# Computer Vision II: Multiple View Geometry Final Exam S20

Computer Vision II: Multiple view geometry (Technische Universität München)

Chair of Computer Vision and Artificial Intelligence
Department of Informatics
Technical University of Munich

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- During the attendance check a sticker containing a unique code will be put on this exam.
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# Computer Vision II: Multiple View Geometry

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| **Exam:** | IN2228 / Final | **Date:** | Tuesday 28$^{th}$ July, 2020 |
| **Examiner:** | Florian Bernard | **Time:** | 08:00 – 10:00 |



## Working instructions

- This exam consists of **18 pages** with a total of **10 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 120 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - This is an open book graded exercise: you are allowed to use any source that does not involve communication. In particular, you are allowed to use all lecture material (including solutions that we provide), textbooks, research papers, Matlab or existing webpages, etc. **Any type of communication, including actively seeking for help from others (e.g. in forums or chatrooms, etc.), is not allowed**.
- This graded exercise uses the TUMexam platform which offers a student manual on their webpage[1].
- The boxes on the sides of the subproblems (those with numbers) are used for correction and ticking them is prohibited. A ticked box that is not part of a multiple choice question may result in zero points for the problem.
- In the multiple choice questions there is always exactly one correct answer. A correct answer (comprising of exactly one tick at the correct position) will give the indicated credits, whereas a wrong answer will result in 0 credits.

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| *Mark correct answers with a cross* | ☒ |
| *To undo a cross, completely fill out the answer option* | ■ |
| *To re-mark an option, use a human-readable marking* | ×■ |

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| Left room from ________ | to ________ | / | Early submission at ________ |

[1] `https://www.tumexam.de/links.html`

## Problem 1 Personal Information (1 credit)

0
1

Please enter your matriculation number with leading zero. **(1 credit(s))**

This field should contain your matriculation number.

## Problem 2 MATLAB Operations (11 credits)

Let `B,C` be given, where for nonnegative integers `m,n` we have `size(B) = size(C) = [m,n]`.

a) What is the dimension of the output of `kron(B,C')`? **(2 credit(s))**

- [ ] `[m, m]`
- [ ] `[m*m, m*n]`
- [ ] `[m*n, n*n]`
- [ ] `[n, m]`
- [ ] `[m, n]`
- [ ] The expression will lead to an error
- [x] `[m*n, m*n]`
- [ ] `[n, n]`

b) What is the dimension of the output of `B*C'`? **(2 credit(s))**

- [ ] `[m*n, m*n]`
- [ ] The expression will lead to an error
- [ ] `[n, n]`
- [ ] `[n, m]`
- [ ] `[m*m, m*n]`
- [ ] `[m, n]`
- [x] `[m, m]`
- [ ] `[m*n, n*n]`

c) How many (scalar) multiplication operations are necessary to compute `B.*C` for general `B,C`? **(2 credit(s))**

- [x] `m*n`
- [ ] `m*m*n*n`
- [ ] `n*n*n*m`
- [ ] `n`
- [ ] `m*m*m*n`
- [ ] `m`
- [ ] The expression will lead to an error
- [ ] `m*m*m*n*n*n`

**— The problem continues on the next page —**

d) How many (scalar) multiplication operations are necessary to compute `B(:)*C(:)'` for general `B,C`? **(2 credit(s))**

- ☐ `n*n*n*m`
- ☐ `m*m*m*n*n*n`
- ☐ `m*n`
- ☐ `m*m*m*n`
- ☐ `m`
- ☐ `n`
- ☐ The expression will lead to an error
- ☒ `m*m*n*n`

0 ☐ 1 ☐ 2 ☐ 3 ☐

e) Write down one line of MATLAB code that computes `trace(B(:)*C(:)')` quickly, i.e. without explicitly constructing `B(:)*C(:)'`. **(3 credit(s))**

There are several ways, e.g. `B(:)'*C(:)`, or `sum(sum(B.*C))`

## Problem 3 Matrix Algebra in MATLAB (12 credits)

Consider the matrix $A$ and vectors $b, c$ given by

$$A = \begin{pmatrix} 4 & 6 \\ 9 & 2 \\ 7 & 2 \end{pmatrix}, \qquad b = \begin{pmatrix} 1 \\ 8 \\ 6 \end{pmatrix}, \qquad c = \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$

a) Let `y = A\b`. For which Matlab code is `x` equal to `y` (up to numerical precision)? **(3 credit(s))**

