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CogSci Methods 2 – F24

Solutions to class 4 exercises

Exercises

RULES USED

(Approximately on the line where it has been applied)

1.1 Simplify the expressions as much as possible

a. $(-x^4y^2)^2$

$$(-x^4y^2)^2$$
$$= (-1)^2(x^4y^2)^2$$

Factorising -1 out and using that $(ab)^2 = a^2b^2$

$$= (x^4y^2)^2$$

Using that $(-1)^2 = 1$

$$= (x^4)^2(y^2)^2$$

Using that $(ab)^2 = a^2b^2$

$$= x^{4*2}y^{2*2}$$

Using that $(a^n)^m = a^{nm}$

$$= x^8y^4$$

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b. $9(3^0)$

$$9(3^0)$$
$$= 9 * 1$$

Using zero exponent rule $a^0 = 1$

$$= 9$$

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c. $(2a^2)(4a^4)$

$$(2a^2)(4a^4)$$
$$= 2 * 4a^{2+4}$$

Using the exponent rule $a^b * a^c = a^{b+c}$

$$= 2 * 4a^6$$
$$= 8a^6$$

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d. $\frac{x^4}{x^3}$

$$\frac{x^4}{x^3}$$
$$= x^{4-3}$$

Using the exponent rule $\frac{x^a}{x^b} = x^{a-b}$

$$= x^1$$
$$= x$$

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e. $(-2)^{7-4}$

$$(-2)^{7-4}$$
$$= (-2)^3$$
$$= -2^3$$
$$= -8$$

Using the exponent rule $(-a)^n = -a^n$ if n is odd

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f. $\left(\frac{1}{27b^3}\right)^{\frac{1}{3}}$

$$\left(\frac{1}{27b^3}\right)^{\frac{1}{3}}$$

$$= \left(\frac{1^{\frac{1}{3}}}{(27b^3)^{\frac{1}{3}}}\right)$$

Using the exponent rule $\left(\frac{\mathrm{a}}{\mathrm{b}}\right)^c = \frac{a^c}{b^c}$

$$= \frac{1^{\frac{1}{3}}}{27^{\frac{1}{3}}(b^3)^{\frac{1}{3}}}$$

Using exponent rule $(a * b)^n = a^n b^n$ on denominator

$$= \frac{1^{\frac{1}{3}}}{3^{3*\frac{1}{3}}(b^3)^{\frac{1}{3}}}$$

Factorising the number $27 = 3^3$

$$= \frac{1^{\frac{1}{3}}}{3(b^3)^{\frac{1}{3}}}$$

Using exponent rule $(a^b)^c = a^{bc}$ for a ≥0

$$= \frac{1^{\frac{1}{3}}}{3b}$$

Using it again here on $(b^3)^{\frac{1}{3}}$

$$\frac{1}{3b}$$

Finally, using that $1^{\frac{1}{3}} = 1$

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g. $y^7y^6y^5y^4$

$$y^7y^6y^5y^4$$
$$= y^{7+6+5+4}$$
$$= y^{22}$$

Using the exponent rule that

$a^b * a^c = a^{b+c}$

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h. $\frac{\frac{2a}{7b}}{\frac{11b}{5a}}$

$$\frac{\frac{2a}{7b}}{\frac{11b}{5a}}$$
$$= \frac{2a * 5a}{7b * 11b}$$
$$= \frac{10a^2}{77b^2}$$

Using the fraction rule
$$\frac{\frac{\mathrm{a}}{\mathrm{b}}}{\frac{\mathrm{c}}{\mathrm{d}}} = \frac{a * d}{b * c}$$

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i. $(z^2)^4$

$$(z^2)^4$$
$$= z^{2*4}$$
$$= z^8$$

Using that $(a^n)^m = a^{nm}$

RULES USED

(Approximately on the line where it has been applied)

