2.2 Multiplying Polynomials

GOAL

Simplify polynomials by multiplying.

LEARN ABOUT the Math

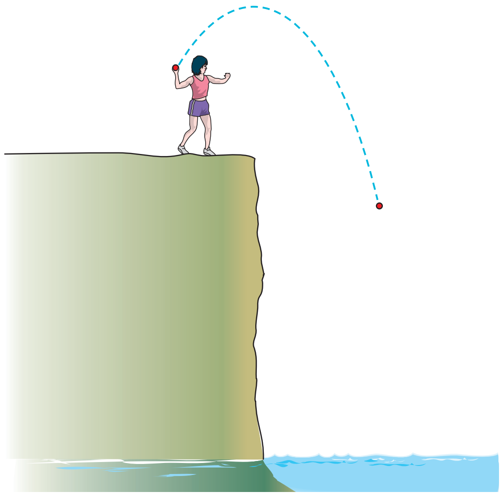
In a physics textbook, Kristina reads about an experiment in which a ball is thrown upward from the top of a cliff, ultimately landing in the water below the cliff. The height of the ball above the cliff, $h(t)$, and its velocity, $v(t)$, at time t are respectively given by

$$h(t) = -5t^2 + 5t + 2.5$$

and

$$v(t) = -10t + 5.$$

Kristina learns that the product of the two functions allows her to determine when the ball moves away from, and when the ball moves toward, the top of the cliff.



? How can she simplify the expression for $v(t) \times h(t)$?

EXAMPLE 1 Selecting a strategy to simplify a product: The distributive property

Simplify the expression $v(t) \times h(t) = (-10t + 5)(-5t^2 + 5t + 2.5)$.

Sam's Solution

$$v(t)h(t) = (-10t + 5)(-5t^2 + 5t + 2.5)$$

I used the **commutative property** for multiplication to create an equivalent expression.

$$\begin{aligned} &= (-10t)(-5t^2 + 5t + 2.5) + (5)(-5t^2 + 5t + 2.5) \\ &= (50t^3 - 50t^2 - 25t) + (-25t^2 + 25t + 12.5) \\ &= 50t^3 - 50t^2 - 25t^2 - 25t + 25t + 12.5 \\ &= 50t^3 - 75t^2 + 12.5 \end{aligned}$$

I used the **distributive property** to expand the product of the polynomials. I multiplied each of the three terms in the trinomial by each of the terms in the binomial.

Then I grouped and collected like terms.

Reflecting

A. How does the simplified expression differ from the original?

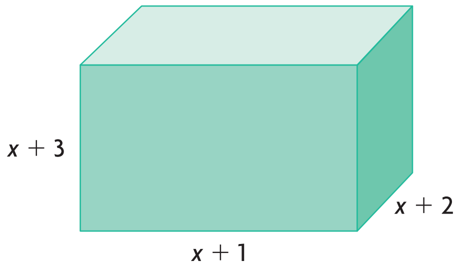
B. Sam grouped together like terms in order to simplify. Is this always necessary? Explain.

C. Would Sam have gotten a different answer if he multiplied $(-5t^2 + 5t + 2.5)(-10t + 5)$ by multiplying each term in the second factor by each term in the first factor? Explain.

WORK WITH the Math

EXAMPLE 2 Selecting a strategy to multiply three binomials

Determine a simplified function that represents the volume of the given box.



Fred's Solution: Starting with the First Two Binomials

$V = lwh$

$V = (x + 1)(x + 2)(x + 3)$

$ = (x^2 + 3x + 2)(x + 3)$

I know that multiplication is **associative,** so I can multiply in any order. I multiplied the first two binomials together and got a trinomial.

$ = (x^2 + 3x + 2)(x) + (x^2 + 3x + 2)(3)$

Then I took the x-term from $(x + 3)$ and multiplied it by the trinomial. Next, I took the 3 term from $(x + 3)$ and multiplied it by the trinomial.

$ = x^3 + 3x^2 + 2x + 3x^2 + 9x + 6$

$ = x^3 + 6x^2 + 11x + 6$

Finally, I simplified by collecting like terms and arranged the terms in descending order.

