Brief theory guide for the *propulsion systems for aircraft* subject

1 Component numeration

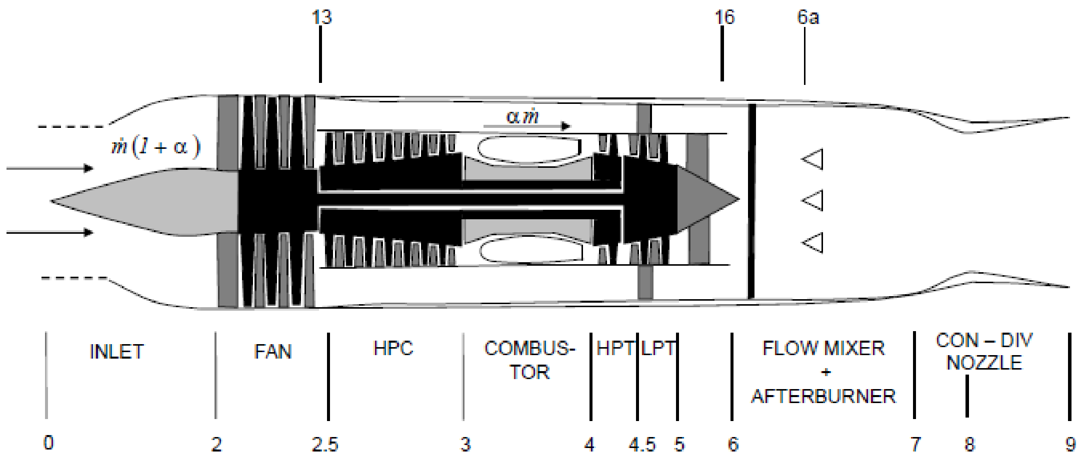

Figure 1: Jet engine sections numeration

$$\pi = \tau^{\frac{\gamma}{\gamma-1}}$$

$$\tau_r = \frac{T_{0t}}{T_0} = 1 + \frac{\gamma-1}{2}M_0^2 \qquad \pi_r = \frac{P_{0t}}{P_0} = \left[1 + \frac{\gamma-1}{2}M_0^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\tau_d = \frac{T_{2t}}{T_{0t}} \qquad \pi_d = \frac{P_{2t}}{P_{0t}}$$

$$\tau_f = \frac{T_{2.5t}}{T_{2t}} \qquad \pi_f = \frac{P_{2.5t}}{P_{2t}}$$

$$\tau_{comp} = \frac{T_{3t}}{T_{2.5t}} \qquad \pi_{comp} = \frac{P_{3t}}{P_{2.5t}}$$

$$\tau_c = \frac{T_{3t}}{T_{2t}} = \tau_f \cdot \tau_{comp} \qquad \pi_c = \frac{P_{3t}}{P_{2t}} = \pi_f \cdot \pi_{comp}$$

$$\tau_\lambda = \frac{T_{4t}}{T_0} \qquad \pi_\lambda = \frac{P_{4t}}{P_0}$$

$$\tau_{t,H} = \frac{T_{4.5t}}{T_{4t}} \qquad \pi_{t,H} = \frac{P_{4.5t}}{P_{4t}}$$

$$\tau_{t,L} = \frac{T_{5t}}{T_{4.5t}} \qquad \pi_{t,L} = \frac{P_{5t}}{P_{4.5t}}$$

$$\tau_{\lambda ab} = \frac{T_{7t}}{T_0} \qquad \pi_{\lambda ab} = \frac{P_{7t}}{P_0}$$

$$\tau_n = \frac{T_{9t}}{T_{5t}} \qquad \pi_n = \frac{P_{9t}}{P_{5t}}$$

2 Thermodynamic equilibrium

Compressor-turbine equilibrium:

$$W_c = W_{t,H} \rightarrow \dot{m}_0 \cdot C_P \cdot (T_{3t} - T_{2.5t}) = \dot{m}_0 \cdot (1+f) \cdot C_P \cdot (T_{4t} - T_{4.5t})$$

$$\tau_r \cdot (\tau_c - \tau_f) = (1+f) \cdot \tau_\lambda \cdot (1 - \tau_{t,H})$$

