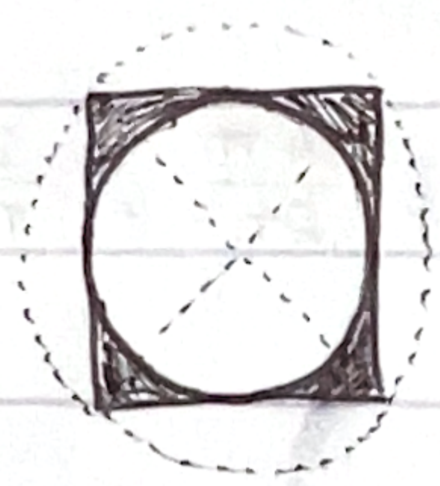
Cyclic Polygons.

— perp bisectors, concurrent.
regular $\Rightarrow$ cyclic.

Tangential/circumscribed polygon $\rightarrow$ circle inside polygon.
(tangency points are concyclic on inscribed circle)

$$\frac{\text{area of circle}}{\text{area of square}} = \frac{\text{perimeter of circle.}}{\text{perimeter of square.}}$$

$x = \frac{\pi}{4+\pi}$



$A_c = \frac{x^2}{4\pi} = 0.0154$

$A_s = \frac{(1-x)^2}{16} = 0.0196$

$\frac{A_c}{A_s} = \frac{11}{14} = 0.7857$

inscribed circle, largest fit inside.
$\rightarrow r = \frac{l}{2}$ radius of incircle.

Excircle $\rightarrow$ $r = \frac{l}{\sqrt{2}}$

Icosagon: 98.36% of circumcircle (excircle)

$x = 0.49793$

$A_c = 0.019729$

$A = 5t^2(1+\sqrt{5}+\sqrt{5+2\sqrt{5}}) = 0.019894$

$t = \frac{1-x}{20}.$

$\frac{A_c}{A_i} = 0.9917$

% of icosagon filled by incircle.

inradius.

$r = 3.1569\, s$
$= 3.1569\left(\frac{1-x}{20}\right)$

$\frac{A_i - A_c}{A_i} = 0.0083\%$ of icosagon not filled by incircle.

proves relationship found from base case gives incircle

$\frac{P_s}{P_c}$

Square: $r = \frac{1}{2}s$
+ incircle.

$\frac{P_c}{P_s}$

$P_c = 2\pi r = a$

$r = \frac{a}{2\pi}$

$A_c = \frac{a^2}{4\pi}$

$P_s = 4s = b$

$s = \frac{b}{4}$

$A_s = s^2 = \frac{b^2}{16}$

Square + excircle: $\underbrace{r = \frac{1}{\sqrt{2}}s}$

$P_c = a \quad P_s = b$

$\frac{1}{\sqrt{2}}s = \frac{a}{2\pi}$

$s = \frac{\sqrt{2}a}{2\pi} = \frac{a}{\sqrt{2}\pi}$

$\frac{b}{4} = \frac{a}{\sqrt{2}\pi}$

$\frac{b}{a} = \frac{4}{\pi\sqrt{2}} = \frac{P_s}{P_c} = \frac{2\sqrt{2}}{\pi}$

$$\left.\begin{aligned} A_c &= \frac{a^2}{4\pi} \\ A_s &= \frac{b^2}{16} = \frac{a^2}{2\pi^2} \end{aligned}\right\} \quad \frac{A_s}{A_c} = \frac{\cancel{a^2}}{2\pi^2} \cdot \frac{4\pi}{\cancel{a^2}} = \frac{4\pi}{2\pi^2} = \frac{2}{\pi}$$

$$\boxed{\frac{A_s}{A_c} = \frac{1}{\sqrt{2}}\frac{P_s}{P_c}}$$