8.6 Using Technology to Investigate Financial Problems

GOAL

Use technology to investigate the effects of changing the conditions in financial problems.

YOU WILL NEED

- graphing calculator
- spreadsheet software

INVESTIGATE the Math

Tina wants to buy a stereo that costs \$566.47 after taxes. The store allows her to buy the stereo by making payments of \$29.95 per month for 2 years.

? What annual interest rate, compounded monthly, is the store charging?

A. Draw a timeline for this situation. Will you be calculating present values or future values?

B. Use a spreadsheet to set up an **amortization schedule** as shown.

	A	B	C	D	E	F
1	Interest Rate	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2						\$566.47
3	0.01	1	\$29.95	"=F2*A3"	"=C3-D3"	"=F2-E3"
4		"=B3+1"	\$29.95	"=F3*A3"	"=C4-D4"	"=F3-E4"

amortization schedule

a record of payments showing the interest paid, the principal, and the current balance on a loan or investment

C. Use the COPY DOWN command to complete the spreadsheet so that 24 payments are showing. The spreadsheet shown here is set up with an interest rate of 1% per compounding period. Adjust the value of the interest rate to solve the problem.

Tech Support

For help creating an amortization schedule using a spreadsheet, see Technical Appendix, B-22.

D. Enter the formula for the present value of the annuity into a graphing calculator, where Y is the (unknown) present value and X is the annual interest rate compounded monthly.

E. Graph the equation in part D, as well as $y = 566.47$. Use these graphs to solve the problem.

F. On your graphing calculator, activate the TVM Solver.

G. Enter the corresponding values and then solve the problem.

Tech Support

For help using the TVM Solver on a graphing calculator, see Technical Appendix, B-19.

Reflecting

H. Why could you not solve this problem easily with pencil and paper?

I. Which of the three methods (the spreadsheet in parts B and C, the graphs in parts D and E, or the TVM Solver in parts F and G) used to solve the problem do you prefer? Why?

APPLY the Math

EXAMPLE 1 Selecting a tool to investigate the effects of varying the interest rate

Jamal has \$10 000 to invest. Bank of North America offers an interest rate of 4.2%/a compounded monthly. Key Bank offers an interest rate of 5%/a compounded quarterly. How much longer will it take the money invested to grow to \$50 000 if Jamal chooses Bank of North America?

Lina's Solution: Using Guess-and-Check

$$i = \frac{0.042}{12} = 0.0035$$

I first looked at the Bank of North America. Since interest is paid monthly, I divided the annual interest rate by 12 to get the interest rate per month.

$$P = \$10\ 000$$

$$A = P(1 + i)^n$$
$$= 10\ 000(1.0035)^n$$

I substituted the values of i and P into the compound-interest formula. I thought 10 years might be a good guess. That would give $n = 120$ compounding periods.

$$A = 10\ 000(1.0035)^{120}$$
$$\doteq 15\ 208.46$$

$$A = 10\ 000(1.0035)^{480}$$
$$\doteq 53\ 498.41$$

My guess was way too small, so I tried 40 years, which gives $n = 480$ compounding periods.

Try $n = 460$:
$$A = 10\ 000(1.0035)^{460}$$
$$\doteq 49\ 887.68$$

Try $n = 461$:
$$A = 10\ 000(1.0035)^{461}$$
$$\doteq 50\ 062.29$$

That guess was much closer. Eventually, I tried 460 months. It was a little low, so I tried 461 months.

$$\frac{461}{12} \doteq 38.417$$

I determined how long 461 months is in terms of years. First I divided 461 by 12 to get 38 years.

$$0.417 \times 12 \text{ months} \doteq 5 \text{ months}$$

Then I multiplied 0.417 by 12 to get 5 months.

$$n = 38 \text{ years and } 5 \text{ months}$$

$$i = \frac{0.05}{4} = 0.0125$$

Next, I looked at Key Bank. Since interest is paid quarterly, I divided the annual interest rate by 4 to get the interest rate per quarter.

$$P = \$10\ 000$$

$$A = P(1 + i)^n$$
$$= 10\ 000(1.0125)^n$$

I substituted the values of i and P into the compound-interest formula.

$$A = 10\ 000(1.0125)^{140}$$
$$\doteq 56\ 925.19$$

Since it took Bank of North America 38 years to grow to \$50 000, I used 35 years as my first guess for Key Bank because the interest rate is higher. 35 years is $35 \times 4 = 140$ quarters.

