MAILAM ENGINEERING COLLEGE

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(Approved by AICTE, New Delhi, Affiliated to Anna University Chennai& Accredited by TCS)

Department of Electrical & Electronics Engineering

SUB CODE / NAME: IC 6501/ CONTROL SYSTEMS

YEAR / SEC : III / A & B

SYLLABUS

UNIT I SYSTEMS AND THEIR REPRESENTATION

Basic elements in control systems – Open and closed loop systems – Electrical analogy of mechanical and thermal systems – Transfer function – Synchros – AC and DC servomotors – Block diagram reduction techniques – Signal flow graphs.

UNIT II - TIME RESPONSE

Time response – Time domain specifications – Types of test input – I and II order system response –Error coefficients – Generalized error series – Steady state error – Root locus construction- Effects of P, PI, PID modes of feedback control – Time response analysis.

UNIT III - FREQUENCY RESPONSE

Frequency response – Bode plot – Polar plot – Determination of closed loop response from open loopresponse - Correlation between frequency domain and time domain specifications- Effect of Lag, leadand lag-lead compensation on frequency response- Analysis.

UNIT IV STABILITY AND COMPENSATOR DESIGN

Characteristics equation – Routh Hurwitz criterion – Nyquist stability criterion- Performance criteria –Lag, lead and lag-lead networks – Lag/Lead compensator design using bode plots

UNIT V - STATE VARIABLE ANALYSIS

Concept of state variables – State models for linear and time invariant Systems – Solution of state and output equation in controllable canonical form – Concepts of controllability and observability – Effect of state feedback.

TEXT BOOKS:

1. M. Gopal, 'Control Systems, Principles and Design', 4thEdition, Tata McGraw Hill, New Delhi, 2012
2. S.K.Bhattacharya, Control System Engineering, 3rd Edition, Pearson, 2013.

REFERENCE BOOKS:

1. Arthur, G.O.Mutambara, Design and Analysis of Control; systems, CRC Press, 2009.
2. Richard C. Dorf and Robert H. Bishop, "Modern Control systems", Pearson Prentice Hall, 2012.

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PART-A

1. **Define transfer function?(NOV-DEC 2014)**

   The transfer function of a system is defined as the ratio of Laplace transform of output to Laplace transform of input with zero initial condition. It is also defined as the Laplace transform of the impulse response of system with zero initial condition.

   $$G(s) = C(s)/R(s)$$

2. **Define open loop and closed loop control system.(Nov/Dec 2011) (Nov/Dec 2017)**

   *The control system in which the output quantity has no effect upon the input quantity is called open loop control system. This means that the output is not fed back to the input for correction.
   *The control system in which the output has an effect upon the input quantity in order to maintain the desired output value are called closed loop control systems.

3. Distinguish between open loop and closed loop system?(MAY-JUNE 2013), Compareopen loop and closed loop systems.(May/June 2016,17)

Open loop	**Closed loop**
1. Inaccurate and unreliable.	Accurate and reliable
2. Simple and economical	Complex and costly
3. Changes in output due to external disturbance are not corrected automatically.	Changes in output due to external disturbances are corrected automatically.
4. They are generally stable.	Great efforts are needed to design a stable system.

5. **Why negative feedback is preferred in control system? (Nov/Dec 2016,May-17)**

   The negative feedback results in better stability in steady state and rejects any disturbance signals. It also has low sensitivity to parameters variations. Hence negative feedback is preferred in control system.

6. **Give an example for open loop and closed loop control system? (Nov/Dec 2015)**

   Open loop system:
   *A man walking on a road with closed eyes due to absence of eyes it is very difficult to walk on the desired path. Eye is used to measure actual path and compare with desired path.
   *D.C. Shunt motor for a give field current a voltage is applied to the armature to produce the desired value of motor speed. If the motor speed is changed to change in mechanical load on the shaft, there is no way in the open loop system to change the value of the applied armature voltage to maintain the desired speed.
   Closed loop system:
   *A man walking from a starting point to the destination point along a prescribed path. The eye performs a function of a sensor, brain compares the actual path of movement with prescribed path and generates error signal. The error signal and transmits a control signal to the legs to connect the actual path of movement to the desired path.

7. **What are the three basics elements in electrical and mechanical system?(May-June 2013), (NOV-Dec 2010, 2014) (or) what are thebasic elements in control systems?(May/June 2016) (Nov/Dec 2017)**

Electrical system:

1. Resistor 2. Inductor 3. Capacitors are the basic elements for electrical system.

Mechanical system:

a) Translational system can be obtained by using three basic elements mass, spring and dashpot.
b) Rotational system can be obtained by using three basic elements mass with moment of inertia I, dash pot with rotational coefficient B, and torsional spring with stiffness K.

8. **What are the important features of feedback?**

*The controlled variable accurately follows the desired value.
*Feedback in the control loop allows accurate control of output even when process or controlled plant parameters are not known accurately.
*Feedback in a control system greatly improves the speed of its response compared to response speed capability of the plant.
*It reduces the sensitivity of the system to parameter variations.
*Feedback signal gives this system the capability to act as self correcting mechanism
*It reduced the disturbance signals.

9. **What are the advantages of closed loop systems? (Apr-May 2015)**

*Accuracy of such system is always very high because controller modifies and manipulates the actuating signal such that the error in the system will be zero.
*Such system senses environmental changes as well as internal disturbances and accordingly modifies the error.
*In such system, there is reduced effect of non linearity's and disturbances.
*Bandwidth of such system ie. Operating frequency zone for such system is very high.

10. **What are the disadvantages of closed loop system?(Apr-May 2014)**

*such systems are complicated and time consuming from design point of view and hence costlier.
*Due to feedback system tries to correct the error time to time. Tendency to overcorrect the error may cause oscillation without bound in the system.
*The system has to be designed taking into consideration problem of instability due to feedback.
*The stability problems are severe and must be taken care of while designing the system.

11. **What are the advantages of open loop system?(Apr-May 2014)**

*System are simple in construction
*Very much convenient when output is difficult to measure.
*Such system are easy from maintenance point of views.
*Generally these are not troubled with the problems of stability.
*Such system are simple to design and hence economical.

12. **What are the disadvantages of open loop system?**

*System are inaccurate and unreliable because accuracy of such systems are totally dependent on the accurate precalibration of the controller.
*Such systems give inaccurate results if there are variations in the external environment.
*Similarly they cannot sense internal disturbance in the system after the controller stage.
*To maintain the quality and accuracy recalibration of the controller is necessary time to time.

13. Give an example for open loop and closed loop system?

Open loop system:

*Electric switch, automatic door opening and closing.
*Automatic washing machine, automatic coffee maker.
*Traffic signal controller, theater light dinner.
*Automatic toaster system, Automatic dryer.
*Room heater, fan regulator, electric lift.

Closed loop system:

*Human being, home heating system.
*Ship stabilization system, D.C motor speed control.
*Voltage stabilizer, missile launching system.
*Temperature control system, optical telescopes.
*Machine tool position control, positioning of radio.
*Audio pilots for aircrafts, sun seeker solar system, railway reservation station display.

14. What do you mean by sensitivity of the control system?

Sensitivity is a measure of the effectiveness of feedback in reducing the influence of these variations on system performance.

Sensitivity=percentage change in T(s)/percentage change in G(s).

15. What are the analogous systems?

The systems for which the differential equations have similar forms are known as analogous system. **E.g. 1.Force-voltage analogy or direct analogy. 2. Force-current analogy or inverse analogy.**

16. What is feedback? What type of feedback is employed in control system?(APR-MAY 2011)

The feedback is a control action in which the output is sampled and a proportional signal is given to input for automatic correction of any changes is desired output. Negative feedback is employed in control system.

17. What is the effect of positive feedback on stability?

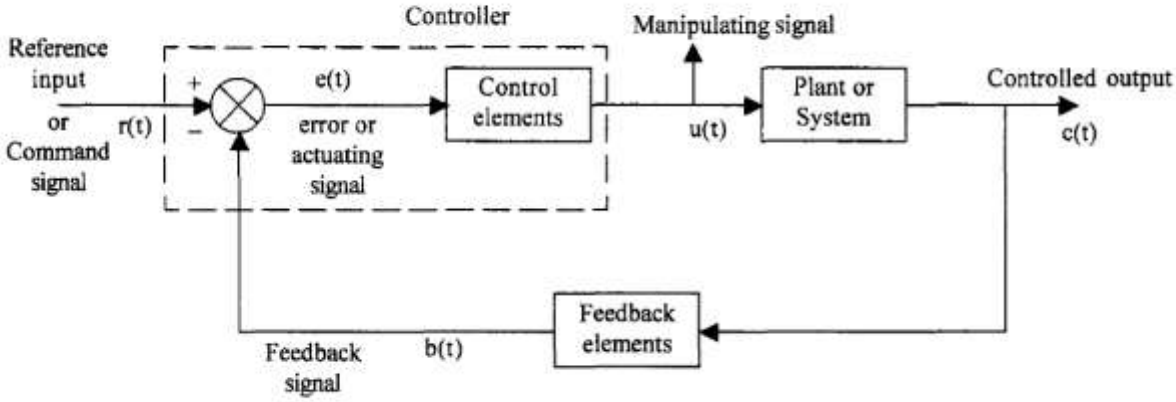
The positive feedback increases the error signal and desires the output to instability. But sometime the positive feedback is used in minor loops n control systems to amplify certain internal signals or parameters.

18. List the characteristics of negative feedback in control system.(April/May2018)

*Accuracy in tracking steady state value.
*Rejection of disturbance signals.
*Low sensitivity to parameter variations.
*Reduction is gain at the expense of better stability.

19. What are the components of feedback control system?

The components of feedback control systems are plant, feedback path elements, error detector and controller.



20. What is system?

When a number of elements or components are connected in a sequence to perform a specific function, the group thus formed is called system.

21. What is control system?

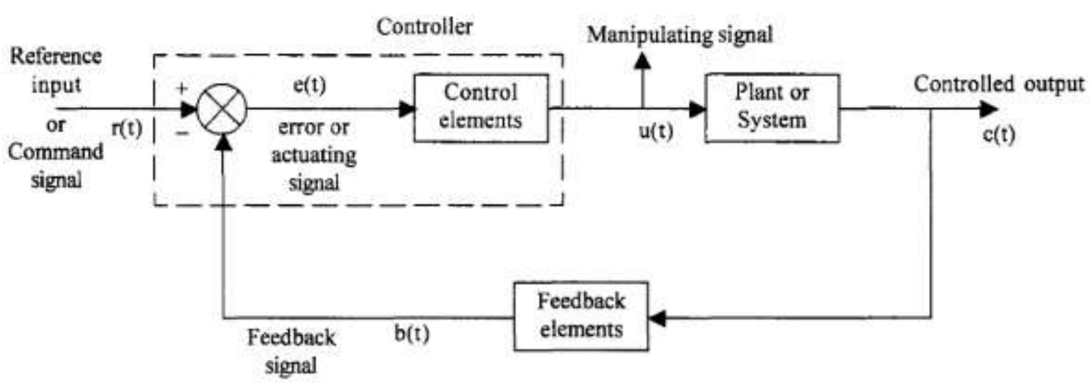
A system consists of a number of components connected together to perform a specific functions in a system when the output quantity is controlled by varying the input quantity, then the system is called control system.

22. What are the different classifications of control system?

*Open loop control system.
*Closed loop control system.
*Linear and non linear control system.
*Time variant and time invariant control system.
*Analog and digital control system.

23. What are the basic components of an automatic control system?

Basic components of an automatic control systems are, Reference input, error detector, actuating signal, control elements, plants, feedback path elements, controlled output forward path element.



24. **State whether transfer function is applicable to non linear system and whether the transfer function is independent of the input of a system.**

*The transfer function technique is not applicable to non linear system.
*The transfer function of a system is independent of input and depends only on system parameters but the output of a system depends on input.

25. What is meant by multivariable system?

A control system with multiple inputs and multiple outputs is called a multivariable system. The system have strong coupling if one input affects more than one output appreciably.

26. What is electrical element analogous to mass in mechanical system?

*Inductance is element for mass of force voltage analogy.
*Capacitance is the element for mass of force current analogy.

27. What is state variable model?

The set of equations that describe the relationship among input, output and the state variables is called the state variable model.

28. Define state and state variable of a system?

State is defined as initial velocity and initial displacement as describing the status of the system at t=to.
The state of the system at any time t is given variable x(t) and v(t) which are called the state variable of the system.

29. What are the advantages of state variable model?

*Transfer function model gives the relationship between input and output variables only like voltage across capacitor and applied voltage only.
*It is not possible to find the value of other variables like voltage across inductance and current through inductance in the circuit.
*Transfer function modeling is a terminal approach where we can find only the output and not the state of other variable inside the system.
*This drawback can be eliminated using state variable model in which we can find the state of other variables of the system at any time.

30. What is an error detector?

Error detector is an element of automatic controller which compares the reference input with feedback signal to produce an error signal if there is a difference between them. The error signal is used to correct the output if there is a deviation from the desired value.
Eg: potentiometer, LVDT ,Synchros.

31. What do you understand by a regulator system and servomechanism?

The servomechanism is a feedback control system in which the output is mechanical position
Eg. Velocity and acceleration
Regulator system is a feedback control system in which for a preset value of the reference input, the output is kept constant at its desired value.
Eg. Servo stabilizer, temperature regulator, frequency controllers, speed governers.

32. What is block diagram? What are the basic components of block diagram?(May-17)

A block diagram of a system is a pictorial representation of the functions performed by each component of the system and shows the flow of signals. The basic elements of block diagram are block, branch, point and summing point.

33. Give the statement of mason's gain formula. (Nov-Dec 2010), (Apr-May 2011)
Write the expression for mason's gain formula. (April/May2018)

Mason's gain formula states that the overall gain of the system (transfer function) as follows

$$overall\ gain, T = \frac{1}{\Delta}\sum_{K} P_k\, \Delta_k$$

T= T(s) = Transfer function of the system
K= Number of forward path
P_k = Forward path gain of k^{th} forward path

$$\Delta = 1 - \begin{bmatrix} sum of individual \\ loop gains \end{bmatrix} + \begin{bmatrix} sum of gain products of all possible \\ combinations of two non-touching loops \end{bmatrix} - \begin{bmatrix} sum of gain products of all possible \\ combinations of three non-touching loops \end{bmatrix} + \cdots$$

$$\Delta_k = \Delta\ for\ that\ part\ of\ the\ graph\ which\ is\ not\ touching\ K^{th}\ forward\ path$$

34. What are the two assumptions to be made while deriving transfer function of electrical systems?

- It is assumed that there is no loading, ie. No power us drawn at the output of the system. If the system has more than one non loading element in tandem, then the transfer function of each element can be determined independently and the overall transfer function of the physical system is determining by multiplying the individual transfer functions.
- The system should be approximated by a linear lumped constant parameters model by making suitable assumptions.

35. Define signal flow graph.

A signal flow graph is diagram that represents a set of simultaneous linear algebraic equations. By taking Laplace transform the time domain differential equations governing a control system can be transferred to a set of algebraic equations in s-domain. This signal flow graph of the system can be constructed using these equations.

36. What are the advantages of SFG approach of determining transfer function? (Apr-May 2011)

- SFG does not require any reduction process because of availability of a flow graph gain formula which relates the input and output system variables.
- A signal flow graph is graphical representation of the relationships between the variable of a set of linear algebraic equations.
- SFG method is simple and less time consuming
- Instead of feedback various feedback loops are considered for analysis.

37. What do you mean by linear time invariant systems?

A mathematical model is linear; if the coefficients of the describing differential equations are functions of time then the system is called linear time invariant systems.
E.g. Frequency response analysis of single input single output linear systems.

38. What is meant by robust control?

Robust control is a branch of control system which deals with the uncertainty because of disturbance and other factors in its approach to the controller design. Its aim is to achieve robust performance and stability in the presence of the small modeling errors. It measures performance changes of a control system with changing system parameters for designing proper controllers.

39. What is a synchrony OR synchro? Where it is used? (Apr-May 2014)
Define synchros.(May/June 2016)

Synchrony is basically a rotary device, an electromagnetic transducer which operates on same principle as that of transformer. It converts angular position of shaft into an electrical signal.

Synchros are used widely in control systems as detectors and encoders because of their rigidness in construction and high reliability.

40. **Explain the disadvantages and advantages of block diagram reduction process over signal flow graph?**

Advantages:

- Block diagrams can be easily visualized.
- It is identical to physical system.
- It consists of blocks and each block represents a function of each components of a system.

Disadvantages:

- Block diagram reduction is not a generalized procedure like signal flow graph.
- Block diagram reduction is difficult to simplify than signal flow graph if the system has complex structures.
- Many rules block diagram reduction depends upon the complexity of the system on the other hand signal flow graph has systematic approach.

41. How D.C servomotor differs from D.C. motor?

D.C. servomotor is more or less same as normal D.c motor. There are some minor differences between the two. All D.c servomotors are essentially separately excited type. This ensures linear torque speed characteristics. General D.c motor operated as self and separately excited method.

42. What is a servomotor?

The servo systems is one in which the output is some mechanical variable like position, velocity or acceleration. Such systems are generally automatic control systems, which work on the error signals. The error signals are amplified to drive the motors used in such systems. These motors used in servo systems are called servomotors.

43. What are the applications of D.C. servomotors?

These are widely used in air craft control systems, electromechanical actuators, process controllers, robotics, machine tools etc.

44. What are the applications of A.C. servomotors?

A.C. servomotors widely used in instruments servomechanisms, remote positioning devices, process control systems, self balancing recorders computers, tracking and guidance systems, robotics, machine tools etc.

45. What are the features of A.C. servomotors?

- Light in weight.
- Robust construction
- Reliable and stable operation
- Smooth and noise free operation
- Large torque to weight ratio
- Large R to X ratio
- No brushes or slip rings maintenance free
- Simple driving circuits.

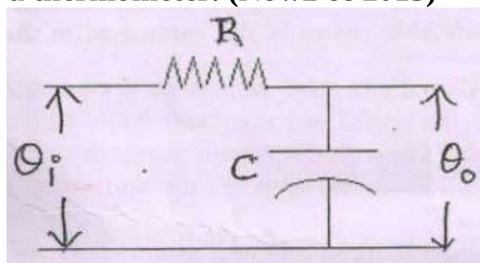
46. What are the features of D.C. servomotors?

- Suitable for large rated motors.
- It has small time constant hence its response is fast to the control signal
- It is closed loop system
- The back e.m.f provides internal damping which makes motor operation motor stable
- Efficiency and overall performance is better.

47. Write the force balance equations of ideal dashpot and ideal spring?(APR-MAY 2015)

$$f_{b1} = B_1 \frac{dX_1}{dt};\ f_{k1} = K_1 X_1;$$

48. Draw the electrical analog of a thermometer. (Nov/Dec 2015)



49. What is electrical zero position of a synchro transmitter. (Nov/Dec 2015)

The electrical zero position of a synchro transmitter is a position of its rotor at which one of the coil-to-coil voltages is zero. Any angular motion of the rotor is measured with respect to the electrical zero position of the rotor.

50. What are the difference between a synchro transmitter and a synchro control transformer.Dec-16

The synchro control transformer is similar in construction to a synchro transmitter. The only one difference is the rotator of synchro control transformer is made cylindrical in shape.so that the air gap is practically uniform.

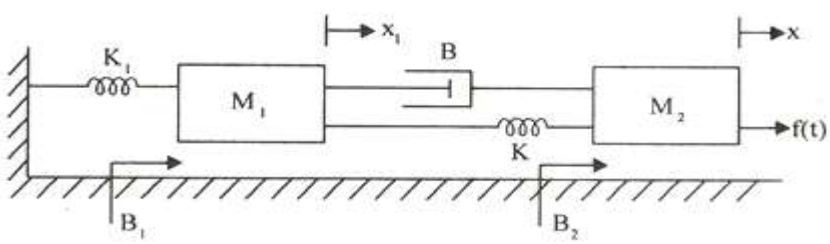
PART-B

1. Write the differential equations governing the mechanical system shown in fig .And determine the transfer function? (Apr-May 2015,Nov/Dec-2016)

SOLUTION

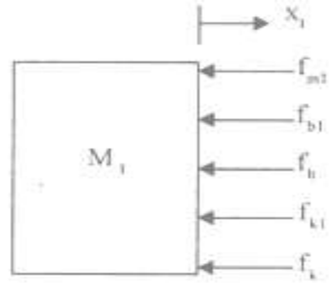
In the given system, applied force f(t) is the input and displacement X is the output. **(Apr-May 2011)**

Let, Laplace transfer of f(t)=$\pounds\{f(t) = F(s)\}$
Laplace transfer of x =)=$\pounds\{X\} = X(s)$
Laplace transfer of X_1= $\pounds\{X_1\} = X_1(s)$



Hence the required transfer function is $\frac{X(s)}{F(s)}$

Free body diagram-1



$$f_{m1} = M_1 \frac{d^2X_1}{dt^2};\ f_{b1} = B_1 \frac{dX_1}{dt}; f_{k1} = K_1X_1;$$

$$f_b = B\frac{d}{dt}(X_1 - X); f_k = K(X_1 - X);$$

By Newton’s second law, $f_{m1} + f_{b1} + f_b + f_{k1} + f_k = 0$

$$\therefore M_1 \frac{d^2X_1}{dt^2} + B_1 \frac{dX_1}{dt} + B\frac{d}{dt}(X_1 - X) + K_1X_1 + K(X_1 - X);$$

On taking Laplace transform of above equation with zero initial conditions we get,

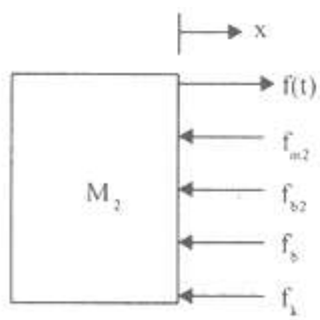
$$M_1s^2X_1(s) + B_1sX_1(s) + Bs[X_1(s) - X(s)] + K_1X_1(s) + K[X_1(s) - X(s)] = 0$$

$$X_1(s)[M_1s^2 + (B_1 + B)s + (K_1 + K)] - X(s)[Bs + K] = 0$$

$$X_1(s)[M_1s^2 + (B_1 + B)s + (K_1 + K)] = X(s)[Bs + K]$$

$$\therefore X_1(s) = X(s)\frac{Bs + K}{M_1s^2 + (B_1 + B)s + (K_1 + K)} \dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

Free body diagram-2



$$f_{m2} = M_2 \frac{d^2X}{dt^2}; f_{b2} = B_2 \frac{dX}{dt}; f_b = B\frac{d}{dt}(X - X_1); f_k = K(X - X_1)$$

By Newton’s second law, $f_{m2} + f_{b2} + f_b + f_k = f(t)$

$$M_2 \frac{d^2X}{dt^2} + B_2 \frac{dX}{dt} + B\frac{d}{dt}(X - X_1) + K(X - X_1) = f(t)$$

On taking Laplace transform of above equation with zero initial conditions we get,

$$M_2s^2X(s) + B_2sX(s) + Bs[X(s) - X_1(s)] + K[X(s) - X_1(s)] = F(s)$$

$$X(s)[M_2s^2 + (B_2 + B)s + K] - X_1(s)[Bs + K] = F(s) \dots\dots\dots\dots\dots..(2)$$

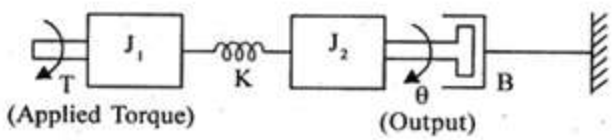
Substituting $X_1(s)$ from equation (1) in equation (2) we get,

$$X(s)[M_2s^2 + (B_2 + B)s + K] - \frac{(Bs + K)^2}{M_1s^2 + (B_1 + B)s + (K_1 + K)} = F(s)$$

$$X(s)\left[\frac{[M_1s^2 + (B_2 + B)s + K][M_2s^2 + (B_1 + B)s + (K_1 + K)] - (Bs + K)^2}{M_1s^2 + (B_1 + B)s + (K_1 + K)}\right] = F(s)$$

$$\therefore\ \frac{\mathbf{X(s)}}{\mathbf{F(s)}} = \frac{\mathbf{M_1s^2 + (B_1 + B)s + (K_1 + K)}}{\mathbf{[M_1s^2 + (B_1 + B)s + (K_1 + K)][M_2s^2 + (B_1 + B)s +] - (Bs + K)^2}}$$

2. Write the differential equations governing the mechanical rotational system as shown in fig, obtain the transfer function of the system.(Nov-Dec 2010), (Nov-Dec 2011)



<u>SOLUTION</u>

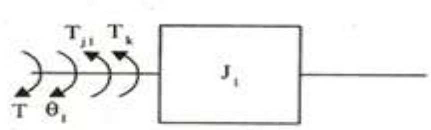
In the given system, applied force f(t) is the input and displacement X is the output.
Let, Laplace transfer of T =£{T} = T(s)
Laplace transfer of x =)=£{θ} = θ(s)
Laplace transfer of X_1= £{θ_1} = θ_1(s)

Hence the required transfer function is $\frac{\theta(s)}{T(s)}$

The system has two nodes and they are mass $J_1$and J_2, the differential equations governing the system are given by torques balance equations at these nodes.

Let the displacement of mass J_1 be θ_1 .The free body diagram of J_1 is shown in fig. the opposing forces acting on $J_1$are marked as T_j, and T_k.

Free body diagram-1



$$T_{j1} = J_1\frac{d^2\theta_1}{dt^2};\ T_k = K(\theta_1 - \theta)$$

By Newton's second law,$T_{j1} + T_k = T$

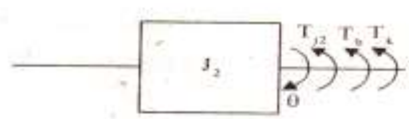
$$J_1\frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) = T$$

On taking Laplace transform of above equation we get,

$$J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta(s) = T(s)$$

$$(J_1 s^2 + K)\theta_1(s) - K\theta(s) = T(s) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..(1)$$

Free body diagram-2



$$T_{j2} = J_2 \frac{d^2\theta}{dt^2}; T_b = B\frac{d\theta}{dt};\ T_k = K(\theta - \theta_1)$$

By Newton's second law,$T_{j2} +\ T_b +\ T_k = 0$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0$$

On taking Laplace transform of above equation we get,

$$J_2 s^2 \theta(s) + Bs\theta(s) + K\theta(s) - K\theta_1(s) = 0$$

$$(J_2 s^2 + Bs + K)\theta(s) - \theta_1(s) = 0$$

$$\theta_1(s) = \frac{(J_2 s^2 + Bs + K)}{K}\theta(s) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.(2)$$

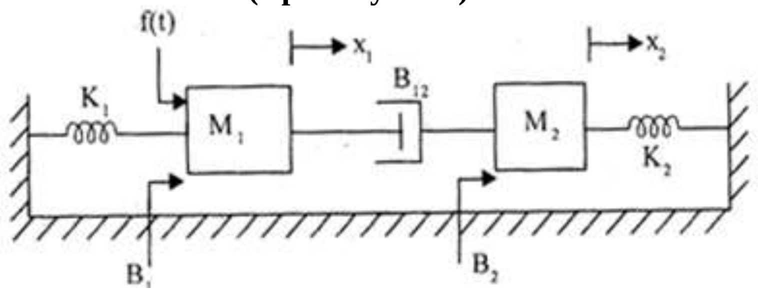
Substitute $\theta_1(s)$from equation 2 in equation 1 we get,

$$(J_1 s^2 + K)\frac{(J_2 s^2 + Bs + K)}{K}\theta(s) -\ K\theta(s) = T(s)$$

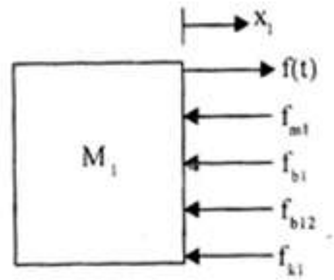
$$\left[\frac{(J_1 s^2 + K) + (J_2 s^2 + Bs + K) - K^2}{K}\right]\theta(s) =\ T(s)$$

$$\boxed{\frac{\boldsymbol{\theta}(\mathbf{s})}{\mathbf{T}(\mathbf{s})} = \frac{\mathbf{K}}{(\mathbf{J_1 s^2 + K}) + (\mathbf{J_2 s^2 + Bs + K}) - \mathbf{K^2}}}$$

3. Write the differential equations forthe mechanical system in fig. Also obtain an analogues electrical circuit based on force-current angels and force voltages angels (Dec 2011), (Apr-May 2014)



Free body diagram-1



$$f_{m1} = M_1\frac{d^2X_1}{dt^2};\ f_{b1} = B_1\frac{dX_1}{dt};\ f_{b12} = B_{12}\frac{d}{dt}(X_1 - X_2);\ f_{k1} = K_1X_1$$

By Newton's second law, $f_{m1} + f_{b1} + f_{b12} + f_{k1} = f(t)$

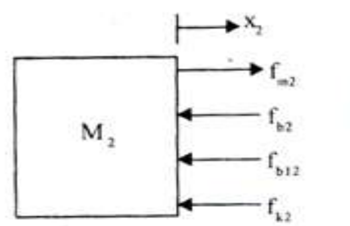
$$M_1\frac{d^2X_1}{dt^2} + B_1\frac{dX_1}{dt} + \frac{d}{dt}(X_1 - X) + K_1X_1 = f(t)$$

On takingLaplace transform of above equation we get,

$$M_1s^2X_1(s) + B_1sX_1(s) + B_{12}s[X_1(s) - X_2(s)] + K_1X_1(s) = F(s)$$

$$X_1(s)[M_1s^2 + (B_1 + B_{12})s + K_1] - \quad B_{12}sX_2(s) = F(s) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Free body diagram-2



$$f_{m2} = M_2\frac{d^2X_2}{dt^2};\ f_{b2} = B_2\frac{dX_2}{dt};\ f_{b12} = B_{12}\frac{d}{dt}(X_2 - X_1);\ f_{k1} = K_1X_1$$

By Newton's second law,

$$M_2\frac{d^2X_2}{dt^2}+B_2\frac{dX_2}{dt}+B_{12}\frac{d}{dt}(X_2-X_1)+K_1X_1=0$$

On taking Laplace transform of above equation we get,

$$M_2s^2X_2(s)+B_2sX_2(s)+B_{12}s[X_2(s)-X_1(s)]+K_2X_2(s)=0$$

$$X_2(s)[M_2s^2+(B_2+B_{12})s+K_2]\text{-}B_{12}sX_1(s)=0$$

$$X_2(s)[M_2s^2+(B_2+B_{12})s+K_2]\text{=}B_{12}sX_1(s)$$

$$X_2(s)=\frac{\mathrm{B_{12}sX_1(s)}}{[\mathrm{M_2s^2+(B_2+B_{12})s+K_2}]}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2)$$

Substituting $X_2(s)$ from equation 2 in equation 1 we get,

$$X_1(s)[M_1s^2+(B_1+B_{12})s+K_1]-\frac{(B_{12}s)^2X_1(s)}{[M_2s^2+(B_2+B_{12})s+K_2]}=F(s)$$

$$\frac{\left[X_1(s)[M_1s^2+(B_1+B_{12})s+K_1]\right]\left[[M_2s^2+(B_2+B_{12})s+K_2]-(B_{12}s)^2\right]}{[M_2s^2+(B_2+B_{12})s+K_2]}=F(s)$$

$$\frac{X_1(s)}{F(s)}=\frac{[M_2s^2+(B_2+B_{12})s+K_2]}{\left[[M_1s^2+(B_1+B_{12})s+K_1]\right]\left[[M_2s^2+(B_2+B_{12})s+K_2]-(B_{12}s)^2\right]}$$

From equation 2 we get,

$$\mathrm{X_1(s)}=\frac{[\mathrm{M_2s^2+(B_2+B_{12})s+K_2}]X_2(s)}{\mathrm{B_{12}s}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

Substitute equation3 in equation1 we get,

$$\frac{[\mathrm{M_2s^2+(B_2+B_{12})s+K_2}]X_2(s)}{\mathrm{B_{12}s}}[M_1s^2+(B_1+B_{12})s+K_1]\text{-}B_{12}sX_2(s)\text{=}F(s)$$

$$X_2(s)\left[\frac{[\mathrm{M_2s^2+(B_2+B_{12})s+K_2}][M_1s^2+(B_1+B_{12})s+K_1]-(B_{12}s)^2}{\mathrm{B_{12}s}}\right]=F(s)$$

$$\boxed{\frac{\boldsymbol{X_2(s)}}{\boldsymbol{F(s)}}=\frac{\mathbf{B_{12}s}}{[\mathbf{M_2s^2+(B_2+B_{12})s+K_2}][\boldsymbol{M_1s^2+(B_1+B_{12})s+K_1}]-(\boldsymbol{B_{12}s})^2}}$$

The different equations of two nodes are

$$M_1\frac{d^2X_1}{dt^2}+B_1\frac{dX_1}{dt}+\frac{d}{dt}(X_1-X)+K_1X_1=f(t)$$

$$M_2\frac{d^2X_2}{dt^2}+B_2\frac{dX_2}{dt}+B_{12}\frac{d}{dt}(X_2-X_1)+K_1X_1=0$$

On replacing the displacements by velocity in the differential equations

$$\therefore\left(\frac{d^2X}{dt^2}=\frac{dv}{dt};\ \frac{dX}{dt}=v\ and\ X=\int v\,dt\right)$$

$$M_1\frac{dv_1}{dt}+B_1v_1+B_{12}(v_1-v_2)+K_1\int(v_1-v_2)\,dt=f(t)$$

$$M_1\frac{dv_1}{dt}+B_1v_1+B_{12}(v_1-v_2)+K_1\int(v_1-v_2)\,dt=f(t)\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(4)$$

$$M_2\frac{dv_2}{dt}+B_2v_2+B_{12}(v_2-v_1)+K_1\int(v_2-v_1)\,dt+K_2\int v_2\,dt=0\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

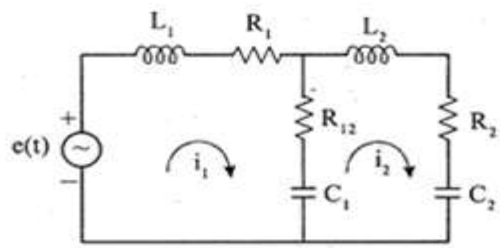
Force-voltage analogous circuit

The electrical analogous elements for the elements of mechanical system are given below.

