

1.6

Exploring Transformations of Parent Functions

YOU WILL NEED

- graphing calculator or graphing software

Communication *Tip*

The function defined by $g(x) = af(x - d) + c$ describes a transformation of the graph of f .

When $f(x) = x^2$,
 $g(x) = a(x - d)^2 + c$.

When $f(x) = \sqrt{x}$,
 $g(x) = a\sqrt{x - d} + c$.

When $f(x) = \frac{1}{x}$,
 $g(x) = \frac{a}{x - d} + c$.

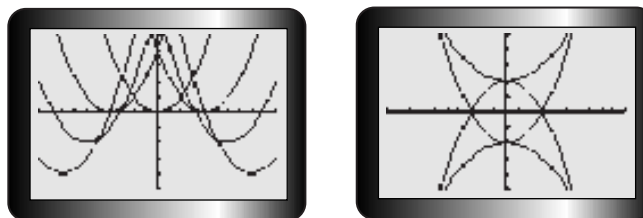
When $f(x) = |x|$,
 $g(x) = a|x - d| + c$.

GOAL

Investigate transformations of parent functions.

EXPLORE the Math

Anastasia and Shelby made patterns with parabolas by applying **transformations** to the graph of the parent quadratic function $y = x^2$.



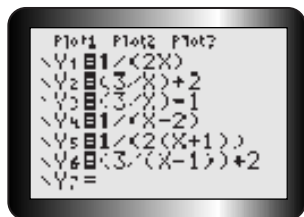
Anastasia thinks they could make more interesting patterns by applying transformations to other parent functions as well. Shelby wonders whether the transformations will have the same effect on the other functions as they do on quadratic functions.

? Do transformations of other parent functions behave in the same way as transformations of quadratic functions?

- Graph the parent functions $f(x) = x^2$, $g(x) = \sqrt{x}$, $h(x) = \frac{1}{x}$, and $j(x) = |x|$. Sketch and label each graph.
- Without using a calculator, use what you know about transformations of quadratic functions to sketch the graphs of $y = 3x^2$, $y = \frac{1}{2}x^2$, and $y = -2x^2$. Describe the transformations in words.
- Predict what the graphs of $y = 3\sqrt{x}$, $y = \frac{1}{2}\sqrt{x}$, and $y = -2\sqrt{x}$ will look like. Use a graphing calculator to verify your predictions. Sketch and label each curve on the same axes, along with a sketch of the parent function. Compare the effect of these transformations with the effect of the same transformations on quadratic functions.
- Repeat part C for $y = \frac{3}{x}$, $y = \frac{1}{2x}$, and $y = -\frac{2}{x}$, and for $y = 3|x|$, $y = \frac{1}{2}|x|$, and $y = -2|x|$.
- Sketch $y = 3x^2 + 2$ and $y = 3x^2 - 1$ without a calculator. Describe the transformations in words. Predict what the graphs of $y = 3f(x) + 2$ and $y = 3f(x) - 1$ for each of the other parent functions will look like. Verify your predictions with a graphing calculator. Make labelled sketches and compare them with transformations on quadratic functions as before.

Tech *Support*

Use brackets when entering transformed versions of $y = \frac{1}{x}$:



To enter $f(x) = |x|$, press



- F. Repeat part E for $y = f(x - 2)$, $y = \frac{1}{2}f(x + 1)$, and $y = 3f(x - 1) + 2$.
- G. Examine your sketches for each type of transformation. Did the transformations have the same effect on the new parent functions as they had on quadratic functions? Explain.

Reflecting

- H. How did the effect of transformations on parent functions compare with that on quadratic functions?
- I. When you graphed $y = af(x - d) + c$, what were the effects of c and d ?
- J. How did the graphs with $a \geq 0$ compare with the graphs with $a \leq 0$?
- K. How did the graphs for which $a > 1$ compare with the graph for which $0 < a < 1$?

In Summary

Key Idea

- In functions of the form $g(x) = af(x - d) + c$, the constants a , c , and d each change the location or shape of the graph of $f(x)$. The shape of the graph of $g(x)$ depends on the graph of the parent function $g(x)$ and on the value of a .

FURTHER Your Understanding

- The graph of the equation $y = (x - 1)^2 + 2$ is the graph of a parabola that opens up and has its vertex at $(1, 2)$. What do you know about the graphs of the following equations?
 a) $y = \sqrt{x - 1} + 2$ b) $y = |x - 1| + 2$ c) $y = \frac{1}{x - 1} + 2$
- The graph of $y = x^2$ opens up and the graph of $y = -x^2$ opens down. How would you compare the graphs of the following pairs of equations?
 a) $y = \sqrt{x}$ and $y = -\sqrt{x}$
 b) $y = |x|$ and $y = -|x|$
 c) $y = \frac{1}{x}$ and $y = -\frac{1}{x}$
- The graph of $y = 2x^2$ is narrower than the graph of $y = x^2$. How do the following graphs compare?
 a) $y = 2\sqrt{x}$ and $y = \sqrt{x}$
 b) $y = 2|x|$ and $y = |x|$
 c) $y = \frac{2}{x}$ and $y = \frac{1}{x}$
- Experiment with each of the parent functions to create patterns on a graphing calculator screen.