Try $n = 129$:

$A = 10\,000(1.0125)^{129}$

$\doteq 49\,654.56$

Try $n = 130$:

$A = 10\,000(1.0125)^{130}$

$\doteq 50\,275.24$

This result was close, but a bit high. Eventually, I tried 129 quarters and then 130 quarters.

$$\frac{130}{4} = 32.5$$

I determined how long 130 quarters is in terms of years. I divided 130 by 4 to get 32 years.

$n = 32$ years and 6 months

I knew that 0.5 years is 6 months.

It will take 38 years and 5 months to get \$50 000 if Jamal chooses Bank of North America. It will take 32 years and 6 months if he chooses Key Bank. So it will take almost 6 years longer to reach his goal if Jamal chooses Bank of North America.

George's Solution: Using a Graphing Calculator

$A = P(1 + i)^n$

Bank of North America:

$$i = \frac{0.042}{12} = 0.0035$$

$A = 10\,000(1.0035)^n$

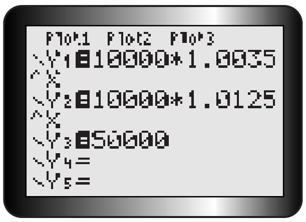
Key Bank:

$$i = \frac{0.05}{4} = 0.0125$$

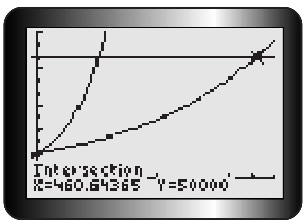
$A = 10\,000(1.0125)^n$

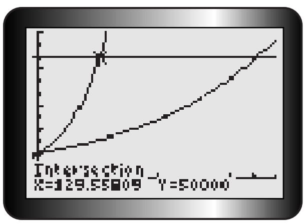
At Bank of North America, interest is compounded monthly. At Key Bank, interest is compounded quarterly. I calculated the interest rate per compounding period at each bank.

Then I used the compound-interest formula to calculate the amounts.



I entered the equations for the amounts into my graphing calculator, using Y1 and Y2 for the amounts for Bank of North America and Key Bank, respectively, and X for the number of compounding periods. I entered Y3 = 50 000.





I graphed the three equations and used the calculator to find the point of intersection of each exponential function with the horizontal line, which indicated when the amount of the investment had reached \$50 000.

It will take about 460 months, or 38 years and 5 months, to get \$50 000 if Jamal chooses Bank of North America. It will take about 129 quarters, or 32 years and 6 months, if he chooses Key Bank. So it will take almost 6 years longer to reach his goal if Jamal chooses Bank of North America.

Coco's Solution: Using the TVM Solver

I entered the information on the investment with Bank of North America into the TVM Solver. Jamal pays into the account at the start, so the present value is −$10 000. Also, no regular payments are being made. This is a lump-sum investment, so I set PMT on my calculator to 0. I determined that it would take a bit more than 460 months, or 38 years and 5 months, to reach his goal with Bank of North America.

I entered the information on the investment with Key Bank into the TVM Solver. I determined that it would take a bit more than 129 quarters, or 32 years and 6 months, to reach his goal with Key Bank.

If Jamal chooses Bank of North America, it will take about 6 years longer to reach his goal.

EXAMPLE 2 Selecting a tool to investigate the effects of increasing the monthly payment

Lia borrows $180 000 to open a restaurant. She can afford to make monthly payments between $1000 and $1500 at 4.8%/a compounded monthly. How much sooner can she pay off the loan if she makes the maximum monthly payment?

Teresa's Solution: Using a Spreadsheet

	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2					$180 000.00
3	1	$1 000 .00	"=E2*(0.048/12)"	"=B3-C3"	"=E2-D3"
4	"=A3+1"	$1 000 .00	"=E3*(0.048/12)"	"=B4-C4"	"=E3-D4"

I set up a spreadsheet to solve the problem. Since the interest is compounded monthly, I divided 4.8% by 12 to get the interest rate per month. For the $1000 minimum payment, I calculated the proportion of the principal paid for each payment. Then I subtracted that proportion from the previous balance to get the balance at the end of the next month.

