$$C = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$\begin{matrix} 1-\lambda & 0 & -1 & 1-\lambda & 0 \\ 0 & 1-\lambda & 0 & 0 & 1-\lambda \\ -1 & 0 & 1-\lambda & -1 & 0 \end{matrix}$$

a) $\chi_C(\lambda) = \det(C - \lambda E)$

$$= (1-\lambda)^3 - (-1)(-1)(1-\lambda)$$

$$= -\lambda^3 + 3\lambda^2 - 2\lambda$$

$$= \lambda(-\lambda^2 + 3\lambda - 2) \overset{!}{=} 0$$

$$\Leftrightarrow \lambda_1 = 0 \quad \vee \quad 0 = -\lambda^2 + 3\lambda - 2$$

$$\Rightarrow \lambda_2 = 2, \quad \lambda_3 = 1$$

b) $aV(\lambda_1) = aV(\lambda_2) = aV(\lambda_3) = 1$

Beispiel: $\lambda_2 = 2$

$$C - 2\cdot E = \begin{pmatrix} -1 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{pmatrix}$$

Kern berechnen:

$$\left(\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \begin{matrix} z_1 \\ z_2 \\ z_3 - z_1 \end{matrix}$$

RWE:

Sei $x_3 = \mu \in \mathbb{C}$ bel.

$-x_2 = 0 \Rightarrow x_2 = 0$

$-x_1 - x_3 = 0$

$\Rightarrow x_1 = -\mu$

$$E_{C,2} = \left\{\mu \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \mu \in \mathbb{C} \text{ bel}\right\}$$

$gV(\lambda_2) = 1$

$gV(\lambda_1) = gV(\lambda_3) = 1$

Ist C diagonalisierbar? Ja, weil

$gV(\lambda_1) = aV(\lambda_1)$

$gV(\lambda_2) = aV(\lambda_2)$

$gV(\lambda_3) = aV(\lambda_3)$