Atish's Solution: Starting with the Last Two Binomials

$V = (x + 1)(x + 2)(x + 3)$

I multiplied the last two binomials together and got a trinomial.

$= (x + 1)(x^2 + 5x + 6)$

Then I took the x-term from $(x + 1)$ and multiplied it by the trinomial. Next, I took the 1 term from $(x + 1)$ and multiplied it by the trinomial.

$= x^3 + 5x^2 + 6x + x^2 + 5x + 6$

$= x^3 + 6x^2 + 11x + 6$

Finally, I simplified by collecting like terms and arranged the terms in descending order.

EXAMPLE 3 Selecting a strategy to determine non-equivalence

Is $(2x + 3y + 4z)^2 = 4x^2 + 9y^2 + 16z^2$?

Mathias's Solution: Using Substitution and then Evaluating

Let $x = y = z = 1$

L.S. $= (2x + 3y + 4z)^2$	R.S. $= 4x^2 + 9y^2 + 16z^2$
$= (2 + 3 + 4)^2$	$= 4 + 9 + 16$
$= 9^2$	$= 29$
$= 81$	

I substituted 1 for each of the variables in each expression to see if the results would be different.

Since the left side did not equal the right side, the expressions are not equivalent.

Lee's Solution: Expanding and Simplifying

$(2x + 3y + 4z)^2 = (2x + 3y + 4z)(2x + 3y + 4z)$

I wrote the left side of the equation as the product of two identical factors. I multiplied directly, multiplying each term in one polynomial by each term in the other.

$= 4x^2 + 6xy + 8xz + 6xy + 9y^2 + 12yz + 8xz + 12yz + 16z^2$

$= 4x^2 + 12xy + 16xz + 9y^2 + 24yz + 16z^2$

I simplified by collecting like terms.

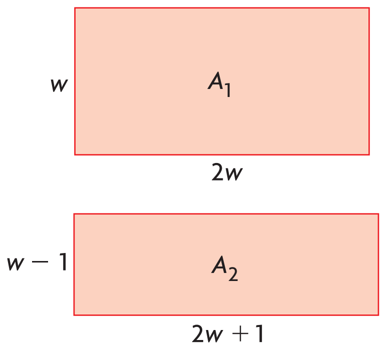
The expressions are not equivalent.

The simplified expression does not result in $4x^2 + 9y^2 + 16z^2$.

EXAMPLE 4 Representing changes in area as a polynomial

A rectangle is twice as long as it is wide. Predict how the area will change if the length of the rectangle is increased by 1 and the width is decreased by 1. Write an expression for the change in area and interpret the result.

Kim's Solution



Since we are increasing the length and decreasing the width by the same amount, I predict that there will be no change in the area.

$$A_1 = (2w)w$$
$$= 2w^2$$

To check my prediction, I let w represent the width and $2w$ the length. Their product gives the original area, A_1.

$$A_2 = (2w + 1)(w - 1)$$
$$= 2w^2 - w - 1$$

I increased the length by 1 and decreased the width by 1 by adding and subtracting 1 to my previous expressions. The product gives the area of the new rectangle, A_2.

$$\text{change in area} = A_2 - A_1$$
$$= (2w^2 - w - 1) - (2w^2)$$
$$= -w - 1$$

I took the difference of the new area, A_2, and the original area, A_1, to represent the change in area.

My prediction was wrong.

w represents width, which is always positive. Substituting any positive value for w into $-w - 1$ results in a negative number. This means that the new rectangle must have a smaller area than the original one.

In Summary

Key Idea

- The product of two or more expressions, one of which contains at least two terms, can be found by using the distributive property (often called expanding) and then collecting like terms.

Need to Know

- Since polynomials behave like numbers, the multiplication of polynomials has the same properties:
  For any polynomials *a*, *b*, and *c*:

$$ab = ba \text{ (commutative property)}$$
$$(ab)c = a(bc) \text{ (associative property)}$$
$$a(b + c) = ab + ac \text{ (distributive property)}$$

  With the use of the distributive property, the product of two polynomials can be found by multiplying each term in one polynomial by each term in the other.

