

# AMA3020 Pairs Project Formula Derivations

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## 1 Initial Problem

From the equation for the area of a segment;

$$A = \frac{1}{2}r^2(\theta - \sin(\theta)) \quad (1)$$

where  $A$  is the area of the segment,  $r$  is the radius of the circle and  $\theta$  is the central angle.

The central angle can be found using Pythagoras' Theorem to be

$$\theta = 2 \cos^{-1} \left( \frac{h_2}{r} \right) \quad (2)$$

where  $h_2$  is the distance between the center of the circle and the top of the segment.

Using the fact that  $h_2 = r - h_1$ , where  $h_1$  is the height of the segment, we obtain,

$$\theta = 2 \cos^{-1} \left( \frac{r - h_1}{r} \right) \quad (3)$$

We can then combine Eq. 1 and Eq. 3 to obtain a formula for the area,

$$A = r^2 \cos^{-1} \left( \frac{r - h_1}{r} \right) - (r - h_1) \sqrt{2rh_1 - h_1^2} \quad (4)$$

To obtain a formula for the volume we then multiply by the length,

$$V(h) = L \left( r^2 \cos^{-1} \left( \frac{r - h}{r} \right) - (r - h) \sqrt{2rh - h^2} \right) \quad (5)$$

where  $V$  is the volume,  $L$  is the length,  $r$  is the radius and  $h$  is the height of the oil in the tank.