- [ ] `x = inv(A)*b`
- [x] `[U,S,V] = svd(A); x = V*pinv(S)*U'*b`
- [ ] `x = A'/b`
- [ ] `x = [-1; 0.5]`
- [ ] `x = inv(A*A')*A*b`
- [ ] `[U,S,V] = svd(A); x = U'*pinv(S)*V*b`
- [ ] `x = pinv(A)'*b`

b) Which of the following Matlab expressions computes `b'*A*c` (up to numerical precision)? **(3 credit(s))**

- [ ] `b*c' + A`
- [ ] `A*kron(b,c)`
- [ ] `sum(sum(b*c'*A))`
- [ ] `kron(b,c')*A'`
- [ ] `b'*svd(A)*c`
- [x] `kron(c,b)'*A(:)`
- [ ] `trace(A*b'*c)`

c) Let `[U,S,V] = svd(D)`. Which of the following holds for *any* choice of `D` with `size(D) = [3,3]` (up to numerical precision)? **(2 credit(s))**

- [ ] `U*V'` is an element of $se(3)$
- [ ] `expm(U*V')` is an element of $SO(3)$
- [ ] `expm(U*V')` is an element of $so(3)$
- [x] `U*V'` is an element of $O(3)$
- [ ] `U*V'` is an element of $SO(3)$
- [ ] `U*V'` is an element of $SE(3)$
- [ ] `U*V'` is an element of $so(3)$

d) Let `[U,S,V] = svd(A)`, and let `X*L*X'` denote the eigenvalue decomposition of `A'*A`. How can you compute `X` and `L` from `U`, `S` and `V`? Write down your derivation symbolically (do not compute any numerical values). **(4 credit(s))**

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

We have that $A = USV^\top$, so $A^\top A = (USV^\top)^\top USV^\top = VS^\top U^\top USV^\top = VS^\top SV^\top$,
thus we need to identify $X$ and $L$ as follows:
`X = V`
`L = S'*S`(since `S` is diagonal, `L=S*S` is also correct)

## Problem 4 Definitions (13 credits)

a) Let $\lambda_1, \ldots, \lambda_k \in \mathbb{R}$. The set of vectors $\{v_1, \ldots, v_k\} \subset \mathbb{R}^n$ is called linearly independent if **(2 credit(s))**

- ☐ $\sum_{i=1}^{k} \lambda_i v_i = 0 \quad \Rightarrow \quad v_i = 0 \quad \forall\, i = 1, \ldots, k$
- ☒ $\sum_{i=1}^{k} \lambda_i v_i = 0 \quad \Rightarrow \quad \lambda_i = 0 \quad \forall\, i = 1, \ldots, k$
- ☐ $v_i \neq v_j \quad \forall\, i, j = 1, \ldots, k$ with $i \neq j$

0 ☐
1 ☐

b) Let $M \in \mathbb{R}^{n\times n}$ be a matrix for which $M^\top = -M$ holds. The matrix $M$ is called **(1 credit(s))**

skew-symmetric.

c) Let $w \in \mathbb{R}^3$ and $\|w\| = 1$. Which of the following holds for the cross product matrix $\hat{w}$ where $\mathbb{I}_{3\times 3} \in \mathbb{R}^{3\times 3}$ denotes the identity matrix. **(2 credit(s))**

- ☐ $\hat{w}^2 = \mathbb{I}_{3\times 3} - ww^\top$
- ☒ $\hat{w}^2 = ww^\top - \mathbb{I}_{3\times 3}$
- ☐ $\hat{w}^3 = -\hat{w}^2$
- ☐ $\hat{w}^3 = \hat{w}$

0 ☐
1 ☐

d) Let $A \in \mathrm{GL}(n)$, $b \in \mathbb{R}^n$, $c \in \mathbb{R}^n$ with $c = 0$. Consider the group that contains all matrices of the form

$$\begin{pmatrix} A & b \\ c^\top & 1 \end{pmatrix}.$$

and that uses matrix multiplication as group operation. State the name of this group. **(1 credit(s))**

The affine group $\mathrm{A}(n)$.

