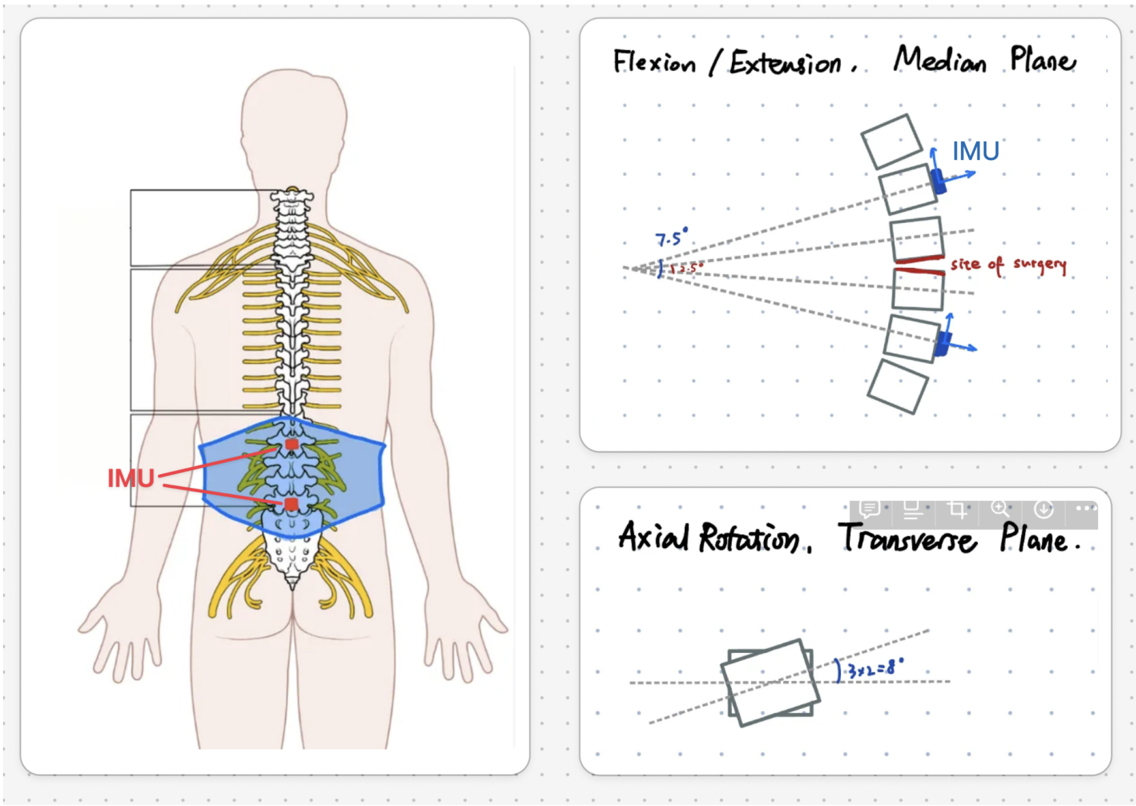
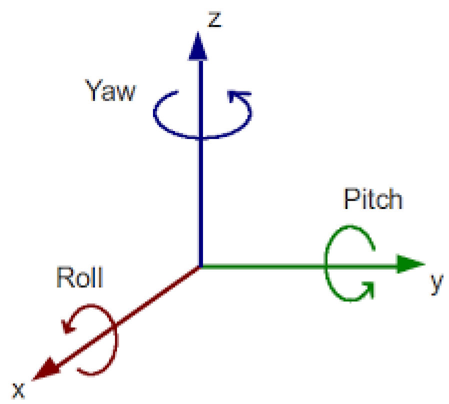
Wearable Sensor for Spine Movement

General Information about this Project

- **Goal:** Using two `IMU sensors` (in red) placed on the spine to monitor the spinal motion and curvature.



- We've done some previous work to have the IMU sensors measure the rotational angles on the `x`, `y`, `z` axis. The angles of rotation are called roll, pitch, yaw, respectfully.

