Simulated Heat Loss to the Environment

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1 Introduction

In this study we executed two different experiments focusing on heat loss to the environment. In the first experiment, we were presented with two different types of thermos cups. The first cup was a normal Bodum thermos, and the second was a product called the Temperfect cup. The Temperfect cup is special in that it brings your hot drink down to drinkable temperature and keeps the drink at the desirable temperature over an extended period of time. In the second experiment, we had two metal blocks stacked on top of each other, both at different temperatures. We analyzed the heat loss from one to the other and the environment until they all were in thermal equilibrium.

2 Method

In our first experiment, we placed thermometers in both cups, filled them up with 3 dl boiling water, and measured the temperature every 0.5 seconds for roughly 112 minutes. Both of the cups have a vacuum insulation, but what makes the Temperfect mug special is that it also has a "Temperfect insulation." The temperfect insulation is a phase-change material(PCM), which changes from solid to liquid as it absorbs heat. During a phase change, the temperature is constant, and in order to change the phase, one must add or remove a large amount of energy from the system. This is what keeps the beverage at a constant temperature over such a long time, when you add a hot liquid to the cup the required energy is removed from the beverage and stored in the PCM.

We wished to numerically approximate the normal bodum mug temperature change behavior, as it is simply only heat loss to the environment. By using these two formulas and setting them together, along with some simplifications and assumptions, we can get a quite nice approximation.

$$AJ_q = \frac{dQ}{dt} = mc_V \frac{dT}{dt} \quad , \quad \vec{J_q} = -\lambda \nabla T$$

Where A is the area(it doesn't apply for us in this example), $\vec{J_q}$ is the heat flux through the wall, dQ the change in heat, dT the change in temperature, c_V is the specific heat capacity of the material at a constant volume, per unit mass, and m is the total mass of the material.

To simplify the problem, we imagined it as a stationary situation in the wall, and replaced ∇T with a ratio of differences $\frac{\Delta T}{\Delta x}$. We then ended up with:

$$-\lambda \frac{\Delta T}{\Delta x} = mc_V \frac{dT}{dt}$$

$\Delta T = T_w - T_a$, the difference between the temperature of the water and air. After a decent amount of reordering and integrating on both sides, we ended up with this:

$$\frac{-\lambda t}{\Delta x mc_V} = ln \frac{\Delta T}{\Delta T_0}$$

And to simplify further:

$$e^{\frac{t}{\tau}} = \frac{\Delta T}{\Delta T_0} \quad , \quad \tau = \frac{\Delta x mc_V}{\lambda}$$

We could then choose a fitting tau and solve for ΔT.

In our second experiment, we visualized the recorded data from the experiment and were given an algorithm to help us approximate the heat transfer in the two block system, but the approximation did not take into account heat loss from the environment. In order to add this to the algorithm, we used some statistical physics and randomly took out some energy from the blocks every step in the program. The amount of energy was decided by the ratio of the specific heat to both of the blocks and from each block and the environment.

3 Results

As one can see in Figure 1, the temperature of the temperfect thermos drops at a much higher rate than the regular at the beginning, and after the initial drop, the temperature almost plateaus. Whereas the bodum cup just gradually drops. After about 40 minutes the temperature of both cups are practically the same and their heat loss is the same until the end of the measurement period due to heat loss to the environment.

If you look at the graph for the Temperfect cup, you can see that the temperature rises until it reaches just under 80 degrees and then quickly drops and flattens out at roughly 63 degrees this is due to the fact that the PCM starts to phase change and then begins to store energy in the form of latent heat, this means that the melting temperature of the PCM is around 63 degrees. It melts for about 4 minutes, and then the temperature drops again and flattens out at around 60.5 degrees, it is then freezing for around 20 minutes. The heat loss to the outside air is exactly the same in both cups because once the PCM freezes again, the two cups lose heat at the exact same rate, and that is solely lost to the air. We did some calculations to find out how much energy the PCM stored.

$$c_V m = \frac{dQ}{dT}, \quad dQ = mc_V dT$$

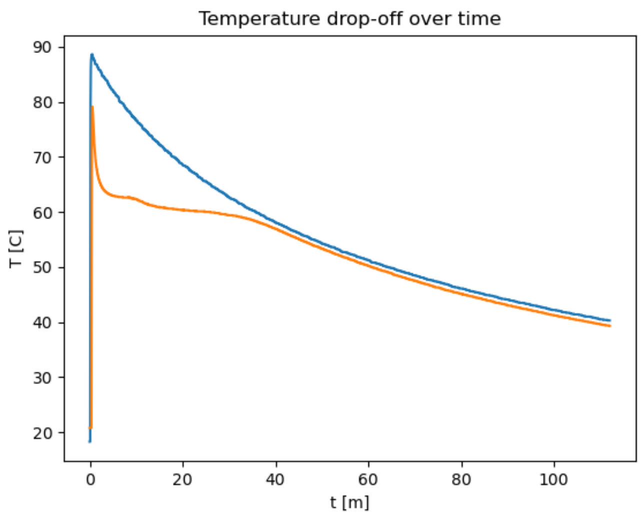

Figure 1:

The specific heat capacity of water is $4186 \frac{kJ}{kg^{\circ}C}$ and we have $0.3kg$ of water in the cup, this means that $c_V m = 4186 \cdot 0.3 = 1,255 \frac{kJ}{^{\circ}C}$. We said that the temperature change was from $80^{\circ}C$ to $63^{\circ}C$

$$dT = 80 - 63 = 17^{\circ}C$$

$$dQ = 1,255 \cdot 17 = 21,235kJ$$

The energy stored in the PCM is then $dQ = 21,235kJ$.