$f(t)\rightarrow e(t)\quad M_1\rightarrow L_1 B_1\rightarrow R_1\qquad K_1\rightarrow\frac{1}{C_1}$

$v_1\rightarrow i_1 M_2\rightarrow L_2\qquad B_2\rightarrow R_2\qquad K_2\rightarrow\frac{1}{C_2}$

$v_2\rightarrow i_2\qquad\qquad B_{12}\rightarrow R_{12}$



The mesh basis equations using Kirchhoff’s voltage law for the circuit shown in fig are given below .

$$L_1\frac{di_1}{dt}+R_1i_1+R_{12}(i_1-i_2)+\frac{1}{C_1}\int(i_1-i_2)\,dt=e(t)\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

$$L_2\frac{di_2}{dt}+R_2i_2+R_{12}(i_2-i_1)+\frac{1}{C_1}\int(i_2-i_1)\,dt+\frac{1}{C_2}\int i_2\,dt=0\ \ldots\ldots\ldots\ldots\ldots\ldots\ (7)$$

It is observed that the mesh basis equations are similar to the differerential equations governing the mechanical system.

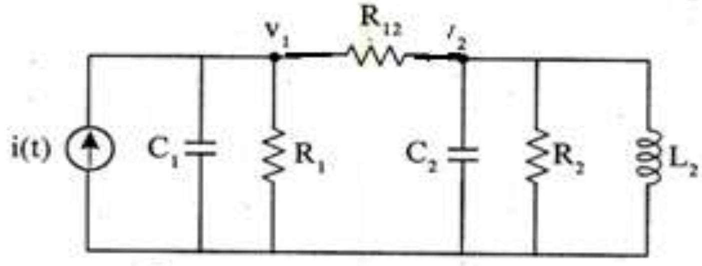
Force –current analogous circuit

The electrical analogous elements for the elements of mecha given below.

$f(t)\rightarrow i(t)\qquad M_1\rightarrow C_1 B_1\rightarrow\frac{1}{R_1}\qquad K_1\rightarrow\frac{1}{L_1}$

$v_1\rightarrow v_{1(voltage)} M_2\rightarrow C_2\qquad B_2\rightarrow\frac{1}{R_2}\qquad K_2\rightarrow\frac{1}{L_2}$

$v_2 \to v_2$(voltage) $\quad\quad B_{12} \to \dfrac{1}{R_{12}}$



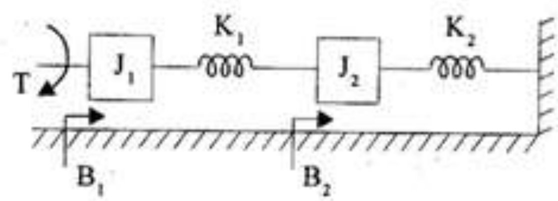
The node basis equations using Kirchhoff's current lawfor the circuit shown in fig are given below .

$$C_1 \frac{dv_1}{dt} + \frac{1}{R_1} v_1 + \frac{1}{R_{12}}(v_1 - v_2) + \frac{1}{L_1}\int (v_1 - v_2)\, dt = i(t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

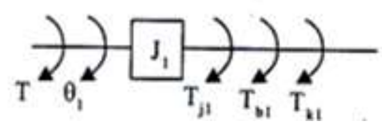
$$C_2 \frac{dv_2}{dt} + \frac{1}{R_2} v_2 + \frac{1}{R_{12}}(v_2 - v_1) + \frac{1}{L_1}\int (v_2 - v_1)\, dt + \frac{1}{L_2}\int v_2\, dt = 0 \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

It is observed that the node basis equations are similar to the differential equations governing the mechanical system.

4. Write the different equations governing the mechanical rotational system shown in fig draw the both the electrical analogues circuits. (May/June2016)



Free body diagram-1

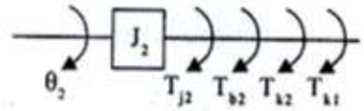


$$T_{j1} = J_1 \frac{d^2\theta_1}{dt^2};\ T_{b1} = B_1 \frac{d\theta_1}{dt}\ \ T_{k1} = K_1(\theta_1 - \theta_2)$$

By Newton's second law, $T_{j1} + T_{b1} + T_{k1} = T$

$$J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2) = T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Free body diagram-2



$$T_{j2} = J_2 \frac{d^2\theta}{dt^2}; T_{b2} = B_2 \frac{d\theta_2}{dt};\ T_{k1} = K_1(\theta_2 - \theta_1);\quad T_{k2} = K_2\theta_2$$

By Newton's second law, $T_{j2} + T_{b2} + T_{k1} + T_{k2} = 0$

$$J_2 \frac{d^2\theta}{dt^2} + B_2 \frac{d\theta_2}{dt} + K_1(\theta_2 - \theta_1) + K_2\theta_2 = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

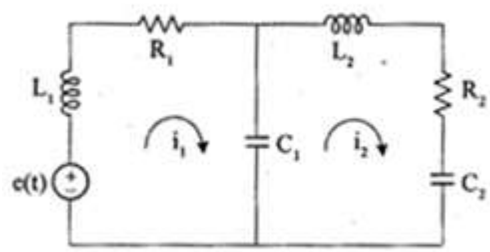
On replacing angular displacement by angular velocity in the differential equations 1 and 2

$$\therefore \left(\frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}; \frac{d\theta}{dt} = \omega \ and\ \theta = \int \omega \, dt\right)$$

$$J_1 \frac{d\omega_1}{dt} + B_1\omega_1 + K_1 \int (\omega_1 - \omega_2) dt = T \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3)$$

$$J_2 \frac{d\omega_2}{dt} + B_2\omega_2 + K_1 \int (\omega_2 - \omega_1)\, dt + K_2 \int \omega_2 dt = 0 \dots\dots\dots\dots\dots (4)$$

Torque- Voltage Analogous Circuit:



The electrical analogous elements for the elements of mechanical system are given below.

$$T \rightarrow e(t) \quad J_1 \rightarrow L_1 B_1 \rightarrow R_1 \qquad K_1 \rightarrow \frac{1}{C_1}$$

$$\omega_1 \rightarrow i_1 J_2 \rightarrow L_2 \qquad B_2 \rightarrow R_2 \qquad K_2 \rightarrow \frac{1}{C_2}$$

$$\omega_2 \rightarrow i_2$$

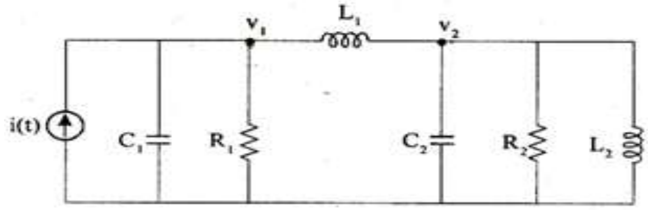
The mesh basis equations using Kirchhoff's voltage law for the circuit shown in fig are given below.

$$L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int (i_1 - i_2)\, dt = e(t) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (5)$$

$$L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_1} \int (i_2 - i_1)\, dt + \frac{1}{C_2} \int i_2 \, dt = 0 \dots\dots\dots\dots\dots\dots \ (6)$$

It is observed that the mesh basis equations 5 and 6 are similar to the differential equations 3 and 4 governing the mechanical system.

Torque- Current Analogous Circuit:



The electrical analogous elements for the elements of mechanical system are given below.

$$T \rightarrow i(t) \quad J_1 \rightarrow C_1 B_1 \rightarrow \frac{1}{R_1} \qquad K_1 \rightarrow \frac{1}{L_1} J_2 \rightarrow C_2 \qquad B_2 \rightarrow \frac{1}{R_2} \qquad K_2 \rightarrow \frac{1}{L_2}$$

$$\omega_1 \rightarrow v_1 \qquad \omega_2 \rightarrow v_2$$

The node basis equations using Kirchhoff's current lawfor the circuit shown in fig are given below

$$C_1 \frac{dv_1}{dt} + \frac{1}{R_1} v_1 + \frac{1}{L_1} \int (v_1 - v_2)\, dt = i(t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$C_2 \frac{dv_2}{dt} + \frac{1}{R_2} v_2 + \frac{1}{L_1} \int (v_2 - v_1)\, dt + \frac{1}{L_2} \int v_2\, dt = 0 \ldots\ldots\ldots\ldots\ldots\ldots \; (8)$$

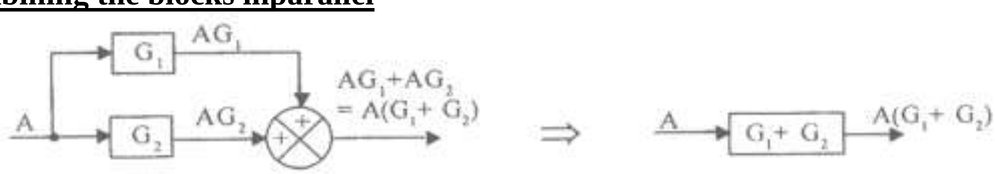
It is observed that the node basis equations 7 and 8 are similar to the differential equations 3 and 4 governing the mechanical system.

BLOCK DIAGRAM REDUCTION

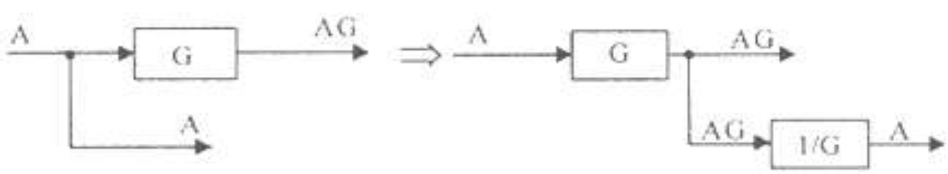
RULES OF BLOCK DIAGRAM ALGEBRA

Rule-1: Combining the blocks in cascade

Rule-2: Combining the blocks inparallel

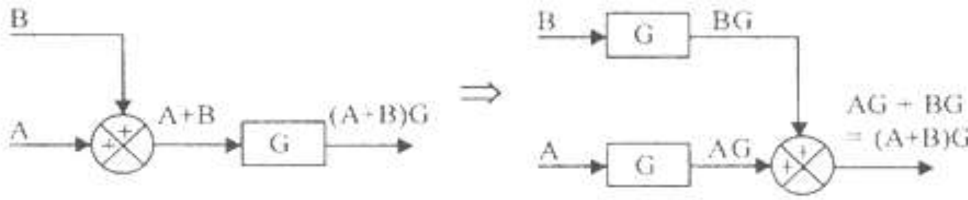


Rule-3: Moving the branch point after the block

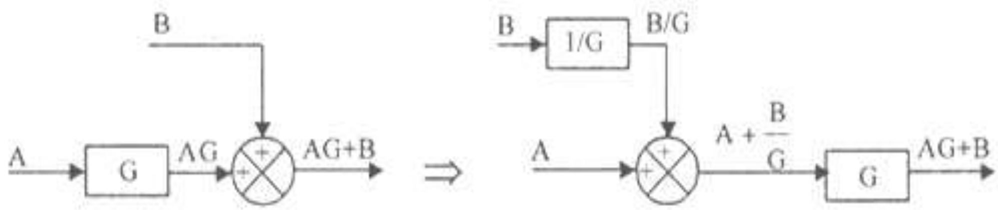


Rule-4: Moving the branch point before the block

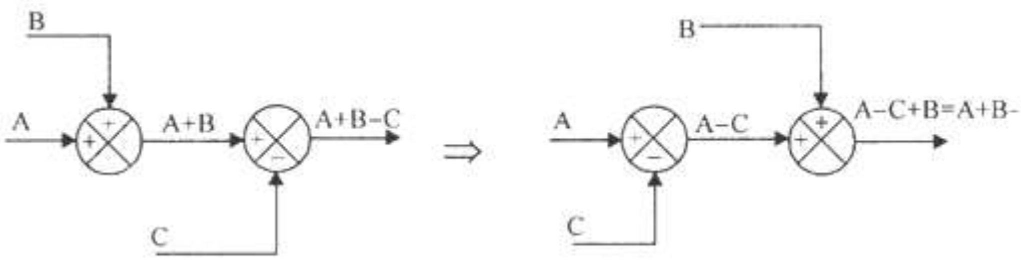
Rule-5: Moving summing point after the block



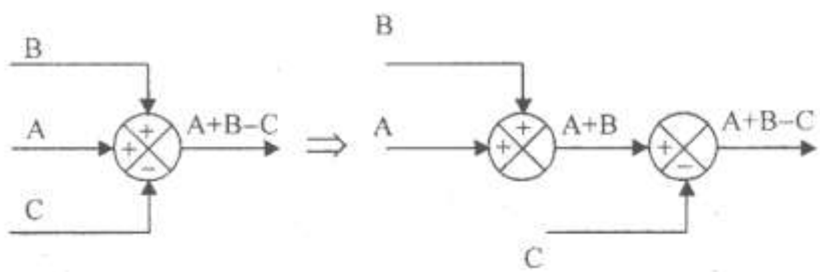
Rule-6: Moving summing point before the block



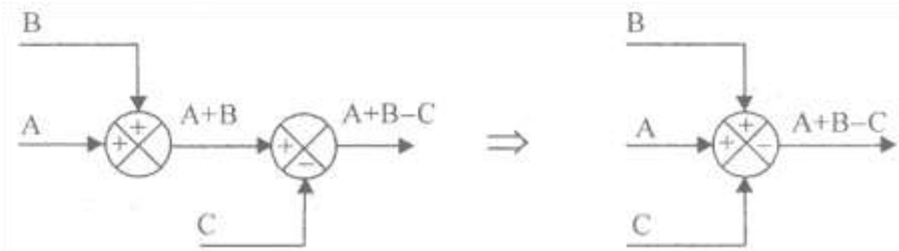
Rule-7: Interchanging summing point



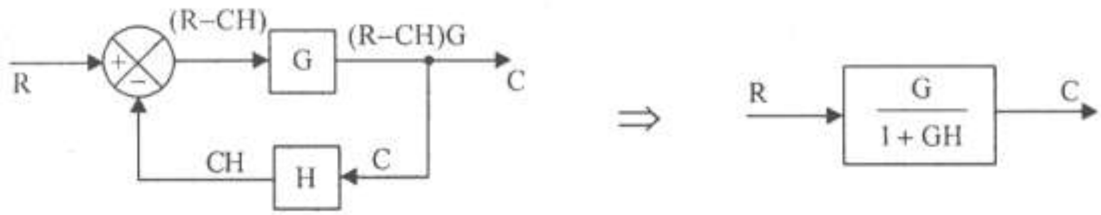
Rule-8: Splitting summing point



Rule-9: Combining summing points



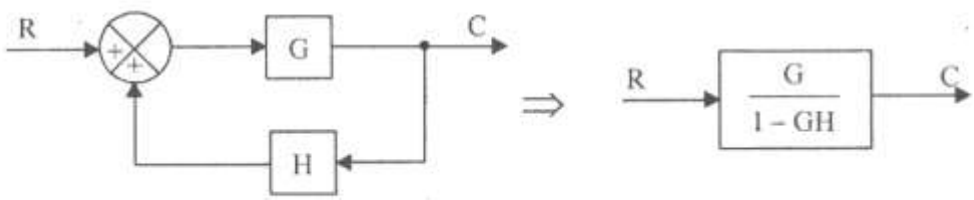
Rule-10: Elimination of negative feedback loop



Proof :

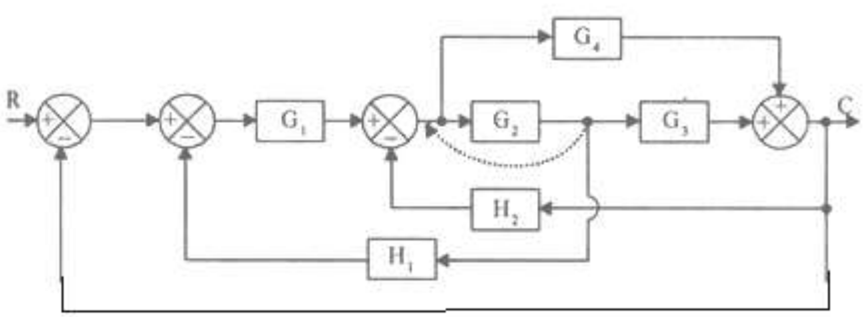
$C = (R - CH)\,G$	$C(1+HG)=RG$
$C = RG - CHG$	$\dfrac{C}{R} = \dfrac{G}{1+GH}$
$C + CHG = RG$	

Rule-11: Elimination of positive feedback loop

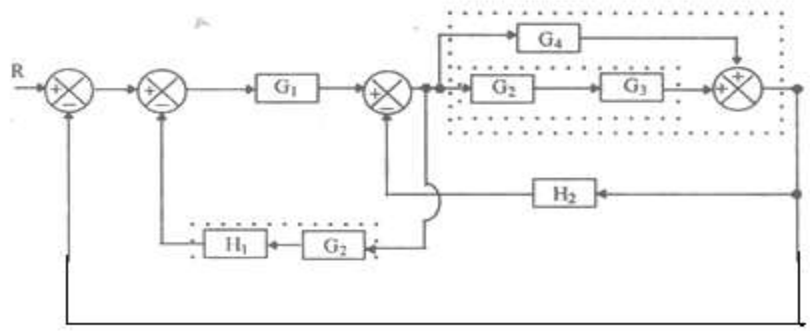


5. Using block diagram reduction technique find closed loop transfer function of the system whose block diagram is shown in fig.(Nov/Dec 2011)

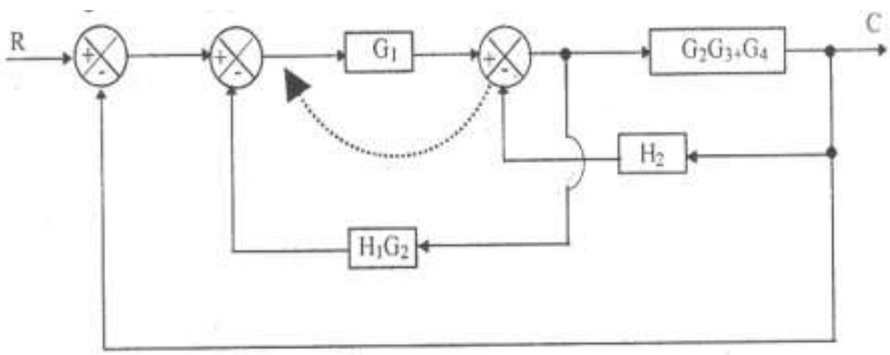
Step 1: Moving the branch point before the block



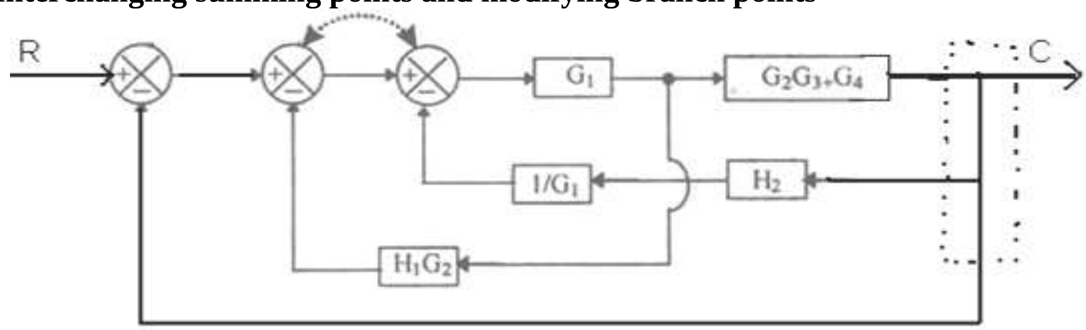
Step 2: Combining the blocks in cascade and eliminating parallel blocks



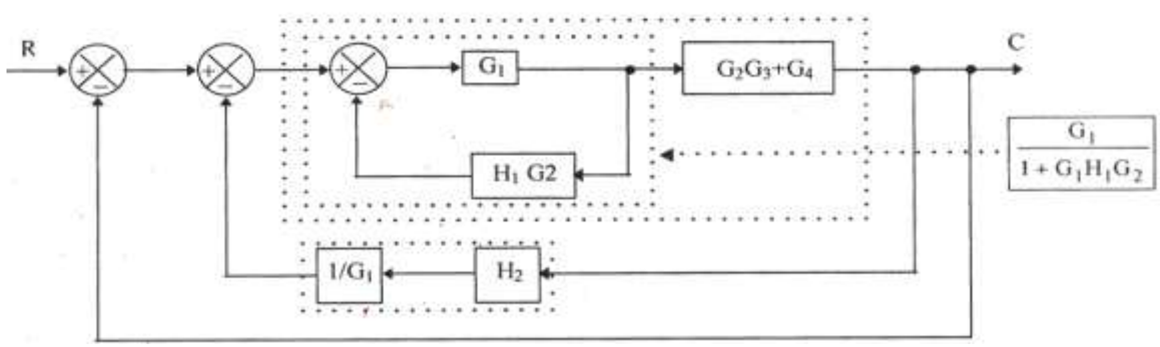
Step 3: Moving summing point before the block



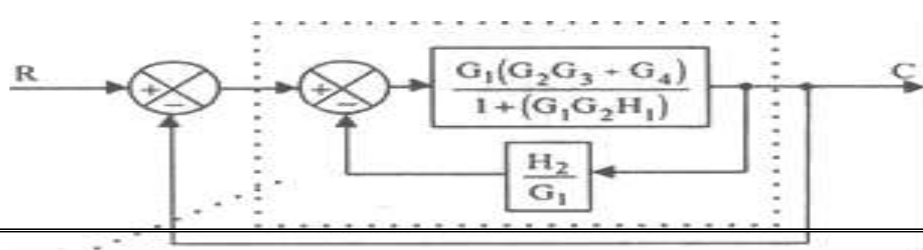
Step 4: Interchanging summing points and modifying branch points



Step 5: Eliminating the feedback path and combining blocks in cascade



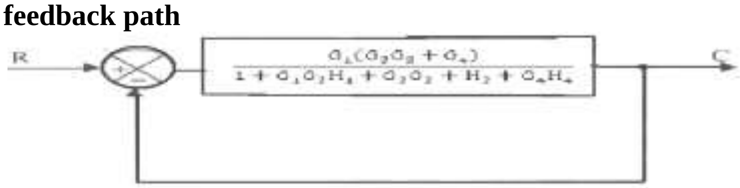
Step 6: Eliminating the feedback path



$$\frac{\frac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}}{1+\frac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}\frac{H_2}{G_1}} \Rightarrow \frac{\frac{G_1(G_2G_3+G_1G_4)}{1+G_1G_2H_1}}{\frac{1+G_1(G_2H_1+G_2G_3H_2+G_4H_2)}{1+G_1G_2H_1}}$$

$$= \frac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1+G_2G_3+H_2+G_4H_4}$$

Step 7:Eliminating the feedback path

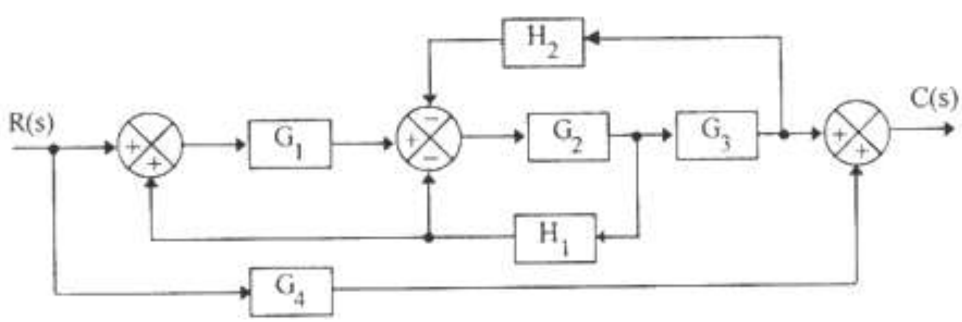


$$\frac{C}{R} = \frac{\frac{G_1(G_2G_3+G_1G_4)}{1+G_1G_2H_1G_2G_3H_2+G_4H_2}}{1+\frac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1+G_2G_3+H_2+G_4H_4}}$$

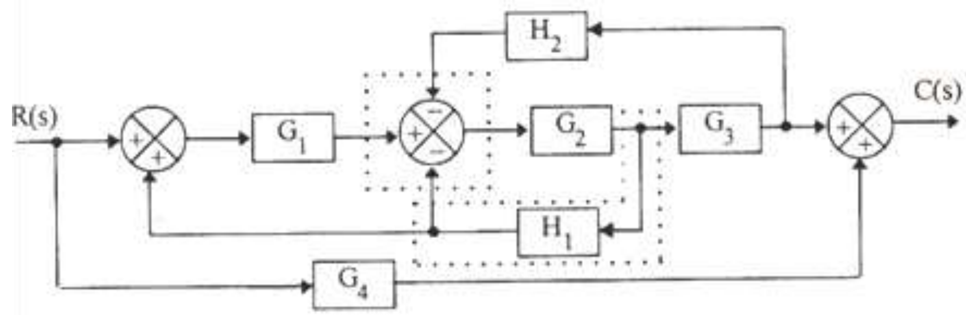
$$= \frac{G_1(G_2G_3+G_1G_4)}{1+G_1G_2H_1+G_2G_3+H_2+G_4H_4+G_1G_2G_3+G_1G_4}$$

$$\boxed{\mathbf{\frac{C}{R} = \frac{G_1(G_2G_3+G_1G_4)}{1+G_1G_2H_1+G_2G_3+H_2+G_4H_4+G_1G_2G_3+G_1G_4}}}$$

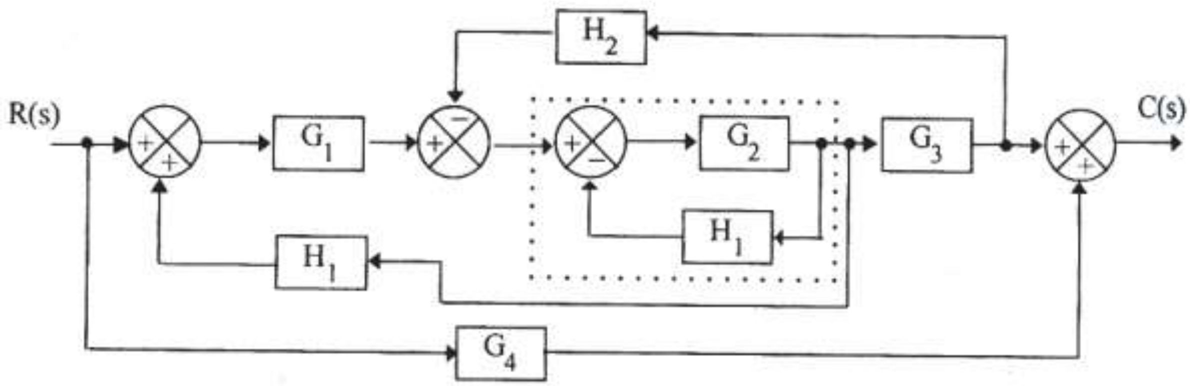
6. Obtain the closed loop transfer function C(s)/R(s) of the system whose block diagram is shown in fig.?(April/May-2010, 2011, Nov/Dec-2014)



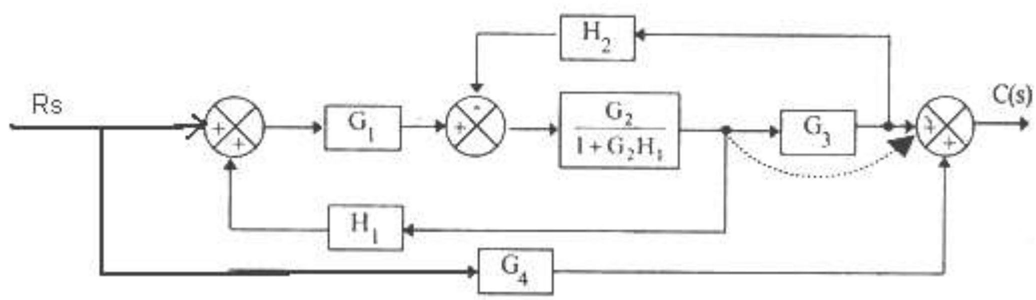
Step 1: Splitting the summing point and rearranging the branch points



