University of Toronto
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Assignment 3: Yen-Chun Wang 1003320934 & Zhijian Wang 1003341010

1. (a) First find the closure for the left side of each dependency
   $LPR^+ = LPQRST$
   $LR^+ = LRST$
   $M^+ = LMO$
   $MR^+ = LMNORST$
   Since none of the closures of the FDs cover every element, all of the FDs violate BCNF.

   (b) Decompose V using FD $MR \rightarrow N$. $MR^+ = LMNORST$
   $T_1 = MPQR$ and $T_2 = LMNORST$
   Since none of the FDs project onto T_1, it is in BCNF and has no FDs.

   Decompose V_2 using FD $M \rightarrow LO$ since M is not a superkey.
   $M^+ = LMO$
   $T_2 = LMO$ and $T_3 = MRST$
   T_2 is in BCNF because the only projected FD is $M \rightarrow LO$
   and M is a superkey for LMO.
   T_3 is also in BCNF because the only projected FD is $MR^+ = ST$
   and MR is a superkey for $MRST$.

   Using $V_1 = T_2, V_2 = T_1, V_3 = T_3$
   Therefore, we end up with 3 tables in BCNF:
   $V_1 = LMO$
   with FD = $\{ M \rightarrow LO \}$
   $V_2 = MPQR$
   with no FDs
   $V_3 = MRST$
   with FD = $\{ MR \rightarrow ST \}$

2. (a) First, split the RHS of each FD in T

1. $AB \to C$
2. $AB \to D$
3. $ACDE \to B$
4. $ACDE \to F$
5. $B \to A$
6. $B \to C$
7. $B \to D$
8. $CD \to A$
9. $CD \to F$
10. $CDE \to F$
11. $CDE \to G$
12. $EB \to D$

Then, check if we can remove elements from left side of FD.

1. $B \to C$ discard A
2. $B \to D$ discard A
3. $ACDE \to B$ no discard
4. $CD \to F$ discard AE
5. $B \to A$ no discard
6. $B \to C$ no discard
7. $B \to D$ no discard
8. $CD \to A$ no discard
9. $CD \to F$ no discard
10. $CD \to F$ discard E
11. $CDE \to G$ no discard
12. $B \to D$ discard E

Then, reduce duplicates to get a new set of FDs.

1. $ACDE \to B$
2. $B \to A$
3. $B \to C$
4. $B \to D$
5. $CD \to A$
6. $CD \to F$
7. $CDE \to G$

Then, check each FD to see if it is redundant.
1. $ACDE \to B$ not redundant.
2. $B \to A$ redundant because $B \to CD$ and $CD \to A$,
   so $B \to A$ is implied even if not specified.
3. $B \to C$ not redundant.
4. $B \to D$ not redundant.
5. $CD \to A$ not redundant.
6. $CD \to F$ not redundant.
7. $CDE \to G$ not redundant.

Therefore, the resulting minimal basis is:
$\{ ACDE \to B, B \to C, B \to D, CD \to A, CD \to F, CDE \to G\}$

(b) Since E never shows up on any right side of the FDs, we know it is part every key.
Since G only shows up on the right side of the FDs, we know it can not be a part of any key.
By calculating the closures of each combination with above conditions, we come up with a key of BE because $BE^+ = ABCDEFG$.

(c) Simply create new tables for each FD of the minimal basis.
$P_1 = ABCDE$
$P_2 = ACDF$
$P_4 = CDEG$
Since the key shows up in P_1, we don't need to add another table.

(d) Since the $B \to CD$ is a non-trivial FD in P_1, but $B^+ = ABCD$ and does not include E, meaning B is not a superkey. This violates BCNF, which means there is redundancy in the table.