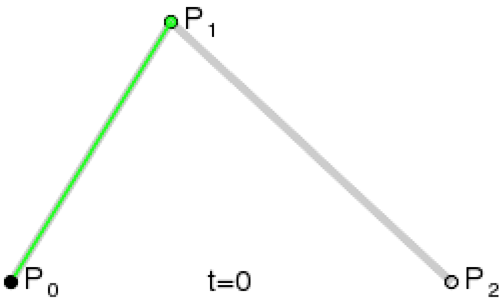


Simulating the Spine Curve

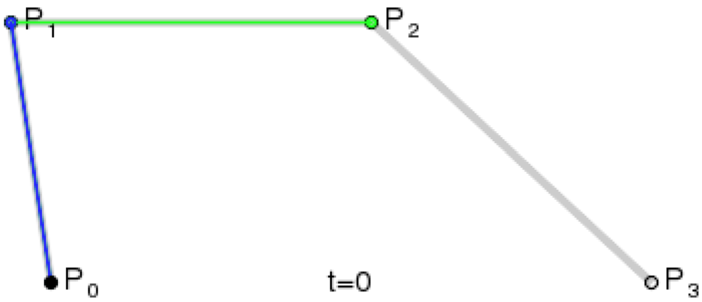
0. Some Background Knowledge -- Bezier Curve

- **What it is:** Parametric Curves defined by a set of **control points**

- **How the curves are drawn:**
  - 3 control points in 2D:



  - 4 control points in 2D:



  - This link demonstrates the Bezier curve output with various control point coordinates using Desmos

- **Mathematical Definition:**
  - The general formula of Bézier curve $\mathbf{B}(t)$ is given by:

    $$\mathbf{B}(t) = \sum_{i=0}^{n} \binom{n}{i} (1 - t)^{n-i} t^i \mathbf{P}_i$$ where t describes the position of the points along the curve, range from 0 to 1.

  - The formula can be expanded as:

    $$\mathbf{B}(t) = (1 - t)^n \mathbf{P}_0 + \binom{n}{1} (1 - t)^{n-1} t \mathbf{P}_1 + \cdots + \binom{n}{n-1} (1 - t) t^{n-1} \mathbf{P}_{n-1} + t^n \mathbf{P}_n, \quad 0 \leq t \leq 1$$

  - The derivative: $$\mathbf{B}'(t) = n \sum_{i=0}^{n-1} b_{i,n-1}(t) (\mathbf{P}_{i+1} - \mathbf{P}_i), \quad 0 \leq t \leq 1$$

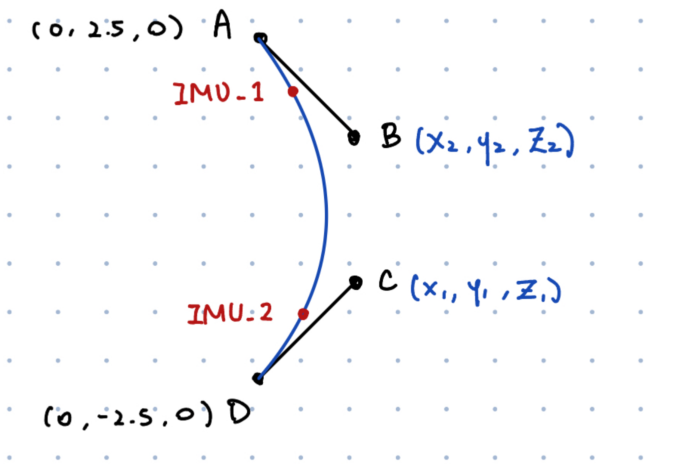
1. Defining Our Problem Space

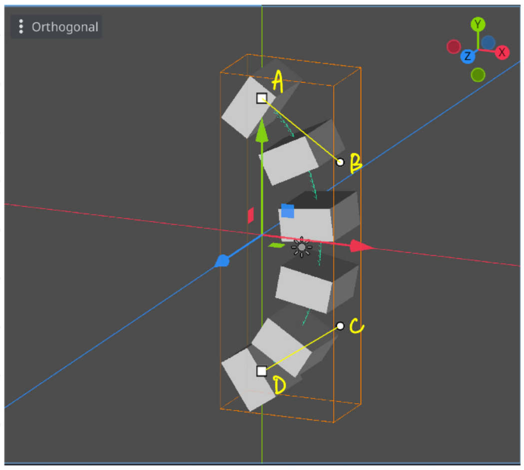
- We aim to simulate the curvature of the spine based on IMU readings.

