0 & \widehat{\mathbf{x}_2^1} T \\ 0 & \widehat{\mathbf{x}_2^2} R \mathbf{x}_1^2 & 0 & \dots & 0 & \widehat{\mathbf{x}_2^2} T \\ 0 & 0 & \ddots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & \widehat{\mathbf{x}_2^{n-1}} R \mathbf{x}_1^{n-1} & 0 & \widehat{\mathbf{x}_2^{n-1}} T \\ 0 & \dots & \dots & 0 & \widehat{\mathbf{x}_2^n} R \mathbf{x}_1^n & \widehat{\mathbf{x}_2^n} T \end{pmatrix}$$

1p for $\widehat{x_2} R x_1$, 1p for $\widehat{x_2} T$, 1p if for matrix structure. -0.5p if hat on x2 is missing. -0.5p if T has a hat

0 1 2 3

5.8 Consider a point $X \in \mathbb{R}^3$ which is observed in $m \geq 2$ images. Let $x_1, \dots, x_m \in \mathbb{R}^3$ denote the respective observations in homogeneous coordinates, and $\Pi_1, \dots, \Pi_m \in \mathbb{R}^{3\times 4}$ the multiple-view projection matrices, projecting a point into the respective image. Let

$$N_p := \begin{pmatrix} \Pi_1 & x_1 & 0 & \cdots & 0 \\ \Pi_2 & 0 & x_2 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Pi_m & 0 & 0 & \cdots & x_m \end{pmatrix} \in \mathbb{R}^{3m\times(m+4)}$$

What does the rank of N_p tell you regarding existence and uniqueness of a reconstruction X?

If $rank(N_p) = m + 4$ (full rank), there are no solutions. (1)
If $rank(N_p) = m + 3$, the solution is unique (up to scale). (1)
If $rank(N_p) < m + 3$, the space of the solutions have a dimension bigger than 1. (1)

Problem 6 Bundle Adjustment with Projective Matrices (13 credits)

Let $P_i \in \mathbb{R}^{3\times 4}$ be the projective camera matrix of camera i, and let $X_j \in \mathbb{R}^4$ denote the homogeneous coordinates of landmark j,

$$X_j = \begin{pmatrix} x_j \\ y_j \\ z_j \\ 1 \end{pmatrix}.$$

For each observation of landmark j by camera i, an image measurement $m_{ij} \in \mathbb{R}^2$ is given.
The projection of X_j onto the image plane of camera i is

$$\Pi(P_i, X_j) = \begin{pmatrix} \frac{P_{i,0:}X_j}{P_{i,2:}X_j} \\ \frac{P_{i,1:}X_j}{P_{i,2:}X_j} \end{pmatrix},$$

where $P_{i,k:}$ denotes the k-th row of P_i.
The residual for observation (i, j) is

$$r_{ij} = \Pi(P_i, X_j) - m_{ij} \quad \in \mathbb{R}^2.$$

Stacking all residuals gives the global residual vector

$$r = \begin{pmatrix} r_{11} \\ r_{12} \\ \vdots \\ r_{ij} \\ \vdots \end{pmatrix}.$$

In this exercise, we consider 10 cameras, 100 landmarks, and each camera sees all the landmarks.

6.1 What is the difference between the residuals defined here, and the residuals of the traditional bundle adjustment problem? In which practical situations should we consider the residuals defined here?

0 1 2 3

- Traditional BA: poses parametrized by $SE(3)$, intrinsics K, metric reconstruction. 1
- Here: full 3×4 projective matrices; unconstrained projective reconstruction. 1
- Use case: uncalibrated structure-from-motion or projective bundle adjustment. 1

6.2 What is the dimension of the residual vector r and the Jacobian

$$J = \begin{bmatrix} J_p & J_\ell \end{bmatrix},$$

where J_p is the Jacobian of r with respect to all camera matrices $\{P_i\}$, and J_ℓ the Jacobian with respect to all landmarks $\{X_j\}$?