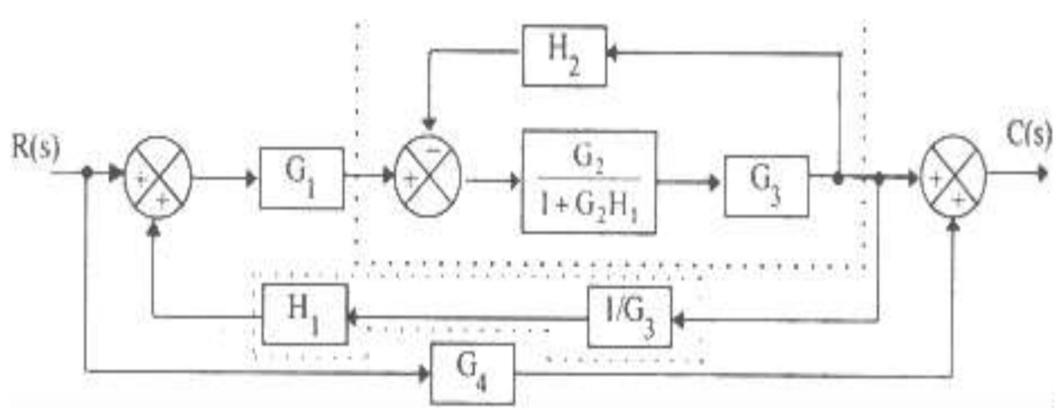
Step 2: Eliminating feedback path



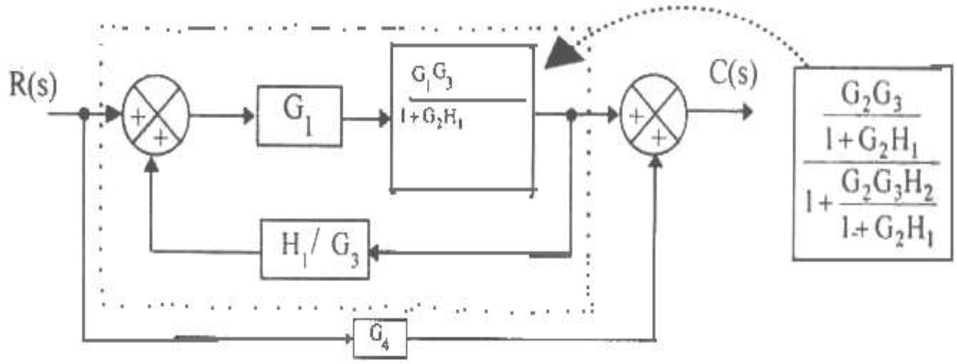
Step 3: Moving the branch point after the block



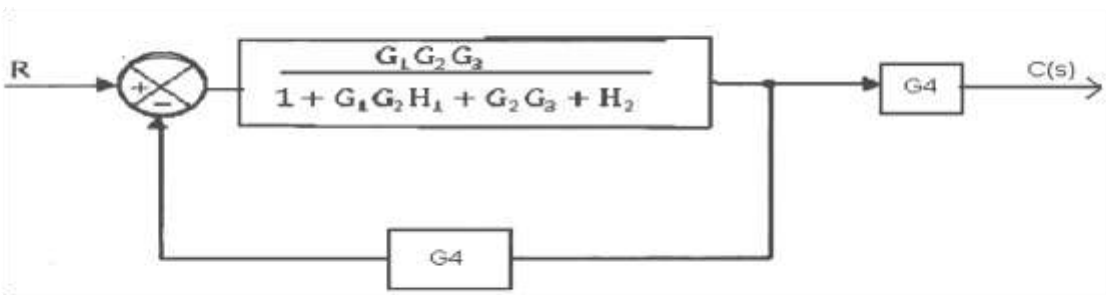
Step 4: combining the blocks in cascade and eliminating feedback path



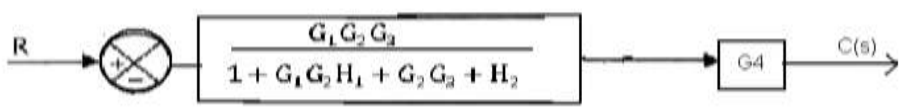
Step 5:Combining the blocks in cascade and eliminating feedback path



Step; 6Eliminatingthe feedback path

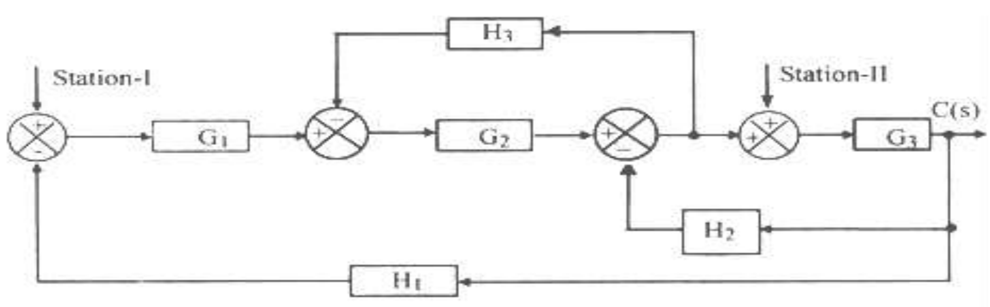


Step7; Combining the blocks in cascade



$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_3G_4H_1 + G_2G_3H_2 + G_1G_2G_3G_4}}$$

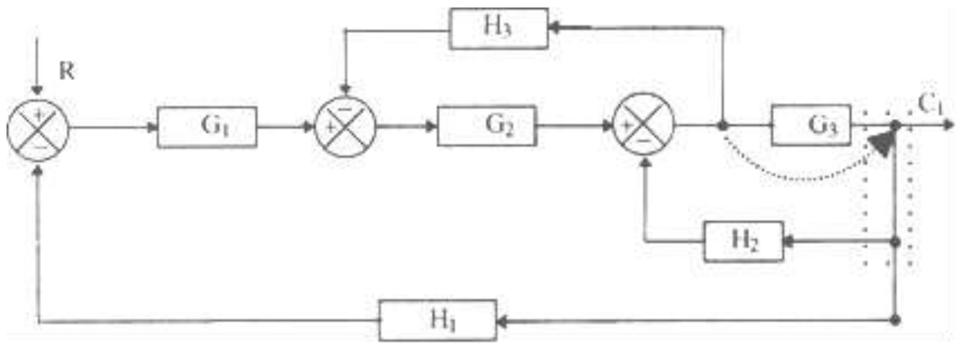
7.For the system represented by the block diagram shown in fig 1. Evaluate the closed loop transfer function when the input R is (i) at station-I (ii) at station-II.(April/May-2010)(Nov/Dec-2010)



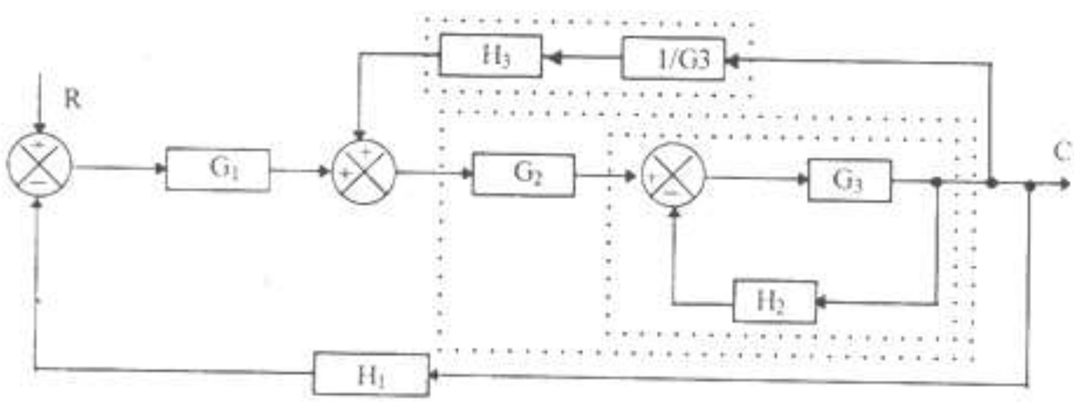
SOLUTION;

Consider the input R is at station-I and so the input at station-II is made zero. Let the output be C_1. Since there is no input at station-II that summing point can be removed and resulting block diagram is shown in fig

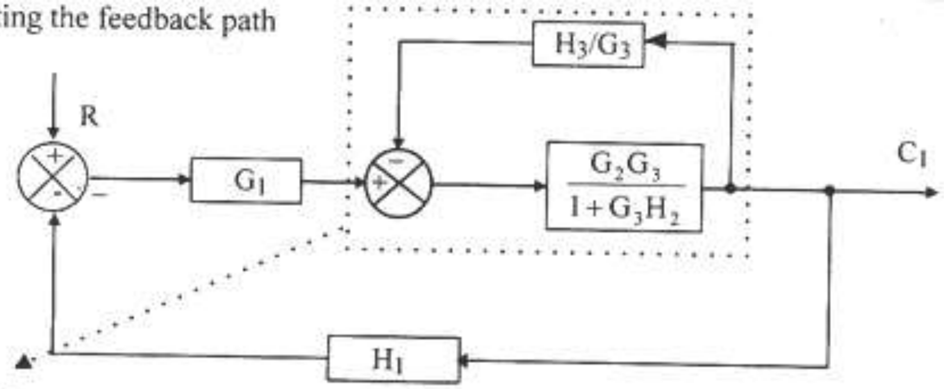
Step1: Shift the take off point of feedback H_3 beyond G_3 and rearrange the branch points



Step2: Eliminating the feedback H_2 and combining blocks in cascade

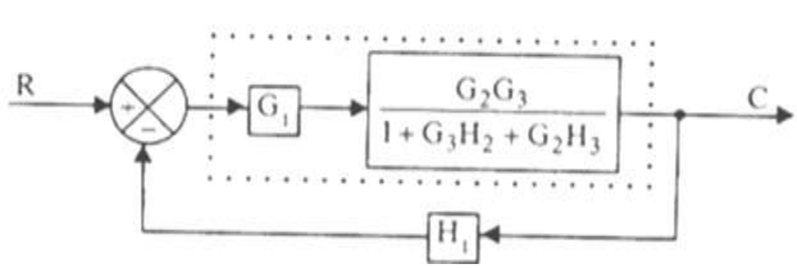


Step 3: Eliminating the feedback path

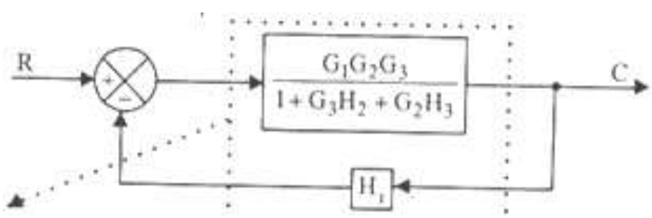


$$\frac{\frac{G_2G_3}{1+G_3H_2}}{1+\frac{G_2G_3}{1+G_3H_2}\times\frac{H_3}{G_3}} = \frac{\frac{G_2G_3}{1+G_3H_2}}{1+\frac{G_2H_3}{1+G_3H_2}} = \frac{G_2G_3}{1+G_3H_2+G_2H_3}$$

Step 4: Combining the blocks in cascade



Step 5: Eliminating the feedback path H1

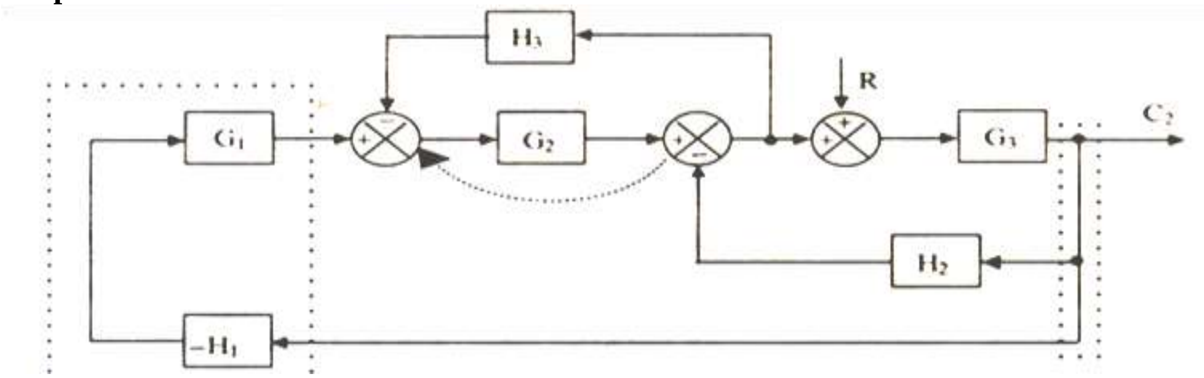


$$\frac{\frac{G_1G_2G_3}{1+G_3H_2+G_2H_3}}{1+\frac{G_1G_2G_3}{1+G_3H_2+G_2H_3}\times H_1} = \frac{G_1G_2G_3}{1+G_3H_2+G_2H_3+G_1G_2G_3H_1}$$

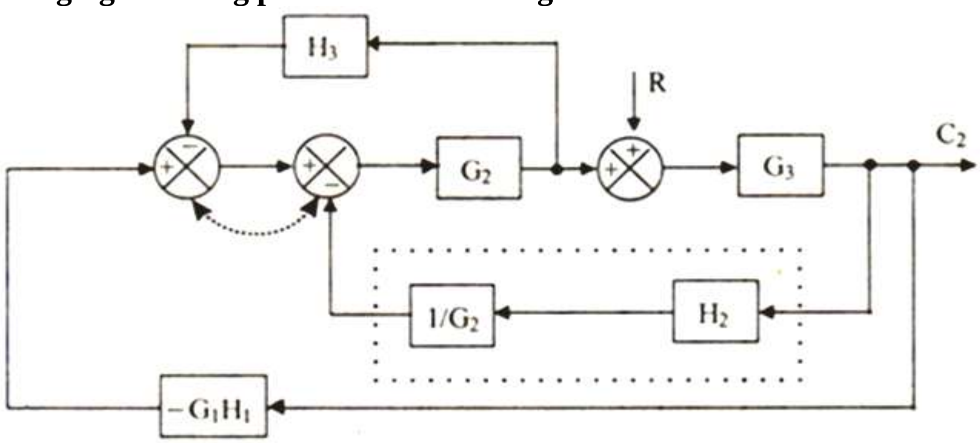
$$\boxed{\frac{\mathbf{C_1(s)}}{\mathbf{R(s)}} = \frac{\mathbf{G_1G_2G_3}}{\mathbf{1+G_3H_2+G_2H_3+G_1G_2G_3H_1}}}$$

(ii) Consider the input R at station II the input at station I is made zero. Let output C there is no input in station I that corresponding summing point can be removed and sign can be attached to the feedback path gain H_1.

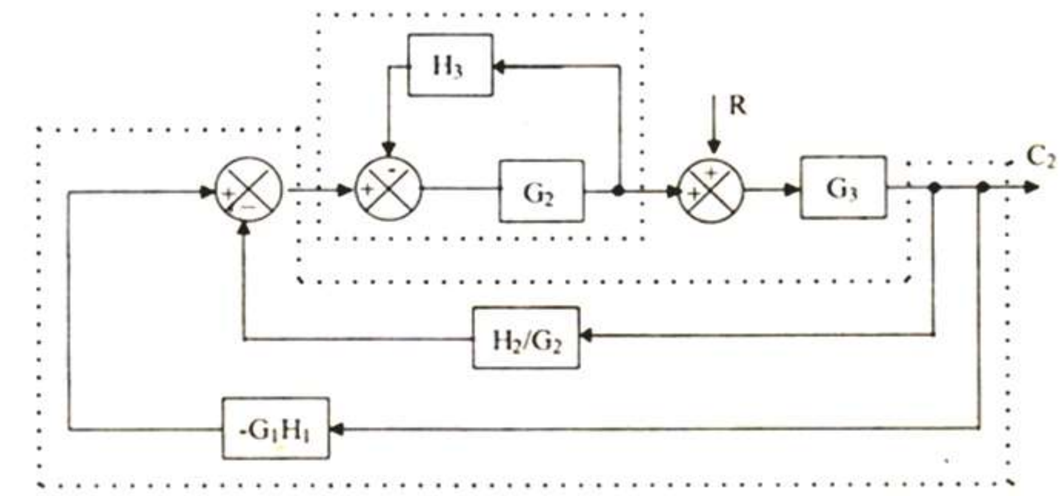
Step1: Combining the blocks in cascade, moving the summing point of H_2 before G_2 and the branch point.



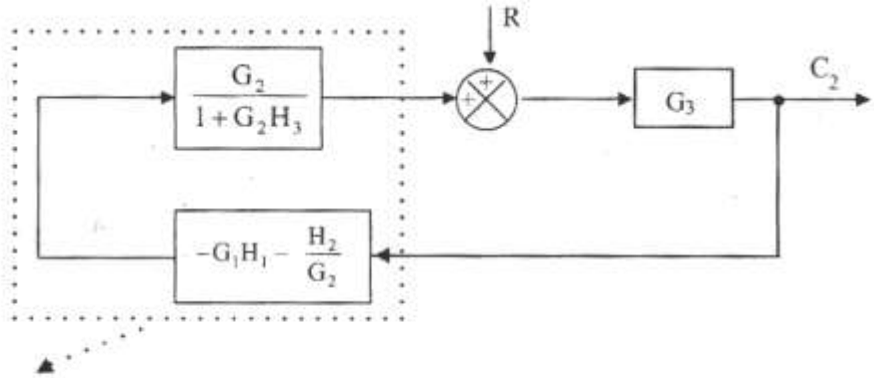
Step 2: Interchanging summing points and combining the blocks in cascade.



Step 3:Combining parallel blocks and eliminating feedback path

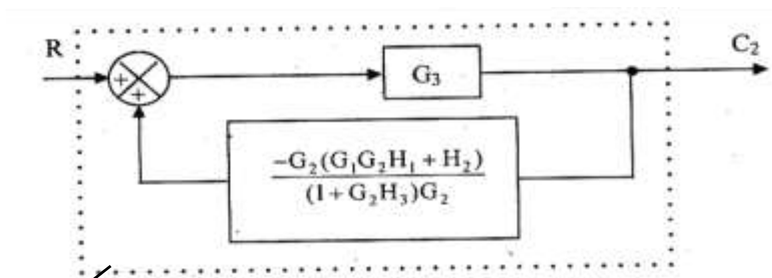


Step4: Combining the blocks in cascade.



$$\left(\frac{G_2}{1+G_2H_3}\right)\times\left(-G_1H_1-\frac{H_2}{G_2}\right)=\left(\frac{G_2}{1+G_2H_3}\right)\times\left(\frac{-G_1H_1G_2-H_2}{G_2}\right)=\frac{-G_2(G_1G_2H_1+H_2)}{(1+G_2H_3)G_2}$$

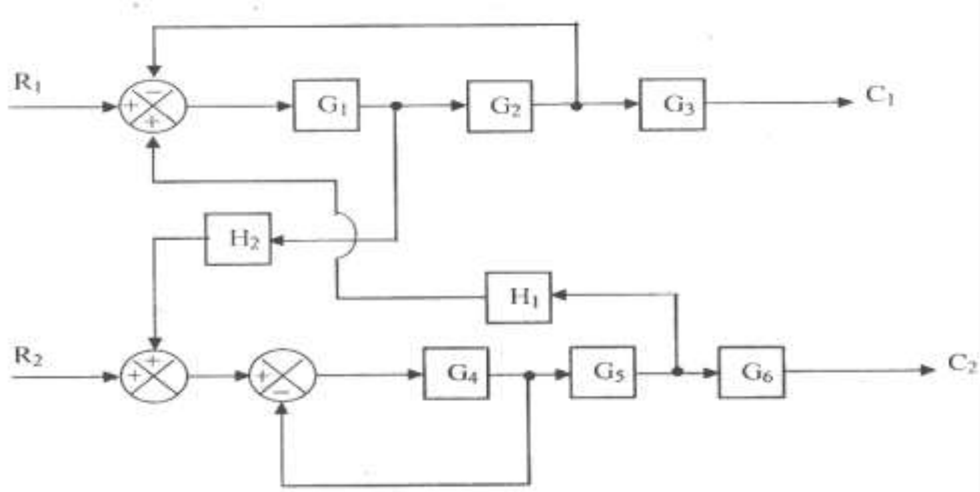
Step 5: Eliminating the feedback path



$$\frac{G_3}{1-\left(\frac{-(G_1G_2H_1+H_2)}{1+G_2H_3}\right)G_3}=\frac{G_3}{\frac{1+G_2H_3+G_3(G_1G_2H_1+H_2)}{1+G_2H_3}}=\frac{G_3(1+G_2H_3)}{1+G_2H_3+G_3(G_1G_2H_1+H_2)}$$

$$\boxed{\frac{C_2(s)}{R(s)}=\frac{\mathbf{G_3(1+G_2H_3)}}{\mathbf{1+G_2H_3+G_3(G_1G_2H_1+H_2)}}}$$

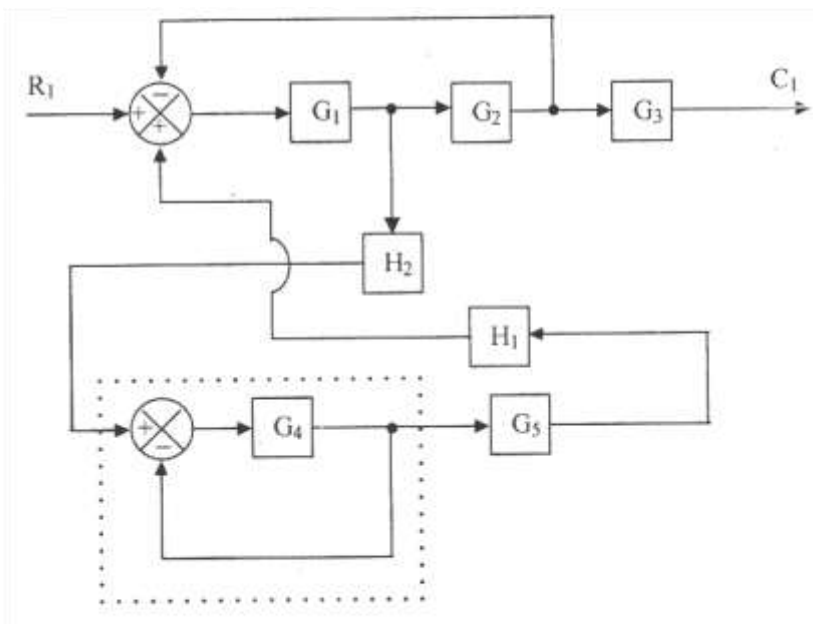
12. For the system represented by the block diagram shown in the fig. determine $\frac{C_1}{R_1}$and $\frac{C_2}{R_2}$(AssimingR2=0) (Apr-May 2011)



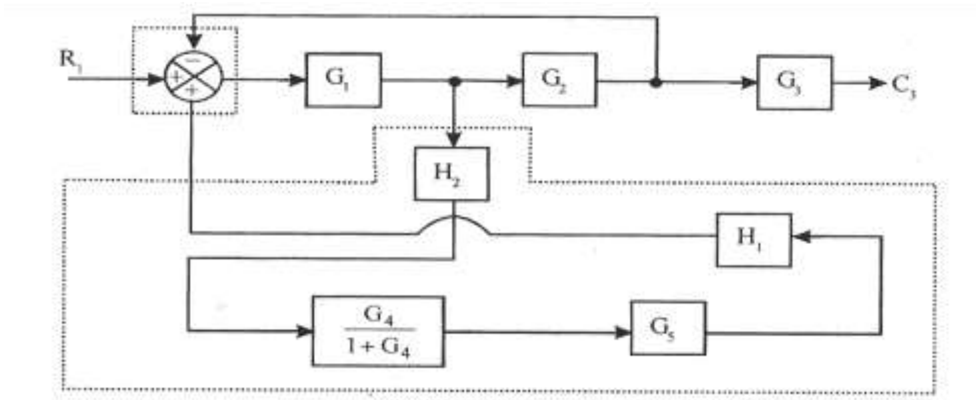
$\boldsymbol{Case(i)\ To find\ \frac{C_1}{R_1}}$

In this case set R_2=0 and consider only one output C_1. Hence we can remove the summing point which adds R_2 and need not consider G_6, since $G_6$is on the open path. The resulting block diagram is shown in fig.

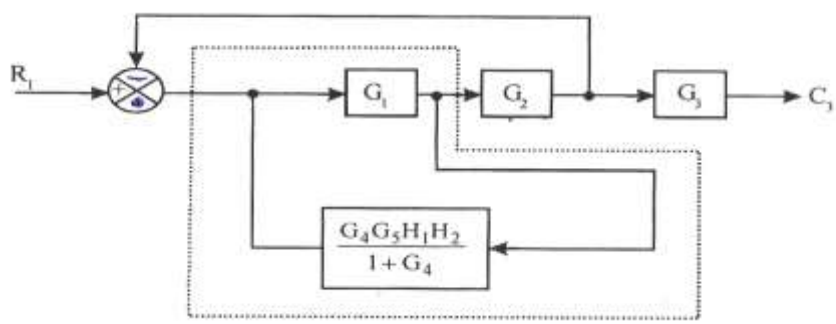
Step 1: Eliminating the feedback path.



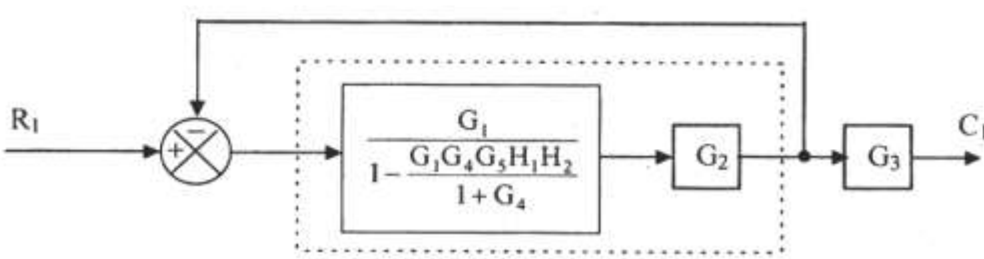
Step2: Combining the blocks in cascade and splitting the summing point.



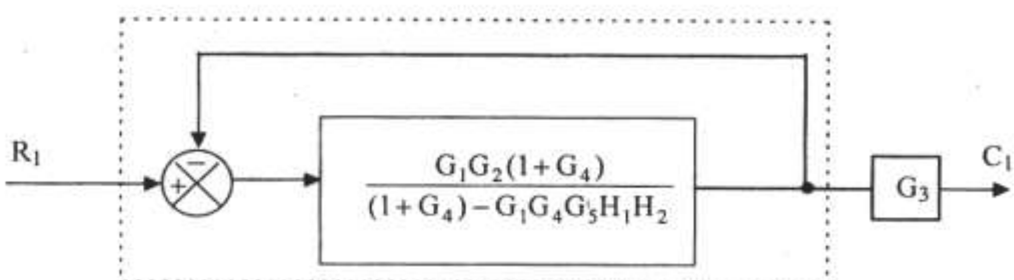
Step3: Eliminating the feedback path.

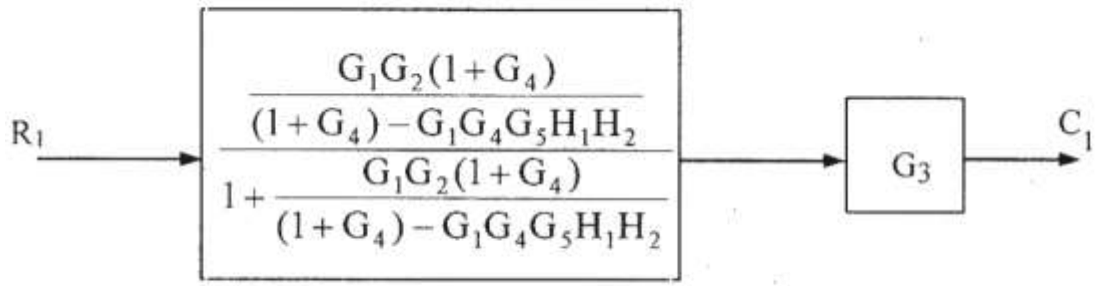


Step4: Combining the blocks in cascade.

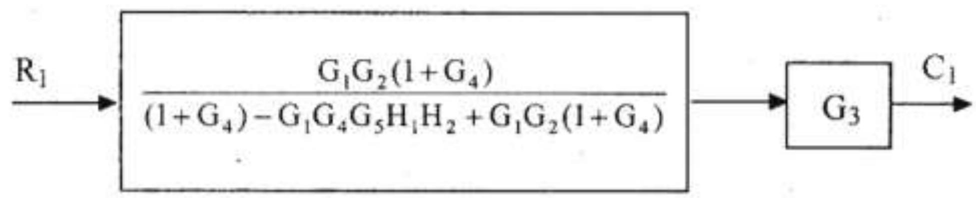


Step5: Eliminating the feedback path.





Step6: Combining the blocks in cascade.

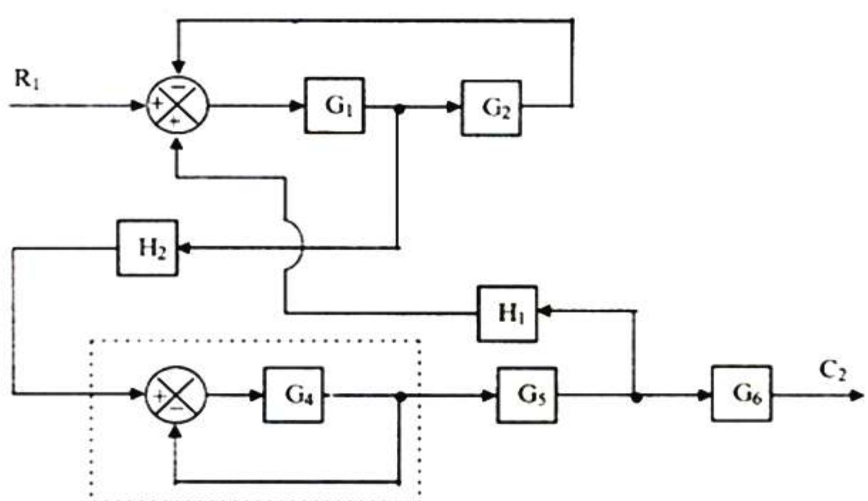


$$\boxed{\frac{\mathbf{C_1}}{\mathbf{R_1}} = \frac{\mathbf{G_1G_2G_3(1+G_4)}}{\mathbf{(1+G_1G_2)(1+G_4)-G_1G_4G_5H_1H_2}}}$$

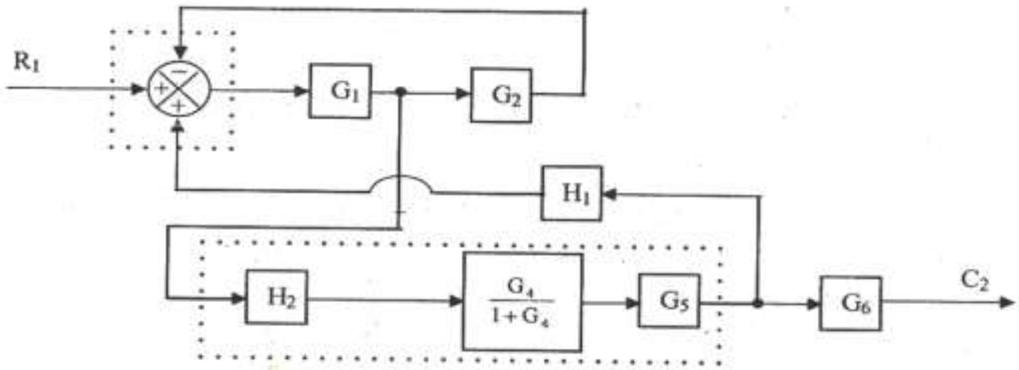
Case*(*ii*) *To find $\boldsymbol{\frac{C_2}{R_1}}$

In this case set R_2=0 and consider only one output C_2. Hence we can remove the summing point which adds R_2 and need not consider G_3, since G_3 is on the open path. The resulting block diagram is shown in fig.

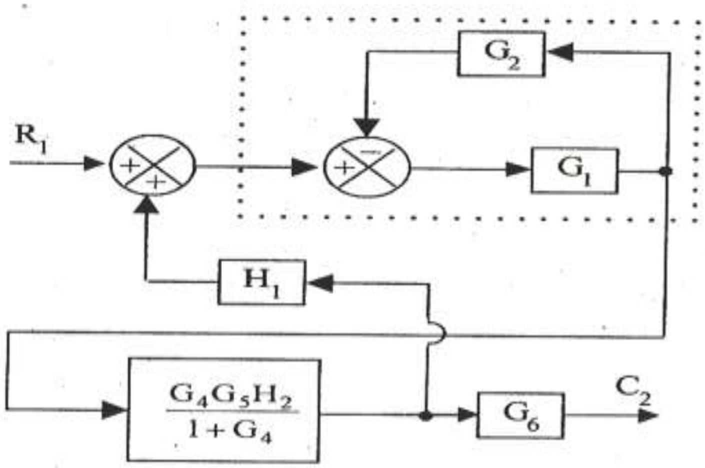
Step1: Eliminating the feedback path.



