# 6.4 Exploring Transformations of Sinusoidal Functions

**GOAL**

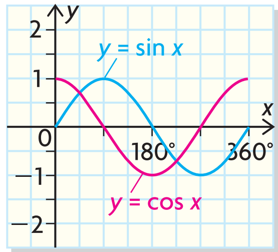
Determine how changing the values of $a$, $c$, $d$, and $k$ affect the graphs of $f(x) = a\sin(k(x - d)) + c$ and $f(x) = a\cos(k(x - d)) + c$

**YOU WILL NEED**

- graphing calculator

## *EXPLORE the Math*

Paula and Marcus know how various transformations affect several types of functions, such as $f(x) = x^2$, $f(x) = \frac{1}{x}$, $f(x) = \sqrt{x}$, and $f(x) = |x|$.




They want to know if these same transformations can be applied to $y = \sin x$ and $y = \cos x$, and if so, how the equations and graphs of these functions change.

**?** Can transformations be applied to sinusoidal functions in the same manner, and do they have the same effect on the graph and the equation?

### Part 1 The graphs of $y = a\sin x$ and $y = a\cos x$

**A.** Predict what the graphs of $y = a\sin x$, $0° \leq x \leq 720°$, will look like for $a = 1, 2,$ and $3$ and for $a = \frac{1}{2}$ and $a = \frac{1}{4}$. Sketch the graphs on the same axes. Verify your sketches using a graphing calculator.

**B.** On a new set of axes, repeat part A for the graphs of $y = a\sin x$, $0° \leq x \leq 720°$, for $a = -1, -2,$ and $-3$.

**C.** How do the graphs in part A compare with those in part B? Discuss how the zeros, amplitude, and maximum or minimum values change for each function.

**D.** Repeat parts A to C using $y = a\cos x$.

**E.** Explain how the value of $a$ affects the graphs of $y = a\sin x$ and $y = a\cos x$.

**Tech *Support***

For Parts 1 and 2, verify your sketches by graphing the parent function ($y = \sin x$ or $y = \cos x$) in Y1 and each transformed function in Y2, Y3, and so on. Use an Xscl = 90°, and graph using ZoomFit by pressing ZOOM 0.

### Part 2 The graphs of $y = \sin x + c$ and $y = \cos x + c$

**F.** Predict what the graphs of $y = \sin x + c$, $0° \leq x \leq 720°$, will look like for $c = -2, -1, 1,$ and $2$. Sketch the graphs on the same axes, and then verify your predictions using a graphing calculator. Discuss which features of the graph have changed.

**G.** Predict what the graphs of $y = \cos x + c$, $0° \leq x \leq 720°$, will look like for $c = -2, -1, 1,$ and $2$. Sketch the graphs on the same axes, and then verify your predictions using a graphing calculator. Discuss which features of the graph have changed.

**H.** Explain how the value of $c$ affects the graphs of $y = a\sin x + c$ and $y = a\cos x + c$.

## Part 3 The graphs of $y = \sin kx$ and $y = \cos kx$

**Tech Support**

For Part 3, verify your sketches by graphing the parent function in Y1 and each transformed function in Y2. Use an Xscl = 90° and graph using ZoomFit.

**I.** Predict what the graphs of $y = \sin kx$ will look like for $k = 2$, 3, and 4, $0° \le x \le 720°$. Sketch each graph, and then verify your predictions using a graphing calculator. Discuss which features of the graph have changed. Clear the previous equation, but not the base equation, from the graphing calculator before entering another equation.

**J.** Repeat part I for $k = \frac{1}{2}$, $k = \frac{1}{4}$, and $k = -1$. Adjust the WINDOW on the graphing calculator so that you can see one complete cycle of each graph.

**K.** Repeat parts I and J using $y = \cos kx$.

**L.** How could you determine the period of $y = \sin kx$ and $y = \cos kx$ knowing that the period of both functions is 360°?

**M.** Explain how the value of $k$ affects $y = \sin kx$ and $y = \cos kx$.

## Part 4 The graphs of $y = \sin(x - d)$ and $y = \cos(x - d)$

**N.** **a)** Predict the effect of $d$ on the graph of $y = \sin(x - d)$.

**b)** Copy and complete the tables of values at the left.

**i)** $y = \sin(x - 60°)$ **ii)** $y = \sin(x + 120°)$

i)

| $x$ | $y$ |
|---|---|
| 60° | |
| 150° | |
| 240° | |
| 330° | |
| 420° | |

ii)

| $x$ | $y$ |
|---|---|
| −120° | |
| −30° | |
| 60° | |
| 150° | |
| 240° | |

**c)** Use your tables to sketch the graphs of the two sinusoidal functions from part (b) on the same coordinate system. Include the graph of $y = \sin x$. Verify your sketches using a graphing calculator, and discuss which features of the graph have changed.

