Prove that $det(M) = det(A) \times det(C)$.

Case 1: Sryfose det (A) = 0.

This means that there exist a column [A] of A that can be expressed as a linear combination of the other columns of A. Since all the columns of Om, q are identical (filled with zeros), the column j of M can be expressed as a linear combination of the other first of columns of M (with the same weights as the previous linear combination for [A]j). So det (M) = 0 and indeed det (M) = det (A) det (C).

Case 2 Suffose det (A) \$ 0. The inverse of A exists.

We remark that M can be written as:

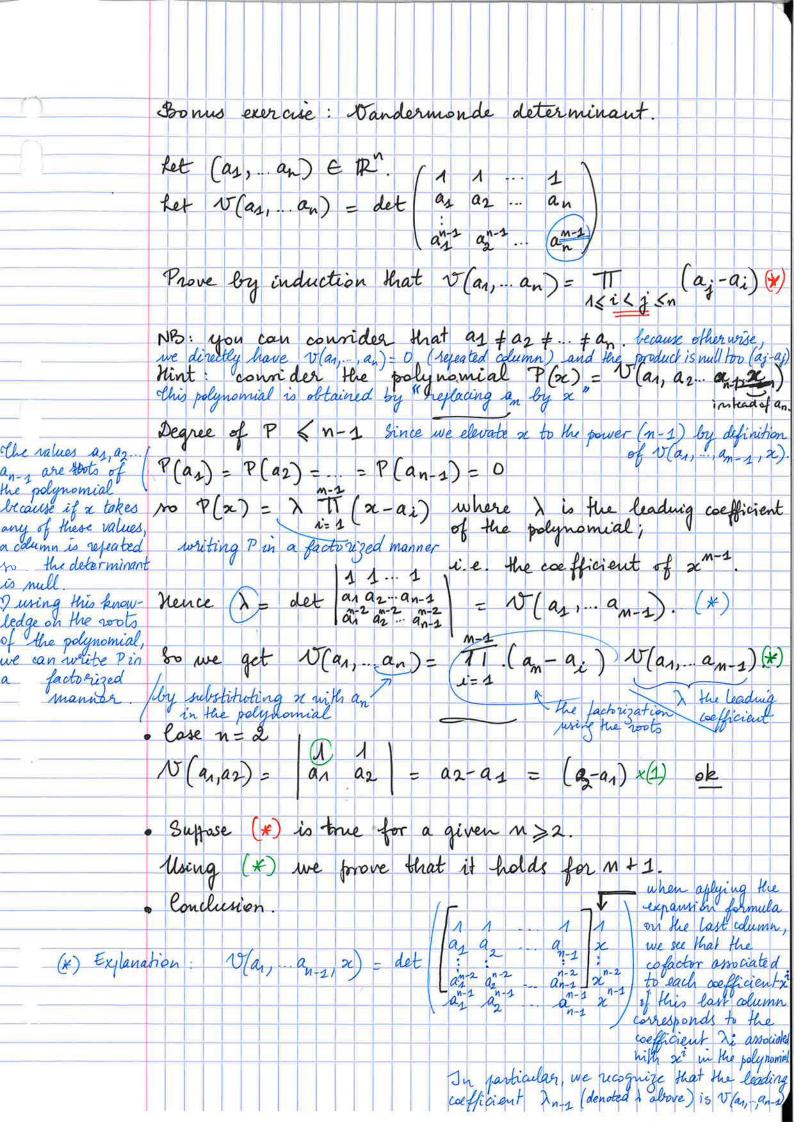
mark that M can be when his.

$$M = \left(\begin{array}{c|c} A & o \\ \hline O & I_P \end{array}\right) \left(\begin{array}{c|c} I_m & A^TB \\ \hline O & C \end{array}\right)$$
, and $det(M) = det\left(\begin{array}{c} A & O \\ \hline O & I_P \end{array}\right) det\left(\begin{array}{c} I_m & A^TB \\ \hline O & C \end{array}\right)$

by expanding along the last column, then forelast column etc. of times, we obtain det (A)

by expanding along the first column, then second colum... in times, we obtain det (c)

so det (M) = def (A) det (C).



Proof by induction Proving the "herioditary" projecty: Reminder: (i) we have proven the property "manually" in the case n=2ii) we suffice that the property is true for a given n > 2 i.e for a given $n \ge 2$, $N(a_1,...,a_n) = \prod_{1 \leq i \leq j \leq n} (a_j \cdot a_i)$. Now we consider of (as, ..., an, an, a). Using (x): 15 (az, an, an+1) = 15 (az, an) 15 (az, az Using . we can write $10(a_1, a_n, a_{n+2}) = 11$ $15(a_j + a_i)$ $15(a_j + a_i)$ $15(a_n + a_i)$ = 1 $(a_j - a_i)$ So the property is true for m+1 hii) finally, we conclude that the property is true for all n > 2