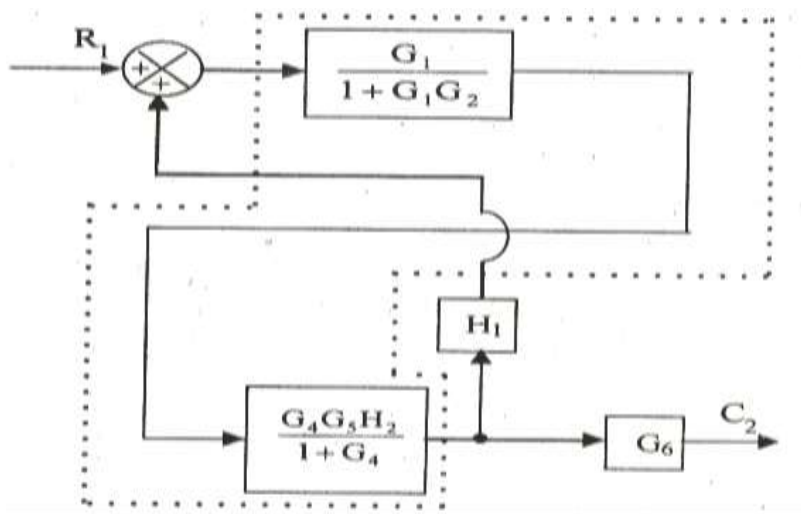
Step2: Combining the blocks in cascade and splitting the summing point.



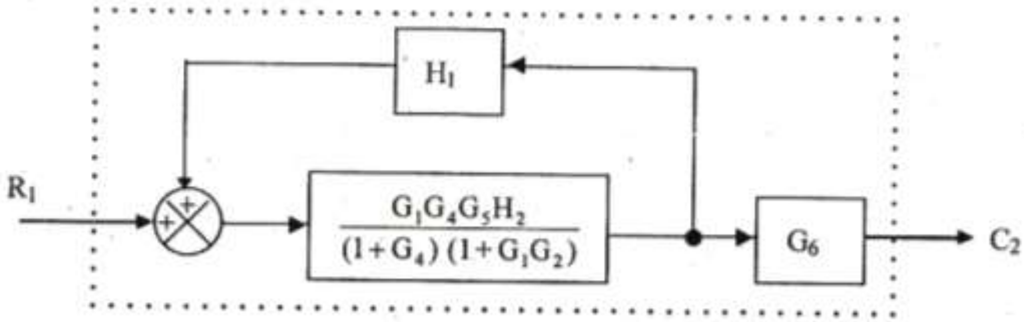
Step3: Eliminating the feedback path.

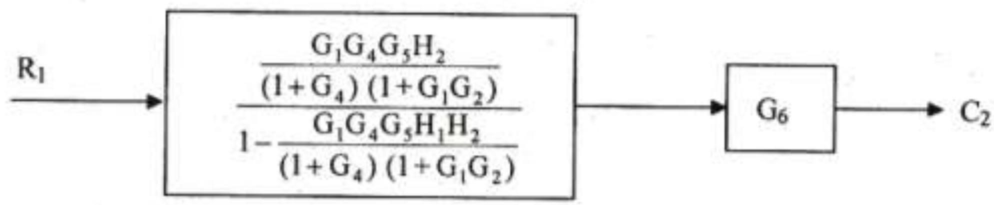


Step4: Combining the blocks in cascade.

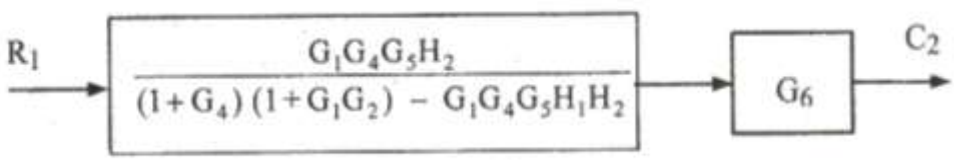


Step5: Eliminating the feedback path.





Step6: Combining the blocks in cascade.



$$\boxed{\frac{C_2}{R_1} = \frac{G_1G_4G_5G_6H_2}{(1+G_4)(1+G_1G_2)-G_1G_4G_5H_1H_2}}$$

SIGNAL FLOW GRAPH ALGEBRA

13. Find the overall transfer function of the system whose signal flow graph is shown in fig. (Apr-May 2011)

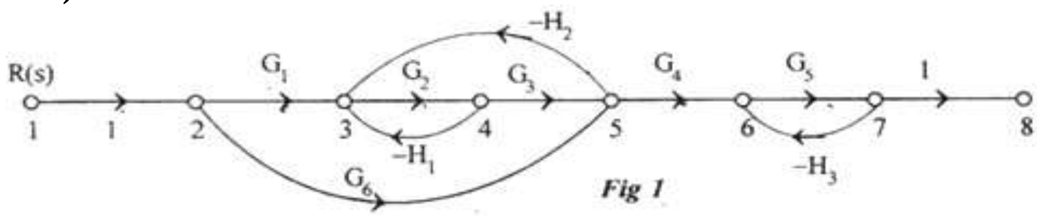
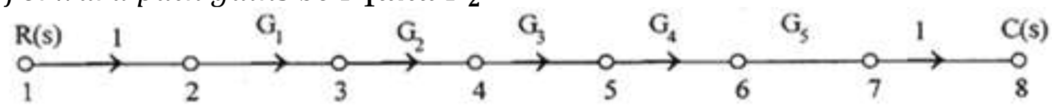


Fig 1

I Forward Path Gains

$There\ are\ two\ forward\ paths. \therefore \boxed{\boldsymbol{K = 2}}$.

$Let\ forward\ path\ gains\ be\ \boldsymbol{P_1 and\ P_2}$



$Gain\ of\ \ forward\ path\ 1, \boxed{\boldsymbol{P_1 = G_1 G_2 G_3 G_4 G_5}}$

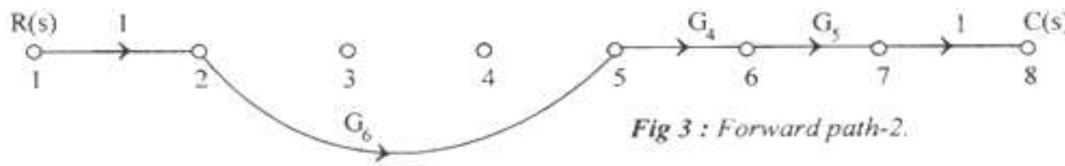


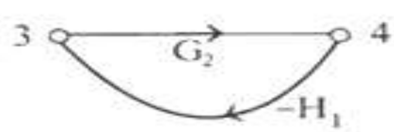
Fig 3 : Forward path-2.

$Gain\ of\ \ forward\ path\ 2, \boxed{\boldsymbol{P_2 = G_4 G_5 G_6}}$

II Individual loop gains

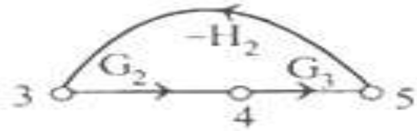
$There\ are\ three\ individual\ loops. let\ individual\ loop\ gains\ be\ \boldsymbol{P_{11} P_{21} P_{31}}$

$Loop\ 1$



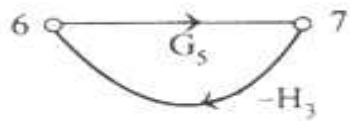
$Gainof\ individual\ loop\ 1, \boxed{\boldsymbol{P_{11} = -G_2 H_1}}$

$Loop\ 2$



Gain of individual loop 2, $\boxed{\boldsymbol{P_{21} = -G_2G_3H_2}}$

Loop 3

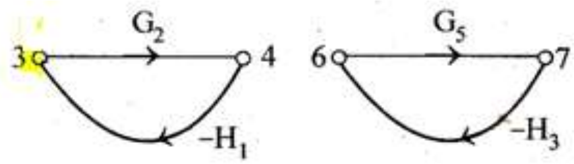


Gain of individual loop 3, $\boxed{\boldsymbol{P_{31} = -G_5H_3}}$

III Gain products of two non touching loops

There are two combinations of two non touching loops. let the gain products of two non touching loops be $\boldsymbol{P_{12}}$ ***and*** $\boldsymbol{P_{22}}$

First combination of two non touching loops.

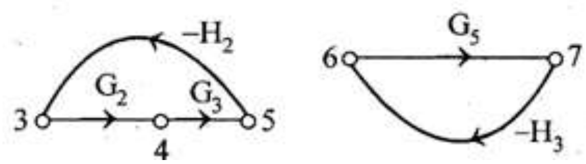


Gain product of first combination of two non touching loops,

$P_{12} = P_{11}P_{31} = (-G_2H_1)(-G_5H_3) = G_2G_5H_1H_3$

$\boxed{\boldsymbol{P_{12} = G_2G_5H_1H_3}}$

Second combination of two non touching loops.



Gain product of second combination of two non touching loops,

$$P_{22} = P_{21}P_{31} = (-G_2G_3H_2)(-G_5H_3) = G_2G_3G_5H_2H_3$$

$$\boxed{\boldsymbol{P_{22} = G_2G_3G_5H_2H_3}}$$

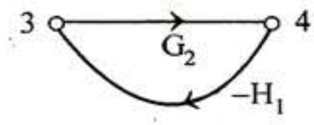
IV Calculation of Δ ***and*** Δ_k

$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$

$= 1 - (-G_2H_1 - G_2G_3H_2 - G_5H_3) + (G_2G_3H_1H_3 + G_2G_3G_5H_2H_3)$

$= 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_3H_1H_3 + G_2G_3G_5H_2H_3$

$\boxed{\boldsymbol{\Delta_1 = 1}}$, *Since there is no part of graph which is not touching with fist forward path.*
The part of the graph which is not touching with second forward path is shown in fig.



$\Delta_2 = 1 - P_{11} = 1 - (-G_2H_1) = 1 + G_2H_1$ $\therefore \boxed{\boldsymbol{\Delta_2 = 1 + G_2H_1}}$

V Transfer function, T

By Mason's gain formula the transfer function, T isgiven by

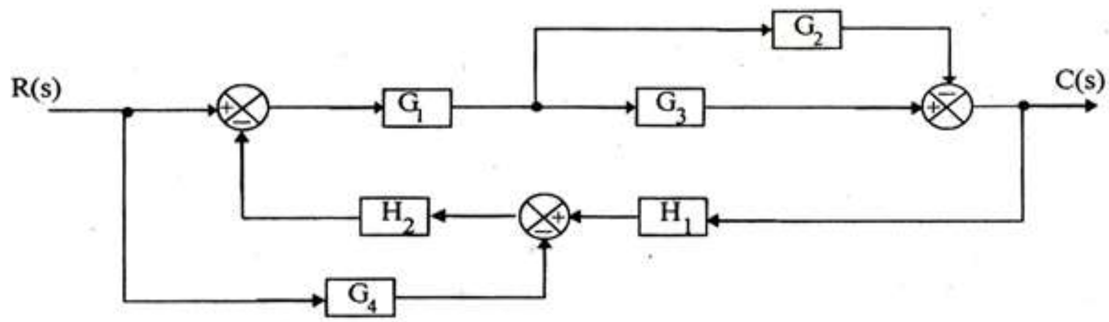
$$\boldsymbol{T} = \frac{\mathbf{1}}{\Delta}\sum_{\boldsymbol{k}} \boldsymbol{P_k}\Delta_{\boldsymbol{k}} = \frac{\mathbf{1}}{\Delta}(\boldsymbol{P_1}\Delta_{\mathbf{1}} + \boldsymbol{P_2}\Delta_{\mathbf{2}})(Number\ of\ forward\ path\ is\ 2\ and\ K = 2)$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6(1 + G_2H_1)}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3}$$

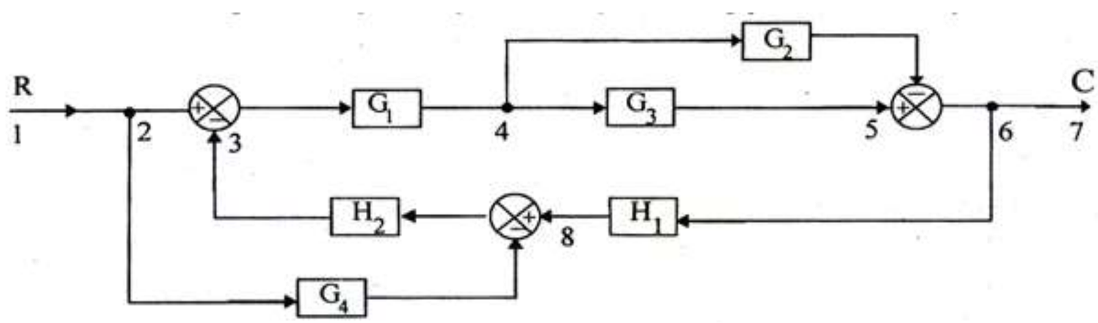
$$\boxed{\boldsymbol{T = \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6 + G_2G_4G_5G_6H_1}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3}}}$$

14. Draw a signal flow graph and evaluate the closed loop transfer function of a system whose block diagram is (Nov/Dec-2008, 2009) (Apr/May-2011)

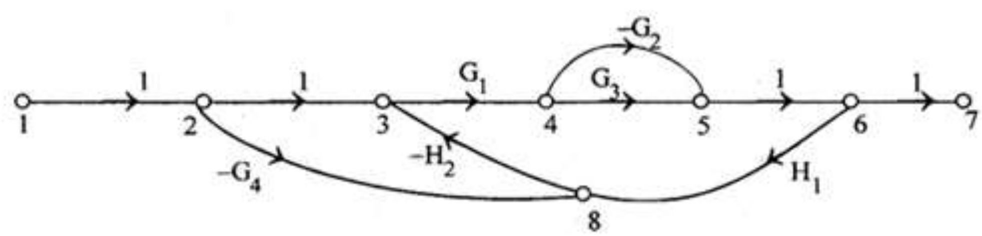
.



$The\ nodes are\ assigned at\ input, output, at\ every\ summing\ point\ and\ branch\ point\ as\ shown\ in fig.$



$The\ signal flow\ graph\ for\ the\ block\ diagram\ is\ shown in fig.$

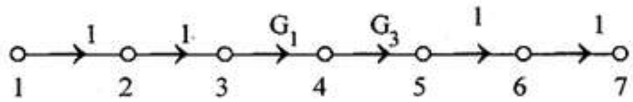


I Forward Path Gains

$There\ are\ four\ forward\ paths. \therefore\ \boxed{\boldsymbol{K = 4}}.$

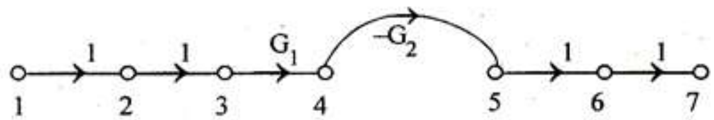
$Let\ forward\ path\ gains\ be$ $\boxed{\boldsymbol{P_1, P_2, P_3 and\ P_4}}$

$Gain\ of\ forward\ path\ 1$



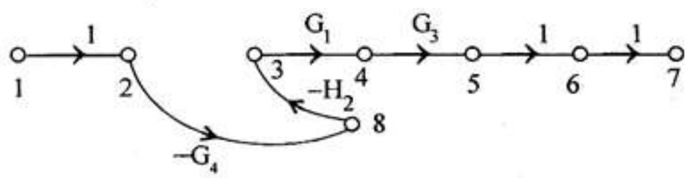
$$\boxed{P_1 = G_1G_3}$$

$Gain\ of\ forward\ path\ 2$



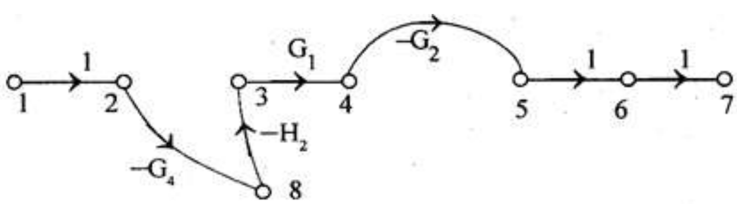
$$\boxed{P_2 = -G_1G_2}$$

$Gain\ of\ forward\ path\ 3$



$$\boxed{P_3 = G_1G_3G_4H_2}$$

$Gain\ of\ forward\ path\ 4$

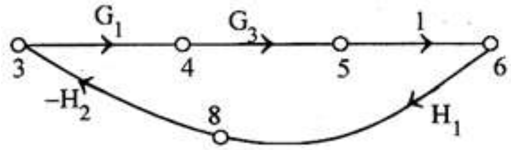


$$\boxed{P_4 = -G_1G_2G_4H_2}$$

II Individual loop gains

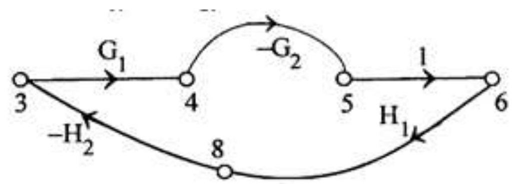
$There\ are\ two\ individual\ loops.\ let\ individual\ loop\ gains\ be\ \boldsymbol{P_{11}P_{21}}$

$Loop\ 1$



$Gain of\ individual\ loop\ 1,$ $\boxed{\boldsymbol{P_{11} = -G_1G_3H_1H_2}}$

$Loop\ 2$



$Gain of\ individual\ loop\ 2,$ $\boxed{\boldsymbol{P_{21} = G_1G_2H_1H_2}}$

III Gainproducts of two non touching loops

$There\ are\ no\ possible\ combinations\ of\ two\ nontouching\ loops, three\ \ nontouching\ loops.$

IV Calculation of Δ and Δ_k

$\Delta = 1 - (P_{11} + P_{21})$

$= 1 - (-G_1G_3H_1H_2 + G_1G_2H_1H_2) = 1 + G_1G_3H_1H_2 - G_1G_2H_1H_2$

$$\boxed{\Delta = \boldsymbol{1 + G_1G_3H_1H_2 - G_1G_2H_1H_2}}$$

$Since\ there\ is\ no\ graph\ is\ non\ touching\ with\ the\ forward\ paths,$

$$\boxed{\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \boldsymbol{1}}$$

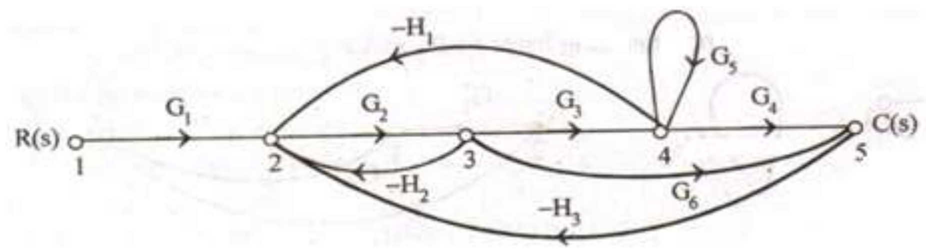
V Transfer function, T

$By\ Mason's\ gain\ formula\ the\ transfer\ function, T\ isgiven\ by,$

$$\boldsymbol{T} = \frac{\boldsymbol{1}}{\Delta}\sum_{\boldsymbol{k}} \boldsymbol{P_k}\Delta_{\boldsymbol{k}} = \frac{\boldsymbol{1}}{\Delta}(\boldsymbol{P_1}\Delta_1 + \boldsymbol{P_2}\Delta_2 + \boldsymbol{P_3}\Delta_3 + \boldsymbol{P_4}\Delta_4)(Since\ K = 4)$$

$$\boxed{\boldsymbol{T = \frac{G_1G_3 - G_1G_2 + G_1G_3G_4H_2 - G_1G_2G_4H_2}{1 + G_1G_3H_1H_2 - G_1G_2H_1H_2}}}$$

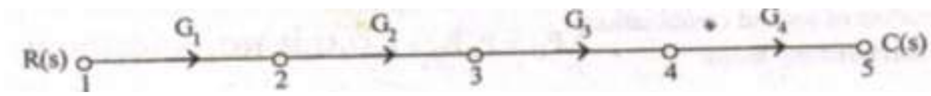
15.Find the overall gain C(s)/R(s)for the signal flowgraph asshown in fig.(May/June 2013) (Nov/Dec2017)



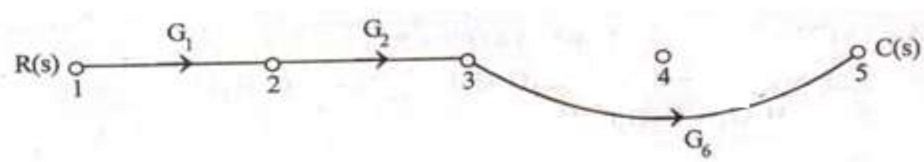
I Forward Path Gains

$There\ are\ two\ forward\ paths. \therefore \boxed{\boldsymbol{K = 2}}.$

$Let\ forward\ path\ gains\ be\ \boldsymbol{P_1 and\ P_2}$



$Gain\ of\ forward\ path\ 1, \boxed{\boldsymbol{P_1 = G_1G_2G_3G_4}}$

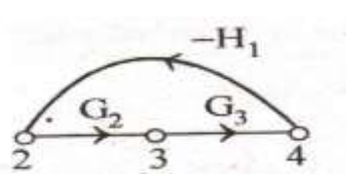


$Gain\ of\ forward\ path\ 2, \boxed{\boldsymbol{P_2 = G_1G_2G_6}}$

$\boldsymbol{II\ Individual\ loop\ gains}$

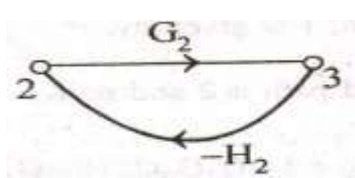
$There\ are\ five\ individual\ loops. let\ individual\ loop\ gains\ be\ \boldsymbol{P_{11}, P_{21}, P_{31}, P_{41} and\ P_{51}}$

$Loop\ 1$



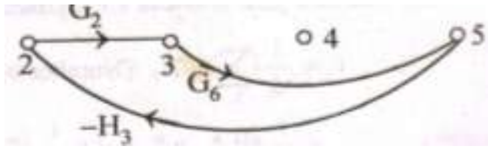
$Gainof\ individual\ loop\ 1, \boxed{\boldsymbol{P_{11} = -G_2G_3H_1}}$

$Loop\ 2$



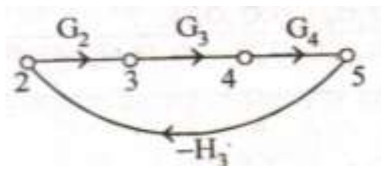
$Gainof\ individual\ loop\ 2, \boxed{\boldsymbol{P_{21} = -G_2H_2}}$

Loop 3



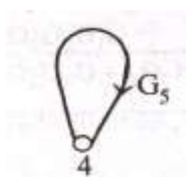
$$Gain of\ individual\ loop\ 3, \boxed{\boldsymbol{P_{31} = -G_2G_6H_3}}$$

Loop 4



$$Gain of\ individual\ loop\ 4, \boxed{\boldsymbol{P_{41} = -G_2G_3G_4H_3}}$$

Loop 5

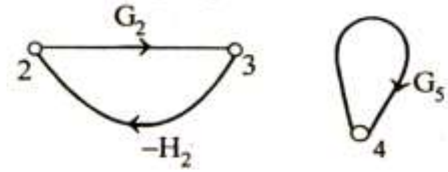


$$Gain of\ individual\ loop\ 5, \boxed{\boldsymbol{P_{51} = G_5}}$$

III Gain products of two non touching loops

There are two combinations of two non touching loops. let the gain products of two non touching loopsbe $\boldsymbol{P_{12}}$ ***and*** $\boldsymbol{P_{22}}$

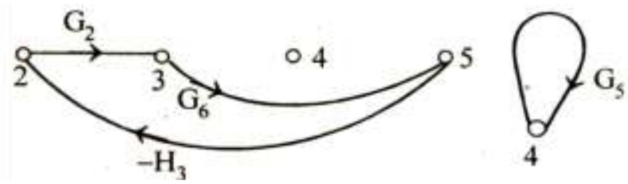
First combination of two non touching loops.



Gain product of first combination of two non touching loops,

$$P_{12} = P_{21}P_{51} = (-G_2H_2)(G_5) \boxed{\boldsymbol{P_{12} = -G_2G_5H_2}}$$

Second combination of two non touching loops.



Gain product of first combination of two non touching loops,

$$P_{22} = P_{31}P_{51} = (-G_2G_6H_3)(G_5) \boxed{\boldsymbol{P_{22} = -G_2G_5G_6H_3}}$$

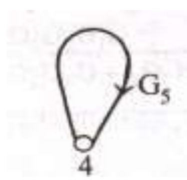
IV Calculation of Δ and Δ_k

$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51}) + (P_{12} + P_{22})$$

$$\boxed{\boldsymbol{\Delta = 1 - (-G_2G_3H_1 - H_2G_2 - G_2G_3G_4 + G_5 - G_2G_6H_3) + (-G_2G_5H_2 - G_2G_5G_6H_3)}}$$

Since there is no part of graph which is not touchingforward path1, $\boxed{\boldsymbol{\Delta = 1}}$.

The part of graph which is not touching forward path 2 is shown infig.



$$\therefore \boxed{\boldsymbol{\Delta_2 = 1 - G_5}}$$

V Transfer function, T

By Mason's gain formula the transfer function, T isgiven by,

$$T = \frac{1}{\Delta}\sum_k P_k\Delta_k = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)(Number\ of\ forward\ path\ is\ 2\ and\ K = 2)$$

$$= \frac{(G_1G_2G_3G_4 \times 1) + G_1G_2G_6(1 - G_5)}{1 - (-G_2G_3H_1 - H_2G_2 - G_2G_3G_4 + G_5 - G_2G_6H_3) + (-G_2G_5H_2 - G_2G_5G_6H_3)}$$

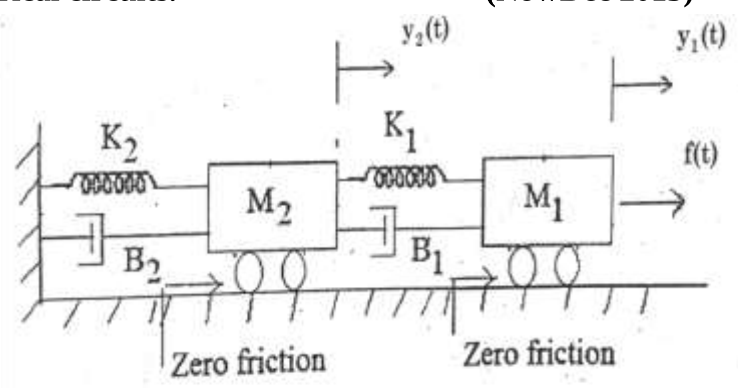
$$\boxed{\boldsymbol{T = \frac{G_1G_2G_3G_4 + G_1G_2G_6 - G_1G_2G_5G_6}{1 + G_2G_3H_1 + H_2G_2 + G_2G_3G_4 - G_5 + G_2G_6H_3 - G_2G_5H_2 - G_2G_5G_6H_3}}}$$

16. DiferentiateDC and AC servo motor. (May/June 2016)

Following are main difference between AC servo motor & DC servo motor

A.C. Servo Motor	D.C. Servo Motor
Low power output of about 0.5 W to 100 W.	Deliver high power output.
Efficiency is less about 5 to 20%.	High efficiency.
Due to absence of commentator maintenance is less.	Frequent maintenance required due to commentator.
Stability problems are less	More problems of stability.
No radio frequency noise.	Brushes produce radio frequency noise.
Compare to DC servo motor it is relatively stable and smooth operation.	It is Noisy operation.
AC amplifier used has no drift.	DC Amplifier used has a drift.

17. For the mechanical system shown in fig .Draw the mechanical network diagram and hence write the differential equations describing the behavior of the system. Draw the force-voltage and force-current analogous electrical circuits. (Nov/Dec 2015)



Solution:

For the given diagram, writing the differential equation for mass M1

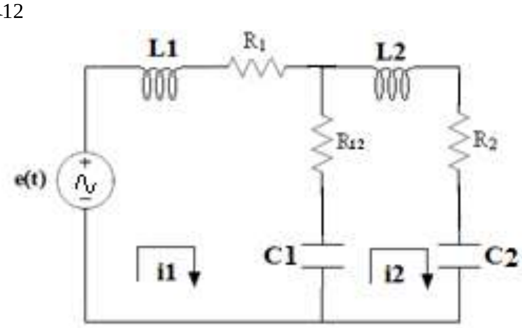
Force voltage analogous circuit

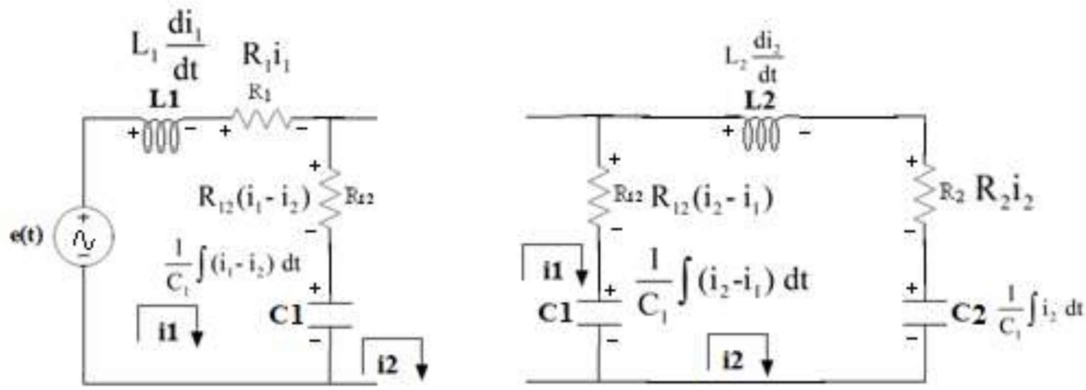
The electrical analogous elements for the elements of mechanical system is given by

$f(t) = e(t)$ $M_1 \rightarrow L_1$ $B_1 \rightarrow R_1$ $K_1 \rightarrow 1/C_1$

$v_1 = i_1$ $M_2 \rightarrow L_2$ $B_2 \rightarrow R_2$ $K_2 \rightarrow 1/C_2$

$B_{12} \rightarrow R_{12}$





The mesh basis equations using Kirchhoff's voltage law for the circuit shown is

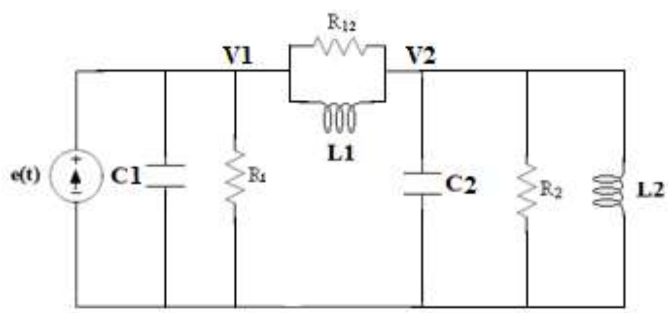
$$L_1 \frac{di_1}{dt} + R_1 i_1 + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\, dt = e(t)$$

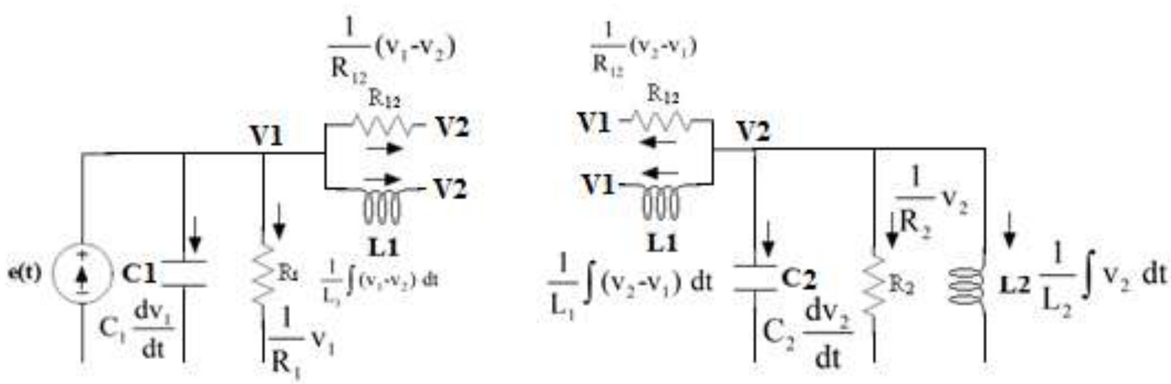
$$L_2 \frac{di_2}{dt} + R_2 i_2 + R_{12}(i_2 - i_1) + \frac{1}{C_1}\int (i_2 - i_1)\, dt = 0$$

Force current analogous circuit

The electrical analogous elements for the elements of mechanical system is given by

$f(t) = i(t)$	$M_1 \to C_1$	$B_1 \to 1/R_1$	$K_1 \to 1/L_1$
$v_1 \to v_1$	$M_2 \to C_2$	$B_2 \to 1/R_2$	$K_2 \to 1/L_2$
$v_2 \to v_2$	$B_{12} \to 1/R_{12}$		



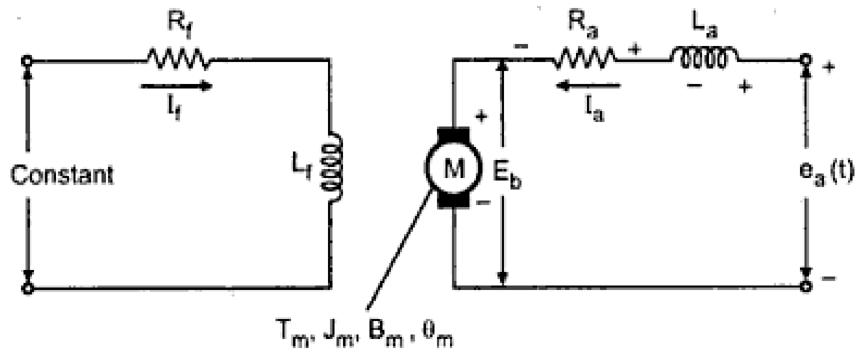


The node basis equations using Kirchhoff's current law for the circuit is

$$C_1 \frac{dv_1}{dt} + \frac{1}{R_1} v_1 + \frac{1}{R_{12}}(v_1 - v_2) + \frac{1}{L_1}\int (v_1 - v_2)\, dt = i(t)$$

$$C_2 \frac{dv_2}{dt} + \frac{1}{R_2} v_2 + \frac{1}{R_{12}}(v_2 - v_1) + \frac{1}{L_2}\int v_2\, dt + \frac{1}{L_1}\int (v_2 - v_1)\, dt = 0$$

18. Derive the transfer function of armature controlled DC motor. (May/June 2014,17) (Nov/Dec 2015,16)



R_a = Armature resistance, Ω

L_a = Armature Inductance, H

I_a = Armature current, A

V_a = armature voltage, V

E_b = back emf, V,K_t= Torque constant, N-m/A

T = Torque developed by motor, N-m

θ = Angular displacement of shaft, rad

J = Moment of inertia of motor and load, Kg-m^2

B = Frictional coefficient of motor and load, N-m/(rad/sec)

K_b = Back emf constant, V/(rad/sec)

By Kirchhoff's voltage law we can write

$$i_a R_a + L_a \frac{di_a}{dt} + e_b = v_a$$

Torque of a dc motor is proportional to the product of flux and current.

