Chapter 6

INTRODUCTION TO VECTORS

Have you ever tried to swim across a river with a strong current or run into a head wind? Have you ever tried sailing across a windy lake? If your answer is yes, then you have experienced the effect of vector quantities. Vectors were developed in the late nineteenth century as mathematical tools for studying physics. In the following century, vectors became an essential tool for anyone using mathematics including social sciences. In order to navigate, pilots need to know what effect a crosswind will have on the direction in which they intend to fly. In order to build bridges, engineers need to know what load a particular design will support. In this chapter, you will learn more about vectors and how they represent quantities possessing both magnitude and direction.

CHAPTER EXPECTATIONS

In this chapter, you will

- represent vectors as directed line segments, **Section 6.1**
- recognize a vector as a quantity with both magnitude and direction, **Section 6.1**
- perform mathematical operations on geometric vectors, **Sections 6.2, 6.3**
- determine some properties of the operations performed on vectors, **Section 6.4**
- determine the Cartesian representation of a vector in two- and three-dimensional space, **Sections 6.5, 6.6, 6.7, 6.8**
- perform mathematical operations on algebraic vectors in two- and three-dimensional space, **Sections 6.6, 6.7, 6.8**

Review of Prerequisite Skills

In this chapter, you will be introduced to the concept of a vector, a mathematical entity having both magnitude and direction. You will examine geometric and algebraic representations of vectors in two- and three-dimensional space. Before beginning this introduction to vectors, you may wish to review some basic facts of trigonometry.

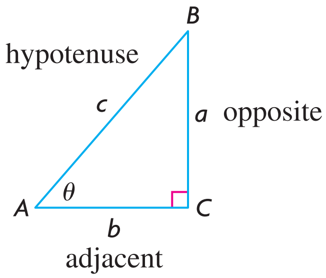
TRIGONOMETRIC RATIOS

In a right-angled triangle, as shown, $a^2 + b^2 = c^2$.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

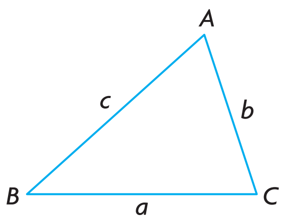
$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$



THE SINE LAW

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



THE COSINE LAW

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{or} \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

SOLVING A TRIANGLE

- To solve a triangle means to find the measures of the sides and angles whose values are not given.
- Solving a triangle may require the use of trigonometric ratios, the Pythagorean theorem, the sine law, and/or the cosine law.

Exercise

1. State the exact value of each of the following:

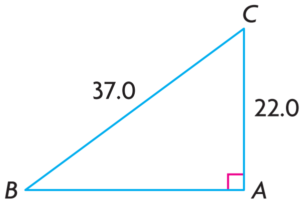
a. $\sin 60°$ c. $\cos 60°$ e. $\sin 135°$

b. $\tan 120°$ d. $\cos 30°$ f. $\tan 45°$

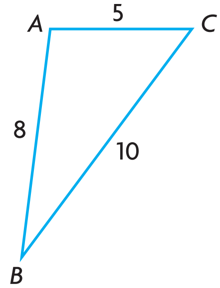
2. In $\triangle ABC$, $AB = 6$, $\angle B = 90°$, and $AC = 10$. State the exact value of $\tan A$.

3. Solve $\triangle ABC$, to one decimal place.

a.



b.



4. In $\triangle XYZ$, $XY = 6$, $\angle X = 60°$, and $\angle Y = 70°$. Determine the values of XZ, YZ, and $\angle Z$, to two digit accuracy.

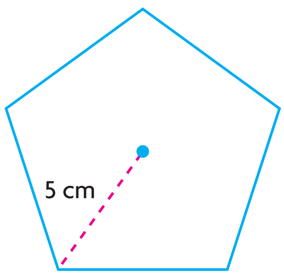
5. In $\triangle RST$, $RS = 4$, $RT = 7$, and $ST = 5$. Determine the measures of the angles to the nearest degree.

6. An aircraft control tower, T, is tracking two airplanes at points A, 3.5 km from T, and B, 6 km from T. If $\angle ATB = 70°$, determine the distance between the two airplanes to two decimal places.

7. Three ships are at points P, Q, and R such that $PQ = 2$ km, $PR = 7$ km, and $\angle QPR = 142°$. What is the distance between Q and R, to two decimal places.

8. Two roads intersect at an angle of 48º. A car and truck collide at the intersection, and then leave the scene of the accident. The car travels at 100 km/h down one road, while the truck goes 80 km/h down the other road. Fifteen minutes after the accident, a police helicopter locates the car and pulls it over. Twenty minutes after the accident, a police cruiser pulls over the truck. How far apart are the car and the truck at this time?

9. A regular pentagon has all sides equal and all central angles equal. Calculate, to the nearest tenth, the area of the pentagon shown.



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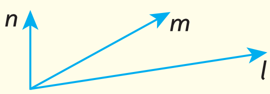
CHAPTER 6: FIGURE SKATING

Figure skaters are exceptional athletes and artists. Their motion while skating is also an illustration of the use of vectors. The ice they skate on is a nearly frictionless surface—so any force applied by the skater has a direct impact on speed, momentum, and direction. Vectors can be used to describe a figure skater's path on the ice. When the skater starts moving in a direction, she will continue moving in that direction and at that speed until she applies a force to change or stop her motion. This is more apparent with pairs figure skaters. To stay together, each skater must skate with close to the same speed as their partner in the same direction. If one skater uses less force or applies the force in a different direction, the skaters will either bump into each other or separate and fly away from each other. If they don't let go of each other, the opposing forces may cause them to spin.

Case Study—Throwing a Triple Salchow

The Triple Salchow throw is one of the more difficult moves in pairs figure skating. Both partners skate together in one direction with a lot of speed. Next, the male skater plants his feet to throw his partner and add his momentum to that of the female skater. She applies force with one skate to jump into the air. In order to make herself spin, she applies force at an angle to the initial direction and spins three times in the air before landing. There are three main vectors at work here. These vectors are the initial thrust of both skaters, the force the male skater applies to the female skater, and the vertical force of the jump.

Vector	Magnitude (size of the force)
Both skaters' initial thrust (l)	60
Female skater's change in direction to cause spin (m)	40
Female skater's vertical leap (n)	20



DISCUSSION QUESTIONS

1. What operation on vectors l and m do you think should be done to find the resulting thrust vector (a) for the female skater?
2. If we performed the same operation as in problem 1 to the vectors a and n, what would the resulting vector represent? You may assume that the angle between a and n is 90°.
3. Can you think of a three-dimensional figure that would represent all of the vectors l, m, and n at the same time, as well as the vectors found in the previous two problems? Give as complete a description as possible for this figure, including any properties you notice. For example, is it constructed from any familiar two-dimensional objects?