4.5 Exploring the Properties of Exponential Functions

YOU WILL NEED

- graphing calculator
- graph paper

GOAL

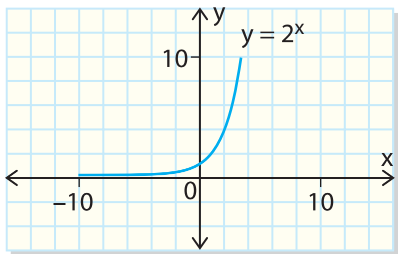
Determine the characteristics of the graphs and equations of exponential functions.

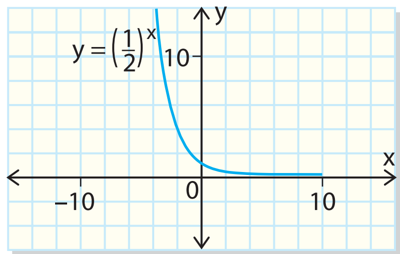
EXPLORE the Math

Functions such as $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$ are examples of **exponential functions**. These types of functions can model many different phenomena, including population growth and the cooling of a liquid.

exponential function

a function of the form $y = a(b^x)$





? What are the characteristics of the graph of the exponential function $f(x) = b^x$, and how does it compare with the graphs of quadratic and linear functions?

A. Create a tables of values for each of the following functions. $g(x) = x$, $h(x) = x^2$, and $k(x) = 2^x$, where $-3 \leq x \leq 5$

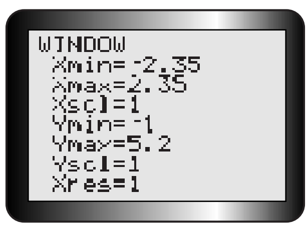
B. In each of your tables, calculate the first and second differences. Describe the difference patterns for each type of function.

C. Graph each function on graph paper and draw a smooth curve through each set of points. Label each curve with the appropriate equation.

D. State the domain and range of each function.

E. For each function, describe how values of the dependent variable, y, change as the values of the independent variable, x, increase and decrease.

F. Use a graphing calculator to graph the functions $y = 2^x$, $y = 5^x$, and $y = 10^x$. Graph all three functions on the same graph. Use the WINDOW settings shown.



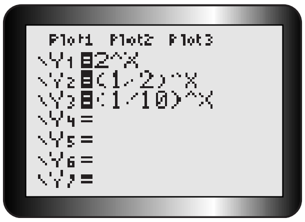
G. For each function, state
- the domain and range
- the intercepts
- the equations of any **asymptotes**, if present

H. Examine the y-values as x increases and decreases. Which curve increases faster as you trace to the right? Which one decreases faster as you trace to the left?

I. Delete the second and third functions $(y = 5^x$ and $y = 10^x)$, and replace them with $y = \left(\frac{1}{2}\right)^x$ and $y = \left(\frac{1}{10}\right)^x$ (or $y = 0.5^x$ and $y = 0.1^x$).

J. For each new function, state
- the domain and range
- the intercepts
- the equations of any asymptotes

K. Describe how each of the graphs of $y = \left(\frac{1}{2}\right)^x$ and $y = \left(\frac{1}{10}\right)^x$ differs from $y = 2^x$ as the x-values increase and as they decrease.



L. Investigate what happens when the base of an exponential function is negative. Try $y = (-2)^x$. Discuss your findings.

M. Compare the features of the graphs of $f(x) = b^x$ for each group. Think about the domain, range, intercepts, and asymptotes.
i) different values of b when $b > 1$
ii) different values of b when $0 < b < 1$
iii) values of b when $0 < b < 1$, compared with values of $b > 1$
iii) values of $b < 0$ compared with values of $b > 0$

Reflecting

N. How do the differences for exponential functions differ from those for linear and quadratic functions? How can you tell that a function is exponential from its differences?

O. The base of an exponential function of the form $f(x) = b^x$ cannot be 1. Explain why this restriction is necessary.

P. Explain how you can distinguish an exponential function from a quadratic function and a linear function by using
- the graphs of each function
- a table of values for each function
- the equation of each function

Tech Support

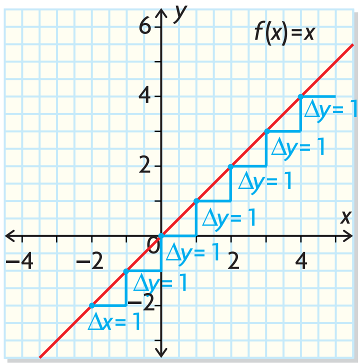
For help tracing functions on the graphing calculator, see Technical Appendix, B-2.