0 1 2 3 4

- Residual vector: $r \in \mathbb{R}^{2\cdot 10\cdot 100} = \mathbb{R}^{2000}$. 1
- Camera Jacobian: $J_p \in \mathbb{R}^{2000\times 120}$ (10 cameras × 12 entries each). 1
- Landmark Jacobian: $J_\ell \in \mathbb{R}^{2000\times 300}$ (100 landmarks × 3 unknowns each, as homogeneous coordinate fixed). 1
- Full Jacobian: $J = [J_p \;\; J_\ell] \in \mathbb{R}^{2000\times 420}$. 1

0
1
2
3
4

6.3 Derive the expressions of the per-observation Jacobians

$$J_{ij}^{(p)} = \frac{\partial r_{ij}}{\partial \operatorname{vec}(P_i)}, \qquad J_{ij}^{(\ell)} = \frac{\partial r_{ij}}{\partial X_j},$$

where $\operatorname{vec}(P_i)$ denotes the 12-vector obtained by stacking the entries of P_i.

Consider one observation (i, j) with camera matrix $P_i \in \mathbb{R}^{3\times 4}$ and landmark $X_j = (x_j, y_j, z_j, 1)^T$. Denote the rows of P_i by $p_0, p_1, p_2 \in \mathbb{R}^4$ and let

$$x_c = p_0 \cdot X_j, \quad y_c = p_1 \cdot X_j, \quad z_c = p_2 \cdot X_j.$$

The residual is

$$r_{ij} = \Pi(P_i, X_j) - m_{ij} = \begin{pmatrix} u - u_{ij}^{\text{obs}} \\ v - v_{ij}^{\text{obs}} \end{pmatrix}, \quad \Pi(P_i, X_j) = \begin{pmatrix} x_c/z_c \\ y_c/z_c \end{pmatrix}.$$

Jacobian w.r.t. landmark X_j (3 unknowns):

$$J_{ij}^{(\ell)} = \frac{\partial r_{ij}}{\partial X_j} = \begin{pmatrix} \frac{p_0^{(xyz)} z_c - p_2^{(xyz)} x_c}{z_c^2} \\ \frac{p_1^{(xyz)} z_c - p_2^{(xyz)} y_c}{z_c^2} \end{pmatrix} \in \mathbb{R}^{2\times 3},$$

where $p_k^{(xyz)}$ denotes the first three components of row p_k (since the last coordinate of X_j is fixed).

Jacobian w.r.t. camera P_i (vectorized 12 entries):

$$J_{ij}^{(p)} = \frac{\partial r_{ij}}{\partial \operatorname{vec}(P_i)} = \begin{pmatrix} \frac{\partial u}{\partial p_0} & \frac{\partial u}{\partial p_1} & \frac{\partial u}{\partial p_2} \\ \frac{\partial v}{\partial p_0} & \frac{\partial v}{\partial p_1} & \frac{\partial v}{\partial p_2} \end{pmatrix} = \begin{pmatrix} X_j^T/z_c & 0 & -x_c X_j^T/z_c^2 \\ 0 & X_j^T/z_c & -y_c X_j^T/z_c^2 \end{pmatrix} \in \mathbb{R}^{2\times 12}.$$

- The first block of $J_{ij}^{(p)}$ corresponds to derivatives w.r.t. p_0, the second to p_1, and the third to p_2.

0
1
2

6.4 In the Levenberg–Marquardt algorithm, how is the approximate Hessian of the minimization problem associated to the residual r expressed in terms of the Jacobian J?

$$H \approx J^\top J + \lambda \operatorname{diag}(J^\top J) = \begin{pmatrix} J_p^\top J_p & J_p^\top J_\ell \\ J_\ell^\top J_p & J_\ell^\top J_\ell \end{pmatrix} + \lambda \operatorname{diag}(\cdot)$$

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.