Since flux is constant in this system the torque is propotional to i_a alone

$T \alpha i_a$

$T = K_t i_a$

The mechanical system of the motor is shown in the fig.

The differential equation governing the mechanical system of motor is given by

$$J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = T$$

The back emf of DCmachine is pproportional to speed (angular velocity) of shaft.

$$e_b \ \alpha \ \frac{d\theta}{dt} \text{or, } e_b = K_b \frac{d\theta}{dt}$$

The Laplace Transform of various time domain signals involved in this system are

$L\{v_a\} = V_a(s); L\{e_b\} = E_b(s); \ L\{T\} = T(s); L\{i_a\} = I_a(s); \ L\{\theta\} = \theta(s)$

The differential equations governing the armature controlled DC motor speed control are

$$i_a R_a + L_a \frac{di_a}{dt} + e_b = v_a; \ T = K_t i_a; \ J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = T; \ e_b = K_b \frac{d\theta}{dt}$$

Taking Laplace transform of above equations with zero initial condictions

$I_a(s)R_a + L_a sI_a(s) + E_b(s) = V_a(s)$ ....(1)

$T(s) = K_t I_a(s)$ ...(2)

$Js^2\theta(s) + Bs\theta(s) = T(s)$ ...(3)

$E_b(s) = K_b s\theta(s)$ ...(4)

On equating equations (2) and (3) we get

$K_t I_a(s) = (Js^2 + Bs)\theta(s)$

$$I_a(s) = \frac{(Js^2 + Bs)}{K_t}\theta(s) \qquad ...(5)$$

Eqn (1) can be written as

$(R_a + sL_a)I_a(s) + E_b(s) = V_a(s)$ ...(6)

Sub $E_b(s)$ and $I_a(s)$ from eqn (4) and (5) in (6) we get

$$(R_a + sL_a)\frac{(Js^2 + Bs)}{K_t}\theta(s) + K_b(s)\theta(s) = V_a(s)$$

$$\left[\frac{(R_a + sL_a)(Js^2 B_s) + K_b K_t s}{K_t}\right]\theta(s) = V_a(s)$$

The required transfer function is $\frac{\theta(s)}{V_a(s)}$

$$\frac{\theta(s)}{V_a(s)} = \frac{K_t}{(R_a + sL_a)(Js^2 B_s) + K_b K_t s}$$

$$= \frac{K_t}{R_a J s^2 + R_a B_s + L_a J s^3 + L_a B s^2 + K_b K_t s}$$

$$= \frac{K_t}{s\left[JL_a s^2 + (JR_a + BL_a)s + (BR_a + K_b K_t)\right]}$$

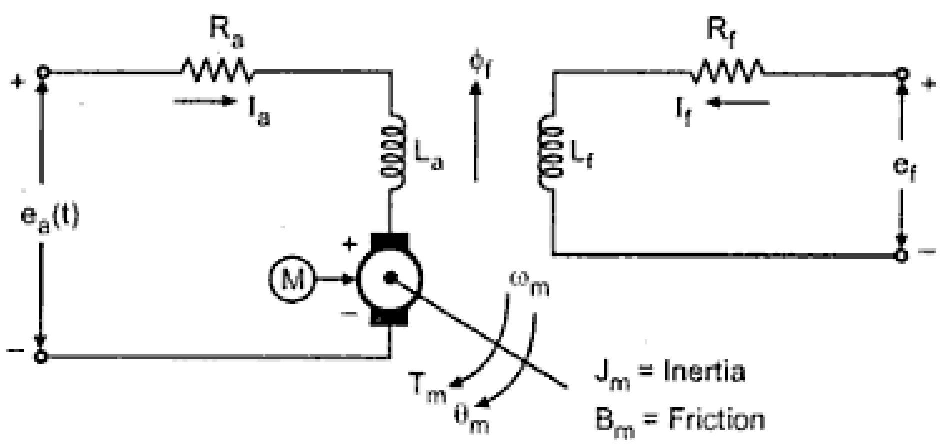
$$= \frac{\dfrac{K_t}{JL_a}}{s\left[s^2 + \left(\dfrac{JR_a + BL_a}{JL_a}\right)s + \left(\dfrac{BR_a + K_b K_t}{JL_a}\right)\right]}$$

The transfer function can be expressed in another standard form as

$$\frac{\theta(s)}{V_a(s)} = \frac{K_t}{(R_a + sL_a)(Js^2 + Bs)K_b K_t s} = \frac{K_t}{R_a\left(\dfrac{sL_a}{R_a} + 1\right)Bs\left(1 + \dfrac{Js^2}{B_s}\right) + K_b K_t s}$$

$$= \frac{\dfrac{K_t}{R_a B}}{s\left[(1 + sT_a)(1 + sT_m) + \dfrac{K_b K_t}{R_a B}\right]}$$

19. Derive the transfer function of field controlled DC motor.



Let

R_f = Field resistance, Ω

L_f = Field inductance, H

I_f = Field current, A

V_f = Field voltage, V

T = Torque developed by motor, N-m

K_{tf} = Torque constant, N-m/A

J = Moment of inertia of rotor and load, Kg-m^2/rad

B = Frictional coefficient of rotor and load, N-m/(rad/sec)

By Kirchoffs voltage law we can write

$$R_f i_f + L_f \frac{di_f}{dt} = v_f$$

The torque of DC motor is proportional to product of flux and armature current.

Since armature current is constant in this system the torque is proportional to flux alone, but flux is proportional to field current.

$T \ \alpha \ i_f$; Torque, $T = K_{tf} i_f$

The mechanical system of motor is shown in figure. The differential equations governing the mechanical system of motor is given by

$$J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = T$$

The Laplace transform of various time domain signals involved in this system are

$L\{i_f\} = I_f(s)$; $L\{T\} = T(s)$; $L\{V_f\} = V_f(s)$ $L\{\theta\} = \theta(s)$

The differential equations governing the field controlled DC motor are

$$K_f i_f + L_f \frac{di_f}{dt} = v_f; \ T = K_{tf} i_f; \ J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = T$$

On taking Laplace Transform of the above equations with zero initial conditions we get

$R_f I_f(s) + L_f s I_f(s) = V_f(s)$ ...(1)

$T(s) = K_{tf} I_f(s)$ ...(2)

$Js^2\theta(s) + Bs\theta(s) = T(s)$ ...(3)

Equating equatio (2) and (3)

$K_{tf} I_f(s) = Js^2\theta(s) + Bs\theta(s)$

$I_f(s) = s \frac{(Js+B)}{K_{tf}} \theta(s)$ ...(4)

Then equation (1) can be written as

$(R_f + sL_f) I_f(s) = V_f(s)$ ...(5)

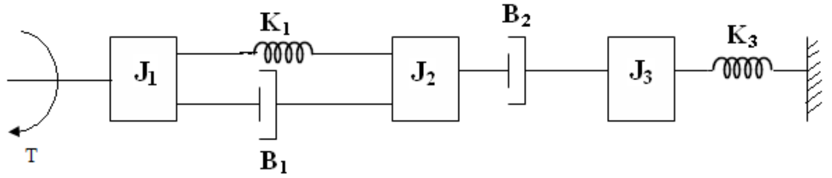
Sub $I_f(s)$ from equation (4) and (5)

$$(R_f + sL_f) s \frac{(Js+B)}{K_{tf}} \theta(s) = V_f(s)$$

$$\frac{\theta(s)}{V_f(s)} = \frac{K_{tf}}{s(R_f + sL_f)(B + sJ)}$$

$$= \frac{K_{tf}}{sR_f \left(1 + \frac{sL_f}{R_f}\right) B \left(1 + \frac{sJ}{B}\right)} = \frac{K_m}{s(1+sT_f)(1+sT_m)}$$

20. Write the differential equations governing the mechanical rotational system shown in fig. Draw the torque-voltage and torque-current electrical analogous circuits and verify by writing mesh and node equations. **(May/ June 2012,17)(Nov/Dec 2015)**



Solution

The given mechanical rotational system has three nodes. The differential equations governing the mechanical rotational system are given by torque balance equations at these nodes.

Let the angular displacements J_1, J_2 and J_3 be θ_1, θ_2 and θ_3 respectively. The corresponding angular velocities be ω_1, ω_2 and ω_3

Consider J_1.

$$T_{j1} = J_1 \frac{d^2\theta_1}{dt^2};\ T_{b1} = B_1 \frac{d(\theta_1 - \theta_2)}{dt};\ T_{k1} = K_1(\theta_1 - \theta_2)$$

By Newtons second law $T_{j1} + T_{b1} + T_{k1} = T$

$$J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d(\theta_1 - \theta_2)}{dt} + K_1(\theta_1 - \theta_2) = T \qquad \text{............(1)}$$

Consider J_2

$$T_{j2} = J_2 \frac{d^2\theta_2}{dt^2};\ T_{b2} = B_2 \frac{d(\theta_2 - \theta_3)}{dt};T_{k1} = K_1(\theta_2 - \theta_1);\ T_{b1} = B_1 \frac{d(\theta_2 - \theta_1)}{dt}$$

By Newtons second law, $T_{j2} + T_{b2} + T_{k1} + T_{b1} = 0$

$$J_2 \frac{d^2\theta_2}{dt^2} + \frac{d(\theta_2 - \theta_3)}{dt} + K_1(\theta_2 - \theta_1) + B_1 \frac{d(\theta_2 - \theta_1)}{dt} = 0 \qquad \text{.........(2)}$$

Consider J_3

$$T_{j3} = J_3 \frac{d^2\theta_3}{dt^2};\ T_{b2} = B_2 \frac{d(\theta_3 - \theta_2)}{dt};T_{k3} = K_3\theta_3$$

By Newtons Second law, $T_{j3} + T_{b2} + T_{k3} = 0$

$$J_3 \frac{d^2\theta_3}{dt^2} + B_2 \frac{d(\theta_3 - \theta_2)}{dt} + K_3\theta_3 = 0 \qquad \text{.............(3)}$$

On replacing the angular displacement by angular velocity in the differential equation (1) and (2) governing the mechanical rotationa system we get

$$\left(\text{i.e } \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt};\ \frac{d\theta}{dt} = \omega \text{ and } \theta = \int \omega\, dt \right)$$

$$J_1 \frac{d\omega_1}{dt} + B_1(\omega_1 - \omega_2) + K_1 \int (\omega_1 - \omega_2)dt = T$$

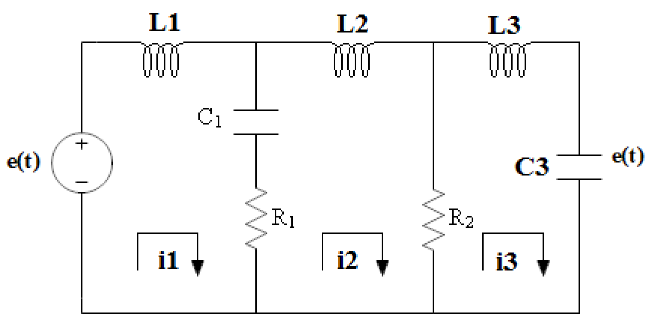
$$J_2 \frac{d\omega_2}{dt} + B_1(\omega_2 - \omega_1) + B_2(\omega_2 - \omega_3) + K_1 \int (\omega_2 - \omega_1)dt = 0$$

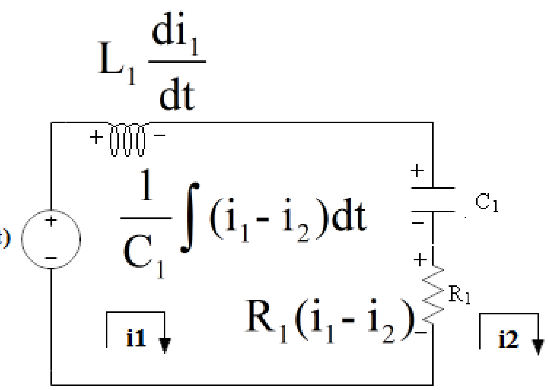
$$J_3 \frac{d\omega_3}{dt} + B_2(\omega_3 - \omega_2) + K_3 \int \omega_3 dt = 0$$

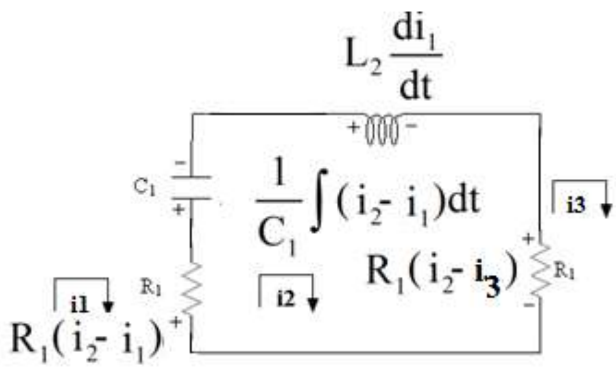
Torque voltage analogous circuit

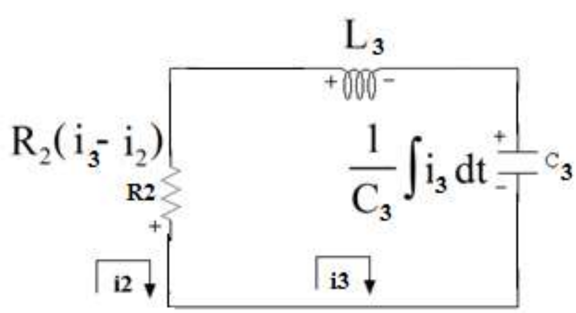
The electrical analogous elements for the elements of mechanical rotational systems are given below

$\omega_1 \rightarrow i_1$	$J_1 \rightarrow L_1$	$B_1 \rightarrow R_1$	$K_1 \rightarrow 1/C_1$
$\omega_1 \rightarrow i_2$	$J_2 \rightarrow L_2$	$B_2 \rightarrow R_2$	$K_3 \rightarrow 1/C_3$
$\omega_3 \rightarrow i_3$	$J_3 \rightarrow L_3$		









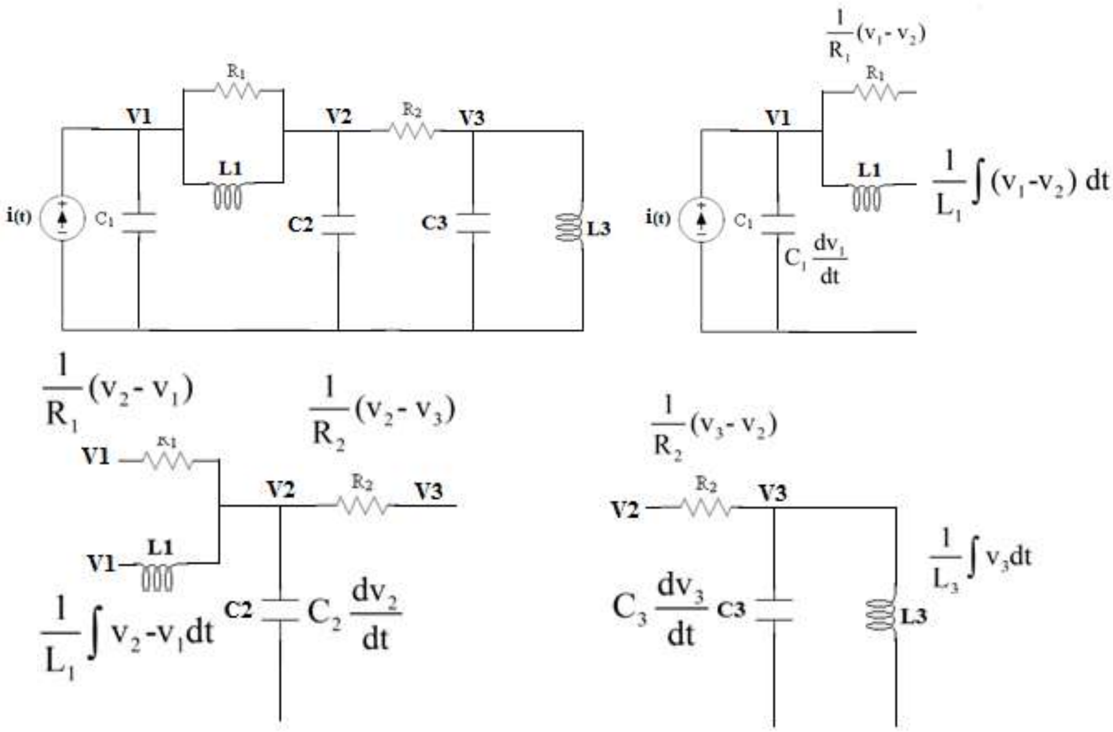
The Mesh basis equations using Kirchhoff's voltage law for the circuit is given by

$$L_1 \frac{di_1}{dt} + R_1(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\,dt = e(t)$$

$$L_2 \frac{di_2}{dt} + R_1(i_2 - i_1) + R_2(i_2 - i_3) + \frac{1}{C_1}\int (i_2 - i_1)\,dt = e(t)$$

$$L_3 \frac{di_3}{dt} + R_2(i_3 - i_2) + \frac{1}{C_3}\int i_3\,dt = 0$$

Torque current analogous circuit



The electrical analogous elements for the elements of mechanical rotational systems are

$T \rightarrow i(t)$ $\omega_1 \rightarrow v_1$ $J_1 \rightarrow C_1$ $B_1 = 1/R_1$ $K_1 \rightarrow 1/L_1$

$\omega_2 \rightarrow v_2$ $J_2 \rightarrow C_2$ $B_2 = 1/R_2$ $K_3 \rightarrow 1/L_3$

$\omega_3 \rightarrow v_3$ $J_3 \rightarrow C_3$

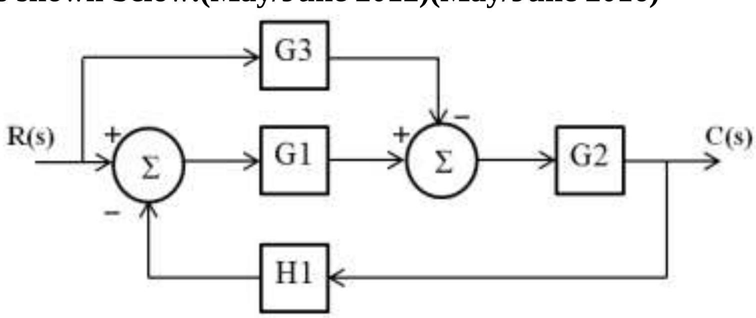
The node basis equations using Kirchhoff's current law for the circuit is

$$C_1 \frac{dv_1}{dt} + \frac{1}{R_1}(v_1 - v_2) + \frac{1}{L_1}\int (v_1 - v_2)dt = i(t)$$

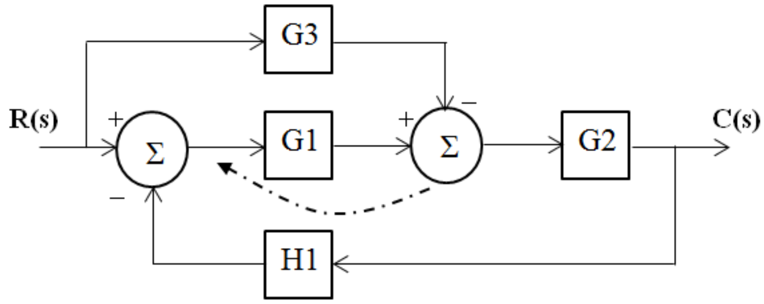
$$C_2 \frac{dv_2}{dt} + \frac{1}{R_1}(v_2 - v_1) + \frac{1}{R_2}(v_2 - v_3) + \frac{1}{L_1}\int (v_2 - v_1)dt = 0$$

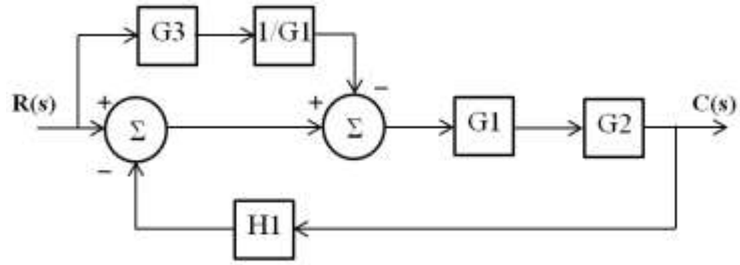
$$C_3 \frac{dv_3}{dt} + \frac{1}{R_2}(v_3 - v_2) + \frac{1}{L_3}\int v_3 dt = 0$$

21. Using block diagram reduction technique find the closed loop transfer function C/R of the system whose block diagram is shown below.(May/June 2012)(May/June 2016)

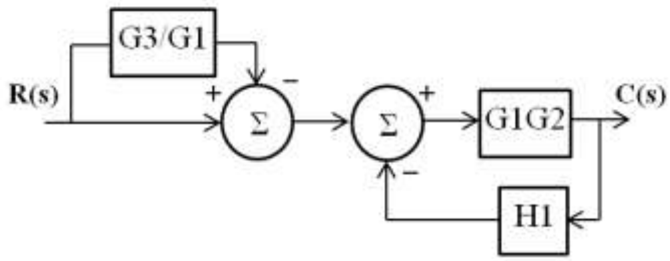


Step 1: Bring the summing point before the block

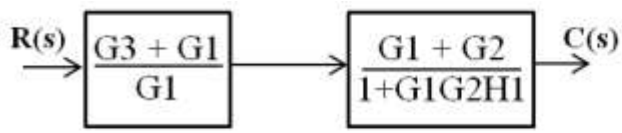




Step 2: Interchanging the summing points



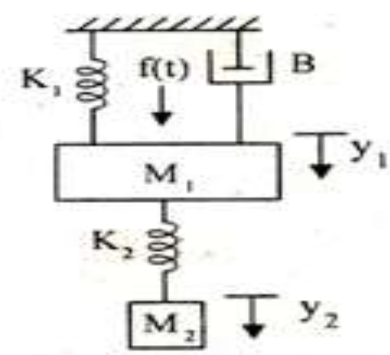
Step 3: Combining the feedback paths



$$\frac{C(s)}{R(s)} = \frac{G1G2\ (G1+G3)}{G1G1^{2}G2H1}$$

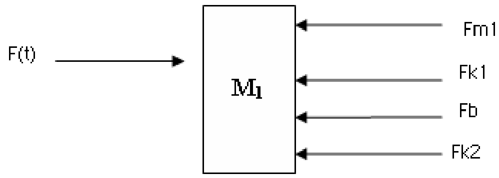
22. Determine the transfer function $Y_2(s)/F(s)$ of the system shown in fig (Nov/Dec2015) (Nov/Dec2017)

(April/May 2018)



Soln:

STEP1: Free hand drawing from Mass (M1)



To find differential equation:

By newton's law,

$$f(t) = f_{m_1} + f_{k_1} + f_b + f_{k_2}$$

$$f(t) = M_1 \frac{d^2 y_1}{dt^2} + k_1 y_1 + B\frac{dy_1}{dt} + k_2(y_1 - y_2)$$

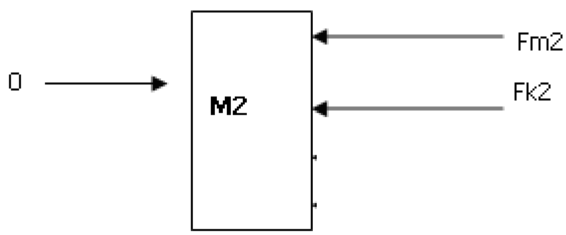
$$f(t) = M_1 \frac{d^2 y_1}{dt^2} + k_1 y_1 + B\frac{dy_1}{dt} + k_2 y_1 - k_2 y_2 \longrightarrow (1)$$

Take laplace transform for (1)

$$F(s) = M_1 s^2 y_1(s) + k_1 y_1(s) + Bs y_1(s) + k_2 y_1(s) - k_2 y_2(s)$$

$$F(s) = (M_1 s^2 + k_1 + Bs + k_2) y_1(s) - k_2 y_2(s) \longrightarrow (2)$$

STEP2: Free hand drawing from Mass (M2)



To find differential equation:

By newton's law,

$$0 = f_{m_2} + f_{k_2}$$

$$0 = M_2 \frac{d^2 y_2}{dt^2} + k_2(y_2 - y_1)$$

$$0 = M_2 \frac{d^2 y_2}{dt^2} + k_2 y_2 - k_2 y_1 \longrightarrow (3)$$

Take laplace transform for (3)

$$0 = M_2 s^2 y_2(s) + k_2 y_2(s) - k_2 y_1(s)$$

$$0 = (M_2 s^2 + k_2) y_2(s) - k_2 y_1(s)$$

$$y_1(s) = \frac{(M_2 s^2 + k_2) y_2(s)}{k_2} \longrightarrow (4)$$

SUB/: (2) in (4)

$$F(s) = (M_1 s^2 + k_1 + Bs + k_2)\frac{(M_2 s^2 + k_2) y_2(s)}{k_2} - k_2 y_2(s)$$

$$F(s) = \frac{(M_1 s^2 + k_1 + Bs + k_2)(M_2 s^2 + k_2) y_2(s) - k_{22} y_2(s)}{k_2}$$

$$\frac{y_2(s)}{F(s)} = \frac{k_2}{(M_1 s^2 + k_1 + Bs + k_2)(M_2 s^2 + k_2) - k_2^{\,2}}$$

23. The block diagram of a closed loop system is shown in fig. using block diagram reduction technique, determine the closed loop transfer function. (April/May 2018)

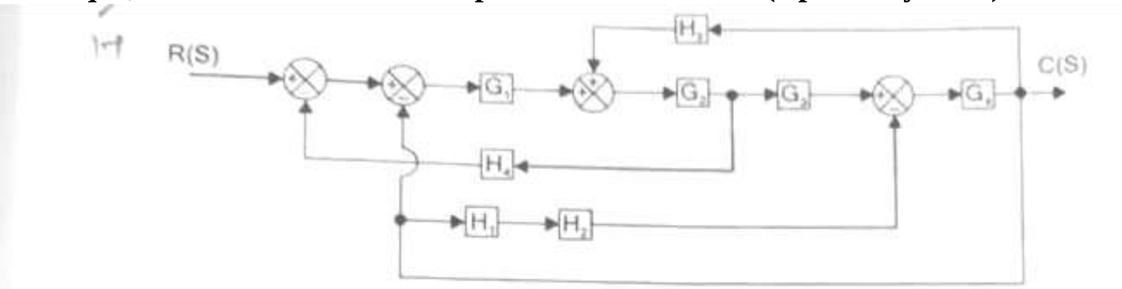
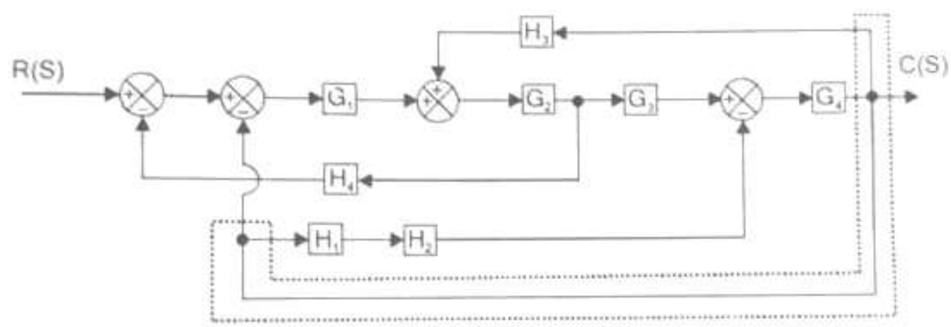


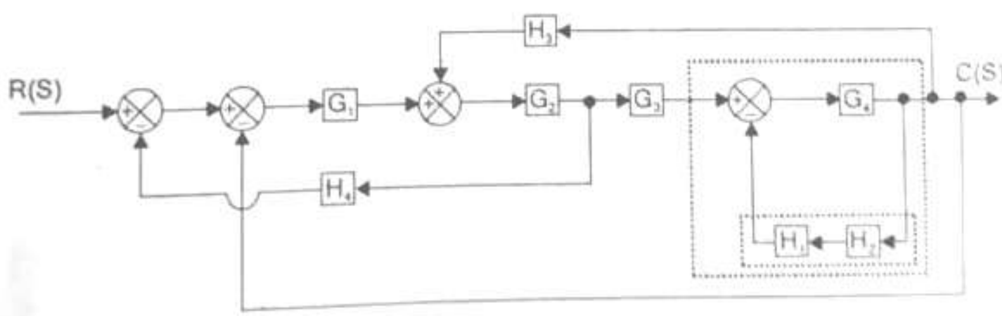
Fig 1.