If you look at Figure 2, you can see the experimental data on the top and our approximation on the bottom. In our approximation, the total energy of the system doesn't decrease as it should. We made some adjustments to the code to account for heat loss to the environment, and our approximation instantly looked similar to the experimental data (See Figure 3).

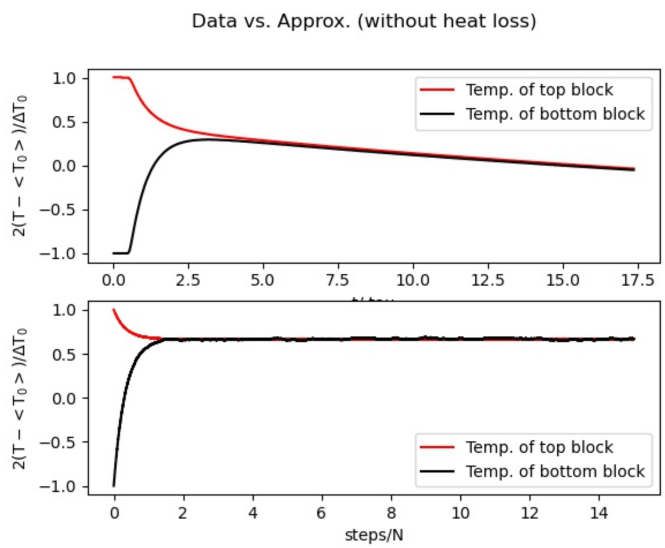

Figure 2:

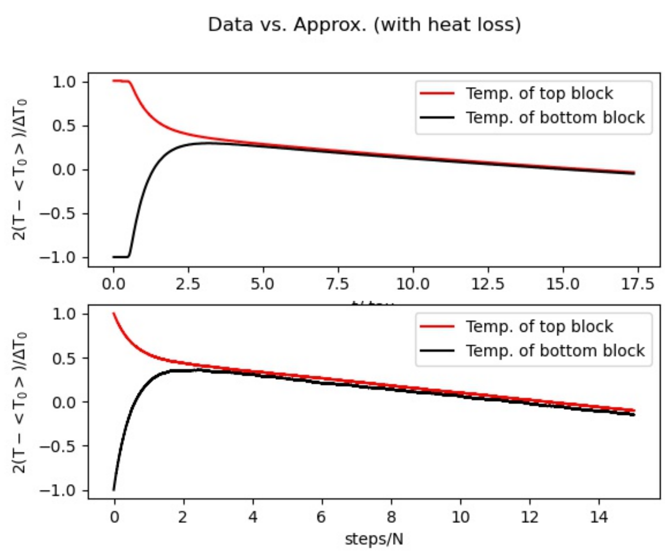

Figure 3:

4 Discussion

In this study, when making our approximations, we had to take many liberties, many of our constants in our algorithms were arbitrarily chosen. We could have put a little more thought into why we were choosing the numbers we did, instead of just choosing the number that made the graph more similar to the actual data graph. Unfortunately, our original idea to simulate the bodum cup's heat loss to the environment never came to fruition, time simply didn't allow it. It also would have been an interesting exercise to use our heat loss algorithm and some if-statements to try to approximate the temperfect mug's behavior.

5 Conclusion

We have analyzed the temperature data from two different types of thermos over time and learned something about phase change materials(PCM). We have conducted an experiment with two metal blocks at different temperatures and watched how they first came into thermal equilibrium with each other, and then with their environment. We then added to the additional model by using statistical models to approximate the heat loss from the environment.

6 Appendix

1)
a) The pressure must increase since the piston is pushing down and decreasing the volume in the thermos.
b) The temperature must also increase since the piston is doing work on the gas and therefore increasing the total energy.
c) If the pressure of the gas increases, and we assume that the gas is not ideal,the particles are then closer together and interacting with each other more, and therefore the temperature increase.
d) As the temperature increases, the randomness increases and with that, the entropy.

2)
a) The pressure of the particles will change while the temperature will stay fixed, so the particles will get closer together without moving faster or increasing temperature.
b) As mentioned previously, the pressure will increase in the system, due to the volume decrease caused by the piston.
c) Again, the piston is decreasing the volume and therefore increasing the pressure.

3)
a) According to the equipartition theorem, if we assume that both the water and the air have the same amount of total energy, and with the fact that water has a higher N amount of particles, the total energy is then spread more throughout those particles, and therefore, the air particles have more energy per molecule.
b) Water has 4 kinetic degrees of freedom, and 3 potential degrees of freedom, a 4:3 ratio, and air has 4 kinetic to 1 potential, a 4:1 ratio, and therefore, water has a higher potential energy per molecule.
c) Due to the same logic in the previous question, air has more kinetic energy per molecule.
d) Due to the concept of thermal equilibrium, the two mediums will always fall into thermal equilibrium.
e) Due to the relationship between pressure, volume, and temperature, since the volume is constant, and the temperature is being increased, the pressure must be proportionally increased, and by definition of pressure, the molecules are being pushed closer together.
f) The ice in the water will remain at 0 degrees throughout the entire phase change, while the water will continue to be heated by the environment. The cold water that is coming out of the ice will sink to the bottom and start a mixing flow with the already heated water. The ice melts because that's what ice does when it's heated.