	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2					$180 000.00
3	1	$1 000 .00	$720.00	$280.00	$179 720.00
4	2	$1 000 .00	$718.88	$281.12	$179 438.88
5	3	$1 000 .00	$717.76	$282.24	$179 156.64
6	4	$1 000 .00	$716.63	$283.37	$178 873.26
7	5	$1 000 .00	$715.49	$284.51	$178 588.76
8	6	$1 000 .00	$714.36	$285.64	$178 303.11
9	7	$1 000 .00	$713.21	$286.79	$178 016.32
10	8	$1 000 .00	$712.07	$287.93	$177 728.39
11	9	$1 000 .00	$710.91	$289.09	$177 439.30
12	10	$1 000 .00	$709.76	$290.24	$177 149.06

Next, I used the FILL DOWN command to complete the other rows.

314	312	$1 000 .00	$30.96	$969.04	$6 770.39
315	313	$1 000 .00	$27.08	$972.92	$5 797.48
316	314	$1 000 .00	$23.19	$976.81	$4 820.67
317	315	$1 000 .00	$19.28	$980.72	$3 839.95
318	316	$1 000 .00	$15.36	$984.64	$2 855.31
319	317	$1 000 .00	$11.42	$988.58	$1 866.73
320	318	$1 000 .00	$7.47	$992.53	$874.20
321	319	$1 000 .00	$3.50	$996.50	–$122.31

I continued until the balance became negative, indicating that the loan was paid off.

	A	B	C	D	E
1	**Payment Number**	**Payment**	**Interest Paid**	**Principal Paid**	**Balance**
2					$180 000.00
3	1	$1 500 .00	$720.00	$780.00	$179 220.00
4	2	$1 500 .00	$716.88	$783.12	$178 436.88
5	3	$1 500 .00	$713.75	$786.25	$177 650.63
6	4	$1 500 .00	$710.60	$789.40	$176 861.23
7	5	$1 500 .00	$707.44	$792.56	$176 068.67
8	6	$1 500 .00	$704.27	$795.73	$175 272.95
9	7	$1 500 .00	$701.09	$798.91	$174 474.04
10	8	$1 500 .00	$697.90	$802.10	$173 671.94
11	9	$1 500 .00	$694.69	$805.31	$172 866.63
12	10	$1 500 .00	$691.47	$808.53	$172 058.09

I replaced the $1000 minimum payment with the $1500 maximum payment and used the FILL DOWN command in all the cells under Payment.

159	157	$1 500 .00	$46.04	$1 453.96	$10 054.91
160	158	$1 500 .00	$40.22	$1 459.78	$8 595.13
161	159	$1 500 .00	$34.38	$1 465.62	$7 129.51
162	160	$1 500 .00	$28.52	$1 471.48	$5 658.03
163	161	$1 500 .00	$22.63	$1 477.37	$4 180.66
164	162	$1 500 .00	$16.72	$1 483.28	$2 697.39
165	163	$1 500 .00	$10.79	$1 489.21	$1 208.17
166	164	$1 500 .00	$4.83	$1 495.17	–$286.99

I continued until the balance became negative, indicating that the loan was paid off.

At the minimum payment of $1000, Lia's loan would be paid off after 319 months, or 26 years and 7 months. At the maximum payment of $1500, the loan would be paid off after 164 months, or 13 years and 8 months. So Lia can pay off the loan almost 13 years sooner if she makes the maximum payment.

Mike's Solution: Using the TVM Solver

I entered the information on the loan into the TVM Solver on a graphing calculator. I entered the minimum monthly payment of $1000 and then used the calculator to determine the number of payments needed.

I then changed the monthly payment to the maximum amount of $1500, and used the calculator to determine the number of payments needed.

At the minimum payment of $1000, Lia's loan would be paid off after 319 months, or 26 years and 7 months. At the maximum payment of $1500, the loan would be paid off after 164 months, or 13 years and 8 months. So Lia can pay off the loan almost 13 years sooner if she makes the maximum payment.

Communication | ***Tip***

Sometimes you can make a large purchase by paying a small portion of the cost right away and financing the rest with a loan. The portion paid right away is called a **down payment**.

EXAMPLE 3 Selecting a tool to investigate the effects of paying more frequently

Sarah and John are both saving for a down payment on their first home. Both plan to save $2400 each year by depositing into an account that earns 4%/a.

- John makes monthly deposits of $200 into an account on which the interest is compounded monthly.
- Sarah makes annual payments of $2400 into an account on which the interest is compounded annually.

