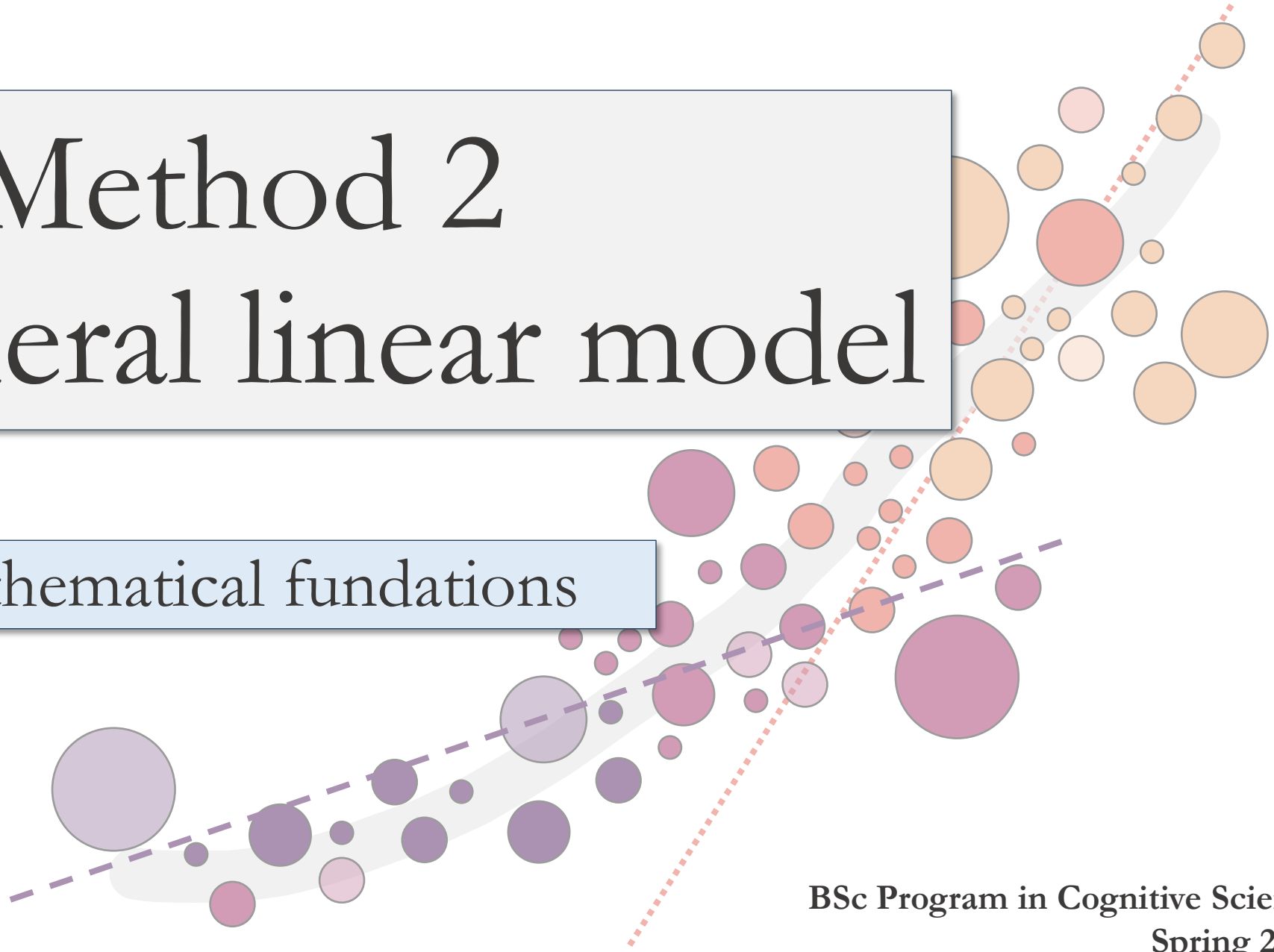
Method 2
The general linear model

Mathematical fundations



BSc Program in Cognitive Science
Spring 2024

Nicolas Legrand – Assistant professor – ILAB (Interacting Minds Centre)
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Math basics