1.2 Simplify

a. $(a+b)^2+(a-b)^2+2(a+b)(a-b)-3a^2$

$$(a+b)^2+(a-b)^2+2(a+b)(a-b)-3a^2$$

We can take them part by part first:

$$(a+b)^2=a^2+b^2+2ab$$

$$(a-b)^2=a^2+b^2-2ab$$

$$2(\mathrm{a}+\mathrm{b})(\mathrm{a}-\mathrm{b})=2a^2-2b^2$$

Now we can continue:

$$=\mathrm{a}^2+\mathrm{b}^2+2\mathrm{ab}+\mathrm{a}^2+\mathrm{b}^2-2\mathrm{ab}+2\mathrm{a}^2-2\mathrm{b}^2-3\mathrm{a}^2$$

$$=2\mathrm{a}^2+2\mathrm{b}^2+2\mathrm{a}^2-2\mathrm{b}^2-3\mathrm{a}^2$$

$$=4\mathrm{a}^2-3\mathrm{a}^2$$

$$=\mathrm{a}^2$$

1.3 Solve the following expressions

a. $\sqrt[3]{2^3}$

$$\sqrt[3]{2^3}$$

$$=2$$

Using radical rule: $\sqrt[\mathrm{n}]{\mathrm{a}^{\mathrm{n}}}=a$ for $a\geq 0$

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b. $\sqrt[3]{27}$

$$\sqrt[3]{27}$$

$$=\sqrt[3]{3^3}$$

Factorising $27=3^3$

$$=3$$

Using radical rule: $\sqrt[\mathrm{n}]{\mathrm{a}^{\mathrm{n}}}=a$ for $a\geq 0$

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c. $\sqrt[4]{625}$

$$\sqrt[4]{625}$$

$$=\sqrt[4]{5^4}$$

Factorising $27=3^3$

$$=5$$

Using radical rule: $\sqrt[\mathrm{n}]{\mathrm{a}^{\mathrm{n}}}=a$ for $a\geq 0$

1.4 The relationship between Fahrenheit and Centigrade can be expressed as 5f – 9c = 160. Show that this is a linear function by putting it in y = mx + b format with c = y. Graph the line indicating slope and intercept.

To express the relationship $5f - 9c = 160$ in the y=mx+b format, where c=y, we first aim to solve for c in terms of f. This will allow us to clearly see the linear relationship between Fahrenheit (f) and Centigrade (c).

Given:

$$5f - 9c = 160$$

We can rearrange this equation to solve for c like this:

$$9c = 5f - 160$$

Now, dividing both sides by 9 to isolate c:

$$c = \frac{5}{9}f - \frac{160}{9}$$

This puts the equation in the form y=mx+b, where c=y, the slope $m = \frac{5}{9}$, and the y-intercept $b = -\frac{160}{9}$

To graph this relationship in R, we can create a sequence of f values and compute the corresponding c values. Then, we can use **ggplot** from the **ggplot2** package to create the graph, illustrating the line with slope $m = \frac{5}{9}$, and intercept $b = -\frac{160}{9}$. See code in Class 4 solutions markdown.

1.15 Using the change of base formula for logarithms, change log6(36) to log3(36).

So we debated with some of yall how much was enough to solve this exercise. I Originally stopped way earlier than this solution, but thinking more of it and working it out in the class, I think this is a better answer than my original – goal is to simplify until it stops making sense (i.e. until it stops becoming simpler and starts becoming more complex again ☺)

Let's go:

We have

$$\log_6(36)$$

Which we can rewrite with the formula:

Change of Base Rule	$\log_a b = \frac{\log_c b}{\log_c a}$ (OR) $\log_a b \,.\, \log_c a = \log_c b$

Where we define a = 6, b = 36, c = 3 (our new base).

$$\log_6 36 = \frac{\log_3 36}{\log_3 6} \Leftrightarrow$$

$$\log_3 36 = \log_6 36 * \log_3 6 \Leftrightarrow$$

Now we can factorise a bit, using that $36 = 6^2$:

$$\log_3 36 = \log_6 6^2 * \log_3 6 \Leftrightarrow$$

And now we can use

Power Rule	$\log_b m^n = n \log_b m$

:

$$\log_3 36 = 2 * \log_6 6 * \log_3 6 \Leftrightarrow$$

And now we can use

Log of the same number as base	$\log_b b = 1$

:

$$\log_3 36 = 2 * \log_3 6$$

I think this is as far as I would go :3 Now we've changed the base and expressed it with the term with the new base on its own side of =, so yea. Nice!!