5.3 Exploring Trigonometric Ratios for Angles Greater than 90°

GOAL

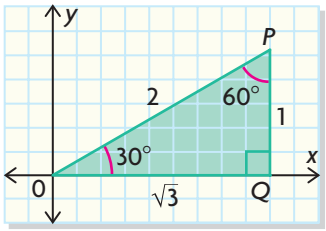
Explore relationships among angles that share related trigonometric ratios.

YOU WILL NEED

- graph paper
- dynamic geometry software (optional)

EXPLORE the Math

Raj is investigating trigonometric ratios of angles greater than 90°. He drew one of the special triangles on a Cartesian grid as shown.

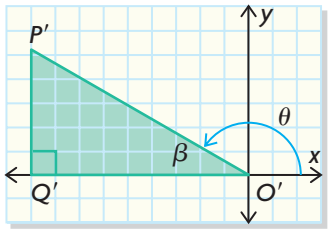


Next he performed a series of reflections in the y- and x-axes.

? **Which angles in the Cartesian plane, if any, have primary trigonometric ratios related to those of a 30° angle?**

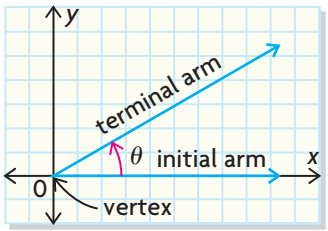
A. Use Raj's sketch of a 30° angle in standard position in the Cartesian plane to record the lengths of all sides and the primary trigonometric ratios for 30° to four decimal places.

B. Reflect the triangle from part A in the y-axis. $\angle P'O'Q'$ is now called the related acute angle β. What is its angle measure? What is the size of the principal angle θ and in which quadrant does the terminal arm lie?



standard position

an angle in the Cartesian plane whose vertex lies at the origin and whose initial arm (the arm that is fixed) lies on the positive x-axis. Angle θ is measured from the initial arm to the terminal arm (the arm that rotates).

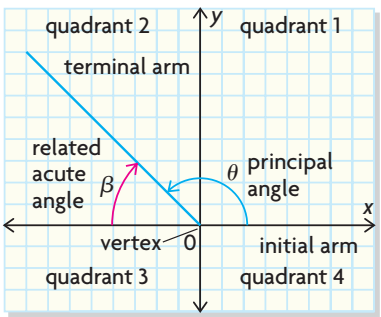


related acute angle

the acute angle between the terminal arm of an angle in standard position and the x-axis when the terminal arm lies in quadrants 2, 3, or 4

principal angle

the counterclockwise angle between the initial arm and the terminal arm of an angle in standard position. Its value is between 0° and 360°.

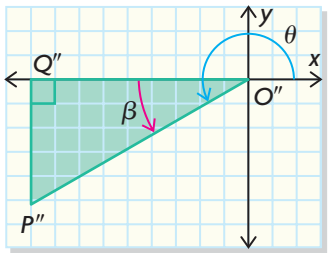


C. Use a calculator to determine the values of the primary trigonometric ratios for the principal angle and the related acute angle. Round your answers to four decimal places and record them in a table similar to the one shown.

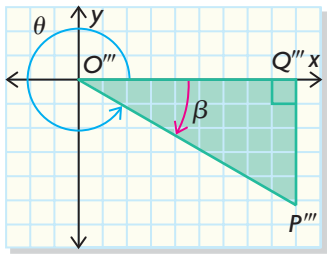
Angles	Quadrant	Sine Ratio	Cosine Ratio	Tangent Ratio
principal angle $\theta =$ _____				
related acute angle $\beta =$ _____				

How are the primary trigonometric ratios for the related acute angle related to the corresponding ratios for the principal angle?

D. Reflect the triangle from part B in the x-axis. What is the size of the related acute angle β? What is the size of the principal angle θ, and in which quadrant does the terminal arm lie? Use a calculator to complete your table for each of these angles. How are the primary trigonometric ratios for the related acute angle related to the corresponding ratios for the principal angle?

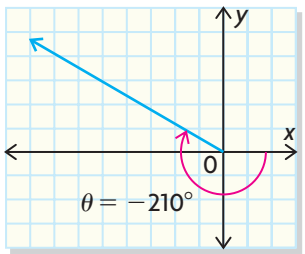


E. Repeat part D, but this time, reflect the triangle from part D in the y-axis.



negative angle

an angle measured *clockwise* from the positive x-axis



F. Repeat parts A to E, but this time start with a 45° and then a 60° angle in quadrant 1. Use **negative angles** for some of your trials.