  Uncertainty 1: I thought that we would have enough information to draw a 4-point Bezier curve yet this might not be true. If this turns out to be causing the issue, I would gladly turn the order of the curve to 3-points.

- For simplification, we treat the selected spine segment as a curved path defined by **four** control points: A, B, C, and D, with IMUs placed at positions corresponding to 20% and 80% along the path.

```
t1 = 0.2, t2 = 0.8
```





- We define the start and end points of the path, A and D, which gives us two points to start with. Then, we work backwards using properties derived from the IMU readings to determine the coordinates of the intermediate control points B and C.
- To summerize the given and unknowns:
    - Given:
        1. The IMU readings `roll, pitch, yaw` at two positions along the curve.
        2. The distances or relative positions of the IMUs (20% and 80% along the curve).
        3. The coordinates of the start and end points, A and D.
            - (the coordinates can be anything, I just happened to be picking `(0, 2.5, 0)` and `(0, -2.5,0)` for simplicity of graphing, we can change this)
    - To find: The coordinates of the intermediate control points B and C.

2. Thought Process in Solving the Problem

- For a 4-point Bezier Curve with points, $\mathbf n=4$, the equation with respect to the four points A, B, C,D is given by:

  $$ \mathbf{B}(t) = (1-t)^3\mathbf{A}+3(1-t)^2t\mathbf{B}+3(1-t)t^2\mathbf{C}+t^3\mathbf{D},\ 0 \le t \le 1 $$

  with derivative:

  $$ \frac{d\mathbf{B}}{dt} = -3(1 - t)^2 \mathbf{A} + 3(1 - t)(1 - 3t) \mathbf{B} + 3t(2 - 3t) \mathbf{C} + 3t^2 \mathbf{D} \ (*) $$

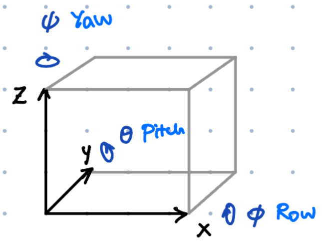
- Here, A and D are known.
- If we are able to get $\frac{d\mathbf{B}}{dt}$ at $\mathbf t=0.2, t=0.8$ through the row, pitch, yaw angles provided by the two IMUs, then the only unknows we are left with will be B, and C.
- We should then be able to solve the problem by solving the two quadratic equations at $\mathbf t=0.2, t=0.8$

Uncertainty 2: The assumption made in here was that the row, pitch, yaw angles we got from the IMU sensors will give us enough information to construct dB/dt, yet this might be false.

3. Current Attempt / Work in Solving the Problem

step 1: Turning roll, pitch, yaw angles from each sensor into $\frac{d\mathbf{B}}{dt}$

1. Construct a rotational matrix from the elementry rotation on each idividual axis



$$\text{Row } R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$\text{Pitch } R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\text{Yaw } R_z(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Combined Rotational Matrix $R = R_z(\psi) \cdot R_y(\theta) \cdot R_x(\phi)$

$$= \begin{bmatrix} \cos(\theta)\cos(\psi) & \cos(\psi)\sin(\phi)\sin(\theta) - \cos(\phi)\sin(\psi) & \sin(\phi)\sin(\psi) + \cos(\phi)\cos(\psi)\sin(\theta) \\ \cos(\theta)\sin(\psi) & \cos(\phi)\cos(\psi) + \sin(\phi)\sin(\theta)\sin(\psi) & \cos(\phi)\sin(\theta)\sin(\psi) - \cos(\psi)\sin(\phi) \\ -\sin(\theta) & \cos(\theta)\sin(\phi) & \cos(\phi)\cos(\theta) \end{bmatrix}$$