0 ☐
1 ☐

e) Tick the correct statement. **(1 credit(s))**

- ☐ $\mathrm{SE}(n) \subset \mathrm{A}(n) \subset \mathrm{GL}(n)$
- ☒ $\mathrm{SE}(n) \subset \mathrm{A}(n) \subset \mathrm{GL}(n+1)$
- ☐ $\mathrm{GL}(n+1) \subset \mathrm{O}(n)$
- ☐ $\mathrm{GL}(n) \subset \mathrm{O}(n)$

— The problem continues on the next page —

f) Let $M \in \mathbb{R}^{m \times n}$ be a matrix and consider the set

$$\{y \in \mathbb{R}^m | \exists x \in \mathbb{R}^n : Mx = y\} .$$

0
1

What is the name of this set? **(1 credit(s))**

It is the range/image of the matrix $M$.

g) Let $w \in \mathbb{R}^3$ and let $\mathbb{I}_{3\times3} \in \mathbb{R}^{3\times3}$ denote the identity matrix. Consider the following equation:

0
1
2

$$? = \mathbb{I}_{3\times3} + \frac{\hat{w}}{\|w\|}\sin(\|w\|) + \frac{\hat{w}^2}{\|w\|^2}(1 - \cos(\|w\|))$$

Name the equation and complete the left hand side s.t. the equation holds for arbitrary $w$. The left hand side can only depend on $\hat{w}$ not $w$. **(2 credit(s))**

The formula is called "Rodrigues' Formula"and the left hand side is given by $e^{\hat{w}}$.

h) Let $A \in \mathbb{R}^{n\times n}$ with rank$(A) = n$ and let $A = U\Sigma V^\top$ be the SVD (singular value decomposition) of $A$, where the singular values in $\Sigma$ are ordered, i.e. $\Sigma = \text{diag}(s_1, \dots, s_n) \in \mathbb{R}^{n\times n}$ with $s_1 \geq s_2 \geq \dots \geq s_n$. How do we find $x \in \mathbb{R}^n$ that minimizes $\|Ax\|$ subject to $\|x\| = 1$? **(2 credit(s))**

- [x] $x$ is the last column of $V$.
- [ ] $x$ is the first column of $V$.
- [ ] $x$ is the last column of $U$.
- [ ] $x$ is the first column of $U$.

i) Given the same conditions as in subproblem h) what is the value of

$$\nu = \min_{x\in\mathbb{R}^n:\ \|x\|=1} \|Ax\| \ ?$$

**(1 credit(s))**

- [x] $\nu = s_n$
- [ ] $\nu = \sqrt{s_n}$
- [ ] $\nu = s_1$
- [ ] $\nu = \sqrt{s_1}$
- [ ] $\nu = \sum_{i=1}^n s_i$
- [ ] $\nu = \sum_{i=1}^n \sqrt{s_i}$

## Problem 5 Linear Algebra (17 credits)

The Euclidean projection $R^*$ of the matrix $A \in \mathbb{R}^{n\times n}$ onto the set $O(n) = \{Q \in \mathbb{R}^{n\times n} \mid Q^\top Q = I\}$ is defined as

$$R^* = \operatorname*{argmin}_{R\in O(n)} \|A - R\|_f ,$$

i.e. we want to find a minimiser $R^*$ of $\|A - R\|_f$ subject to $R \in O(n)$. It can be computed as $R^* = UV^\top$ for $U\Sigma V^\top$ being the singular value decomposition of $A$. Briefly explain the individual steps of the derivation below.
Hints:

- for two matrices $X, Y$ of the same size, $\langle X, Y\rangle := \operatorname{trace}(X^\top Y)$ denotes an inner product.
- for matrices $X, Y, Z$ of appropriate dimensions, the trace is invariant under cyclic permutations: $\operatorname{trace}(XYZ) = \operatorname{trace}(YZX) = \operatorname{trace}(ZXY)$.

**Note: Even if you cannot answer a particular step you can still solve subsequent steps.**

0 1 a) **(1 credit(s))**

$$\operatorname*{argmin}_{R\in O(n)} \|A - R\|_f = \operatorname*{argmin}_{R\in O(n)} \|A - R\|_f^2$$

Squaring the norm does not change its minimiser(monotonicity over non-negative reals)

0 1 2 b) **(2 credit(s))**

$$= \operatorname*{argmin}_{R\in O(n)} \langle A, A\rangle - 2\langle A, R\rangle + \langle R, R\rangle$$

Expanding the squared Frobenius norm gives $\langle A, A\rangle - \langle A, R\rangle - \langle R, A\rangle + \langle R, R\rangle$
$\langle A, R\rangle = \operatorname{trace}(A^T R) = \operatorname{trace}((A^\top R)^\top) = \operatorname{trace}(R^\top A) = \langle R, A\rangle$, so we get $\langle A, A\rangle - 2\langle A, R\rangle + \langle R, R\rangle$