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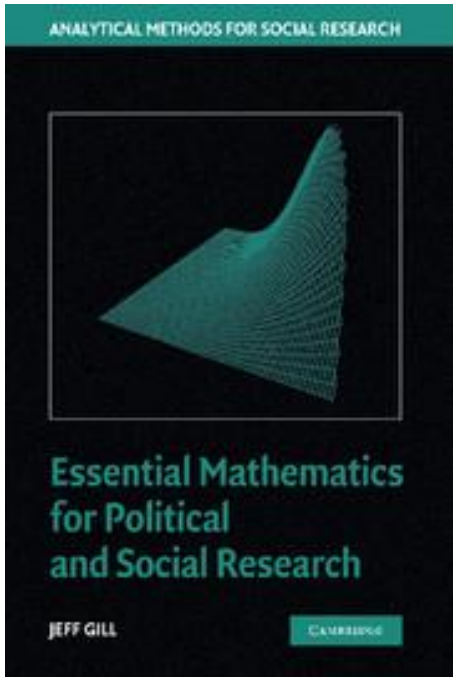
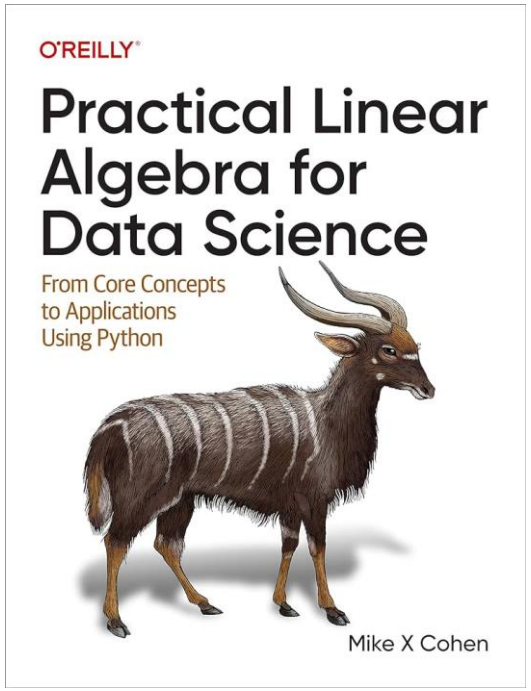
Resources

[3blue1brown]

The Bright Side of Mathematics





Textbook:
Gill, J. (2006). Essential Mathematics for Political and Social Research (Analytical Methods for Social Research). Cambridge: Cambridge University Press. doi:10.1017/CBO9780511606656
No need to buy it! You have access to PDFs of the chapters via the Royal Library.

- This course is on YouTube!

https://www.youtube.com/playlist?list=PLvJwKACYy5_MTdnrzxx_1sN389dS9OB3S

Powers

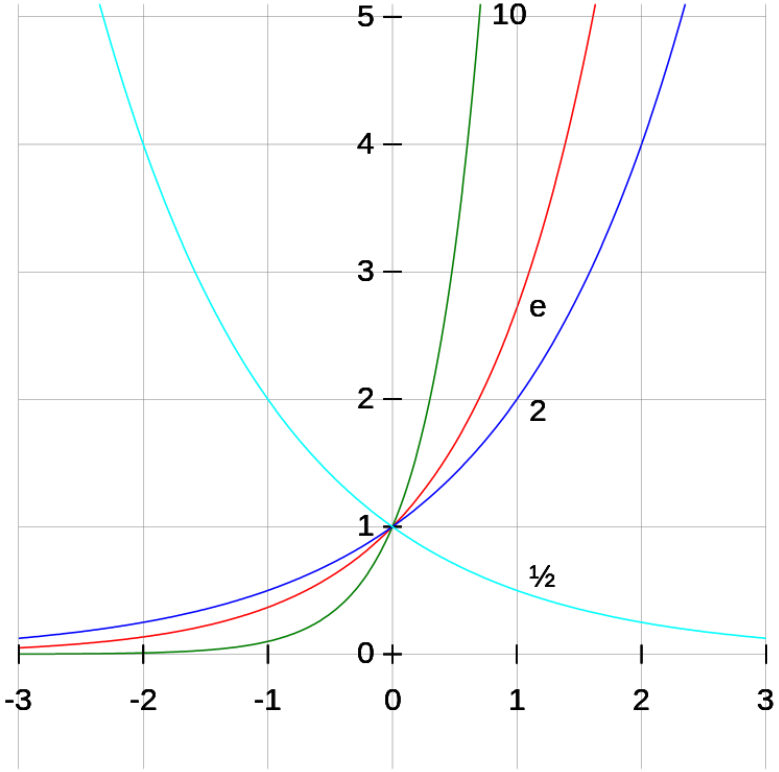
power

b^n

base

When *n* is a positive integer, exponentiation corresponds to repeated multiplication of the base.

$$b^n = \underbrace{b \times b \times \cdots \times b \times b}_{\text{n times}}$$



Zero property

$$x^0 = 1$$

One property

$$x^1 = x$$

Power notation

$$power(x, a) = x^a$$

Nested Exponents

$$(x^a)^b = x^{ab}$$

Product Property

$$x^a * x^b = x^{a+b}$$

Fraction Property

$$\left(\frac{x}{y}\right)^a = \left(\frac{x^a}{y^a}\right) = x^a y^{-a}$$

Distributive Property

$$(xy)^a = x^a y^a$$

Ratio Property

$$x^{\frac{a}{b}} = (x^a)^{\frac{1}{b}} = \left(x^{\frac{1}{b}}\right)^a = \sqrt[b]{x^a}$$

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Logarithms

The logarithm of a number x to the base b is the exponent to which b must be raised to produce x.