2. Multiple the rotation matrix with a basis vector: $$ \mathbf{v}_{\text{rotated}} = R \cdot \mathbf{v},\ \ v = \begin{bmatrix} 1 \ 0 \ 0 \end{bmatrix} $$`

3. I then used $\mathbf{v}_{\text{rotated}}$ as the gradient vector $\frac {d\mathbf{B}}{dt}$

   Uncertainty 3: I'm aware that this step may be wrong but I don't know how to fix it: the roll, pitch, yaw angles gives only the direction of the gradient vector but not the magnitude. I tried normalizing the gradient vector got from this step and use it as dB/dt, but it did't work out and made no logical sense.

step 2: Solving for B and C knowing A, D, $\frac{d\mathbf{B}}{dt}$

1. pluged in $ t = 0.2 $ and $ t=0.8 $ into equation $(*)$

$t = 0.2$:

$$\frac{dB}{dt} = (1-0.2)^3\vec{A} + 3(1-0.2)^2(0.2)\vec{B} + 3(1-0.2)(0.2)^2\vec{C} + (0.2)^3\vec{D}$$

$$\frac{dB}{dt} = -1.92\vec{A} + 0.96\vec{B} + 0.84\vec{C} + 0.12\vec{D}$$

$t = 0.8$:

$$\frac{dB}{dt} = -0.12\vec{A} - 0.84\vec{B} - 0.96\vec{C} + 1.92\vec{D}$$

2. Solved B and C using matrices:

① Isolate $\vec{B}$ & $\vec{C}$

$$0.96\vec{B} + 0.84\vec{C} = \overset{t=0.2}{\frac{dB}{dt}} + 1.92\vec{A} - 0.12\vec{D} = \vec{J}$$

$$-0.84\vec{B} - 0.96\vec{C} = \underset{t=0.8}{\frac{dB}{dt}} + 0.12\vec{A} - 1.92\vec{D} = \vec{K}$$

② Regard equations as matrices:

$$Ax = B, \quad x = A^{-1}B$$

$$A = \begin{bmatrix} 0.96 & 0.84 \\ -0.84 & -0.96 \end{bmatrix} \quad x = \begin{bmatrix} \vec{B} \\ \vec{C} \end{bmatrix} \quad B = \begin{bmatrix} \vec{J} \\ \vec{K} \end{bmatrix}$$

③ Find A^{-1}

$$A^{-1} = \cdots = \begin{bmatrix} 4.44 & 3.89 \\ -3.89 & -4.44 \end{bmatrix}$$

④ Finally solve for $\vec{B}$ & $\vec{C}$ with $x = A^{-1}B$.

4. Current (irrational) Results

- right now, when I test my code by setting the roll, pitch, yaw angles as shown in the picture:

```
func _ready():

    # imu1_readings
    var roll1 = 0 # Example roll angle in radians
    var pitch1 = 0  # Example pitch angle in radians
    var yaw1 = -PI/6 # Example yaw angle in radians

    #imu2_readings
    var roll2 = 0
    var pitch2 = 0
    var yaw2 = PI/6

    var a = Vector3(0, -2.5, 0)  # Example start point
    var d = Vector3(0, 2.5, 0)  # Example end point
```

the gradient vector $\frac {d\mathbf{B}}{dt}$ I got for t = 0.2 and 0.8 are:

tangent_t0_2	dB/dt at t=0.2	
x 0.866	y 0.5	z 0
tangent_t0_8	at t=0.8	
x 0.866	y -0.5	z 0

the final answer calculated for vector B and C are:

b		
x 7.217	y -42.222	z 0
c		
x -7.217	y 42.778	z 0

- This answer is way off, because given the start point and end point, the y-axis value for B and C should be within [-2.5, 2.5].