0 1 2 c) **(2 credit(s))**

$$= \operatorname*{argmin}_{R\in O(n)} -2\langle A, R\rangle$$

The term $\langle A, A\rangle$ is independent of $R$ and does not affect the minimisation w.r.t. $R$, so it can be dropped
$R^\top R = \mathbb{I}_{n\times n}$, so the term $\langle R, R\rangle = \operatorname{trace}(R^\top R) = \operatorname{trace}(\mathbb{I}_{n\times n})$ is also independent of $R$ and can be dropped

0 1 2 d) **(2 credit(s))**

$$= \operatorname*{argmax}_{R\in O(n)} \langle A, R\rangle$$

the factor 2 does not affect the minimiser
we can remove the minus and replace argmin with argmax

**— The problem continues on the next page —**

e) **(1 credit(s))** 0 1

$$= \underset{R \in O(n)}{\text{argmax}} \langle U\Sigma V^\top, R \rangle$$

we plugin the singular value decomposition $U\Sigma V^\top$ for $A$

f) **(3 credit(s))** 0 1 2 3

$$= \underset{R \in O(n)}{\text{argmax}} \langle \Sigma, U^\top RV \rangle$$

we rearrange the terms in the inner product:$\langle U\Sigma V^\top, R \rangle$ = trace($(U\Sigma V^\top)^\top R$) = trace($V\Sigma^\top U^\top R$)= trace($\Sigma^\top U^\top RV$) = $\langle \Sigma, U^\top RV \rangle$

g) **(4 credit(s))** 0 1 2 3 4

$$= \underset{R \in O(n),\ \mathbb{I}_{n\times n} = U^\top RV}{\text{argmax}} \langle \Sigma, \mathbb{I}_{n\times n} \rangle$$

Note: $\mathbb{I}_{n\times n}$ is the $n \times n$ identity matrix.
Hints: your answer should address the following:

- There are two relevant properties of $\Sigma$, name them.
- What property does $U^\top RV$ have?
- What effect do these three properties have on the maximiser of $\text{argmax}_{R\in O(n)} \langle \Sigma, U^\top RV \rangle$?

$\Sigma$ is a diagonal matrix
$\Sigma$ is nonnegative
$U^\top RV$ is an orthogonal matrix
thus the maximiser is achieved for the identity matrix $\mathbb{I}_{n\times n}$(which is also orthogonal)

h) **(2 credit(s))** 0 1 2

$$= UV^\top$$

$U$ and $V$ are orthogonal
thus $\mathbb{I}_{n\times n} = U^\top RV \Leftrightarrow U\mathbb{I}_{n\times n}V^\top = UU^\top RVV^\top \Rightarrow R^* = UV^\top$

## Problem 6 Image Formation (16 credits)

A 3D point is given in the camera coordinate frame by $P = (1, 2, 4)^\top$. The intrinsic parameter matrix $K$ is given by

$$K = \begin{pmatrix} 400 & 0 & 250 \\ 0 & 400 & 200 \\ 0 & 0 & 1 \end{pmatrix}.$$

Calculate the pixel-position of the projected point in the image and tick the correct answer.

a) What is the value of $u$? **(2 credit(s))**

- [ ] u = 650
- [x] u = 350
- [ ] u = 380
- [ ] u = 1000

b) What is the value of $v$? **(2 credit(s))**

- [x] v = 400
- [ ] v = 340
- [ ] v = 170
- [ ] v = 960

A 3D point is given in the world coordinate frame by $P = (0, 1, 2)^\top$. The point is observed by a camera, which has its optical center at $C = (1, 0, 0)^\top$ and no rotation ($R = \mathbb{I}_{3\times3}$, where $\mathbb{I}_{3\times3} \in \mathbb{R}^{3\times3}$ is the identity matrix). The intrinsic parameter matrix $K$ is given by

$$K = \begin{pmatrix} 500 & 0 & 320 \\ 0 & 400 & 240 \\ 0 & 0 & 1 \end{pmatrix}.$$

Calculate the pixel-position of the projected point in the image and tick the correct answer.

