2 Chapter Review

FREQUENTLY ASKED Questions

Q: What is a rational function and how do you determine its simplified form?

A: A rational function is a function that can be expressed as a quotient of two polynomials.

The domain of a rational function is the set of all real numbers, except the zeros of the denominator.

To simplify, divide out common factors of the numerator and denominator.

EXAMPLE

$$f(x) = \frac{4\cancel{(x+1)}^{1}(x+2)}{2\cancel{(x+1)}_{1}(x+3)} = \frac{2(x+2)}{(x+3)}; x \neq -1, -3$$

Study Aid

- See Lesson 2.4, Examples 1 to 5.
- Try Chapter Review Questions 9, 10, and 11.

Q: How do we add, subtract, multiply, and divide rational expressions?

A: Rules for adding, subtracting, multiplying, and dividing rational expressions are the same as those for rational numbers.

EXAMPLE

$$\begin{aligned}
&\frac{2x^2}{(x-1)^2} \div \frac{4x}{x^2-1} + \frac{7}{2x-2} \\
&= \frac{2x^2}{(x-1)(x-1)} \div \frac{4x}{(x+1)(x-1)} + \frac{7}{2(x-1)} \\
&= \frac{2x^2}{(x-1)(x-1)} \times \frac{(x-1)(x+1)}{4x} + \frac{7}{2(x-1)} \\
&= \frac{x(x+1)}{2(x-1)} + \frac{7}{2(x-1)} \\
&= \frac{x^2+x+7}{2(x-1)}; x \neq 1, -1, 0
\end{aligned}$$

Study Aid

- See Lesson 2.6, Examples 1 to 4 for multiplication and division.
- See Lesson 2.7, Examples 1 to 4 for addition and subtraction.
- Try Chapter Review Questions 12 to 17.

Q: Why are there sometimes restrictions on the variables in a rational expression, and how do you determine these restrictions?

A: The restrictions occur because division by zero is undefined. To determine restrictions, set all denominators equal to zero before simplifying and solve, usually by factoring.

In the preceding example, set

$(x-1)^2 = 0$, $x^2 - 1 = 0$, $2x - 2 = 0$, and $4x = 0$

Solve by factoring:

$(x-1)^2 = 0$, $(x-1)(x+1) = 0$, $2(x-1) = 0$, and $4x = 0$

Solving gives the restrictions $x \neq 1, -1, 0$.

Study Aid

- See Lessons 2.4, 2.6, and 2.7, all Examples.
- Try Chapter Review Questions 9 to 17.

PRACTICE Questions

Lesson 2.1

1. Simplify.

a) $(7x^2 - 2x + 1) + (9x^2 - 4x + 5) - (4x^2 + 6x - 7)$

b) $(7a^2 - 4ab + 9b^2) - (-a^2 + 2ab + 6b^2)$

2. Determine two non-equivalent polynomials $f(x)$ and $g(x)$, such that $f(0) = g(0)$ and $f(1) = g(1)$.

3. Ms. Flanagan has three daughters: Astrid, Beatrice, and Cassandra. Today, January 1, their ages are, respectively,

$$A(n) = -(n + 30) + (2n + 5)$$
$$B(n) = (7 - n) - (32 - 2n)$$
$$C(n) = (n - 26) - (n + 4) + (n - 3)$$

All ages are expressed in years, and n represents Ms. Flanagan's age.

a) Are the daughters triplets? Explain.

b) Are any of them twins? Explain.

c) How old was Ms. Flanagan when Cassandra was born?

Lesson 2.2

4. Expand and simplify.

a) $-3(7x - 5)(4x - 7)$

b) $-(y^2 - 4y + 7)(3y^2 - 5y - 3)$

c) $2(a + b)^3$

d) $3(x^2 - 2)^2(2x - 3)^2$

5. The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$. Determine the volume of the cone in simplified form if the radius is increased by x and the height is increased by $2x$.