SOLUTION

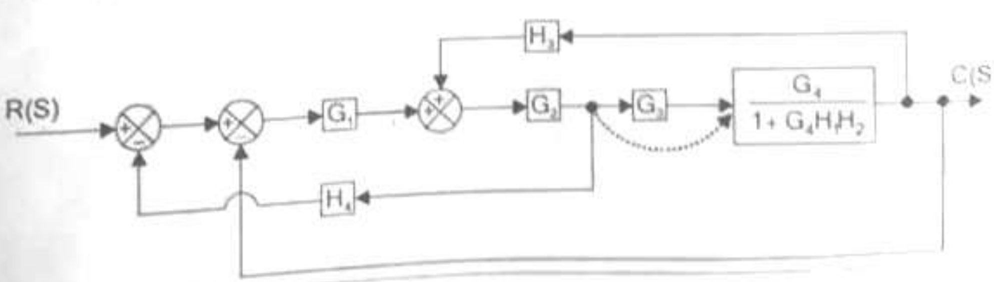
Step 1 : Rearranging the branch points



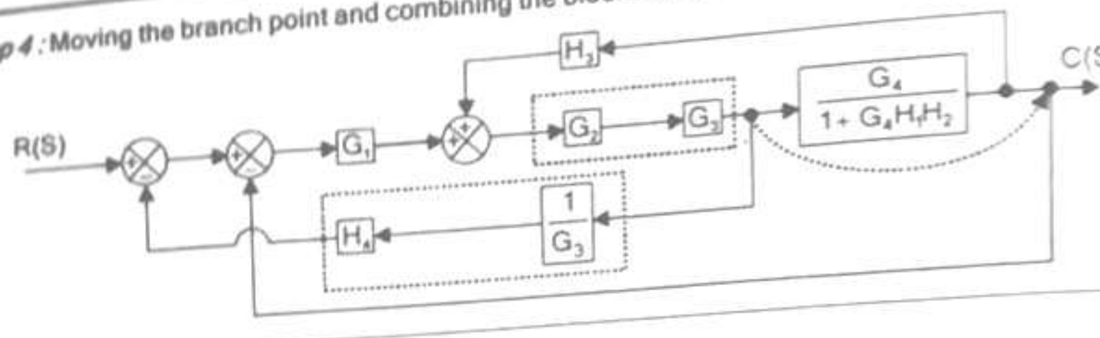
Step 2 : Combining the blocks in cascade and eliminating the feedback path.



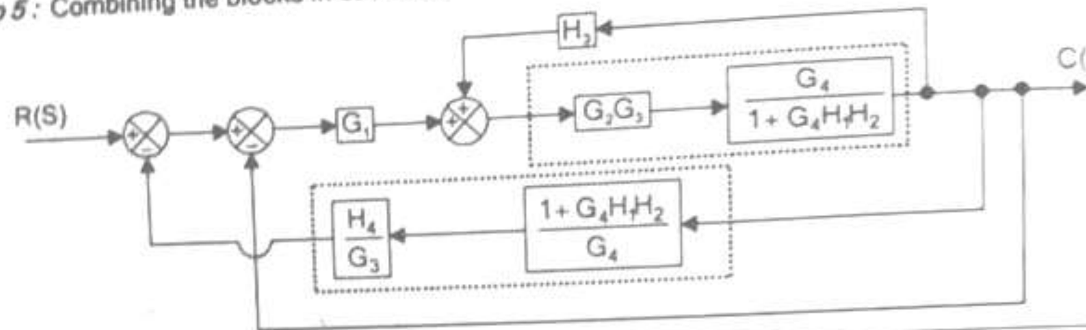
Step 3 : Moving the branch point after the block.



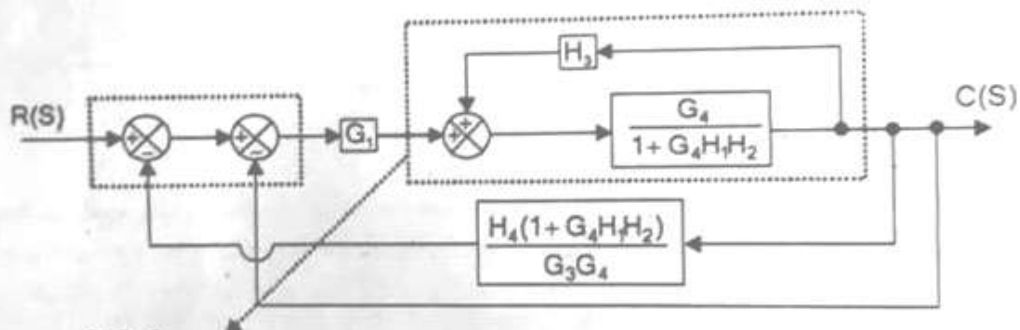
Step 4 : Moving the branch point and combining the blocks in cascade.



Step 5 : Combining the blocks in cascade

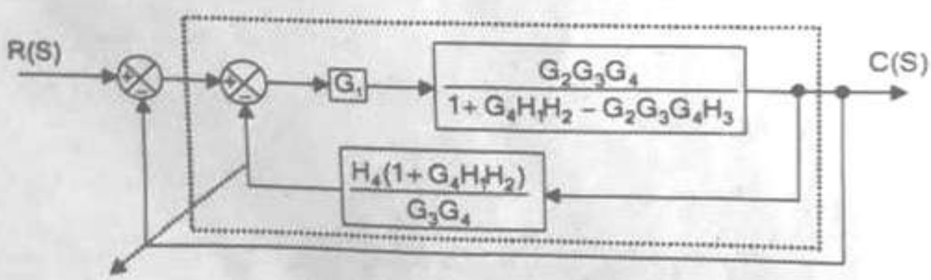


Step 6 : Eliminating feedback path and interchanging the summing points.



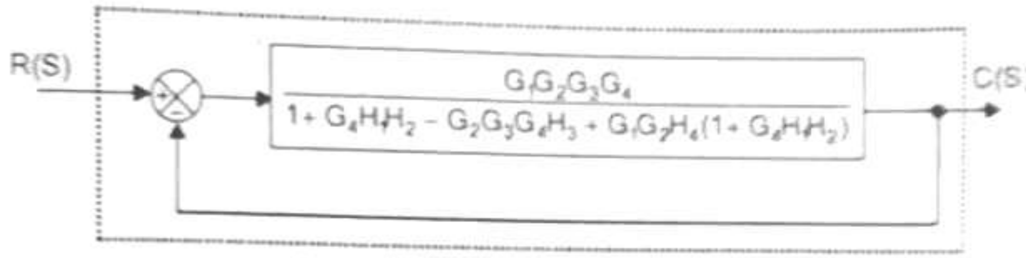
$$\frac{\dfrac{G_2G_3G_4}{1+G_4H_1H_2}}{1-\dfrac{G_2G_3G_4H_3}{1+G_4H_1H_2}} = \frac{G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3}$$

Step 7 : Combining the blocks in cascade and eliminating the feedback path



$$\frac{\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3}}{1+\left(\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3}\right)\left(\dfrac{H_4(1+G_4H_1H_2)}{G_3G_4}\right)} = \frac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3+G_1G_2H_4(1+G_4H_1H_2)}$$

ep 8 : Eliminating the unity feedback path.



$$\therefore \frac{C(s)}{R(s)}=\frac{\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3+G_1G_2H_4(1+G_4H_1H_2)}}{1+\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3+G_1G_2H_4(1+G_4H_1H_2)}}$$

$$=\frac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3+G_1G_2H_4(1+G_4H_1H_2)+G_1G_2G_3G_4}$$

$$=\frac{G_1G_2G_3G_4}{1+H_1H_2(G_4+G_1G_2G_4H_4)+G_1G_2(H_4+G_3G_4)-G_2G_3G_4H_3}$$

RESULT

The transfer function of the system is,

$$\frac{C(s)}{R(s)}=\frac{G_1G_2G_3G_4}{1+H_1H_2(G_4+G_1G_2G_4H_4)+G_1G_2(H_4+G_3G_4)-G_2G_3G_4H_3}$$

Part-A

1. Define rise time?(may-2010,dec-2011)

It is the time taken for response to raise from 0-100% the very first time. For under damped system, the rise time is calculated form 0-100%. But for over damped system it is the time taken by response to raise form 10-90%. For critically damped system, it is the time taken for response to raise form 5-95%.

2. Define maximum peak overshoot?(Dec-2010,May-17)

It is defined as the ratio of the maximum peak value to final value, where maximum peak value is measured from final value.

Let final value= c(∞),maximum value=c(tp)

Therefore peak overshoot Mp=$\frac{(tp)-c(\infty)}{c(\infty)}$

3. What is time response?

The time response is the output of the closed loop system as a function of time. It is denoted by c(t). it is given by inverse laplace of the product of input and transfer function of the system.

The closed loop transfer function, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)}$

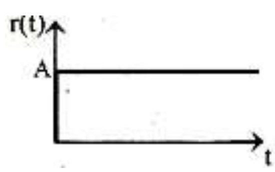
Response in s-domain, $C(s) = \frac{R(s)\,G(s)}{1+G(s)\,H(s)}$

Response in time domain, $c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{R(s)\,G(s)}{1+G(s)\,H(s)}\right\}$

4. Define step signal?(dec-2009)

The step signal is signal whose value changes from 0 to A and remain constant at A for t>0. The mathematical representation of step signal is

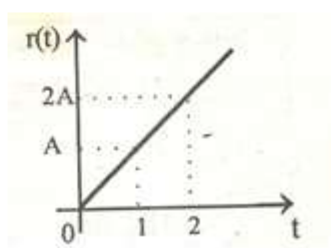
r(t)=A t>=0, o t<0.



5. Define ramp signal?

A ramp signal is a signal whose value increases linearly with time form an initial value of zero at t=0. Mathematical expression of ramp signal is

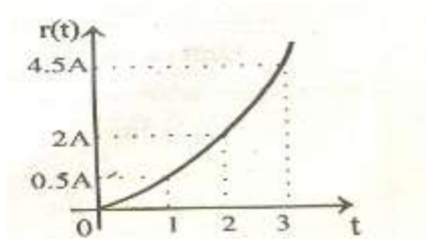
r(t)=A.t t>=0, 0 t<0.



6. Define parabolic signal?

It is signal in which the instantaneous value varies an square of the time form an initial value of zero at t=0. The mathematical representation of parabolic signal is

r(t)=A.t.t/2 t>=0, 0 t<0.



7. What is weighting function?

The impulse response of system is called weighting function. It is given by inverse laplace transform of system transfer function

8. What is an impulse signal?

A signal which is available for every short duration is called impulse signal is a unit impulse signal which is defined as a signal having zero values at all time except at t=0. At t=0 the magnitude becomes infinite. It is denoted by and mathematically expressed as

$$\delta(t) = \infty; \quad t = 0 \qquad \text{and} \qquad \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$
$$= 0; \quad t \neq 0$$

9. Define pole?

The pole of a function, F(s) is the value at which the function, F(s) becomes infinite, where F(s) is a function of complex variable s.

10. Define zero?

The zero of a function, F(s) is the value at which the function F(s) becomes zero, where F(s) is a function of complex variable s.

11. List out the various test signal used for the analysis of system?(May/June 2014)

The standard test signals are:

1. a) step signal

b) unit step signal.

2. a) ramp signal

b)unit ramp signal

3. a) parabolic signal

b)unit parabolic signal

- ❖ Impulse signal
- ❖ Sinusoidal signal.

12. How a control system is classified depending on the value of damping? (April/May2018)

There are four cases:

- ❖ Under damped system
- ❖ Un damped system
- ❖ Critically damped system
- ❖ Over damped system.

13. What is type and order of the system?(Dec-2011,May-2011)

The type number is given by number of poles of loop transfer function at the origin. The type of the system decides the steady state error.

The order of the system is given by the order of the differential equation governing the system.

It is also given by the maximum power of s in the denominator polynomial of transfer function.

14. Define damping ratio?

The damping ratio is defined as the ratio of actual damping to critical damping

15. What is damped frequency of oscillation?

In under damped system the response is damped oscillatory. The frequency of damped oscillation is given by, $\omega_d = \omega_n\sqrt{1-\zeta^2}$.

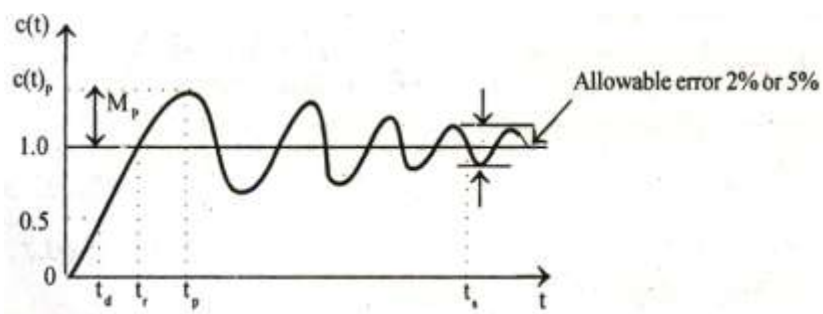
16. Distinguish between type and order of system.(May-2011)(April/May 2015) (Nov/Dec 2017)

- ❖ Type number is specified for loop transfer function but order can be specified for any transfer function
- ❖ The type number is given by number of poles of loop transfer function lying at origin of s-plane but the order is given by the number of poleis of transfer function.

17. List the importance of type and order of a system?(Dec-2012)

- ❖ The type number of the system decides the steady state error
- ❖ The maximum power of s also given the number of poles of the system and so the order of the system is also given by number of poles of transfer function.

18. Sketch the response of a second order under damped system?



19. List the time domain specification?(Dec-2003,2008,2009,May-2011)(May/June 2016)

- ❖ delay time(td)
- ❖ Rise time(tr)
- ❖ Peak time(tp)
- ❖ Maximum Overshoot(Mp)
- ❖ Settling time(ts).

20. Define Peak time?

It is the time taken for response of reach the peak value, the very first time or it is the time taken for the response of reach peak over shoot Mp.

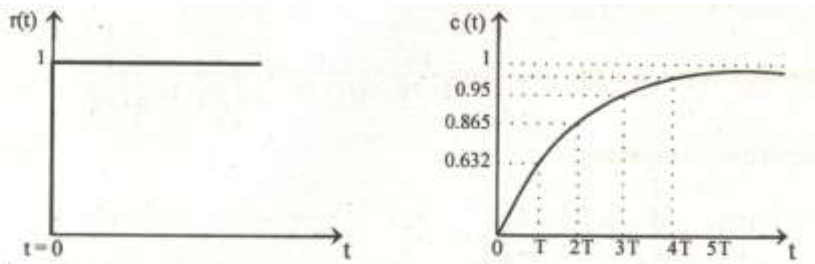
21. Define delay time?

It is the time taken for response to reach 50% of the final value, the very first time.

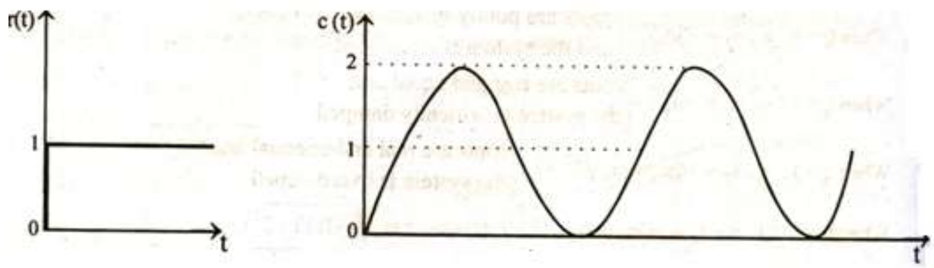
22. Define setting time?(May-2005)

It is defined as the time taken by the response to reach and stay with in a specified error and the error is usually specified as % of final value. The usual tolerable error is 2% or 5% of the final value.

23. Draw the response of a first order system step input?(Dec-2007,2008,2011,June-2006)



24. Draw the response of un damped second order system for unit step input?

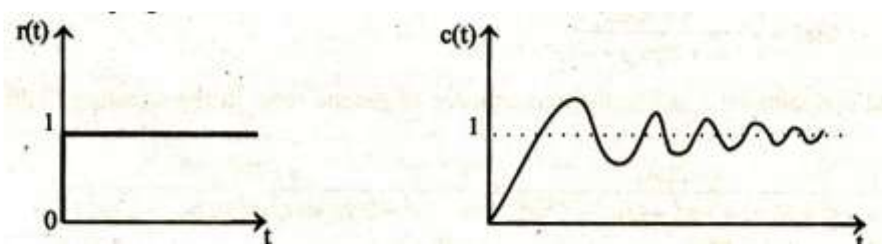


∴ For closed loop undamped second order system,

Unit step response $= 1 - \cos \omega_n t$

Step response $= A(1 - \cos \omega_n t)$

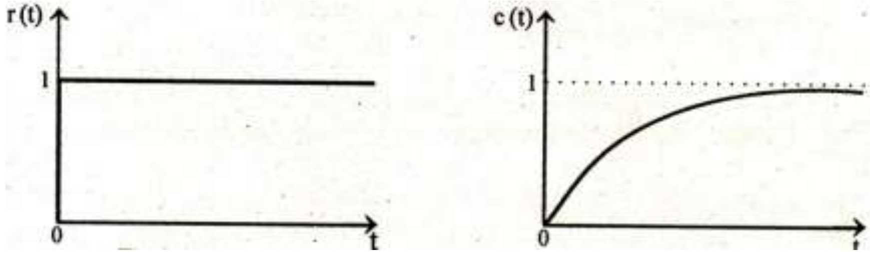
25. Draw the response of under damped second order system for unit step input?



$$\text{Unit step response} = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)\ ; \quad \theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$\text{Step response} = A\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)\ ; \quad \theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right]$$

26. Draw the response of critically damped second order system for unit step input?



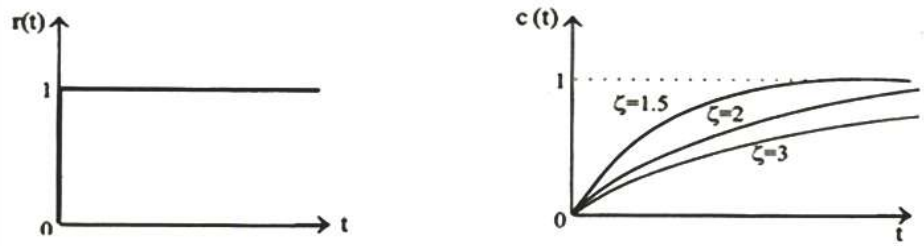
$$\text{Unit step response} = 1 - e^{-\omega_n t}(1+\omega_n t)$$

$$\text{Step response} = A\left[1 - e^{-\omega_n t}(1+\omega_n t)\right]$$

27. Draw the response of over damped second order system for unit step input?

$$\text{Unit step response} = 1 - \frac{\omega_n}{2\sqrt{\zeta^2-1}}\,\frac{1}{s_1}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right) \quad \text{where, } s_1 = \zeta\omega_n - \omega_n\sqrt{\zeta^2-1}$$

$$\text{Step response} = A\left[1 - \frac{\omega_n}{2\sqrt{\zeta^2-1}}\,\frac{1}{s_1}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right)\right] \quad s_2 = \zeta\omega_n + \omega_n\sqrt{\zeta^2-1}$$



28. What is steady state error? (April/May2018)

The steady state error is the value of error signal e(t), when t tends to infinity. The steady state error is a measure of system accuracy. These errors arise from the nature of inputs, type of system and from non linearity of system components.

29. What are static error constants?(Dec-2007,2009,2011)

The Kp, Kv and Ka are called static error constants. These constants are associated with steady state error in a particular type of system and for a standard input.

30. Define positional error constant?

The positional error constant $Kp = \underset{s\to 0}{\alpha\gamma} G(s)H(s)$The steady state error in type 0 system when the input is unit step is given by$\frac{1}{1+Kp}$.

31. Define velocity error constant?

The velocity error constant $Kv = \underset{s\to 0}{\alpha\gamma} G(s)H(s)$The steady state error in type 1 system for unit ramp is given by $\frac{1}{kv}$

32. Define acceleration error constant?

The acceleration error constant $Ka = \underset{s\to 0}{\alpha\gamma} s^2 G(s)H(s)$

The steady state error in type 2 system for unit parabolic input is given by $\frac{1}{ka}$

34. A unity feedback system has a open loop transfer functionG(s)=10/(s+1).(s+2). Determine the steady state error for unit step input?

The steady state error for unit step input, $e_{ss} = \frac{1}{1+K_p}$, where $K_p = \underset{s\to 0}{Lt}\ G(s)\ H(s)$ ·

For unity feedback system H(s) = 1.

$\therefore K_p = \underset{s\to 0}{Lt}\ G(s) = \underset{s\to 0}{Lt}\ \frac{10}{(s+1)(s+2)} = 5$ and $e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+5} = \frac{1}{6}$

35. A unity feedback system has an open loop transfer function ofG(s)=$\frac{25(s+4)}{s(s+0.5)(s+2)}$.

Determine the steady state error for unit ramp input?

The steady state error for unit ramp input is $e_{ss} = \frac{1}{K_v}$, where $K_v = \underset{s\to 0}{Lt}\ s\,G(s)\,H(s)$. For unity feedback system H(s) = 1.

$\therefore K_v = \underset{s\to 0}{Lt}\ sG(s) = \underset{s\to 0}{Lt}\ s\left[\frac{25(s+4)}{s(s+0.5)(s+2)}\right] = \frac{25\times 4}{0.5\times 2} = 100$ and $e_{ss} = \frac{1}{K_v} = \frac{1}{100} = 0.01$

36. A unity feedback system has a open loop transfer function ofG(s)=20(s+5)/s.s(s+0.1).(s+3). Determine the steady state error for parabolic input?

The steady state error for unit ramp input is $e_{ss} = \frac{1}{K_a}$, where $K_a = \underset{s\to 0}{Lt}\ s^2G(s)\,H(s)$. For unity feedback system H(s) = 1.

$\therefore K_a = \underset{s\to 0}{Lt}\ s^2\left[\frac{20(s+5)}{s^2(s+0.1)(s+3)}\right] = \frac{20\times 5}{0.1\times 3} = \frac{100}{0.3} = 333.33$ and $e_{ss} = \frac{1}{K_a} = \frac{1}{333.33} = 0.003$

37. What are generalized error coefficients?

They are the coefficients of generalized error series.The generalized error series is given by,

$$e(t) = C_0 r(t) + C_1\dot{r}(t) + \frac{C_2}{2!}\ddot{r}(t) + \frac{C_3}{3!}\dddot{r}(t) + \ldots\ldots + \frac{C_n}{n!}\overset{n}{r}(t)\ldots.$$

The coefficients C_0, C_1, C_2......C_n are called generalized error coefficients or dynamic error coefficients.

The n^{th} coefficient, $C_n = \underset{s\to 0}{Lt}\ \frac{d^n}{ds^n}\ F(s)$, where $F(s) = \frac{1}{1+G(s)\,H(s)}$.

38. Give the relation between generalized and static error coefficients?(Nov/Dec-2016)

The following expression shows the relation between generalized and static error coefficient.

$C_0 = \frac{1}{1+K_p}$; $C_1 = \frac{1}{K_v}$; $C_2 = \frac{1}{K_a}$

39. Mention two advantages of generalized error constants over static error constant?

- ❖ Generalized error series gives error signal as a function time.
- ❖ Using generalized error constants the steady state error can be determined for any type of input but static error constants are used to determine steady state error when the input is any one of the standard input.

40. State the effect of PI controller on the system performance.(May/June 2016)

The PI controller increases the order of the system by one, which results in reducing, the steady state error. But the system becomes less stable than the original system.

41. What is the effect of PD controller on the system performance?(May/June 2014)

The effect of PD controller is to increase the damping ratio of the system and so the peak overshoot is reduced.

42. Why derivative controller is not used in control system?

The derivative controller produces a control action based on rate of change of error signal and it does not produce corrective measures for any constant error. Hence derivative controller is not used in control systems.

43. What are the disadvantages of proportional controller?

The disadvantages in proportional controller are that it produces a constant steady state error.

44. A second order system has a damping ratio of 0.6 and natural frequency of oscillation is 10rad/sec. determine the damped frequency of oscillation?

Damped frequency of oscillation, $\omega_d = \omega_n\sqrt{1-\zeta^2} = 10\sqrt{1-(0.6)^2} = 10 \times 0.8 = 8 \text{ rad / sec}$

45. The damping ratio of a system is 0.75 and the natural frequency of oscillation is 12rad/sec. determine the peak overshoot and the peak time?

Peak overshoot, $M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = e^{-\frac{0.75\times\pi}{\sqrt{1-(0.75)^2}}} = 0.028$; $\therefore \%M_p = 0.028 \times 100 = 2.8\%$

Damped frequency of oscillation, $\omega_d = \omega_n\sqrt{1-\zeta^2} = 12\sqrt{1-(0.75)^2} = 7.94 \text{ rad / sec}$

Peak time, $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{7.94} = 0.396 \text{ sec}$

46. Write the PID controller equation.(Nov/Dec 2014)

$\mathbf{m(t)=K_p e(t) + K_p T_d de(t)/dt + K_p/T_i \int_0^t e(t)\,dt}$

47. What is the effect on the system performance when a proportional controller is introduced in a system? (April/May 2015) (Nov/Dec 2017)

The proportional controller improves the steady-state tracking accuracy, disturbance signal rejection and relative stability of the system. It also increases the loop gain of the system which results in reducing the sensitivity of the system to parameter variations.

48. ***What is root locus?***

The path taken by a root of characteristic equation when open loop gain K is varied from 0 to ∞ is called root locus.

49. ***What is magnitude criterion?***

The magnitude condition states that $s = s_a$ will be a point on root locus if for that value of s magnitude of G(s)H(s) is equal to 1, (i.e. $|G(s)H(s)| = 1$).

$$\text{Let, } G(s)\,H(s) = \frac{K\,(s+z_1)\,(s+z_2)\,(s+z_3)\,.....}{(s+p_1)\,(s+p_2)\,(s+p_3)\,.....}$$

$\therefore$ For $s = s_a$ be a point in root locus,

$$|G(s)\,H(s)| = \frac{K\,|s_a+z_1|\,|s_a+z_2|\,|s_a+z_3|\,.....}{|s_a+p_1|\,|s_a+p_2|\,|s_a+p_3|\,.....} = 1$$

$$\left[\text{or } \ |G(s)\,H(s)| = K = \frac{\text{Product of length of vectors from open loop zeros to the point } s_a}{\text{Product of length of vectors from open loop poles to the point } s_a} = 1\right]$$

50. ***What is angle criterion?***

The angle criterion states that $s = s_a$ will be a point on root locus if for that value of s the argument or phase of G(s)H(s) is equal to an odd multiple of 180°, [i.e., $\angle G(s)H(s) = \pm 180°\,(2q+1)$].

$$\text{Let, } G(s)H(s) = K\,\frac{(s+z_1)\,(s+z_2)\,(s+z_3)\,.....}{(s+p_1)\,(s+p_2)\,(s+p_3)\,.....}$$

$\therefore$ For $s = s_a$ be a point on root locus,

$$\angle G(s)\,H(s) = \angle(s_a+z_1) + \angle(s_a+z_2) + \angle(s_a+z_3)\,.....-\angle(s_a+p_1) - \angle(s_a+p_2)....\ = \pm 180°(2q+1)$$

$$\left[\text{or} \begin{pmatrix}\text{Sum of angles}\\ \text{of vectors from zeros}\\ \text{to the point } s = s_a\end{pmatrix} - \begin{pmatrix}\text{Sum of angles}\\ \text{of vectors from poles}\\ \text{to the point } s = s_a\end{pmatrix} = \pm 180°(2q+1)\right]$$

51. ***How will you find the gain K at a point on root locus?***

The gain K at a point $s = s_a$ on root locus is given by,

$$K = \frac{\text{Product of length of vector from open loop poles to the point } s_a}{\text{Product of length of vector from open loop zeros to the point } s_a}$$

52. ***How will you find root locus on real axis?***

To find the root locus on real axis, choose a test point on real axis. If the total number of poles and zeros on the real axis to the right of this test point is odd number, then the test point lies on the root locus. If it is even then the test point does not lie on the root locus.

53. ***What are asymptotes? How will you find the angle of asymptotes?***

Asymptotes are straight lines which are parallel to root locus going to infinity and meet the root locus at infinity.

$$\text{Angles of asymptotes} = \frac{\pm 180°\ (2q+1)}{n-m}\ ; \ q = 0,\ 1,\ 2,......\ (n-m)$$

54. ***What is centroid? How the centroid is calculated?***

The meeting point of asymptotes with real axis is called centroid. The centroid is given by,

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$$

55. ***What is breakaway and breakin point? How to determine them?***

At breakaway point the root locus breaks from the real axis to enter into the complex plane. At breakin point the root locus enters the real axis from the complex plane.

To find the breakaway or breakin points, form an equation for K from the characteristic equation, and differentiate the equation of K with respect to s. Then find the roots of equation dK/ds = 0. The roots of dK/ds = 0 are breakaway or breakin points, provided for this value of root, the gain K should be positive and real.

56. ***How to find the crossing points of root locus in imaginary axis.***

Method (i) : By Routh hurwitz criterion.

Method (ii) : By letting $s = j\omega$ in the characteristic equation and separate the real and imaginary part. These two equations are equated to zero. Solve the two equations for ω and K. The value of ω gives the point where the root locus crosses imaginary axis and the value of K is the gain corresponding to the crossing point.

57. ***What is dominant pole?***

The dominant pole is a pair of complex conjugate pole which decides transient response of the system. In higher order systems the dominant poles are very close to origin and all other poles of the system are widely separated and so they have less effect on transient response of the system.

58. ***How will you fix dominant pole on root locus and find the gain K corresponding to the dominant pole?***

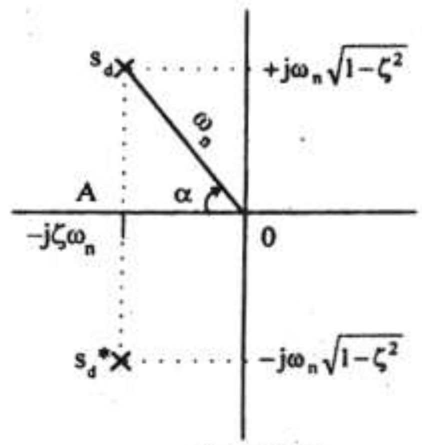
The dominant poles are given by roots of a quadratic factor, $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$.

$$\therefore s = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

The dominant pole can be plotted on the s-plane as shownin fig Q5.28.

In the right angle triangle OAP,

$$\cos\alpha = \frac{\zeta\omega_n}{\omega_n} = \zeta \qquad \therefore \alpha = \cos^{-1}\zeta$$



FigQ 5.28

To fix a dominant pole on root locus draw a line at an angle of $\cos^{-1}\zeta$ with respect to negative real axis. The meeting point of this line with root locus will give the location of dominant pole. The value of K corresponding to dominant pole can be obtained from magnitude condition.

$$K = \frac{\text{Product of length of vectors from open loop poles to dominant pole}}{\text{Product of length of vectors from open loop zeros to dominant pole}}$$

59. Determine type and order of the following system G(s)H(s)=10/[S^3(S^2+2S+1)](May-17)

Type=3, order=5

60. State the basic properties of Root Locus. Dec-16

i)The root locus has a no. of branches that is equal to the no. of open loop poles. Each branches starts from a pole of G(s) for Kc=0. For Kc--+α, m branches terminate on the m zeros of G(s), while n-m roots goes towards infinity.

ii) The root locus is symmetric W.R.T the real axis.