c) What is the value of $u$? **(2 credit(s))**

- [x] u = 70
- [ ] u = 590
- [ ] u = 880
- [ ] u = 890

d) What is the value of $v$? **(2 credit(s))**

- [ ] v = 820
- [ ] v = 40
- [ ] v = 680
- [x] v = 440



**— The problem continues on the next page —**

Given is a camera, which has its optical center at $C = (1, 0, 0)^\top$ and no rotation ($R = \mathbb{I}_{3\times3}$, where $\mathbb{I}_{3\times3} \in \mathbb{R}^{3\times3}$ is the identity matrix). The 3D points $X_1 = (0, 1, 2)^\top$, $X_2 = (1, 0, 1)^\top$ in the world coordinate frame, appear in the image at pixel coordinates $x_1 = (230, 300)^\top$ and $x_2 = (280, 200)^\top$. Assuming a perspective pinhole camera with no skew the intrinsic matrix is given by

$$K = \begin{pmatrix} f_x & 0 & o_x \\ 0 & f_y & o_y \\ 0 & 0 & 1 \end{pmatrix} .$$

What are the values of the intrinsic camera parameters?

e) What is the value of $f_x$? **(2 credit(s))**

- [ ] $f_x = 150$
- [x] $f_x = 100$
- [ ] $f_x = 200$

f) What is the value of $f_y$? **(2 credit(s))**

- [x] $f_y = 200$
- [ ] $f_y = 100$
- [ ] $f_y = 150$

g) What is the value of $o_x$? **(2 credit(s))**

- [ ] $o_x = 200$
- [ ] $o_x = 180$
- [x] $o_x = 280$

h) What is the value of $o_y$? **(2 credit(s))**

- [ ] $o_y = 280$
- [ ] $o_y = 180$
- [x] $o_y = 200$

## Problem 7 Radial Distortion (12 credits)

a) Which of the following statements about radial distortion is true? **(2 credit(s))**

☐ The FOV (or ATAN) model for radial distortion is not invertible in closed form.

☐ Radial distortion is caused by non-rectangular pixels.

☒ A radial distortion model applied after the pinhole projection can not model cameras with a field of view of more than 180°.

b) If the camera lens exhibits radial distortion, one can obtain a rectified, virtual image through undistortion. Assuming perfect undistortion the rectified, virtual image perfectly follows the pinhole camera model. Which of the following statements is true? **(2 credit(s))**

☐ Straight lines in 3D will not appear as straight lines in the rectified, virtual image.

☐ It cannot be decided if straight lines in 3D will appear as straight lines in the rectified, virtual image.

☒ Straight lines in 3D will appear as straight lines in the rectified, virtual image.

c) Consider the radial distortion model

$$x_d = g(r)\frac{x}{\|x\|}\,, \tag{1}$$

which maps coordinates $x \in \mathbb{R}^2$ in the normalized image plane to distorted coordinates $x_d \in \mathbb{R}^2$. The factor $g(r)$ depends on the radius $r = \|x\|$ and is given as

$$g(r) = s\,\log(1 + kr)\,, \tag{2}$$

where $s \in \mathbb{R}^+$, $k \in \mathbb{R}^+$ are model parameters. Choose the function $f$ such that the inverse distortion can be written as

$$x = f(r_d)\frac{x_d}{\|x_d\|}\,, \tag{3}$$

where $r_d = \|x_d\|$. **(3 credit(s))**

☐ $f(r_d) = \frac{1 - \exp(r_d/s)}{k}$

☒ $f(r_d) = \frac{\exp(r_d/s) - 1}{k}$

☐ $f(r_d) = \frac{1 - \exp(r_d\, s)}{k}$

☐ $f(r_d) = \frac{\exp(r_d\, s) - 1}{k}$



**— The problem continues on the next page —**

d) Given are three points in the camera coordinate system $X_1 = (-1, 0, 10)^\top$, $X_2 = (1, 1, 10)^\top$, and $X_3 = (3, 2, 10)^\top$, as well as their pixel coordinates after undistortion $x_1 = (270, 240)^\top$, $x_2 = (370, 280)^\top$, and $x_3 = (470, 310)^\top$ using the pinhole camera model. Which of the following statements is true for a non-degenerate intrinsic matrix? **(2 credit(s))**

- [ ] The undistortion worked perfectly and thus the undistorted image can be perfectly modeled using the pinhole camera model.
- [ ] Without knowing the exact intrinsic parameters it is not possible to decide if the undistortion worked perfectly or not and if the image can be perfectly modeled using the pinhole camera model or not.
- [x] The undistortion did not work perfectly and thus the undistorted image cannot be perfectly modeled using the pinhole camera model.

e) Explain your choice in subproblem d). **(3 credit(s))**

0 1 2 3

The three points $X_1$, $X_2$, and $X_3$ lie on a line in 3D.

