Combing through noisy data

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We were given data from 5 distant stars. This data included the mass of each star, the wavelength of the light received from the stars, the flux, and the time recorded. We wished to investigate these stars for signs of planets in orbit around them. We looked closely at one of the stars we found, and saw that it had a planet with a mass of $1.9 \times 10^{27} kg$ and a very roughly estimated radius of $64,615.2km$.

I. INTRODUCTION

NASA had sent us data from 5 different distant stars, and wanted us to analyse it, the data included the wavelength of light received and the total flux over time, and the mass of each star. Our data is simplified in the fact that we do not take into account the fact that the earth is moving away or towards the star, so all of our calculations here are mere estimations. Our data is also very noisy, there are many obstructions in Earth's atmosphere. We used the wavelength received to measure the velocity, with use of Doppler's formula. We used the flux to see if we could see any drops in light recieved, where we could then assume a planet was transiting in front of the star. We then used the Least Squares method to 'clean up' the data so that we can make more accurate estimations.

II. METHOD

To calculate the velocity of the stars, we used the wavelength of the light received and the Doppler formula:

$$v_r = \frac{(\lambda - \lambda_0)}{\lambda_0} c \quad , \quad v_* = v_r - v_p$$

Where λ is the recorded wavelength, λ_0 is the wavelength of H_2 molecules, and c is the speed of light. This only gives us the radial velocity though, (one part of the stars velocity around its center of mass). We need to subtract the peculiar velocity (the velocity of the center of mass with respect to the observer) in order to find the proper orbital velocity (the velocity at which the star is rotating around its center of mass. We took the average of all the radial velocities to find the peculiar velocity:

Once we had the orbital velocity, we plotted that against time, as well as the flux from the star against time. The data had many fluctuations and inaccuracies so the graphs were expected to be very noisy, or messy. We looked over all the velocity plots to see if we could decide if a planet was orbiting around the stars and from the flux plots we could tell if they were being eclipsed by the planet, if the flux suddenly drops out of nowhere, then you can see that something is moving between the star and the observer.

Using formula (5) from Hansen:

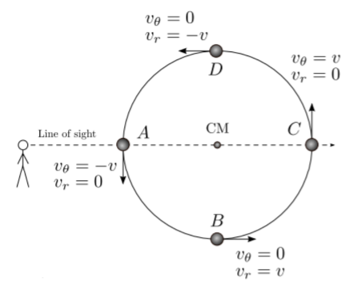


Figure 1.

$$m_p = \frac{m_*^{2/3} v_{*r} T^{2/3}}{(2\pi G)^{1/3} \sin i}$$

We can find the lower limit of the mass of the planets using (5), where m_p is the mass of the planet, m_* is the mass of the star, v_{*r} is the max radial velocity of the star, G being the gravitational constant, and i being the inclination of the solar system in respect to us. Since we have no knowledge of the inclination, we will assume $i = 90°, sin(90°) = 1$ and that gives us the lowest possible mass, since we could only get a smaller number with $i < 90°$, which would inversely make m_p larger.

We found the an estimation of the period of the planet each individual planet T_i by looking at the time between the peaks in the sine-like functions we got from plotting the velocity. We are looking at the peaks because the highest radial velocity represents point B in Figure 1, so the time between the highest velocities is the period. We found the max radial velocity by simply 'zooming in' on the first peak and looking for the highest value.

By looking at the flux of the stars that have planets orbiting, we could look at the drops in received light, and record the amount of time it took to fully eclipse the star Δt. If we know the amount of time it took and the velocity of the planet v_p, we can then discover the radius of the planet R_p. We can find v_p by using the following formula, a form of conservation of momentum:

$$v_p = v_* \frac{m_*}{m_p}$$

Once we had the velocity of the planet, we then know the velocity at which the planet is moving, and the time