Determine the difference in their account balances at the end of 5 years.

Jason's Solution

I used the TVM Solver on my graphing calculator and entered the information on John. I found that his balance would be $13 259.80 at the end of 5 years.

I repeated the same type of calculation, but this time with the information on Sarah. I found that her balance would be $12 999.17 at the end of 5 years.

$\$13\,259.80 - \$12\,999.17 = \$260.63$

I subtracted to calculate the difference in the amounts.

John's account will have $260.63 more than Sarah's after 5 years.

EXAMPLE 4 Connecting the interest paid on a loan with time

You borrow $100 000 at 8.4%/a compounded monthly. You make monthly payments of $861.50 to pay off the loan after 20 years. How long will it take to pay off

a) the first $25 000?
b) the next $25 000?
c) the next $25 000?
d) the last $25 000?
e) Why are the answers to parts (a) through (d) all different?

Mena's Solution

a)

	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2					$100 000.00
3	1	$861 .50	$700.00	$161.50	$99 838.50
4	2	$861 .50	$698.87	$162.63	$99 675.87
5	3	$861 .50	$697.73	$163.77	$99 512.10
6	4	$861 .50	$696.58	$164.92	$99 347.19
7	5	$861 .50	$695.43	$166.07	$99 181.12
104	102	$861 .50	$534.80	$326.70	$76 073.31
105	103	$861 .50	$532.51	$328.99	$75 744.32
106	104	$861 .50	$530.21	$331.29	$75 413.03
107	105	$861 .50	$527.89	$333.61	$75 079.42
108	106	$861 .50	$525.56	$335.94	$74 743.48

I used a spreadsheet to create an amortization schedule. I then used the FILL DOWN feature to complete the spreadsheet.

I noticed that the balance is reduced to $74 743.48 after 106 months, so it took 8 years and 10 months to pay off the first $25 000.

b)

164	162	$861.50	$365.00	$496.50	$51 646.68
165	163	$861.50	$361.53	$499.97	$51 146.71
166	164	$861.50	$358.03	$503.47	$50 643.23
167	165	$861.50	$354.50	$507.00	$50 136.24
168	166	$861.50	$350.95	$510.55	$49 625.69

The balance is reduced to $49 625.69 after 166 months, so it took 60 months, or 5 years, to pay off the next $25 000.

c)

206	204	$861.50	$195.99	$665.51	$27 332.96
207	205	$861.50	$191.33	$670.17	$26 662.79
208	206	$861.50	$186.64	$674.86	$25 987.93
209	207	$861.50	$181.92	$679.58	$25 308.35
210	208	$861.50	$177.16	$684.34	$24 624.00

The balance is reduced to $24 624.00 after 208 months, so it took 42 months, or 3 years and 6 months, to pay off the next $25 000.

d)

239	237	$861.50	$23.72	$837.78	$2 551.46
240	238	$861.50	$17.86	$843.64	$1 707.82
241	239	$861.50	$11.95	$849.55	$858.28
242	240	$861.50	$6.01	$855.49	$2.78
243	241	$861.50	$0.02	$861.48	−$858.70

The loan is paid off after 240 months, or 20 years. It takes 208 months to pay about $75 000, so I subtracted 208 from 240 to determine how long it takes to pay the last $25 000 of the loan. The last $25 000 takes 32 months, or 2 years and 8 months, to pay off.

e) It takes different lengths of time to pay off the same amount of money because the interest paid is greater when the balance owed is greater. Less of the payment goes toward the principal.

In Summary

Key Idea

- Spreadsheets and graphing calculators are just two of the technological tools that can be used to investigate and solve financial problems involving interest, annuities, and amortization schedules.

Need to Know

- The advantage of an amortization schedule is that it provides the history of all payments, interest paid, and balances on a loan.
- More interest can be earned if
  - the interest rate is higher
  - there are more compounding periods per year
- If you increase the amount of the regular payment of a loan, you can pay it off sooner and save a significant amount in interest charges.
- Early in the term of a loan, the major proportion of each regular payment is interest, with only a small amount going toward paying off the principal. As time progresses, a larger proportion of each regular payment goes toward the principal.