$$X_2 - X_1 = (2, 1, 0)^\top, \qquad X_3 - X_2 = (2, 1, 0)^\top.$$

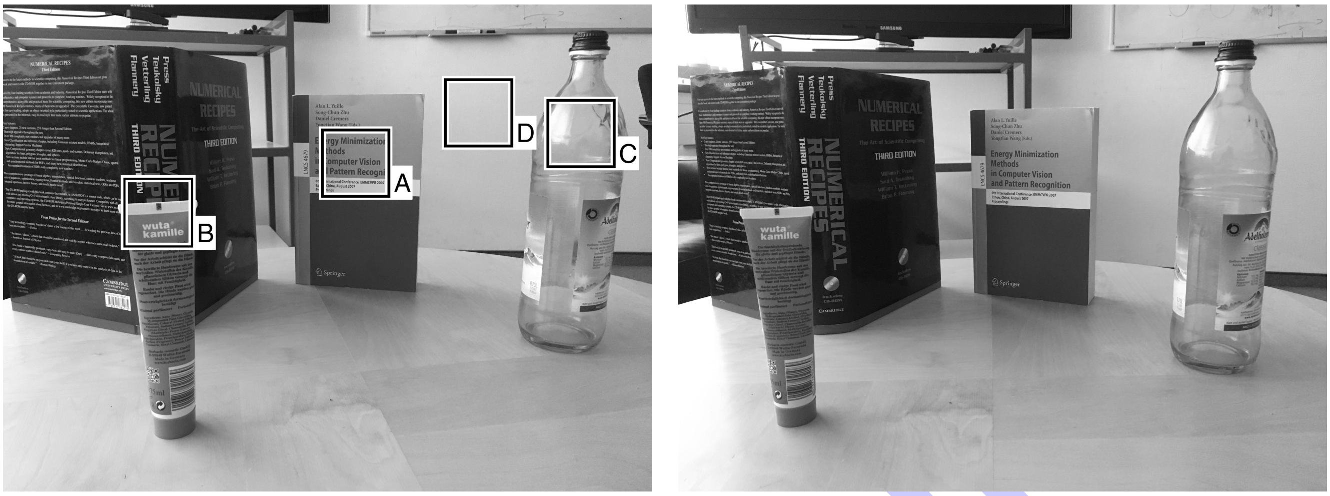
If the image after undistortion could be perfectly modeled by the pinhole camera model, the points $x_1$, $x_2$, and $x_3$ would have to lie on a line as well because the image of a line is a line.The given points $x_1$, $x_2$, and $x_3$ do not lie on a line and therefore the undistortion cannot be perfect.

$$x_2 - x_1 = (100, 40)^\top, \qquad x_3 - x_2 = (100, 30)^\top.$$

Note: Other solutions, e.g., using a general intrinsic matrix and solving the resulting system, are possible as well.

## Problem 8 The Lucas-Kanade Method (14 credits)

Given are the following images where the black and white boxes are annotations used within this exercise and not present in the images used for the Lucas-Kanade method.




The Lucas-Kanade algorithm is used to estimate the optical flow for the two images. For the marked image regions A – D decide and explain if the assumptions of the Lucas-Kanade method are fulfilled. The neighborhood $W(x)$ is set to be the same as the annotation boxes. Use **precise, technical** terms from the lecture for your explanations.

0 1 2

a) Image region A. **(2 credit(s))**

The assumptions are fulfilled.The **brightness constancy assumption** and the **constant motion in a neighborhood assumption** are fulfilled. The neighborhood contains sufficient structure to avoid the **aperture problem**.

0 1 2

b) Image region B. **(2 credit(s))**

The assumptions are not fulfilled.The foreground object (creme) and the background object (book) move differently from left to right image. This can be seen by noting that the alignment of creme and book changes. Thus the **constant motion in a neighborhood assumption**is violated.

0 1 2

c) Image region C. **(2 credit(s))**

The assumptions are not fulfilled.The glass water bottle has a non-Lambertian surface and thus the **brightness constancy assumption**is violated. This can be seen clearly in the top right corner of the annotation box where the black chair is reflected in the left image but not in the right image.

0 1 2

d) Image region D. **(2 credit(s))**

The assumptions are not fulfilled.The wall is uniformly white and thus the **aperture problem**arises.