**O.** **a)** Predict the effect of $d$ on the graph of $y = \cos(x - d)$.

**b)** Copy and complete the tables of values at the left.

**i)** $y = \cos(x + 45°)$ **ii)** $y = \cos(x - 120°)$

i)

| $x$ | $y$ |
|---|---|
| −45° | |
| 45° | |
| 135° | |
| 225° | |
| 315° | |

ii)

| $x$ | $y$ |
|---|---|
| 120° | |
| 210° | |
| 300° | |
| 390° | |
| 480° | |

**c)** Use your tables to sketch the graphs of the two sinusoidal functions from part (b) on the same coordinate system. Include the graph of $y = \cos x$. Verify your sketches with a graphing calculator, and discuss which features of the graph have changed.

**P.** Explain how the value of $d$ affects the graphs of $y = \sin(x - d)$ and $y = \cos(x - d)$.

**Tech Support**

For Part 4, verify your sketches using a domain of $0° \le x \le 360°$ and an Xscl = 30°. Graph using ZoomFit.

## Reflecting

**Q.** What transformation affects the period of a sinusoidal function?

**R.** What transformation affects the equation of the axis of a sinusoidal function?

**S.** What transformation affects the amplitude of a sinusoidal function?

**T.** What transformations affect the location of the maximum and minimum values of the sinusoidal function?

**U.** Summarize how the graphs of $y = a\sin(k(x - d)) + c$ and $y = a\cos(k(x - d)) + c$ compare with the graphs of $y = \sin x$ and $y = \cos x$.

## In Summary

### Key Ideas

- The graphs of the functions $f(x) = a\sin(k(x - d)) + c$ and $f(x) = a\cos(k(x - d)) + c$ are periodic in the same way that the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ are. The differences are only in the placement of the graph and how stretched or compressed it is.
- The values $a$, $k$, $c$, and $d$ in the functions $f(x) = a\sin(k(x - d)) + c$ and $f(x) = a\cos(k(x - d)) + c$ affect the graphs of $y = \sin x$ and $y = \cos x$ in the same way that they affect the graphs of $y = f(k(x - d)) + c$, where $f(x) = x^2$, $f(x) = \frac{1}{x}$, $f(x) = \sqrt{x}$, and $f(x) = |x|$.

### Need to Know

- Changing the value of $c$ results in a vertical translation and affects the equation of the axis, the maximum and minimum values, and the range of the function but has no effect on the period, amplitude, or domain.
- Changing the value of $d$ results in a horizontal translation and slides the graph to the left or right but has no effect on the period, amplitude, equation of the axis, domain, or range unless the situation forces a change in the domain or range.
- Changing the value of $a$ results in a vertical stretch or compression and affects the maximum and minimum values, amplitude, and range of the function but has no effect on the period or domain. If $a$ is negative, a reflection in the $x$-axis also occurs.
- Changing the value of $k$ results in a horizontal stretch or compression and affects the period, changing it to $\frac{360°}{|k|}$, but has no effect on the amplitude, equation of the axis, maximum and minimum values, domain, and range unless the situation forces a change in the domain or range. If $k$ is negative, a reflection in the $y$-axis also occurs.

## *FURTHER Your Understanding*

**1.** State the transformation to the graph of either $y = \sin x$ or $y = \cos x$ that has occurred to result in each sinusoidal function.

**a)** $y = 3\cos x$ **c)** $y = -\cos x$ **e)** $y = \cos x - 6$
**b)** $y = \sin(x - 50°)$ **d)** $y = \sin(5x)$ **f)** $y = \cos(x + 20°)$

**2.** Each sinusoidal function below has undergone one transformation that has affected either the period, amplitude, or equation of the axis. In each case, determine which characteristic has been changed and indicate its value.

**a)** $y = \sin x + 2$ **c)** $y = \cos(8x)$ **e)** $y = 0.25\cos x$
**b)** $y = 4\sin x$ **d)** $y = \sin(2x + 30°)$ **f)** $y = \sin(0.5x)$

**3.** Which two of these transformations do not affect the period, amplitude, or equation of the axis of a sinusoidal function?

**a)** reflection in the $x$-axis
**b)** vertical stretch/vertical compression
**c)** vertical translation
**d)** horizontal stretch/horizontal compression
**e)** horizontal translation