Fan-turbine equilibrium:

$$W_f = W_{t,L} \rightarrow \dot{m}_0 \cdot (1+\alpha) \cdot C_P \cdot (T_{2.5t} - T_{2t}) = \dot{m}_0 \cdot (1+f) \cdot C_P \cdot (T_{4.5t} - T_{5t})$$

$$(1+\alpha) \cdot \tau_r \cdot (\tau_f - 1) = (1+f) \cdot \tau_\lambda \cdot \tau_{t,H} \cdot (1 - \tau_{t,L})$$

Combustion chamber (burner) equilibrium:

$$Q_b = Q_h \rightarrow \dot{m}_0 \cdot (1+f) \cdot C_P \cdot T_{4t} - \dot{m}_0 \cdot C_P \cdot T_{3t} = \dot{m}_0 \cdot f \cdot h$$

$$\dot{m}_f = \frac{C_P \cdot T_0}{h} \cdot \dot{m}_0 \cdot ((1+f) \cdot \tau_\lambda - \tau_c \tau_r)$$

3 Thrust equation

$$F = (\dot{m}_0 + \dot{m}_f) \cdot u_9 + \dot{m}_0 \cdot \alpha \cdot u_{1.9} - \dot{m}_0 \cdot u_0 + A_9 \cdot (P_9 - P_0) + A_{1.9} \cdot (P_{1.9} - P_0)$$

$$\frac{F}{\dot{m}_0 \cdot a_0} = (1+f) M_9 \sqrt{\frac{T_9}{T_0}} - M_0 + \alpha \left[M_{1.9} \sqrt{\frac{T_{1.9}}{T_0}} - M_0 \right] + \frac{1+f}{M_9 \gamma} \sqrt{\frac{T_9}{T_0}} \left[1 - \frac{P_0}{P_9} \right] + \frac{\alpha}{M_{1.9} \gamma} \sqrt{\frac{T_{1.9}}{T_0}} \left[1 - \frac{P_0}{P_{1.9}} \right]$$

4 Nozzle

$$M_9 = \sqrt{\frac{2}{\gamma - 1} \left[\frac{P_{9t}}{P_9} - 1 \right]^{\frac{\gamma-1}{\gamma}}}$$

- $P9 \geq P_0$ is **always** accomplished.
- Simple convergent nozzle:
  - If $M_9 > 1$, the fluid can't be expanded at all. Take $M_9 = 1$ and recalculate $\frac{P_{9t}}{P_9}$
  - If $M_9 < 1$, the fluid is expanded at all and $P_9 = P_0$
  - It's possible that the exhaust velocity becomes $M_9 = 1$ but the fluid not be expanded at all ($P_9 \neg P_0$). This is caused because the nozzle is not large enough.
  - The ideal convergent nozzle expands the fluid until $M_9 = 1$ and $P_9 = P_0$.
- Convergent-divergent nozzle: $M9 > 1$ in the throat (only is the throat area is enough small).

5 MPF

MPF or *mass flow parameter* is a parameter that relates the properties of a confined fluid. This parameter allow us relate the properties of a fluid in different stages of a motor; or relates the properties of the same stage but in different actuations.

$$\dot{m} = \bar{m}\frac{P_t A}{\sqrt{T_t R}} \qquad \bar{m} = \sqrt{\gamma}M\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{-(\gamma+1)}{2(\gamma-1)}}$$

6 Actuations

When the flight regime is modified ($M_0;T_0;P_0$), all the engine parameters are also modified. But it is necessary that we obtain some appropriate results. We can consider that changes on some parameters are very low (quasi-equal parameters); and other parameters suffer major changes.