**— The problem continues on the next page —**

What can we say about the pixel $x$ and its neighborhood region based on the eigenvalues $\lambda_1$ and $\lambda_2$ of its structure tensor $M$ in the following situations.

e) For the case: $\lambda_1 \gg 0, \lambda_2 \gg 0$? **(2 credit(s))**

- [x] The pixel $x$ lies on a corner.
- [ ] The pixel $x$ lies in an untextured region.
- [ ] The pixel $x$ lies on an edge.

f) For the case: $\lambda_1 \approx \lambda_2 \approx 0$? **(2 credit(s))**

- [x] The pixel $x$ lies in an untextured region.
- [ ] The pixel $x$ lies on a corner.
- [ ] The pixel $x$ lies on an edge.

g) For the case: $\lambda_1 \gg 0, \lambda_2 \approx 0$? **(2 credit(s))**

- [ ] The pixel $x$ lies on a corner.
- [ ] The pixel $x$ lies in an untextured region.
- [x] The pixel $x$ lies on an edge.

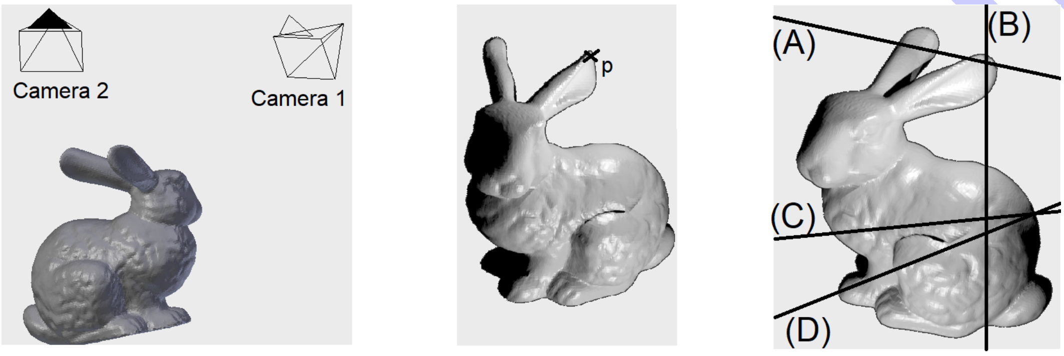
# Problem 9 Epipolar Lines (13 credits)

0 ☐ 1 ☐ 2 ☐

a) Describe briefly how the epipolar line can be used in practice. **(2 credit(s))**

The epipolar line can be used to find correspondences.For any given pixel in the first image, the correspondent pixel in the second image has to be in the epipolar line.

b) In the following figure are given two images coming from two different cameras, the whole scene is shown in the first subfigure.




Scene Image 1 Image 2

Among the different lines $(A), (B), (C), (D)$ drawn in the image 2, which one corresponds to the epipolar line associated to the pixel $p$ in image 1? **(3 credit(s))**

- ☒ $(A)$
- ☐ $(B)$
- ☐ $(C)$
- ☐ $(D)$

We consider in the following two cameras for which there is only a translation $T = (5, 4, 1)^\top$. This means that given a 3D point whose coordinates in the two cameras are $X_1$ and $X_2$, we have that : $X_2 = X_1 + T$. We assume that the intrinsic matrices of both views are the identity : $K_1 = K_2 = \mathbb{I}_{3\times 3}$.

c) What is the equation of the epipolar line associated to the pixel $x_1 = (50, 20)^\top$ of the first image? **(4 credit(s))**

- ☐ $y = \frac{12}{25}x - 5$
- ☐ $y = \frac{1}{2}x + \frac{3}{2}$
- ☒ $y = \frac{16}{45}x + \frac{20}{9}$
- ☐ $y = \frac{3}{2}x - \frac{1}{2}$
- ☐ $y = -\frac{1}{2}x + \frac{2}{5}$

d) Among the following pairs, which one is unlikely to be a pair of correspondent pixels? **(4 credit(s))**

- ☐ $x_1 = (30, 20)^\top$ and $x_2 = (5, 4)^\top$
- ☐ $x_1 = (50, 20)^\top$ and $x_2 = (2, \frac{132}{45})^\top$
- ☒ $x_1 = (30, 20)^\top$ and $x_2 = (10, \frac{31}{5})^\top$
- ☐ $x_1 = (50, 20)^\top$ and $x_2 = (\frac{35}{16}, 3)^\top$

## Problem 10 Bundle Adjustment & Nonlinear Optimization (11 credits)

Which of the following statements concerning Direct Image Alignment, Bundle Adjustment, and the 8-point Algorithm are true?