Lesson 2.3

6. Simplify.

a) $(2x^4 - 3x^2 - 6) + (6x^4 - x^3 + 4x^2 + 5)$

b) $(x^2 - 4)(2x^2 + 5x - 2)$

c) $-7x(x^2 + x - 1) - 3x(2x^2 - 5x + 6)$

d) $-2x^2(3x^3 - 7x + 2) - x^3(5x^3 + 2x - 8)$

e) $-2x[5x - (2x - 7)] + 6x[3x - (1 + 2x)]$

f) $(x + 2)^2(x - 1)^2 - (x - 4)^2(x + 4)^2$

g) $(x^2 + 5x - 3)^2$

7. Factor.

a) $12m^2n^3 + 18m^3n^2$

b) $x^2 - 9x + 20$

c) $3x^2 + 24x + 45$

d) $50x^2 - 72$

e) $9x^2 - 6x + 1$

f) $10a^2 + a - 3$

8. Factor.

a) $2x^2y^4 - 6x^5y^3 + 8x^3y$

b) $2x(x + 4) + 3(x + 4)$

c) $x^2 - 3x - 10$

d) $15x^2 - 53x + 42$

e) $a^4 - 16$

f) $(m - n)^2 - (2m + 3n)^2$

Lesson 2.4

9. Simplify. State any restrictions on the variables.

a) $\dfrac{10a^2b + 15bc^2}{-5b}$

b) $\dfrac{30x^2y^3 - 20x^2z^2 + 50x^2}{10x^2}$

c) $\dfrac{xy - xyz}{xy}$

d) $\dfrac{16mnr - 24mnp + 40kmn}{8mn}$

10. Simplify. State any restrictions on the variables.

a) $8xy^2 + 12x^2y - \dfrac{6x^3}{2xy}$

b) $\dfrac{7a - 14b}{2(a - 2b)}$

c) $\dfrac{m + 3}{m^2 + 10m + 21}$

d) $\dfrac{4x^2 - 4x - 3}{4x^2 - 9}$

e) $\dfrac{3x^2 - 21x}{7x^2 - 28x + 21}$

f) $\dfrac{3x^2 - 2xy - y^2}{3x^2 + 4xy + y^2}$

11. If two rational functions have the same restrictions, are they equivalent? Explain and illustrate with an example.

Lessons 2.6 and 2.7

12. Simplify. State any restrictions on the variables.

a) $\frac{6x}{8y} \times \frac{2y^2}{3x}$

b) $\frac{10m^2}{3n} \times \frac{6mn}{20m^2}$

c) $\frac{2ab}{5bc} \div \frac{6ac}{10b}$

d) $\frac{5p}{8pq} \div \frac{3p}{12q}$

13. Simplify. State any restrictions on the variables.

a) $\frac{x^2}{2xy} \times \frac{x}{2y^2} \div \frac{(3x)^2}{xy^2}$

b) $\frac{x^2 - 5x + 6}{x^2 - 1} \times \frac{x^2 - 4x - 5}{x^2 - 4} \div \frac{x - 5}{x^2 + 3x + 2}$

c) $\frac{1 - x^2}{1 + y} \times \frac{1 - y^2}{x + x^2} \div \frac{y^3 - y}{x^2}$

d) $\frac{x^2 - y^2}{4x^2 - y^2} \times \frac{4x^2 + 8xy + 3y^2}{x + y} \div \frac{2x + 3y}{2x - y}$

14. Simplify. State any restrictions on the variables.

a) $\frac{4}{5x} - \frac{2}{3x}$

b) $\frac{5}{x + 1} - \frac{2}{x - 1}$

c) $\frac{1}{x^2 + 3x - 4} + \frac{1}{x^2 + x - 12}$

d) $\frac{1}{x^2 - 5x + 6} - \frac{1}{x^2 - 9}$

15. Simplify and state any restrictions on the variables.

a) $\frac{1}{2x} - \frac{7}{3x^2} + \frac{4}{x^3}$

b) $\frac{3x}{x + 2} + \frac{4x}{x - 6}$

c) $\frac{6x}{x^2 - 5x + 6} - \frac{3x}{x^2 + x - 12}$

d) $\frac{2(x - 2)^2}{x^2 + 6x + 5} \times \frac{3x + 15}{(2 - x)^2}$

e) $\frac{(x - 2y)^2}{x^2 - y^2} \div \frac{(x - 2y)(x + 3y)}{(x + y)^2}$

f) $\frac{2b - 5}{b^2 - 2b - 15} + \frac{3b}{b^2 + b - 30} \times \frac{b^2 + 8b + 12}{b + 3}$

16. Fred's final mark in an online course was determined entirely by two exams. The mid-term exam was out of x marks and was worth 25% of his final mark. The final exam was out of $2x$ marks and was worth 75% of his final mark. Fred scored 40 marks on the first exam and 60 marks on the second exam. Determine the value of x if Fred earned a final mark of 50% in the course.

17. Sam plays a game in which he selects three different numbers from 1 to n $(n > 3)$. After he selects his numbers, four different winning numbers from 1 to n are chosen, one at a time. Sam wins if all three of his numbers are among the four winning numbers.

The first number chosen is one of Sam's! His probability of winning is now given by

$$P(n) = \frac{24}{n^3 - 3n^2 + 2n} \div \frac{3}{n}$$

a) Simplify $P(n)$ and state the restrictions on n.

b) What would Sam's probability of winning be if

i) $n = 5$? **ii)** $n = 4$?