$g(t) = 2$
$f(t) = x^2+1$

$\langle g(t), f(t)\rangle = \int_{-1}^{1} g(t)\cdot f(t)\, dt$
$= \int_{-1}^{1} 2\cdot(x^2+1)\, dt$
$= \int_{-1}^{1} 2x^2+2\, dt = [\dots]_{-1}^{1}$
$= a \in \Pi_n$

$\begin{pmatrix}1\\t\\t^2\end{pmatrix}$ $f_1(t) = 1$
$f_2(t) = t$
$f_3(t) = t^2$

$x_1 = \begin{pmatrix}1\\1\\0\\0\end{pmatrix}, \quad x_2 = \begin{pmatrix}1\\0\\-1\\1\end{pmatrix}, \quad x_3 = \begin{pmatrix}0\\0\\1\\1\end{pmatrix}$

$\langle v_i, v_j\rangle = 0$, $i \neq j$
$\langle v_i, v_i\rangle = 1$

$x_1(t) = 1$ $\quad x_2(t) = t$ $\quad x_3(t) = t^2$

Orange: GS mit Polynomen

1.1 $\tilde{v}_1 = \begin{pmatrix}1\\1\\0\\0\end{pmatrix}, \quad \tilde{v}_1 \neq \underline{0}$
$\tilde{v}_1(t) = 1 \quad \tilde{v}_1 \neq 0$

1.2 $v_1 = \frac{\tilde{v}_1}{\|\tilde{v}_1\|} = \frac{1}{\sqrt{1^2+1^2+0^2+0^2}}\begin{pmatrix}1\\1\\0\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\\0\end{pmatrix}$

$v_1(t) = \frac{\tilde{v}_1(t)}{\|\tilde{v}_1(t)\|} = \dots = \underline{\underline{\text{"}2t\text{"}}}$ (Beispiel)

$\|\tilde{v}_1(t)\| = \sqrt{\langle \tilde{v}_1(t), \tilde{v}_1(t)\rangle}$

2.1 $\tilde{v}_2 = x_2 - \langle x_2, v_1\rangle v_1$
$\tilde{v}_2(t) = t - \langle t, \text{"}2t\text{"}\rangle \text{"}2t\text{"}$
$= \begin{pmatrix}1\\0\\-1\\1\end{pmatrix} - \langle \begin{pmatrix}1\\0\\-1\\1\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\\0\end{pmatrix}\rangle \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\\0\end{pmatrix}$
$= \begin{pmatrix}1\\0\\-1\\1\end{pmatrix} - 1\cdot\frac{1}{2}\begin{pmatrix}1\\1\\0\\0\end{pmatrix} = \begin{pmatrix}\frac{1}{2}\\-\frac{1}{2}\\-1\\1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\-1\\-2\\2\end{pmatrix}$

2.2 $\tilde{v}_2 \neq \underline{0}, \quad v_2 = \frac{\tilde{v}_2}{\|\tilde{v}_2\|} = \frac{1}{\frac{1}{2}\sqrt{1^2+(-1)^2+(-2)^2+2^2}}\cdot\frac{1}{2}\begin{pmatrix}1\\-1\\-2\\2\end{pmatrix}$
$= \frac{1}{\sqrt{10}}\begin{pmatrix}1\\-1\\-2\\2\end{pmatrix}$

3.1 $\tilde{v}_3 = x_3 - \langle x_3, v_1\rangle v_1 - \langle x_3, v_2\rangle v_2$
$= \begin{pmatrix}0\\0\\1\\1\end{pmatrix} - \langle\begin{pmatrix}0\\0\\1\\1\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\\0\end{pmatrix}\rangle\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\\0\end{pmatrix} - \langle\begin{pmatrix}0\\0\\1\\1\end{pmatrix}, \frac{1}{\sqrt{10}}\begin{pmatrix}1\\-1\\-2\\2\end{pmatrix}\rangle\frac{1}{\sqrt{10}}\begin{pmatrix}1\\-1\\-2\\2\end{pmatrix}$
$= \begin{pmatrix}0\\0\\1\\1\end{pmatrix}$

3.2 $\tilde{v}_3 \neq \underline{0}$, $v_3 = \frac{\tilde{v}_3}{\|\tilde{v}_3\|} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$

$$ONS = (v_1, v_2, v_3) = \left(\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ -1 \\ -2 \\ 2 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} \right)$$

$P_{\mathbb{R}^4 \to W}(y)$ $\quad y = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

$$= \langle y, v_1 \rangle v_1 + \langle y, v_2 \rangle v_2 + \langle y, v_3 \rangle v_3$$

$$= \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\rangle \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$+ \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ -1 \\ -2 \\ 2 \end{pmatrix} \right\rangle \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ -1 \\ -2 \\ 2 \end{pmatrix}$$

$$+ \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} \right\rangle \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

$$= 2 \cdot \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + 0 \cdot \frac{1}{10} \begin{pmatrix} 1 \\ -1 \\ -2 \\ 2 \end{pmatrix} + 2 \cdot \frac{1}{2} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}}} = y$$