Δt, then you know the distance it has travelled, which in this case is the diameter of the planet:

$$2R_p = (v_* + v_p)\Delta t$$

After we calculated the radius of the planet, we could then calculate the density to find out if it is a gas or rock planet:

$$\rho_p = \frac{m_p}{4/3\pi R_p^3}$$

We then wished to use a method that is a little more accurate, The Least Squares method, which is based on finding intervals around certain variables, one then uses these variables in a model function, and finds the difference between the model and their data. The point is to get the least possible difference, and then you have the best possible values for those variables. In this case those variables are the max velocity V_{max}, the time when the velocity could be highest t_0, and the possible period of the star around its center of mass T. We chose these values again by looking at the graph and seeing where the actual values must be. We created a model v_r which we called v_r^{model}:

$$v_p^{model}(t) = v_p cos(\frac{2\pi}{T}(t - t_0))$$

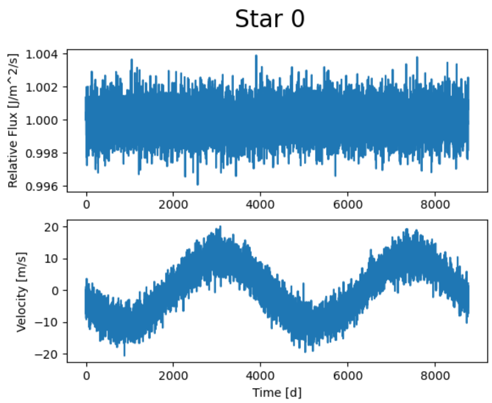
We then subtract each element in our model from the corresponding element in the data we received to see the difference between them. We then square it to get a positive number, and finally we add all the difference together to find the total difference between v_r^{model} and v_r^{data}.

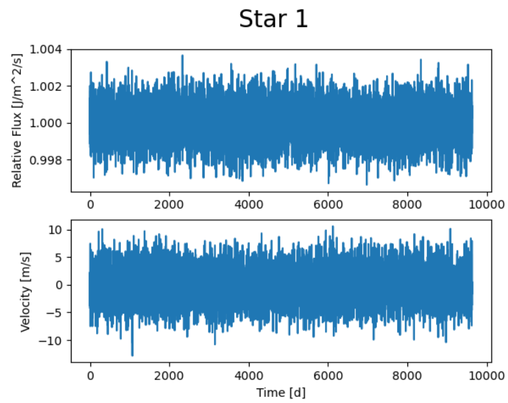
$$\Delta(t_0, T, v_r) = \sum_t (v_r^{data}(t) - v_r^{model}(t, t_0, T, v_r))^2$$

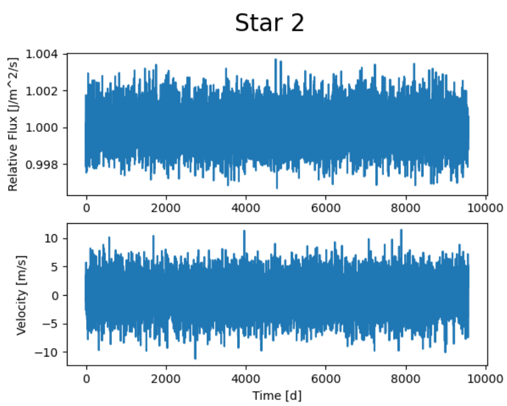
We do this for every possible combination of values (t_0, T, v_r). We wish to find the values that gave us the lowest possible Δ, or to be more specific, the lowest possible difference between every value in v_r^{model} and v_r^{data} squared and summed up, it is squared so that we can't get any negative numbers. Once we had those values we could then use those in Formula (5) from Hansen, what we used to find the mass of the planet first. We then compared those masses to see how accurate our original guess was.

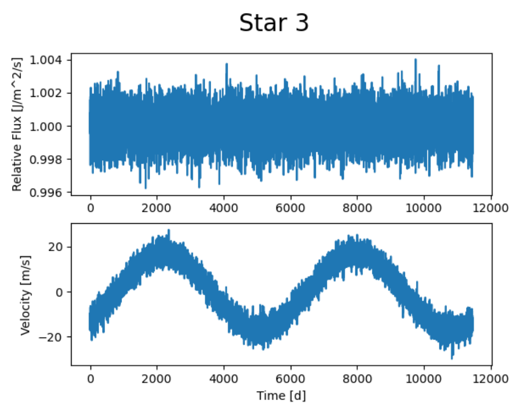
III. RESULTS

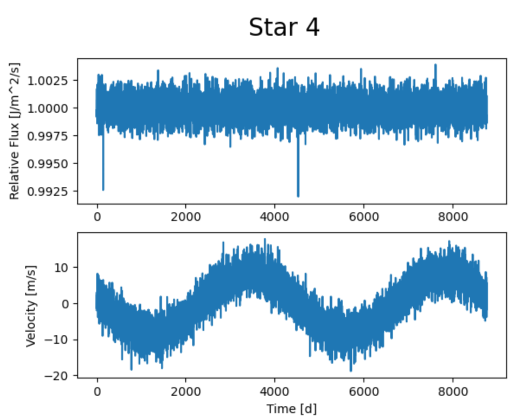
Our plots ended up like this:











As one can see, the velocities of Stars 0, 3, and 4 around their center of mass are oscillating quite a lot which means there must be objects there affecting them. We noticed that there were a couple drops in the flux to Star4, which means that the planet rotating around it must also be eclipsing the star. The rough estimates we made for the periods of Stars 0, 3, and 4 were 4500, 6000, and 5000 days respectfully. The planets around the stars were at least, in order from first to last, 86%, 178%, and 109% the mass of Jupiter, which, for a frame of reference, is $1.89 \times 10^{27} kg$. We only had one star that had a planet fully eclipse it, and that was Star 4. We saw that the flux graph had 2 deep spikes in it, so we zoomed in one of those spikes to see more accurately how long it took for the planet to fully eclipse the star, which was only one day (as you can see in Figure 2), and we didn't have enough data to very accurately predict the planet's radius.

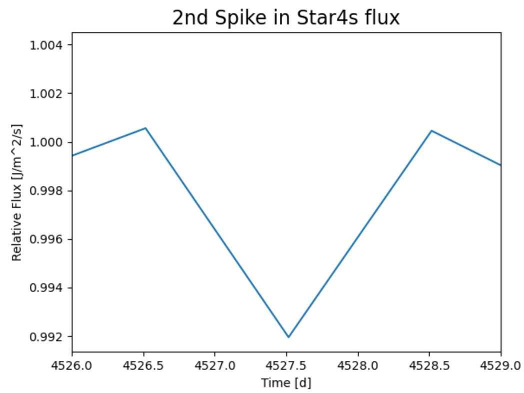


Figure 2.

We did however make a rough estimate, which was $64,615km$. The density of the planet was $1833.33kg/m^3$.

After using the Least Squares method, we found that the best possible values were $t_0 = 3427.59d, T = 4379.31d, v_r = 8.72m/s$. Our least possible delta was $\Delta = 79432.12m/s$. We used the new T and v_r values we found to more accurately estimate the lower limit of the mass of the planet using from Formula (5) from Hansen, which turned out to be $1.9 \times 10^{27}kg$, 7.25% smaller than the first estimate and 101.23% the mass of Jupiter. We then plotted our model over the original graph to see that it looked as it should, and it did, it looked exactly like a nice sinus graph as you can see in Figure (3).

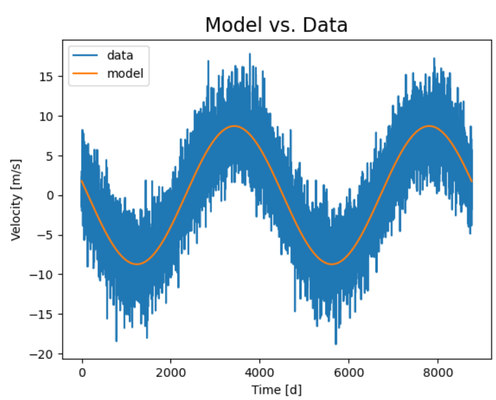


Figure 3.

IV. DISCUSSION

The fact that our planet was 101.23% the mass of Jupiter, had a radius of $64,615km$, which is 92.42% the radius of Jupiter, and that led to a density of $1833.33kg/m^3$,138.26% the density of Jupiter makes a lot of sense. If a planet has a little more mass, and shorter radius, then it will have a higher density by definition. But one must remember that our radius was very roughly estimated (along with the rest of our calculations, since we didn't take into account the velocity the earth is moving in, in relation to the distant star). Also, the fact that $\Delta = 79432.12m/s$ was the best we could get isn't all that great.

V. CONCLUSION

We have used the data we have received to estimate the velocity of stars, the lower limit of the mass of the planet's, and the radius and density of a specific planet. We then used the Least Squares method to more accurately estimate the mass of the specific planet. All of our estimations we very rough estimates due to the fact that we didn't include the velocity at which the earth moves in relation to the stars we were recording.

REFERENCES

[1] Hansen, F. K., 2021, Forelesningsnotat 1c i kurset AST2000