$$y = \log_b x \leftrightarrow x = b^y$$

Exponentiation

$$\log_b(b^x) = b^{log_b(x)} = x$$

Zero/one

$$\log_b(1) = 0$$

Division

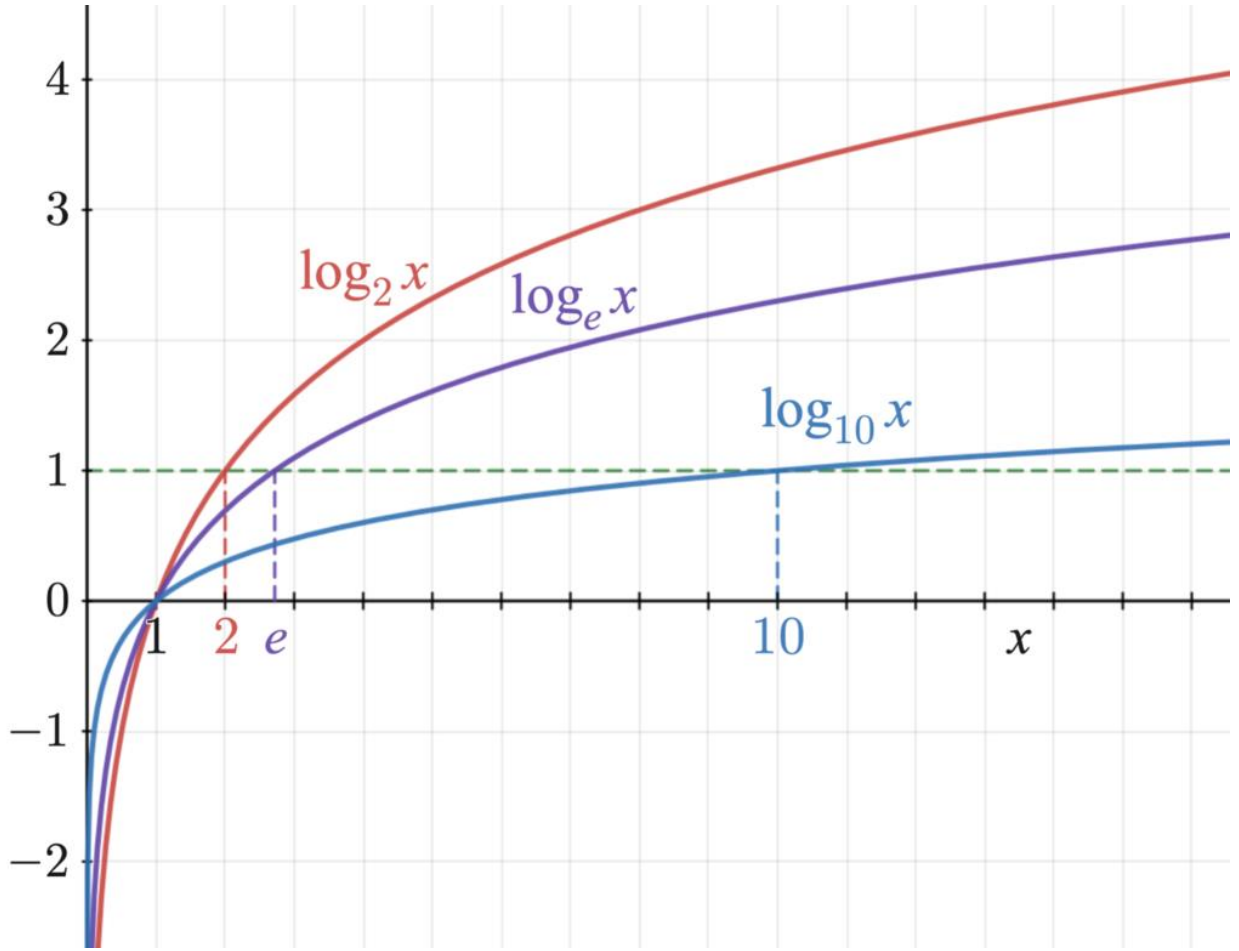
$$\log\left(\frac{x}{y}\right) = \log(x) - \log(y)$$

Multiplication

$$\log(x * y) = \log(x) + \log(y)$$

Changing base

$$\log_a x = \frac{\log_b x}{\log_b a}$$



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Find the value of y

a. $\log_8 \frac{1}{64} = y$ **b.** $\log_6 36 = y$ **c.** $\log_7 1 = y$ **d.** $\log_4 \frac{1}{4} = y$

Write the following expressions in terms of logs of x, y and z.

e. $\log(xy)^{\frac{1}{3}}$ **f.** $\log x\sqrt{z}$ **g.** $\log \frac{x^3y^2}{z}$ **h.** $\log \frac{\sqrt{x}\cdot\sqrt[3]{y^2}}{z^4}$

True or False?

i. $\frac{\log a}{\log b} = \log(a-b)$ **j.** $\log(a-b) = \log a - \log b$ **k.** $-\ln\left(\frac{1}{x}\right) = \ln x$

Answers

a. -2

b. 2

c. 0

d. -1

e. $\frac{1}{3}\log x + \frac{1}{3}\log y$

f. $\log x + \frac{1}{2}\log z$

g. $3\log x + 2\log y - \log z$

h. $\frac{1}{2}\log x + \frac{2}{3}\log y - 4\log z$

i. $FALSE$

j. $FALSE$

k. $TRUE$