G. Based on your observations, which principal angles and related acute angles in the Cartesian plane have the same primary trigonometric ratio?

Reflecting

H. **i)** When you reflect an acute principal angle θ in the y-axis, why is the resulting principal angle $180° - \theta$?

ii) When you reflect an acute principal angle θ in the y-axis and then in the x-axis, why is the resulting principal angle $180° + \theta$?

iii) When you reflect an acute principal angle θ in the x-axis, why is the resulting principal angle $360° - \theta$ (or $-\theta$)?

I. What does your table tell you about the relationships among the sine, cosine, and tangent of an acute principal angle and the resulting reflected principal angles?

J. How could you have predicted the relationships you described in part I?

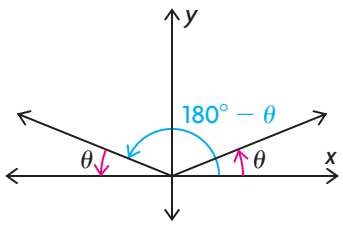
In Summary

Key Idea

- For any principal angle greater than 90°, the values of the primary trigonometric ratios are either the same as, or the negatives of, the ratios for the related acute angle. These relationships are based on angles in standard position in the Cartesian plane and depend on the quadrant in which the terminal arm of the angle lies.

Need to Know

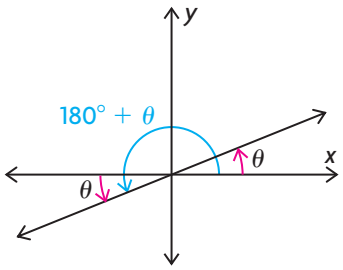
- An angle in the Cartesian plane is in standard position if its vertex lies at the origin and its initial arm lies on the positive x-axis.
- An angle in standard position is determined by a counterclockwise rotation and is always positive. An angle determined by a clockwise rotation is always negative.
- If the terminal arm of an angle in standard position lies in quadrants 2, 3, or 4, there exists a related acute angle and a principal angle.
- If θ is an acute angle in standard position, then
  - the terminal arm of the principal angle $(180° - \theta)$ lies in quadrant 2



$$\sin(180° - \theta) = \sin\theta$$
$$\cos(180° - \theta) = -\cos\theta$$
$$\tan(180° - \theta) = -\tan\theta$$

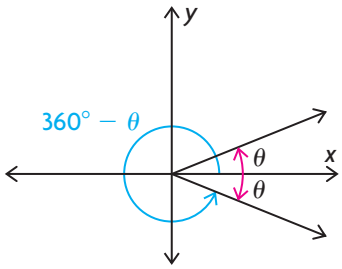
(*continued*)

- the terminal arm of the principal angle $(180° + \theta)$ lies in quadrant 3



$$\sin(180° + \theta) = -\sin\theta$$
$$\cos(180° + \theta) = -\cos\theta$$
$$\tan(180° + \theta) = \tan\theta$$

- the terminal arm of the principal angle $(360° - \theta)$ lies in quadrant 4



$$\sin(360° - \theta) = -\sin\theta$$
$$\cos(360° - \theta) = -\cos\theta$$
$$\tan(360° - \theta) = \tan\theta$$

FURTHER Your Understanding

1. State all the angles between 0° and 360° that make each equation true.

a) $\sin 45° = \sin$ _____

b) $\cos$ _____ $= -\cos(-60°)$

c) $\tan 30° = \tan$ _____

d) $\tan 135° = -\tan$ _____

2. Using the special triangles from Lesson 5.2, sketch two angles in the Cartesian plane that have the same value for each given trigonometric ratio.

a) sine **b)** cosine **c)** tangent

3. Sylvie drew a special triangle in quadrant 3 and determined that $\tan(180° + \theta) = 1$.

a) What is the value of angle θ?

b) What would be the exact value of $\tan\theta$, $\cos\theta$, and $\sin\theta$?

4. Based on your observations, copy and complete the table below to summarize the signs of the trigonometric ratios for a principal angle that lies in each of the quadrants.

Trigonometric Ratio	Quadrant			
	1	2	3	4
sine	+			
cosine	+			
tangent	+			