a) Direct Image Alignment. **(1 credit(s))**

- ☐ Direct Image Alignment minimizes the (re-)projection error (Euclidean point distances).
- ☒ Direct Image Alignment minimizes the photometric error (intensity differences).
- ☐ Direct Image Alignment minimizes neither the (re-)projection error (Euclidean point distances) nor the photometric error (intensity differences).

b) Bundle Adjustment. **(1 credit(s))**

- ☐ Bundle Adjustment minimizes neither the (re-)projection error (Euclidean point distances) nor the photometric error (intensity differences).
- ☐ Bundle Adjustment minimizes the photometric error (intensity differences).
- ☒ Bundle Adjustment minimizes the (re-)projection error (Euclidean point distances).

c) The 8-point Algorithm. **(1 credit(s))**

- ☐ The 8-point Algorithm minimizes the photometric error (intensity differences).
- ☐ The 8-point Algorithm minimizes the (re-)projection error (Euclidean point distances).
- ☒ The 8-point Algorithm minimizes neither the (re-)projection error (Euclidean point distances) nor the photometric error (intensity differences).

— The problem continues on the next page —

In the following, the Gauss-Newton method is used to minimize the energy function

$$E(x) = (r(x))^2\,, \qquad x \in \mathbb{R}\,, \qquad r : \mathbb{R} \to \mathbb{R}\,. \tag{4}$$

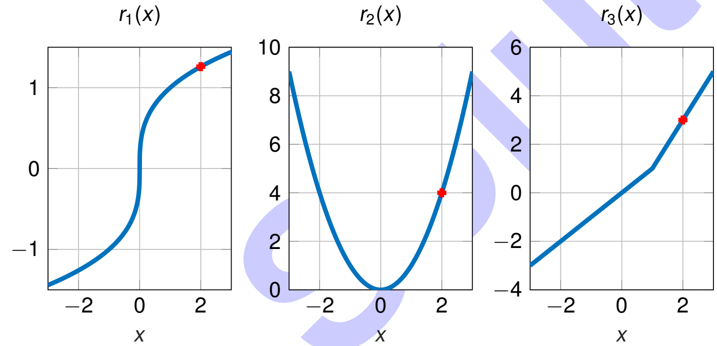
d) For the energy function defined in Equation 4 one step of the Gauss-Newton algorithm is given by $x_{t+1} = x_t + \Delta_t$. We consider points $x_t$ where the derivative $r'(x_t)$ is defined, $r(x_t) \neq 0$, and $r'(x_t) \neq 0$. What is the correct expression for $\Delta_t$ for such points? **(2 credit(s))**

- [ ] $\Delta_t = -\,r'(x_t)\,(r(x_t))^{-1}$
- [ ] $\Delta_t = -\,r'(x_t)\,(r(x_t))^{-2}$
- [ ] $\Delta_t = -\,r(x_t)\,\left(r'(x_t)\right)^{-2}$
- [x] $\Delta_t = -\,r(x_t)\,\left(r'(x_t)\right)^{-1}$

We consider three residual functions given by

$$r_1(x) = \operatorname{sign}(x)\,|x|^{1/3}\,, \qquad r_2(x) = x^2\,, \qquad r_3(x) = \max(x, 2x-1)\,,$$

where sign($x$) gives the sign of $x$. The residual functions and the initial point $x_0 = 2$ are shown below.




The Gauss-Newton algorithm is applied to $E$ in Equation 4 with the three residual functions using the initial point $x_0 = 2$ which yields the following step sequences. Note: For all sequences the increment $\Delta_t$ is well defined.

A: $x_0 = 2\,,\quad x_1 = 1\,,\quad x_2 = 1/2$

B: $x_0 = 2\,,\quad x_1 = -4\,,\quad x_2 = 8$

C: $x_0 = 2\,,\quad x_1 = 1/2\,,\quad x_2 = 0$

e) Which step sequence does the residual function $r_1$ yield? **(2 credit(s))**

- [ ] The step sequence C.
- [ ] The step sequence A.
- [x] The step sequence B.

f) Which step sequence does the residual function $r_2$ yield? **(2 credit(s))**

- [ ] The step sequence C.
- [ ] The step sequence B.
- [x] The step sequence A.

g) Which step sequence does the residual function $r_3$ yield? **(2 credit(s))**

- [ ] The step sequence B.
- [ ] The step sequence A.
- [x] The step sequence C.