- This parameters refers to the flight regime: $T_{t4};\theta_0;\theta_t$
- Consider that this parameters maintains constant it's values: $\tau_{t,H};\tau_{t,L}$
- Consider that this parameters changes it's values: $\tau_f;\tau_c;\alpha$

7 Turbo-propeller

The turboprop is a turbojet that moves a propeller. The main difference is that the thrust is not obtained by the jet exhaust; it is obtained by the propeller. The optimization of a turboprop must be done through the power, not the thrust. We consider total gas expansion ($P_9 = P_0$) and $1+f \approx 1$

$$C_{cin} = \frac{F\cdot v_0}{\dot{m}_0\cdot C_P\cdot T_0} = M_0(\gamma-1)\left[\sqrt{\frac{2}{\gamma-1}\left(\tau_{t,L}(\tau_\lambda+\tau_r-\tau_r\tau_c)\right)-\frac{\tau_\lambda}{\tau_c\tau_r}}-M_0\right]$$

$$C_{prop} = \frac{\dot{W}_{prop}\cdot\eta_{prop}}{\dot{m}_0\cdot C_P\cdot T_0} = \eta_{prop}\cdot(1-\tau_{t,L})(\tau_\lambda+\tau_r-\tau_r\tau_c)$$

$$C_{Tot} = C_{cin}+C_{prop}$$

8 Mixer

The exhaust flux from a gas mixer is the sum of the main and the secondary fluxes:

$$\dot{m}_7 = \dot{m}_5 + \dot{m}_{1.6} = (1+\alpha+f)\dot{m}_0$$

The total temperature at the exhaust of the mixer T_{7t} is obtained as an average temperature from the main and the secondary fluxes temperatures:

$$\dot{m}_7 \cdot C_{P,7} \cdot T_{7t} = \dot{m}_6 \cdot C_{P,6} \cdot T_{6t} + \dot{m}_{1.6} \cdot C_{P,1.6} \cdot T_{1.6t}$$

$$\alpha' = \frac{\dot{m}_{1.6}}{\dot{m}_6} = \frac{\alpha}{1+f} \qquad R_7 = \frac{R_6 + \alpha' R_{1.6}}{1+\alpha'}$$

$$C_{P,7} = \frac{C_{P,6} + \alpha' C_{P1.6}}{1+\alpha'} \qquad \gamma_7 = \frac{C_{P,7}}{C_{P,7} - R_7}$$

The static pressure is considered constant along the mixer:

$$P_7 = P_6 = P_{1.6}$$

The total pressure at the mixer exhaust is obtained through the momentum equilibrium on the flux (equations below are simplified):

$$I = \dot{m}u + PA = \dot{m}\sqrt{\frac{RT_t}{\gamma\Phi}}$$

$$\Phi = \left[\frac{M\sqrt{1+\frac{\gamma-1}{2}M^2}}{1+\gamma M^2}\right]^2$$

$$I_6 + I_{1.6} = I_7$$

From the equation on I we can obtain the value of M_7 and after that calculate the value of P_{7t}

9 Real motor

A real motor has some power loses caused by the thermodynamic approximation (we consider that the mechanical efficiency of the motor is 1). There are two different ways to calculate the efficiency of the motor:

- Design criteria: We want to design a motor that produces the thrust required. In this case, total pressures of all components are calculated as the ideal case, but the temperature is affected by the inefficiency.
- User criteria: The motor is designed and constructed. In this case we must to wear the motor with sensors and calculate the inefficiencies of the components.

$$\eta_c = \frac{T_{3t}^{real} - T_{2.5t}^{id}}{T_{3t}^{id} - T_{2.5t}^{id}}$$

$$\eta_t = \frac{T_{4t}^{id} - T_{4.5t}^{id}}{T_{4t}^{id} - T_{4.5t}^{real}}$$

10 Efficiencies

Thermal efficiency

$$\eta_t = \frac{\Delta E_c}{\dot{m}_f h} = \frac{0.5(1+f)\dot{m}_0 u_9^2 - 0.5\dot{m}_0 u_0^2}{f\dot{m}_0 h} = 1 - \frac{1}{\theta_0 \tau_c}$$

Propulsive efficiency

$$\eta_p = \frac{F u_0}{\Delta E_c} = \frac{u_0(m_0(1+f)u_9 - \dot{m_0}u_0 + A_9(P_9 - P_0))}{0.5(1+f)\dot{m}_0 u_9^2 - 0.5\dot{m}_0 u_0^2} = \frac{2}{u_0 + u_9}$$

Total efficiency

$$\eta = \eta_t \cdot \eta_p$$

Specific consume

$$s = \frac{\dot{m}_f}{F} = \frac{f}{a_0\left(\frac{F}{\dot{m}_0 a_0}\right)}$$