61. What is dominant pole? (April/May 2018)

The dominant pole is a pair of complex conjugate pole which decides transient response of the system. In higher order systems the dominant poles are very close to origin and all other poles of the system are widely separated and so they less effect on transient response of the system.

62. What are the effects adding open loop poles and zeros on the nature of root locus?

(Nov/Dec 2017)

- Addition of poles pulls the root locus to the right.
- Addition of zeros pulls the root locus to the left.

PART-B

1. Explain in detail the system response with PI, PD and PID controller? (Dec-2006, 2011)

Controllers;

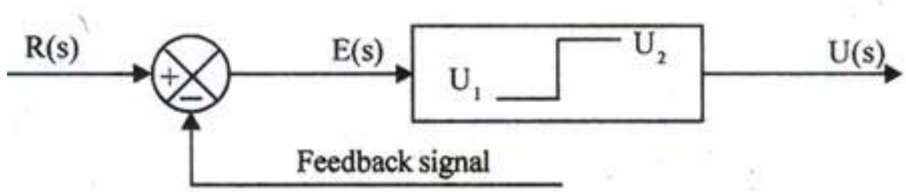
A controller is a device introduced in the system to modify the error signal and to produce a control signal. The manner in which the controller produces the control signal is called the control action.

Type of controllers;

1. ON-OFF control action.
2. Proportional control action.(P controller).
3. Integral control action (I controller).
4. Proportional-plus-integral control action.
5. Proportional-plus-derivative control action.
6. Proportional-plus-integral-plus-derivative control action .

<u>1.ON-OFF control action.</u>

It has only two fixed positions.they are either on or off. The on-off control system is very simple in construction and hence less expensive. For this reason it is very widely used in both industrial and domestic control system.



U (t)=u_1;for e (t) <0

=u_2 fore(t)>0

Let the output signal from the controller be u(t)and the actuating error signal be e(t).in this controller, u(t)Remains at either a maximum or minimum value.

(2). PROPORTIONAL CONTROLLER (P-CONTROLLER)

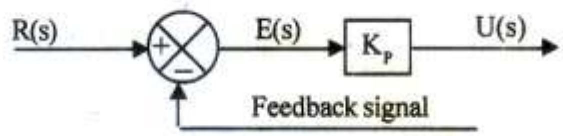
The Proportional controller is a device that produces a control signal, u(t) proportional to the input errorsignal,e(t).

In P-controller, u(t) $\alpha\ e(t)$

$$\therefore u(t) = K_P e(t)$$

Where, K_p=proportional gain or constant

Taking Laplace transform of equation ----------------- (1)



$U(S)=K_P E(S)$

$$\therefore \frac{U_S}{E_S} = K_P$$

Het proportional controller amplifies the error signal by an amount $k_p$the increase in loop gain improves the steady state tracking accuracy, disturbance signal rejection and the relative stability. There are the various Effect of proportional controller.

Draw back;

The increase in loop gain it decreed the sensitivity of the system to parameter variations the draw back in proportional control action in that it produces a constant steady state error.

(3). INTEGRAL CONTROLLER(I-CONTROLLER)

The integral controller is a device that produces a control signal u(t) which is proportional to integral of the input error signal, e(t).

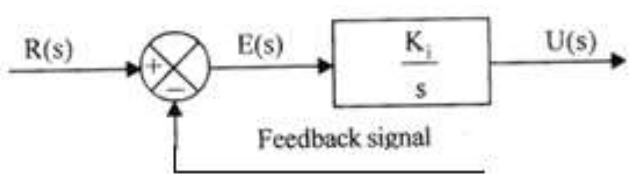
In I-controller, u (t) α $\int e(t)dt$; $\therefore u(t) = k_i \int e(t)dt$ --------------- (1)

Where, k_i=integral gain or constant.

Taking Laplace transformer of equation ------ (1)

$$U(s) = K_i \frac{E(s)}{s} \text{ -------- (2)}$$

$$\therefore \frac{U(s)}{E(s)} = \frac{k_i}{s} \text{-------(2)}$$



The integral controller removes or reduces the steady error without the need for manual reset. Hence the I-controller is sometimes called automatic reset.

Draw back;

The drawback in integral controller is that it may lead to oscillatory response of increasing or decreasing amplitude which is undesirable and the system may become unstable.

4) PROPORTIONAL PULS INTEGRAL CONTROLLER (PI-CONTROLLER)

The proportional plus integral controller (pi-controller) produces an output signal consisting of two terms, one proportional to error signal and the other proportional to the integral of error signal.

$$\text{In PI} - \text{controller}, u(t)\alpha \left[e(t) + \int e(t)dt\right]$$

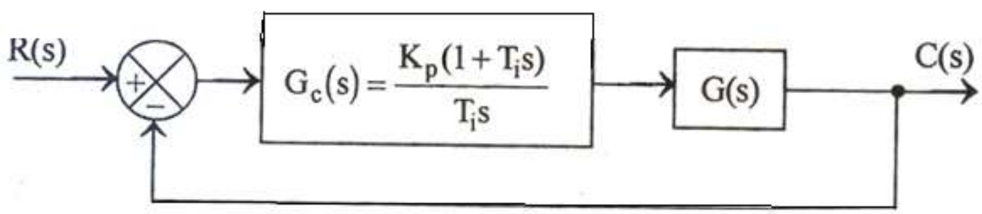
$$\therefore u(t) = K_p\, e(t) + \frac{K_p}{T_i}\int e(t)dt$$

K_p=proportional gain
T_i=Integral time.
Taking Laplace transform of equation ---- (1)

$$U(s) = K_p E(s) + \frac{E(s)}{s}$$

$$\therefore \frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s}\right)$$

Effect of PI- controller;



Open loop transfer function G(s) =$\frac{\omega_{n^2}}{s(s+2\tau\omega n}$
Loop transfer function= $G_c(s)G(s)H(s)$
= $G_c(s)G(s)$
= Kp $\left(\frac{1+Tis}{Tis}\right) \times \frac{\omega_{n^2}}{s(s+2\zeta\omega n)}$
= $\frac{K_p\omega_{n^2}(1+Tis)}{s^2Ti(s+2\zeta\omega n)}$

The closed loop transfer function $\frac{C(s)}{R(s)} = \frac{Gc(s)G(s)}{1+Gc(s)G(s)}$

$$\frac{\frac{K_p\omega_{n^2}(1+Tis)}{s^2Ti(s+2\zeta\omega n)}}{1+\frac{K_p\omega_{n^2}(1+Tis)}{s^2Ti(s+2\zeta\omega n)}} = \frac{K_p\omega_{n^2}(1+Tis)}{s^2Ti(s+2\zeta\omega n) + K_p\omega_{n^2}(1+Tis)}$$

$$= \frac{K_p\omega_{n^2}(1+Tis)}{Tis^3 + 2\zeta\omega nTis^2 + K_p\omega_{n^2}Tis+K_p\omega_{n^2}} \qquad \therefore Ki = \frac{Kp}{Ti}$$

$$= \frac{(\frac{kp}{Ki})\omega_{n^2}(1+Tis)}{S^3 + 2\zeta\omega nTis^2 + K_p\omega_{n^2}S + \frac{Kp}{Ti}\omega_{n^2}}$$

$$= \frac{Ki\omega_{n^2}(1+\mathrm{Tis})}{S^3 + 2\zeta\omega n\mathrm{Tis}^2 + K_p\omega_{n^2}S + Ki\omega_{n^2}}$$

It observed that the PI controller in traduces a zero in the system and increases the order by one. the increases in type number the result in reducing the steady state error. If the steady state error of the original system is constant then the integral controller will reduce the error to zero.

5.PROPORTIONAL PLUS DERIVATIVE CONTROLLER(PD- CONTROLLER)

The proportional plus derivative controller produces an output signal consisting of two term; one proportional to error signal and the other proportional to the derivative of error signal.

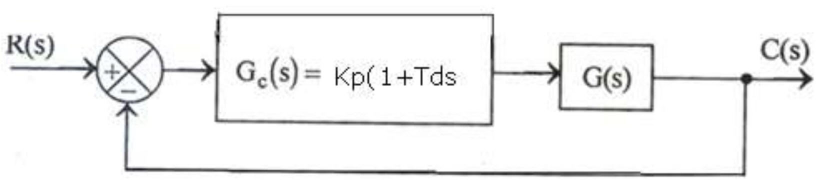
In PD-controller, u(t) α $\left[e(t) + \frac{d}{dt}et\right]$; $\therefore u(t) = k_p e(t) + K_p T_d \frac{d}{dt} e(t) ----(1)$

Where, K_P=proportional gain

T_d=Derivative time.

Taking Laplace transform of equation (1)

$U(s)=K_pE(s)+K_pT_{dsE(s)}$ $\therefore \frac{U(s)}{E(s)} = K_p(1+T_dS)$



Open loop transfer function G(s) =$\frac{\omega_{n^2}}{s(s+2\tau\omega n}$

Loop transfer function= $G_c(s)G(s)H(s)$

= $G_c(s)G(s)$

= $\mathrm{Kp}(1+\mathrm{Td}(s)) \times \frac{\omega_{n^2}}{s(s+2\zeta\omega n)}$

The closed loop transfer function $\frac{C(s)}{R(s)} = \frac{Gc(s)G(s)}{1+Gc(s)G(s)}$

$$\frac{\frac{K_p\omega_{n^2}(1+Tds)}{s(s+2\zeta\omega n)}}{1+\frac{K_p\omega_{n^2}(1+Tds)}{s(s+2\zeta\omega n)}} = \frac{K_p\omega_{n^2}(1+Tds)}{s(s+2\zeta\omega n)+K_p\omega_{n^2}(1+Tds)}$$

$$= \frac{K_p\omega_{n^2}(1+Tds)}{s^2 + 2\zeta\omega ns + K_p\omega_{n^2+K_p\omega_{n^2Tds}}}$$

$$\therefore Kd = Kp\ Td$$

$$= \frac{Kp\omega_{n^2}(1 + \mathrm{Tds})}{s^2 + (2\zeta\omega ns + K_p\omega_{n^2 Td)s + K_p\omega_{n^2}}}$$

$$= \frac{\omega_{n^2}(Kp + \mathrm{Kds})}{s^2 + (2\zeta\omega n + K_d\omega_{n^2)s + K_p\omega_{n^2}}}$$

It is observed that the PD controller introducer a zero in the system and increases the damping ratio. The addition of zero may increase the peak overshoot and reduce the rise time. But the effect of increased damping ultimately reduces the peak overshoot.

It is observed the PD controller introducer does not modify the type number of the system. Hence PD controller will not act modify steady state error.

6. PROPORTIONAL PLUS INTEGRAL PLUS DERIVATIVE CONTROLLER (PID-CONTROLLER)(Nov/Dec-2016)

Derive the tansfer function of PIDcontroller.(April/May2018)

The PID-controller produces an output signal consisting of three terms: one proportional to error signal, another one proportional to integral of error signal and the third one proportional to derivative of error signal.

In PID- controller, u(t)α $\left[e(t) + \int e(t)dt + \frac{d}{dt}e(t)\right]$

$$\therefore u(t) = K_p e(t) + \frac{K_p}{T_i}\int e(t)\,dt + K_P T_d \frac{d}{dt}e(t)$$

Where, K_p=proportional gain

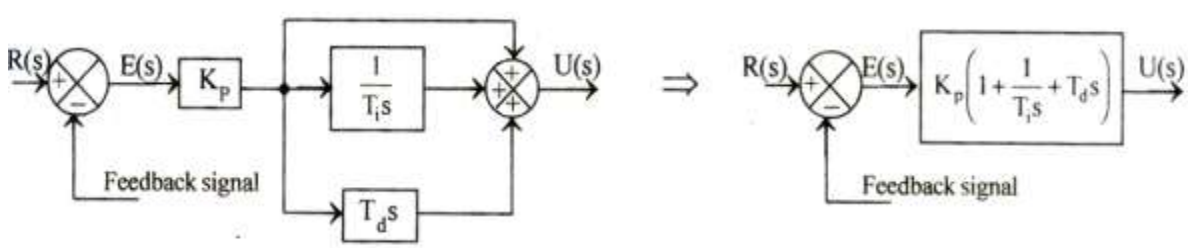
T_i=integral time.

T_d= derivative time.

Taking Laplace transform of equation,

$U(s) = K_p E(S) + \frac{K_p}{T_i}\frac{E(S)}{S} + K_P T_d sE(s)$

$$\therefore \frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i S} + T_d S\right)$$



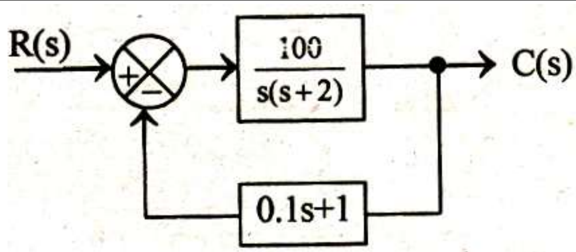
EFFECT OF PID-CONTROLLER;

The proportional controller stabilizes the gain but produces a steady state error. the integral controller reduces or eliminates the steady state error. the derivative controller reduces the rate of change of error. The combined effect of all the three con not be judge from the pentameters Kp, Ki and Kd.

7.Positional control system with velocity feedback is shown in fig what is the response of the system for unit step in put?

SOLUTION

The closed loop transfer function;



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)}$$

Given that , $G(s) = \dfrac{100}{s(s+2)}$ and $H(s) = 0.1s+1$

$$\therefore \frac{C(s)}{R(s)} = \frac{\dfrac{100}{s\,(s+2)}}{1+\left(\dfrac{100}{s\,(s+2)}\right)(0.1s+1)} = \frac{\dfrac{100}{s\,(s+2)}}{\dfrac{s(s+2)+100\,(0.1s+1)}{s\,(s+2)}} = \frac{100}{s^2+2s+10s+100} = \frac{100}{s^2+12s+100}$$

Here $(s^2 + 12s + 100)$ is characteristic polynomial. The roots of the characteristic polynomial are,

$$s_1,\, s_2 = \frac{-12 \pm \sqrt{144-400}}{2} = \frac{-12 \pm j16}{2} = -6 \pm j8$$

The roots are complex conjugate. The system is underdamped and so the response of the system will have damped oscillations.

The response in s-domain, $C(s) = R(s)\,\dfrac{100}{s^2+12s+100}$

Since input is unit step, $R(s) = \dfrac{1}{s}$

$$\therefore C(s) = \frac{1}{s}\,\frac{100}{s^2+12s+100} = \frac{100}{s(s^2+12s+100)}$$

By partial fraction expansion we can write,

$$C(s) = \frac{100}{s\,(s^2+12s+100)} = \frac{A}{s} + \frac{Bs+C}{s^2+12s+100}$$

The residue A is obtained by multiplying C(s) by s and letting s = 0.

$$A = C(s) \times s\Big|_{s=0} = \left.\frac{100}{s^2+12s+100}\right|_{s=0} = \frac{100}{100} = 1$$

The residue B and C are evaluated by cross multiplying the following equation and equating the coefficients of like power of s.

$$\frac{100}{s\,(s^2+12s+100)} = \frac{A}{s} + \frac{Bs+C}{s^2+12s+100}$$

$$100 = A(s^2+12s+100) + (Bs+C)\,s$$

$$100 = As^2 + 12As + 100A + Bs^2 + Cs$$

On equating the coefficients of s^2 we get, $\quad 0 = A + B \qquad \therefore B = -A = -1$

On equating coefficients of s we get, $\quad 0 = 12A + C \qquad \therefore C = -12A = -12$

$$\therefore C(s) = \frac{1}{s} + \frac{-s-12}{s^2+12s+100} = \frac{1}{s} - \frac{s+12}{s^2+12s+36+64} = \frac{1}{s} - \frac{s+6+6}{(s+6)^2+8^2}$$
$$= \frac{1}{s} - \frac{s+6}{(s+6)^2+8^2} - \frac{6}{(s+6)^2+8^2} = \frac{1}{s} - \frac{s+6}{(s+6)^2+8^2} - \frac{6}{8}\,\frac{8}{(s+6)^2+8^2}$$

The time domain response is obtained by taking inverse Laplace transform of C(s).

$$\text{Time response, } c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{s+6}{(s+6)^2+8^2} - \frac{6}{8}\,\frac{8}{(s+6)^2+8^2}\right\}$$
$$= 1 - e^{-6t}\cos 8t - \frac{6}{8}\,e^{-6t}\sin 8t = 1 - e^{-6t}\left[\frac{6}{8}\sin 8t + \cos 8t\right]$$

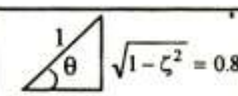
The result can be converted to another standard form by constructing right angle triangle with ζ and $\sqrt{1-\zeta^2}$. The damping ratio ζ is evaluated by comparing the closed loop transfer function of the system with standard form of second order transfer function.

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n+\omega_n^2} = \frac{100}{s^2+12s+100}$$

On comparing we get, $\omega_n^2 = 100 \qquad \therefore \omega_n = 10$

$2\zeta\omega_n = 12 \qquad \therefore \zeta = \frac{12}{2\omega_n} = \frac{12}{2 \times 10} = 0.6$

Constructing right angled triangle with ζ and $\sqrt{1-\zeta^2}$ we get,



$\zeta = 0.6$

$\cos\theta = 0.6$

$\sin\theta = 0.8$

$$\sin\theta = 0.8 \;;\; \cos\theta = 0.6 \;;\; \tan\theta = \frac{0.8}{0.6}$$
$$\therefore \theta = \tan^{-1}\frac{0.8}{0.6} = 53° = 53° \times \frac{\pi}{180°}\text{rad} = 0.925 \text{ rad.}$$

$$\therefore \text{ Time response , } c(t) = 1 - e^{-6t}\left[\frac{6}{8}\sin 8t + \cos 8t\right] = 1 - e^{-6t}\,\frac{10}{8}\left[\frac{6}{10}\sin 8t + \frac{8}{10}\cos 8t\right]$$
$$= 1 - \frac{10}{8}e^{-6t}[\sin 8t \times 0.6 + \cos 8t \times 0.8] = 1 - 1.25\,e^{-6t}[\sin 8t\cos\theta + \cos 8t\sin\theta]$$
$$= 1 - 1.25\,e^{-6t}[\sin(8t+\theta)] = 1 - 1.25\,e^{-6t}\sin(8t + 0.925)$$

Note : θ is expressed in radians

RESULT

The response in time domain,

$$c(t) = 1 - e^{-6t}\left[\frac{6}{8}\sin 8t + \cos 8t\right] \quad \text{or} \quad c(t) = 1 - 1.25\,e^{-6t}\sin(8t + 0.925)$$

3. The response of a servomechanism is, $c(t) = 1 + 0.2\, e^{-60t} - 1.2\, e^{-10t}$ when subject to a unit step input. Obtain an expression for closed loop transfer function. Determine the undamped natural frequency and damping ratio.

SOLUTION

Given that, $c(t) = 1 + 0.2\, e^{-60t} - 1.2\, e^{-10t}$

On taking Laplace transform of c(t) we get,

$$C(s) = \frac{1}{s} + 0.2\frac{1}{(s+60)} - 1.2\frac{1}{(s+10)} = \frac{(s+60)(s+10) + 0.2\,s\,(s+10) - 1.2\,s\,(s+60)}{s\,(s+60)(s+10)}$$

$$= \frac{s^2 + 70s + 600 + 0.2\,s^2 + 2s - 1.2s^2 - 72s}{s\,(s+60)(s+10)} = \frac{600}{s\,(s+60)(s+10)} = \frac{1}{s}\,\frac{600}{(s+60)(s+10)}$$

Since input is unit step, R(s) = 1/s

$$\therefore C(s) = R(s)\,\frac{60}{(s+60)(s+10)} = R(s)\,\frac{600}{s^2 + 70s + 600}$$

$\therefore$ The closed loop transfer function of the system, $\dfrac{C(s)}{R(s)} = \dfrac{600}{s^2 + 70s + 600}$

The damping ratio and natural frequency of oscillation can be estimated by comparing the system transfer function with standard form of second order transfer function.

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{600}{s^2 + 70s + 600}$$

On comparing we get,

$$\omega_n^2 = 600$$

$$\therefore \omega_n = \sqrt{600} = 24.49 \text{ rad / sec}$$

$$2\zeta\omega_n = 70$$

$$\therefore \zeta = \frac{70}{2\omega_n} = \frac{70}{2 \times 24.49} = 1.43$$

RESULT

The closed loop transfer function of the system, $\dfrac{C(s)}{R(s)} = \dfrac{600}{s^2 + 70s + 600}$

Natural frequency of oscillation, $\omega_n = 24.49$ rad/sec

Damping ratio, $\zeta = 1.43$

9.The unity feedback system is characterized by an open loop transfer function G(s)=K/s (s+10).Determine the gain K, such that the system will have a damping ratio of 0.5 for this value of K. Determine time domain specifications for a unit step input.(April/May 2014), (April/May 2015)(Nov/Dec-16)

$$The\ closed\ loop\ transfer\ function, \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$Given\ that, G(s) = \frac{K}{s(s+10)}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+10)}}{1+\frac{K}{s(s+10)}} = \frac{K}{s(s+10)+K} = \frac{K}{s^2+10s+K}$$

The value of Kcanbe evaluated by comparing the system transfer function with standard form of second order transfer function.

$$\frac{C(s)}{R(s)} = \frac{{\omega_n}^2}{s^2+2\zeta\omega_n s+{\omega_n}^2} = \frac{K}{s^2+10s+K}$$

On comparing we get,

$${\omega_n}^2 = K, \therefore \omega_n = \sqrt{K}$$

$$2\zeta\omega_n = 10\ put\ \zeta = 0.5\ and\ , \omega_n = \sqrt{K}$$

$$2 \times 0.5 \times \sqrt{K} = 10, \qquad \sqrt{K} = 10, \qquad K = 100\ and\ \omega_n = \sqrt{K} = 10$$

*The value of gain,**K=100***

$$Percentage\ peak\ overshoot, \%M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$$

$$= e^{\frac{-0.5\pi}{\sqrt{1-0.5^2}}} \times 100 = 0.163 \times 100 = 16.3\%$$

$$\boxed{\boldsymbol{M_p = 16.3\%}}$$

$$Peak\ time, t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{10\sqrt{1-0.5^2}} = 0.363$$

$$\boxed{\boldsymbol{t_p = 0.363\ sec}}$$

10. The open loop transfer function of a unity feedback system is given by G(s) =K/s (sT+1), where K and T are positive constant. By what factor should the amplifier gain K be reduced, so that the peak over shoot of unit step response of the system is reduced from 75% to 25%?

SOLUTION

The unity feedback system is shown in fig 1.

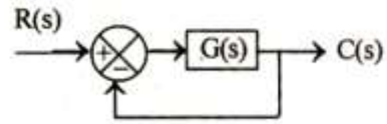


Fig 1 : Unity feedback system.

The closed loop transfer function, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$

Given that, G(s) = K/s (sT +1)

$$\therefore \frac{C(s)}{R(s)} = \frac{K/s(sT+1)}{1+K/s(sT+1)} = \frac{K}{s(sT+1)+K} = \frac{K}{s^2T+s+K} = \frac{K/T}{s^2+\frac{1}{T}s+\frac{K}{T}}$$

Expression for ζ and ω_n can be obtained by comparing the transfer function with the standard form of second order transfer function.

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{K/T}{s^2+\frac{1}{T}s+\frac{K}{T}}$$

On comparing we get,

$$\omega_n^2 = K/T \qquad \therefore \omega_n = \sqrt{K/T}$$

$$2\zeta\omega_n = 1/T \qquad \zeta = \frac{1}{2\omega_n T} = \frac{1}{2\sqrt{\frac{K}{T}}\,T} = \frac{1}{2\sqrt{KT}}$$

The peak overshoot, M_p is reduced by increasing the damping ratio ζ. The damping ratio ζ is increased by reducing the gain K.

When $M_p = 0.75$, Let $\zeta = \zeta_1$ and $K = K_1$

When $M_p = 0.25$, Let $\zeta = \zeta_2$ and $K = K_2$

Peak overshoot, $M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

Taking natural logarithm on both sides, $\ln M_p = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}}$

On squaring we get, $(\ln M_p)^2 = \frac{\zeta^2\pi^2}{1-\zeta^2}$

On cross multiplication we get,

$$(1-\zeta^2)(\ln M_p)^2 = \zeta^2\pi^2$$

$$(\ln M_p)^2 - \zeta^2(\ln M_p)^2 = \zeta^2\pi^2$$

$$(\ln M_p)^2 = \zeta^2\pi^2 + \zeta^2(\ln M_p)^2$$

$$(\ln M_p)^2 = \zeta^2\left[\pi^2 + (\ln M_p)^2\right]$$

$$\therefore \zeta^2 = \frac{(\ln M_p)^2}{\pi^2+(\ln M_p)^2} \quad ....(1)$$

But $\zeta = \frac{1}{2\sqrt{KT}}$, $\therefore \zeta^2 = \frac{1}{4KT}$ ....(2)

On equating, equation (1) and (2) we get,

$$\frac{1}{4KT} = \frac{(ln\,M_p)^2}{\pi^2 + (ln\,M_p)^2}$$

$$\frac{1}{K} = \frac{4T\,(ln\,M_p)^2}{\pi^2 + (ln\,M_p)^2}$$

$$K = \frac{\pi^2 + (ln\,M_p)^2}{4T\,(ln\,M_p)^2}$$

When, $K = K_1$, $M_p = 0.75$, $\therefore K_1 = \frac{\pi^2 + (ln 0.75)^2}{4T\,(ln 0.75)^2} = \frac{9.952}{0.331T} = \frac{30.06}{T}$

When, $K = K_2$, $M_p = 0.25$, $\therefore K_2 = \frac{\pi^2 + (ln 0.25)^2}{4T\,(ln 0.25)^2} = \frac{11.79}{7.68T} = \frac{1.53}{T}$

$$\therefore \frac{K_1}{K_2} = \frac{(1/T)\,30.06}{(1/T)\,1.53} = 19.6$$

$$\mathbf{K_1 = 19.6\,K_2} \quad \text{(or)} \quad \mathbf{K_2 = \frac{1}{19.6}\,K_1}$$

To reduce peak overshoot from 0.75 to 0.25, K should be reduced by 19.6 times (approximately 20 times).

RESULT

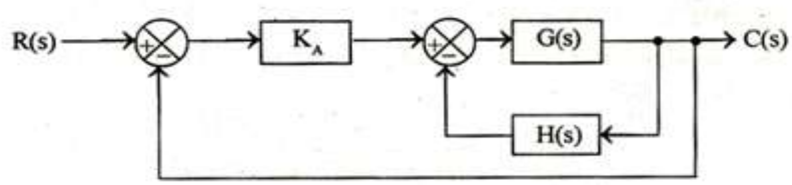
The value of gain, K should be reduced approximately 20 times to reduce peak overshoot from 0.75 to 0.25.

PROBLEM:11

6 A unity feedback control system has an amplifier with gain $K_A = 10$ and gain ratio, $G(s) = 1/s(s+2)$ in the feed forward path. A derivative feedback, $H(s) = sK_o$ is introduced as a minor loop around G(s). Determine the derivative feedback constant, K_o so that the system damping factor is 0.6.

SOLUTION

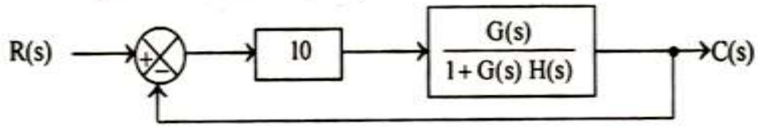
The given system can be represented by the block diagram shown in fig 1.



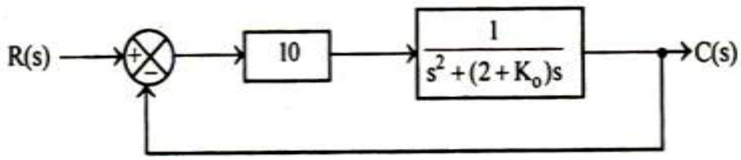
Here, $K_A = 10$; $G(s) = \frac{1}{s(s+2)}$ and $H(s) = sK_o$

The closed loop transfer function of the system can be obtained by block diagram reduction techniques.

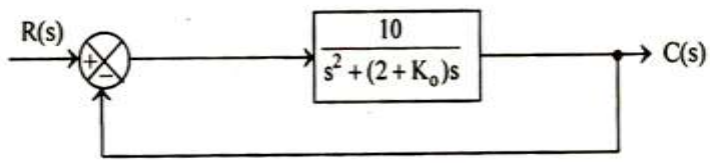
Step 1: Reducing the inner feedback loop.