CHECK Your Understanding

1. Use technology to determine how long it will take to reach each investment goal.

	Principal	Rate of Compound Interest per Year	Compounding Period	Future Value
a)	\$5 000	8.3%	annually	\$13 000
b)	\$2 500	6.8%	semi-annually	\$4 000
c)	\$450	12.4%	quarterly	\$4 500
d)	\$15 000	3.6%	monthly	\$20 000

2. Use technology to determine the annual interest rate, to two decimal places, being charged in each loan. The compounding period corresponds to when the payments are made.

	Principal	Regular Payment	Time
a)	\$2 500	\$357.59 per year	10 years
b)	\$15 000	\$1497.95 every 6 months	6 years
c)	\$3 500	\$374.56 per quarter	3 years
d)	\$450	\$29.62 per month	18 months

PRACTISING

3. Trevor wants to save \$3500. How much will he have to put away each month at 12.6%/a compounded monthly in order to have enough money in $2\frac{1}{2}$ years?

4. **A** Nadia borrows \$120 000 to buy a house. The current interest rate is 6.6%/a compounded monthly, and Nadia negotiates the term of the loan to be 25 years.

a) What will be each monthly payment?

b) After paying for 3 years, Nadia receives an inheritance and makes a one-time payment of \$15 000 against the outstanding balance of the loan. How much earlier can she pay off the loan because of this payment?

c) How much will she save in interest charges by making the \$15 000 payment?

5. **K** Lisa and Karl are deciding to invest \$750 per month for the next 7 years.

- Bank A has offered them 6.6%/a compounded monthly.
- Bank B has offered them 7.8%/a compounded monthly.

How much more will they end up with by choosing the second offer?

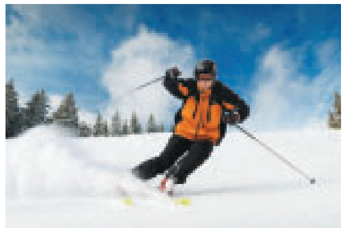
6. Mario decides to pay \$250 per month at 5%/a compounded monthly to pay off a \$25 000 loan. After 2 years, he is making a bit more money and decides to increase the monthly payment. If he pays \$50 extra per month at the end of each 2-year period, how long will it take him to pay off the loan?

7. Natalie borrows \$150 000 at 4.2%/a compounded monthly for a period of 20 years to start a business. She is guaranteed that interest rate for 5 years and makes monthly payments of \$924.86. After 5 years, she renegotiates her loan, but interest rates have gone up to 7.5%/a compounded monthly.

a) If Natalie would like to have the loan paid off after the original 20-year period, what should her new monthly payment be?

b) If she keeps her payments the same, how much extra time will it take her to pay off the loan?

8. Peter buys a ski vacation package priced at \$2754. He pays \$350 down and finances the balance at \$147 per month for $1\frac{1}{2}$ years. Determine the annual interest rate, compounded monthly, being charged. Round your answer to two decimal places.



9. T **a)** Suppose you have a loan where the interest rate doubles. If you want to keep the same amortization period, should you double the payment? Justify your reasoning with examples.

b) Suppose you are borrowing money. If you decide to double the amount borrowed, should you double the payment if you want to keep the same amortization period? Justify your reasoning with examples.

10. Laurie borrows \$50 000 for 10 years at 6.6%/a compounded monthly. How much sooner can she pay off the loan if she doubles the monthly payment after 4 years?

11. C What are the advantages and disadvantages of using each technology to solve financial problems?

- a spreadsheet
- a graphing calculator

Extending

12. A music store will finance the purchase of a rare guitar at 3.6%/a compounded monthly over 5 years, but offers a \$250 reduction if the payment is cash. If you can get a loan from a bank at 4.8%/a compounded annually, how much would the guitar have to sell for to make it worthwhile to take out the loan?

13. The interest on all mortgages is charged semi-annually. You are given a choice of monthly, semi-monthly, bi-weekly, and weekly payments. Suppose you have a mortgage at 8%/a, the monthly payments are \$1000, and the amortization period is 20 years. Investigate the effect on the time to pay off the mortgage if you made each of these payments.

a) \$500 semi-monthly

b) \$500 bi-weekly

c) \$250 weekly

14. Steve decides to pay \$150 per month to pay off a \$6800 loan. In the beginning, the interest rate is 13%/a compounded monthly. The bank guarantees the interest rate for one year at a time. The rate for the next year is determined by the going rate at the time. Assuming that each year the rate drops by 0.5%/a, how long will it take Steve to pay off his loan?