CHECK *Your Understanding*

1. Expand and simplify.

a) $2x(3x - 5x^2 + 4y)$

b) $(3x - 4)(2x + 5)$

c) $(x + 4)^2$

d) $(x + 1)(x^2 + 2x - 3)$

2. a) Is $(3x + 2)^2 = 9x^2 + 4$? Justify your decision.

b) Write the simplified expression that is equivalent to $(3x + 2)^2$.

3. Expand and simplify $(2x + 4)(3x^2 + 6x - 5)$ by

a) multiplying from left to right

b) multiplying from right to left

PRACTISING

4. Expand and simplify.

a) $5x(5x^2 + 3x - 4)$

b) $(x - 6)(2x + 5)$

c) $(x + 3)(x - 3) + (5x - 6)(3x - 7)$

d) $4(n - 4)(3 + n) - 3(n - 5)(n + 8)$

e) $3(2x - 1)^2 - 5(4x + 1)^2$

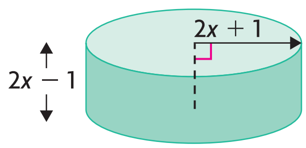
f) $2(3a + 4)(a - 6) - (3 - a)^2 + 4(5 - a)$

5. Expand and simplify.
   a) $4x(x + 5)(x - 5)$
   b) $-2a(a + 4)^2$
   c) $(x + 2)(x - 5)(x - 2)$
   d) $(2x + 1)(3x - 5)(4 - x)$
   e) $(9a - 5)^3$
   f) $(a - b + c - d)(a + b - c - d)$

6. Determine whether each pair of expressions is equivalent.
   a) $(3x - 2)(2x - 1)$ and $3x(2x - 1) - 2(2x - 1)$
   b) $(x - 4)(2x^2 + 5x - 6)$ and
   $2x^2(x - 4) + 5x(x - 4) - 6(x - 4)$
   c) $(x + 2)(3x - 1) - (1 - 2x)^2$ and $x^2 + 9x - 3$
   d) $2(x - 3)(2x^2 - 4x + 5)$ and $4x^3 - 20x^2 + 34x - 30$
   e) $(4x + y - 3)^2$ and $16x^2 - 8xy + 24x + y^2 - 6y + 9$
   f) $3(y - 2x)^3$ and $-24x^3 + 36x^2y - 18xy^2 + 3y^3$

7. Is the equation $(x - 1)(x^4 + x^3 + x^2 + x + 1) = x^5 - 1$ true for all,
T some, or no real numbers? Explain.

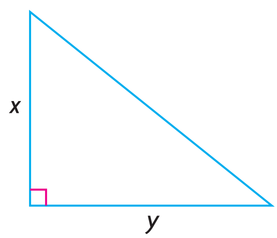
8. Recall that the associative property of multiplication states that $(ab)c = a(bc)$.
K a) Verify this property for the product $19(5x + 7)(3x - 2)$ by expanding and simplifying in two different ways.
   b) Which method did you find easier?



9. A cylinder with a top and bottom has radius $2x + 1$ and height $2x - 1$.
A Write a simplified expressions for its
   a) surface area, where $SA = 2\pi r^2 + 2\pi rh$
   b) volume, where $V = \pi r^2 h$

10. a) Is $(x - 3)^2 = (3 - x)^2$? Explain.
    b) Is $(x - 3)^3 = (3 - x)^3$? Explain.

11. Expand and simplify.
    a) $(x^2 + 2x - 1)^2$
    b) $(2 - a)^3$
    c) $(x^3 + x^2 + x + 1)(x^3 - x^2 - x - 1)$
    d) $2(x + 1)^2 - 3(2x - 1)(3x - 5)$



12. The two sides of the right triangle shown at the left have lengths x and y. Represent the change in the triangle's area if the length of one side is doubled and the length of the other side is halved.

13. The kinetic energy of an object is given by $E = \frac{1}{2}mv^2$, where m represents the mass of the object and v represents its speed. Write a simplified expression for the kinetic energy of the object if
    a) its mass is increased by x
    b) its speed is increased by y