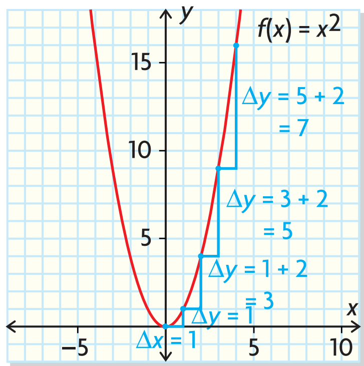
In Summary

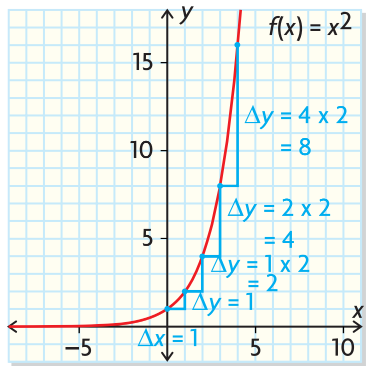
Key Ideas

- Linear, quadratic, and exponential functions have unique first-difference patterns that allow them to be recognized.

Linear	Quadratic	Exponential
Linear functions have constant first differences.	Quadratic functions have first differences that are related by addition. Their second differences are constant.	Exponential functions have first differences that are related by multiplication. Their second finite differences are not constant.







- The exponential function $f(x) = b^x$ is
  - an increasing function representing growth when $b > 1$
  - a decreasing function representing decay when $0 < b < 1$

Need to Know

- The exponential function $f(x) = b^x$ has the following characteristics:
  - If $b > 0$, then the function is defined, its domain is $\{x \in \mathbf{R}\}$, and its range is $\{y \in \mathbf{R} \mid y \geq 0\}$.
  - If $b > 1$, then the greater the value of b, the faster the growth.
  - If $0 < b < 1$, then the lesser the value of b, the faster the decay.
  - The function has the x-axis, $y = 0$, as horizontal *asymptote*.
  - The function has a y-intercept of 1.
- Linear, quadratic, and exponential functions can be recognized from their graphs. Linear functions are represented by straight lines, quadratic functions by parabolas, and exponential functions by quickly increasing or decreasing curves with a horizontal asymptote.
- A function in which the variables have exponent 1 (e.g., $f(x) = 2x$) is linear. A function with a single squared term (e.g., $f(x) = 3x^2 - 1$) is quadratic. A function with a positive base (0 and 1 excluded) and variable exponent (e.g., $f(x) = 5^x$) is exponential.

FURTHER Your Understanding

1. Use differences to identify the type of function represented by the table of values.

a)

x	*y*
−4	5
−3	8
−2	13
−1	20
0	29
1	40

c)

x	*y*
−2	−2.75
0	−2
2	1
4	13
6	61
8	253

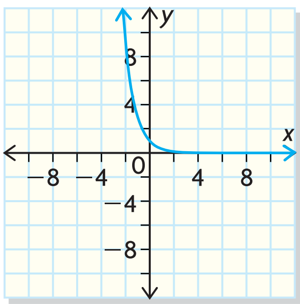
b)

x	*y*
−5	32
−4	16
−3	8
−2	4
−1	2
0	1

d)

x	*y*
0.5	0.9
0.75	1.1
1	1.3
1.25	1.5
1.5	1.7
1.75	1.9

2. What type of function is represented in each graph? Explain how you know.

a)


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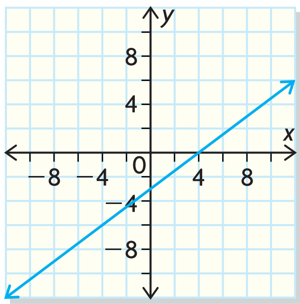
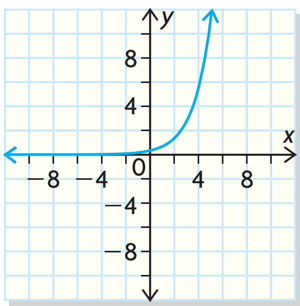


b)


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