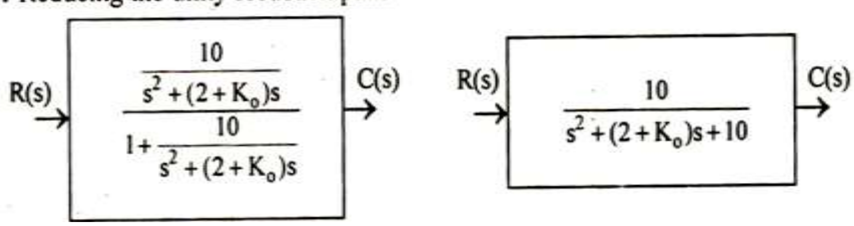
$$\frac{G(s)}{1+G(s)H(s)} = \frac{\frac{1}{s(s+2)}}{1+\frac{1}{s(s+2)}sK_o} = \frac{1}{s(s+2)+sK_o} = \frac{1}{s^2+2s+sK_o} = \frac{1}{s^2+(2+K_o)s}$$



Step 2 : combining blocks in cascade



Step 3 : Reducing the unity feedback path



The closed loop transfer function, $\frac{C(s)}{R(s)} = \frac{10}{s^2+(2+K_o)s+10}$ .....(1)

The given system is a second order system. The value of K_o can be determined by comparing the system transfer function with standard form of second order transfer function given below.

$$\left.\begin{array}{l}\text{Standard form of}\\ \text{Second order transfer function}\end{array}\right\} \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \quad .....(2)$$

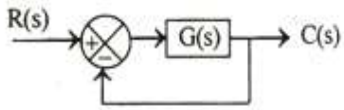
On comparing equation (1) & (2) we get,

$\omega_n^2 = 10$	$2+K_o = 2\zeta\omega_n$
$\therefore \omega_n = \sqrt{10} = 3.162$ rad / sec	$\therefore K_o = 2\zeta\omega_n - 2$
	$= 2\times 0.6\times 3.162 - 2 = 1.7944$

RESULT

The value of constant, $K_o = 1.7944$

12. A unit feedback control system has an open loop transfer function, G(s) =10/s(s+2). Find the rise time percentage over shoot, peak time and settling time for a step input of 12 units?(dec-2003,May-17)



The closed loop transfer function, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$

Given that, G(s) = 10/s (s+2)

$$\therefore \frac{C(s)}{R(s)} = \frac{\frac{10}{s(s+2)}}{1+\frac{10}{s(s+2)}} = \frac{10}{s(s+2)+10} = \frac{10}{s^2+2s+10}$$

The values of damping ratio ζ and natural frequency of oscillation ω_n are obtained by comparing the system transfer function with standard form of second order transfer function.

$$\left.\begin{array}{l}\text{Standard form of}\\ \text{Second order transfer function}\end{array}\right\} \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

On comparing equation (1) & (2) we get,

$$\omega_n^2 = 10 \qquad 2\zeta\omega_n = 2$$

$$\therefore \omega_n = \sqrt{10} = 3.162 \text{ rad / sec} \qquad \therefore \zeta = \frac{2}{2\omega_n} = \frac{1}{3.162} = 0.316$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} = \tan^{-1}\frac{\sqrt{1-0.316^2}}{0.316} = 1.249 \text{ rad}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 3.162\sqrt{1-0.316^2} = 3 \text{ rad / sec}$$

$$\text{Rise time, } t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-1.249}{3} = 0.63 \text{ sec}$$

$$\text{Percentage overshoot, } \%M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100 = e^{\frac{-0.316\pi}{\sqrt{1-0.316^2}}} \times 100$$

$$= 0.3512 \times 100 = 35.12\%$$

$$\text{Peak overshoot} = \frac{35.12}{100} \times 12 \text{ units} = 4.2144 \text{ units}$$

$$\text{Peak time, } t_p = \frac{\pi}{\omega_d} = \frac{\pi}{3} = 1.047 \text{ sec}$$

Time constant, $T=\frac{1}{\zeta\omega_n} = \frac{1}{0.316\times3.162} = 1\text{sec}$

For 5% error, setting time ts=3T=3sec

For 2% error, setting time ts=4T=4sec

RESULT

Rise time,	t_r	=	0.63 sec
Percentage overshoot,	$\%M_p$	=	35.12%
Peak overshoot		=	4.2144 units, (for a input of 12 units)
Peak time,	t_p	=	1.047 sec
Settling time,	t_s	=	3 sec for 5% error
		=	4 sec for 2% error

13.A system is given by differential equation $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 8x$, where y =output , and x= input ,determine all time domain specification for unit step input?(may-2007)

Solution;

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 8x$$

Take Laplace transform for above equation**;**

$$s^2\,Y(s) + 4s\,Y(s) + 8\,Y(s) = 8\,X(s)$$

$$Y(s)\left[s^2 + 4s + 8\right] = 8\,X(s)$$

$$\text{T.F.}\quad \frac{Y(s)}{X(s)} = \frac{8}{s^2 + 4s + 8}$$

Comparing this with standard T.F. of second order system $\frac{\omega_n^2}{s^2 + 2\xi\,\omega_n\,s + \omega_n^2}$

$$\therefore \quad \omega_n^2 = 8$$

$$\therefore \quad \omega_n = \mathbf{2.83\ rad/sec}$$

$$2\,\xi\,\omega_n = 4 \qquad \therefore \quad \xi = \mathbf{0.7067}$$

$$\therefore \quad \omega_d = \omega_n\sqrt{1-\xi^2} = 2.83\sqrt{1-(0.7067)^2} = \mathbf{2.002\ rad/sec}$$

$$\therefore \quad T_p = \text{Time for peak overshoot}$$

$$= \frac{\pi}{\omega_d} = \frac{\pi}{2.002} = \mathbf{1.57\ sec}$$

$$\%\ M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = e^{-\pi \times 0.706/\sqrt{1-(0.706)^2}\times 100}$$

$$= \mathbf{4.33\%}$$

$$T_s = \text{Settling time} = \frac{4}{\xi\,\omega_n} = \frac{4}{0.7067 \times 2.83} = \mathbf{2\ sec}$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d\,t + \theta)$$

where $$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) = 45^\circ = \frac{\pi}{4}\ \text{rad}$$

$$\therefore \quad c(t) = 1 - \frac{e^{-0.7067 \times 2.87\,t}}{\sqrt{1-(0.7067)^2}}\sin\left(2t + \frac{\pi}{4}\right)$$

$$c(t) = 1 - 1.41\,e^{-2t}\sin\left(2t + \frac{\pi}{4}\right)$$

$\omega_n = \mathbf{2.83\ rad/sec}$ $\qquad T_p = \mathbf{1.57\ sec}$

$\omega_d = \mathbf{42.002\ rad/sec}$ $\qquad \%\ M_p = \mathbf{4.33\%}$

$T_s = \mathbf{2\ sec}$ $\qquad \xi = \mathbf{0.7067}$

14.The open loop transfer function of a servo system with unity feedback is G(s) =10/(0.1s+1).Evaluate the static error constants of the system. Obtain the steady state error of the system. When subjected to an input given by the polynomial,r (t) =$a_0 + a_1 t + \frac{a_2}{2} t^2$?(April/May 2014) , (April/May 2015)(Nov/Dec 2017)

SOLUTION

To find static error constant

For unity feedback system, H(s) = 1.

$\therefore$ Loop transfer function, G(s) H(s) = G(s)

The static error constants are K_p, K_v and K_a.

Position error constant, $K_p = \underset{s\to 0}{Lt}\ G(s) = \underset{s\to 0}{Lt} \frac{10}{s(0.1s+1)} = \infty$

Velocity error constant, $K_v = \underset{s\to 0}{Lt}\ sG(s) = \underset{s\to 0}{Lt}\ s\frac{10}{s(0.1s+1)} = 10$

Acceleration error constant, $K_a = \underset{s\to 0}{Lt}\ s^2G(s) = \underset{s\to 0}{Lt}\ s^2\frac{10}{s(0.1s+1)} = 0$

To find steady state error

Method - I

Steady state error for non-standard input is obtained using generalized error series, given below.

The error signal, $e(t) = r(t)C_o + \dot{r}(t)C_1 + \ddot{r}(t)\frac{C_2}{2!} + ... + \overset{n.}{r}(t)\frac{C_n}{n!} + ...$

Given that, $r(t) = a_o + a_1 t + \frac{a_2}{2}t^2$

$$\therefore\ \dot{r}(t) = \frac{d}{dt}r(t) = \frac{d}{dt}\left(a_o + a_1 t + \frac{a_2}{2}t^2\right) = a_1 + a_2 t$$

$$\ddot{r}(t) = \frac{d^2}{dt^2}r(t) = \frac{d}{dt}\left(\frac{d}{dt}r(t)\right) = \frac{d}{dt}(a_1 + a_2 t) = a_2$$

$$\dddot{r}(t) = \frac{d^3}{dt^3}r(t) = \frac{d}{dt}\left(\frac{d^2}{dt^2}r(t)\right) = \frac{d}{dt}(a_2) = 0$$

Derivatives of r(t) is zero after 2nd derivative. Hence, let us evaluate three constants C_0, C_1 & C_2.

The generalized error constants are given by,

$$C_o = \underset{s\to 0}{Lt}\ F(s)\ ; \qquad C_1 = \underset{s\to 0}{Lt} \frac{d}{ds}F(s)\ ; \qquad C_2 = \underset{s\to 0}{Lt} \frac{d^2}{ds^2}F(s)$$

$$F(s) = \frac{1}{1+G(s)H(s)} = \frac{1}{1+G(s)} = \frac{1}{1+\frac{10}{s(0.1s+1)}} = \frac{s(0.1s+1)}{s(0.1s+1)+10} = \frac{0.1\,s^2+s}{0.1s^2+s+10}$$

$$C_o = \underset{s\to 0}{Lt}\ F(s) = \underset{s\to 0}{Lt}\frac{0.1s^2+s}{0.1s^2+s+10} = 0$$

$$= \underset{s\to 0}{\mathrm{Lt}}\left[\frac{\left(0.1s^2+s+10\right)(0.2s+1)-\left(0.1s^2+s\right)(0.2s+1)}{\left(0.1s^2+s+10\right)^2}\right] = \underset{s\to 0}{\mathrm{Lt}}\frac{2s+10}{\left(0.1s^2+s+10\right)^2} = \frac{10}{10^2} = 0.1$$

$$C_2 = \underset{s\to 0}{\mathrm{Lt}}\ \frac{d^2}{ds^2}F(s) = \underset{s\to 0}{\mathrm{Lt}}\frac{d}{ds}\left[\frac{d}{ds}F(s)\right] = \underset{s\to 0}{\mathrm{Lt}}\ \frac{d}{ds}\left[\frac{2s+10}{\left(0.1s^2+s+10\right)^2}\right]$$

$$= \underset{s\to 0}{\mathrm{Lt}}\ \frac{d}{ds}\left[\frac{\left(0.1s^2+s+10\right)^2\times 2-(2s+10)\times 2\left(0.1s^2+s+10\right)(0.2s+1)}{\left(0.1s^2+s+10\right)^4}\right]$$

$$\therefore C_2 = \frac{10^2\times 2-10\times 2\times 10\times 1}{10^4} = 0$$

Error signal, $e(t) = r(t)C_0 + \dot{r}(t)C_1 + \ddot{r}(t)\frac{C_2}{2!} = \dot{r}(t)C_1 + 0 + 0 = (a_1 + a_2 t)\ 0.1$

$\therefore$ Steady state error, $e_{ss} = \underset{t\to\infty}{\mathrm{Lt}}\ e(t) = \underset{t\to\infty}{\mathrm{Lt}}\left[\ (a_1 + a_2 t)\ 0.1\ \right] = \infty$

Method - II

The error signal in s-domain, $E(s) = \frac{R(s)}{1+G(s)H(s)}$

Given that, $r(t) = a_o + a_1 t + \frac{a_2}{2}t^2$; $\quad G(s) = \frac{10}{s(0.1s+1)}$; $\quad H(s) = 1$

On taking Laplace transform of $r(t)$ we get $R(s)$,

$$\therefore R(s) = \frac{a_0}{s} + \frac{a_1}{s^2} + \frac{a_2}{2}\frac{2!}{s^3} = \frac{a_0}{s} + \frac{a_1}{s^2} + \frac{a_2}{s^3}$$

$$\therefore E(s) = \frac{R(s)}{1+G(s)H(s)} = \frac{\frac{a_o}{s} + \frac{a_1}{s^2} + \frac{a_2}{s^3}}{1+\frac{10}{s\ (0.1s+1)}} = \frac{\frac{a_o}{s} + \frac{a_1}{s^2} + \frac{a_2}{s^3}}{\frac{s\ (0.1s+1)+10}{s\ (0.1s+1)}}$$

$$= \frac{a_0}{s}\left[\frac{s\ (0.1s+1)}{s\ (0.1s+1)+10}\right] + \frac{a_1}{s^2}\left[\frac{s\ (0.1s+1)}{s\ (0.1s+1)+10}\right] + \frac{a_2}{s^3}\left[\frac{s\ (0.1s+1)}{s\ (0.1s+1)+10}\right]$$

The steady state error e_{ss} can be obtained from final value theorem.

Steady state error, $e_{ss} = \underset{t\to\infty}{\mathrm{Lt}}\ e(t) = \underset{s\to 0}{\mathrm{Lt}}\ s\,E(s)$

$$\therefore e_{ss} = \underset{s\to 0}{\mathrm{Lt}}\ s\left\{\frac{a_o}{s}\left[\frac{s(0.1s+1)}{s(0.1s+1)+10}\right] + \frac{a_1}{s^2}\left[\frac{s(0.1s+1)}{s(0.1s+1)+10}\right] + \frac{a_2}{s^3}\left[\frac{s(0.1s+1)}{s(0.1s+1)+10}\right]\right\}$$

$$= \underset{s\to 0}{\mathrm{Lt}}\left\{\frac{a_o s(0.1s+1)}{s(0.1s+1)+10} + \frac{a_1(0.1s+1)}{s(0.1s+1)+10} + \frac{a_2(0.1s+1)}{s\left[s(0.1s+1)+10\right]}\right\} = 0 + \frac{a_1}{10} + \infty = \infty$$

Method - III

Error signal in s-domain, $E(s) = \frac{R(s)}{1+G(s)H(s)}$; $\therefore \frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$

Given that, $G(s) = \frac{10}{s(0.1s+1)}$ and $H(s) = 1$.

$$\therefore \frac{E(s)}{R(s)} = \frac{1}{1+\frac{10}{s(0.1s+1)}} = \frac{s(0.1s+1)}{s(0.1s+1)+10} = \frac{0.1s^2+s}{0.1s^2+s+10} = \frac{s+0.1s^2}{10+s+0.1s^2} = \frac{s}{10} - \frac{s^3}{1000} + ...$$

$$\therefore E(s) = \frac{s}{10}R(s) - \frac{s^3}{1000}R(s) + ...$$

On taking inverse Laplace transform,

$$e(t) = \frac{1}{10}\dot{r} - \frac{1}{1000}\dddot{r}(t) + ...$$

Given that, $r(t) = a_0 + a_1 t + \frac{a_2}{2}t^2$

$$\therefore \quad \dot{r} = \frac{d}{dt}r(t) = a_1 + a_2 t$$

$$\ddot{r}(t) = \frac{d}{dt}\dot{r}(t) = a_2$$

$$\dddot{r}(t) = \frac{d}{dt}\ddot{r}(t) = 0$$

$\therefore$ Error signal in time domain, $e(t) = \frac{1}{10}\dot{r}(t) = \frac{1}{10}(a_1 + a_2 t)$

Steady state error, $e_{ss} = \underset{t\to\infty}{Lt}\ e(t) = \underset{t\to\infty}{Lt}\ \frac{1}{10}(a_1 + a_2 t) = \infty$

RESULT

1. Position error constant, $K_p = \infty$
2. Velocity error constant, $K_v = 10$
3. Acceleration error constant, $K_a = 0$
4. When input, $r(t) = a_o + a_1 t + \frac{a_2 t^2}{2}$, Steady state error, $e_{ss} = \infty$

15.A unity feedback system has the forward transfer function G(s)=$\frac{k_1(2s+1)}{s(5s+1)(1+s)^2}$ when the input r(t)=1+6t, determine the minimum value of K_1so that the steady error is less then0.1?

(April/May 2018)

SOLUTION

Given that, input $r(t) = 1 + 6t$

On taking laplace transform of r(t) we get R(s).

$$\therefore R(s) = \mathcal{L}\{r(t)\} = \mathcal{L}\{1+6t\} = \frac{1}{s} + \frac{6}{s^2}$$

The error signal in s-domain E(s) is given by,

$$\therefore E(s)=\frac{R(s)}{1+G(s)H(s)}=\frac{\frac{1}{s}+\frac{6}{s^2}}{1+\frac{K_1(2s+1)}{s(5s+1)(1+s)^2}}=\frac{\frac{1}{s}+\frac{6}{s^2}}{\frac{s\,(5s+1)\,(1+s)^2+K_1(2s+1)}{s\,(5s+1)\,(1+s)^2}}$$

$\boxed{\text{Here } H(s)=1}$

$$=\frac{1}{s}\left[\frac{s\,(5s+1)\,(1+s)^2}{s\,(5s+1)\,(1+s)^2+K_1(2s+1)}\right]+\frac{6}{s^2}\left[\frac{s\,(5s+1)\,(1+s)^2}{s\,(5s+1)\,(1+s)^2+K_1(2s+1)}\right]$$

The steady state error e_{ss} can be obtained from final value theorem.

$$e_{ss}=\underset{t\to\infty}{Lt}\;e(t)=\underset{s\to0}{Lt}\;s\,E(s)$$

$$=\underset{s\to0}{Lt}\;s\left\{\frac{1}{s}\left[\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2+K_1(2s+1)}\right]+\frac{6}{s^2}\left[\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2+K_1(2s+1)}\right]\right\}$$

$$=\underset{s\to0}{Lt}\left\{\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2+K_1(2s+1)}+\frac{6(5s+1)(1+s)^2}{s(5s+1)(1+s)^2+K_1(2s+1)}\right\}=0+\frac{6}{K_1}=\frac{6}{K_1}$$

Given that, $e_{ss}<0.1$, $\quad\therefore 0.1=\frac{6}{K_1}\quad$ or $\quad K_1=\frac{6}{0.1}=60$

RESULT

For steady state error, $e_{ss}<0.1$, the value of K_1 should be greater than 60.

16.A unit feed back system is characterized by the open loop transfer function G(s)=$\frac{1}{s(0.5s+1)\,(0.25+10}$ determine the state error for unit – step, unit-ramp and unit acceleration inputs. Also determined the damping ratio and natural frequency of the detriment roots? (april-2004,dec-2008,may-2009)

The various static error coefficient are $K_p=\lim_{s\to0}$ G(S)H(S)

$= \lim_{s\to0}\frac{1}{S(0.5S+1)(0.2S+1)}$

=1

$K_a=\lim_{s\to0}S^2$G(S)H(S)

$=\lim_{s\to0}\frac{1}{(0.5S+1)(0.2S+1)}$ -5.5156 0.1 0.7 1 1

0 -0.5515 - 0.8187 -0.99

=1

$K_a=\lim_{s\to0}S^2$G(S)H(S)

0.1 -0.1484 0.1812 0

$=\lim_{s\to0}S^2\frac{1}{S(0.5S+1)(0.2S+1)}$=0

I. For unit step input $e_{ss}=\frac{1}{1+k_p}$=0

II. For unit ramp input $e_{ss}=\frac{1}{k_v}$ =0

III. For unit acceleuatione_{ss}=$\frac{1}{k_a}$ =α

For $\delta$and ω_n , the closed loop transfer function is

$\frac{C(s)}{R(s)}$= $\frac{G(s)}{1+G(s)+R(s)}$

$=\frac{\frac{1}{S(0.5S+1)(0.2S+1)}}{1+\frac{1}{S(0.5S+1)(0.2S+1)}}$

=$\frac{1}{S(0.5S+1)(0.2S+1)+1}$

$\frac{1}{0.1s^3+0.7s^2+s+1}$

The characteristics equation is

$0.1s^3+0.7s^2+s+1=0$

The determinant roots are complex conjugates with quadratic factor ($0.1s^2+0.1485s$ +0.1809) is dividing by 0.1 we get

$s^2+0.1485s+\omega_n s+1.809$

Comparing with $s^2+\delta\omega_n s+\omega_n^2$

$\omega_n^2=1.809$

$$\omega_n=1.3449\ rad/sec$$

$$2\zeta\omega_n=1.485$$

$$\zeta=\frac{1.485}{2\omega_n}$$

=0.552

17.A. unit feedback near treatment has G(s)= $\frac{\mathbf{10000}}{\mathbf{(1+s)(1+0.5s)(1+0.2s)}}$

The out put set point is 500^0 c what is the steady state temperature? (May-2007)

Solution;

G(s)= $\frac{10000}{(1+s)(1+0.5s)(1+0.2s)}$H(S) =1

K_p=$\lim_{s\to 0} G(s)H(s)=\frac{10000}{1}$

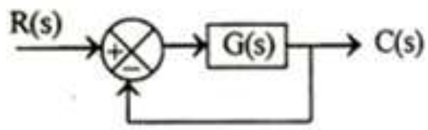
Set point is 500^0c is A=500

$\therefore\ ess$=$\frac{A}{1+k_p}=\frac{500}{1+10000}=0.04995$

∴steady state temperature= setport-ess

= 500-0.04995=499.95^0C .

18. A unity feedback control system has an open-loop transfer function G(s) =$\frac{5}{S(S+1)}$, find the unit time, percentage overshot, peak time and sifting time for a step in put of 10 unit?



G(s) =$\frac{5}{S(S+1)}$, H(S)=1

The closed loop transfer function of the system

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(S)H(s)}=\frac{G(s)}{1+G(S)} =\frac{\frac{5}{S(S+1)}}{1+\frac{5}{S(S+1)}}$$

$$=\frac{\frac{5}{S(S+1)}}{\frac{s(s+1)+5}{s(S+1)}}\,\omega n$$

Compare with second order transfer function =$\frac{5}{s^2+s+5}$

$$\frac{C(s)}{R(s)}=\frac{\omega_n{}^2}{s^2+2\zeta\omega_n s+\omega_n{}^2} =\frac{5}{s^2+s+5}$$

$\omega_n{}^2$=5 ,ω_n =$\sqrt{5}$ =2.236 rad /sec

w_d=$\omega_n\sqrt{1-\zeta^2}$

=2.2236 *$\sqrt{1-(0.223)^2}$ =2.124 rad/ sec

$\Phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$ =$\tan^{-1}\frac{\sqrt{1-(0.223)^2}}{0.223}$ =1.346 rad/sec

Rise time T_r =$\frac{\pi-\Phi}{w_d}$ =$\frac{3.141-1.346}{2.124}$=0.845 s

M_p =$e^{\frac{-\pi s}{\sqrt{1-\zeta^2}}}$ *100

=$e^{\frac{-0.223s}{\sqrt{1-(0.223)^2}}}$ *100

=0.478 *100

=47.8%

Per unit peak overshoot for a unit step input =0.478

For an input of 10 units , the peak overshoot is

0.478*10=4.78

Peak time T_p=$\frac{\pi}{w_d}$=$\frac{\pi}{2.124}$=1.479 s

Time constant T =$\frac{1}{\zeta\omega_n}$=$\frac{1}{0.223*2.236}$ =2s

For 5% error the T_s =3 T =3*2=6 s

For 2% error the T_s =4 T =4*2=8 s

19) Derive the time response of a typical under damped second order system for a unit step input. (Nov/Dec 2014)

Derive the expression for second order system for under damped case and when the input is unit step.

(Nov/Dec 2017)

The standard form of closed loop transfer function of second order system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For underdamped system, $0 < \zeta < 1$ and roots of the denominator (characteristic equation) are complex conjugate.

The roots of the denominator are, $s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$

Since $\zeta < 1$, ζ^2 is also less then 1, and so $1-\zeta^2$ is always positive.

$$\therefore\ s = -\zeta\omega_n \pm \omega_n\sqrt{(-1)(1-\zeta^2)} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

The damped frequency of oscillation, $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$\therefore s = -\zeta\omega_n \pm j\omega_d$$

The response in s-domain, $C(s) = R(s)\,\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

For unit step input, $r(t) = 1$ and $R(s) = 1/s$.

$$\therefore C(s) = \frac{\omega_n^2}{s\,(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

By partial fraction expansion, $C(s) = \dfrac{\omega_n^2}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)} = \dfrac{A}{s} + \dfrac{Bs + C}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ .....(3.25)

A is obtained by multiplying C(s) by s and letting s = 0.

$$\therefore\ A = s \times C(s)\big|_{s=0} = s \times \left.\frac{\omega_n^2}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}\right|_{s=0} = \frac{\omega_n^2}{\omega_n^2} = 1$$

To solve for B and C, cross multiply equation (3.25) and equate like power of s.

On cross multiplication equation (3.25) after substituting A = 1, we get,

$$\omega_n^2 = s^2 + 2\zeta\omega_n s + \omega_n^2 + (Bs + C)s$$
$$\omega_n^2 = s^2 + 2\zeta\omega_n s + \omega_n^2 + Bs^2 + Cs$$

Equating coefficients of s^2 we get, $0 = 1 + B$ $\quad \therefore B = -1$

Equating coefficient of s we get, $0 = 2\zeta\omega_n + C$ $\quad \therefore C = -2\zeta\omega_n$

$$\therefore\ C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad .....(3.26)$$

Let us add and subtract $\zeta^2\omega_n^2$ to the denominator of second term in the equation (3.26).

$$\therefore\ C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2 + \zeta^2\omega_n^2 - \zeta^2\omega_n^2} = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s^2 + 2\zeta\omega_n s + \zeta^2\omega_n^2) + (\omega_n^2 - \zeta^2\omega_n^2)}$$

$$= \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)} = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} \qquad \boxed{\omega_d = \omega_n\sqrt{1 - \zeta^2}}$$

$$= \frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} \quad .....(3.27)$$

Let us multiply and divide by ω_d in the third term of the equation (3.27).

$$\therefore\ C(s) = \frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{\omega_d}\,\frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

$$\boxed{\mathcal{L}\{1\} = \frac{1}{s}}$$

$$\boxed{\mathcal{L}\{e^{-at}\sin\omega t\} = \frac{\omega^2}{(s + a)^2 + \omega^2}}$$

$$\boxed{\mathcal{L}\{e^{-at}\cos\omega t\} = \frac{s + a}{(s + a)^2 + \omega^2}}$$

The response in time domain is given by,

$$c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{\omega_d}\,\frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2}\right\}$$

$$= 1 - e^{-\zeta\omega_n t}\cos\omega_d t - \frac{\zeta\omega_n}{\omega_d}\,e^{-\zeta\omega_n t}\sin\omega_d t = 1 - e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta\omega_n}{\omega_n\sqrt{1 - \zeta^2}}\sin\omega_d t\right) \qquad \boxed{\omega_d = \omega_n\sqrt{1 - \zeta^2}}$$

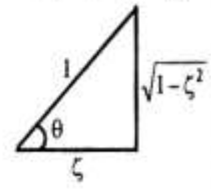
$$= 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}}\left(\sqrt{1 - \zeta^2}\,\cos\omega_d t + \zeta\,\sin\omega_d t\right) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}}\left(\sin\omega_d t \times \zeta + \cos\omega_d t \times \sqrt{1 - \zeta^2}\right)$$

Let us express c(t) in a standard form as shown below.

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}(\sin\omega_d t \times \cos\theta + \cos\omega_d t \times \sin\theta)$$

$$= 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta) \qquad ....(3.28)$$

$$\text{where, } \left(\theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

> ***Note :** On constructing right angle triangle with ζ and $\sqrt{1-\zeta^2}$, we get*
>
> $\sin\theta = \sqrt{1-\zeta^2}$
>
> $\cos\theta = \zeta$
>
> $\tan\theta = \frac{\sqrt{1-\zeta^2}}{\zeta}$
>
> 
>

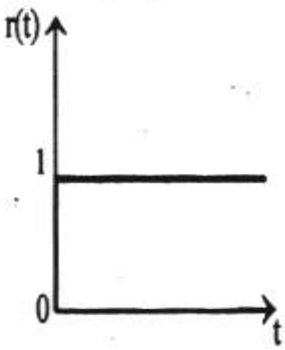
The equation (3.28) is the response of under damped closed loop second order system for unit step input. For step input of step value, A, the equation (3.28) should be multiplied by A.

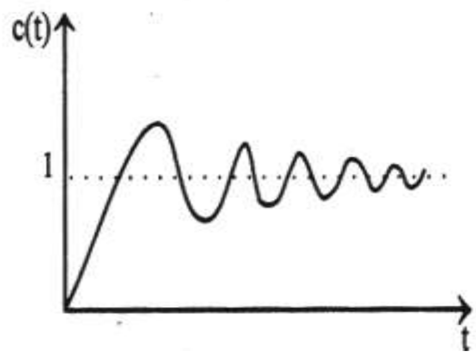
$\therefore$ For closed loop under damped second order system,

$$\text{Unit step response} = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)\ ;\ \theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$\text{Step response} = A\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)\ ;\ \theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right]$$

Using equation (3.28) the response of underdamped second order system for unit step input is sketched and observed that the response oscillates before settling to a final value. The oscillations depends on the value of damping ratio.





20. Write step by step rules for constructing root locus? (April/May 2011)

1. The root locus is symmetrical about the real axis. The first step is to locate the open loop poles and zeros on the complex plane.

2. Let m and n be the number of zeros and poles respectively.

   Number of root locus = n or m whichever is greater.

   Each branch starts from an open loop pole corresponding to K = 0 and terminates either on finite open loop zero or infinite open loop zero corresponding to $K = \infty$.

   If $m > n$, then there will be poles at infinity.

   $n > m$, there will be zeros at infinity.

3. Any point on real axis having the sum of open loop poles and zeros to their right is odd, then those points are the part of root locus.

4. The root locus branches that tend to infinity take their path along the straight line asymptotes which make angle with the real axis given by,

$$\phi_A = \pm \frac{180\,(2\theta + 1)}{n - m}, \quad \text{where } \theta = 0, 1, 2, 3, \ldots..(n - m - 1)$$

5. The asymptotes intersects the real axis at a point called centroid which is given as,

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{\text{Number of poles} - \text{Number of zeros}}$$

6. The breakaway and break in points are determined on the root locus by equating,

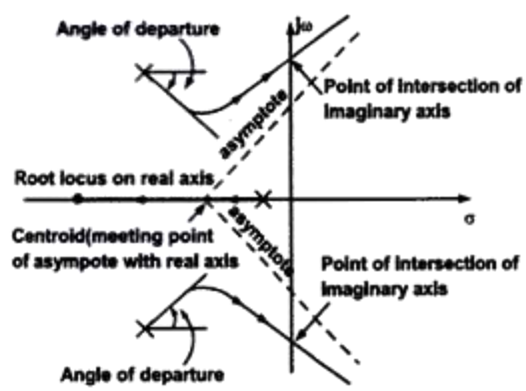
$$\frac{dK}{ds} = 0$$

PROBLEM:21

7. If complex poles are present, we have angle of departure. If complex zeros are present we have angle of arrival.

   a) The angle of departure of the complex open loop pole is given by, $\phi_p = 180 - \phi$

   where ϕ is the net angle contribution at the pole under consideration of all other open loop poles and zeros.



b) The angle of arrival at the complex open loop zero is given by, $\phi_z = 180 + \phi$

where ϕ is the net angle contribution at the zero under consideration of all other open poles and zeros.

8. The intersection of root locus branches with the imaginary axis can be determined by the use of Routh-Hurwitz criterion. It can also be determined by replacing s by $j\omega$ in the characteristic equation. The value of ω is determined by equating imaginary part to zero and the value of K is determined by equating the real to zero. The value of K is the intersection point of root locus on the imaginary axis.

9. The open loop gain K at any point $s = s_a$ on the root locus is given by,

$$K = \frac{\text{Product of phasor lengths from } s_a \text{ to open loop poles}}{\text{Product of phasor lengths from } s_a \text{ to open loop zeros}}$$

22) **Sketch the root locus of the system whose open loop transfer function is,**

$G(s) = \dfrac{K}{s(s+2)(s+4)}$. Find the value of K such that the damping ratio of the closed loop system is 0.5. **(April/May 2012) (April/May 2014) (April/May 2015)(Nov/Dec-16)**

Solution

Step 1: Locate poles and zeros on the complex plane

The open loop poles are the roots of the equation.

$\therefore$ The poles are, s = 0, –2, –4 and they are located on the complex plane.

There are no zeros present in the system.

Step 2: Number of root locus branches, starting and ending points

Here n = 3 and m = 0. $\therefore$ Number of root locus branches = 3. The root locus branches start from s = 0, –2 and –4 and end at infinity.

Step 3: Determination of the root locus on real axis

There are three poles and no zeros on the real axis. A test point on real axis is chosen between s = 0 and s = – 2. To the right of this point it is found that the total number of real poles and zeros is one, which is an odd number. Hence the real axis between s = 0 and s = – 2 will be a part of the root locus.

Again a test point is chosen on real axis between s = – 2 and s = – 4. To the right of this point, the total number of real poles and zeros is two which is an even number. Hence it will not be a part of the root locus.

The same procedure is repeated after each pole on the real axis. It will be found that the segment between s = 0 to s = – 2 and s = – 4 to – ∞ will be part of the root locus.

Step 4: Determination of angle of asymptotes

Since there are three poles and no zeros, all the three root locus branches end at zeros at infinity. The path taken by the root locus are given by asymptotes and their angles are given by,

$$\text{Angles of asymptotes } \phi_A = \frac{\pm 180°(2\theta+1)}{n-m} \quad \text{where } \theta = 0, 1 \ldots (3-0-1)$$

$$\text{If } \theta = 0, \quad \text{angles} = \pm\frac{180°}{3} = \pm 60°$$

$$\text{If } \theta = 1, \quad \text{angles} = \pm\frac{180° \times 3}{3} = \pm 180°$$

$$\text{If } \theta = 2, \quad \text{angles} = \pm\frac{180° \times 5}{3} = \pm\, 300° = 60°$$

Step 5: Determination of centroid

The point of intersection of asymptotes on real axis is given by,

$$\text{Centroid} = \frac{\text{sum of poles} - \text{sum of zeros}}{n-m}$$

$$= \frac{0-2-4-0}{3} = -2$$

The centroid is marked on real axis and from centroid the angles of asymptotes are marked as shown in Figure (a).

Step 6: Determination of breakaway and breakin points

The closed loop transfer function,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$\therefore \qquad \frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+2)(s+4)}}{1+\frac{K}{s(s+2)(s+4)}} = \frac{K}{s(s+2)(s+4)+K}$$

The characteristic equation is given by,

$$s(s+2)(s+4)+K = 0$$

$$s(s^2+6s+8)+K = 0$$

$$s^3+6s^2+8s+K = 0$$

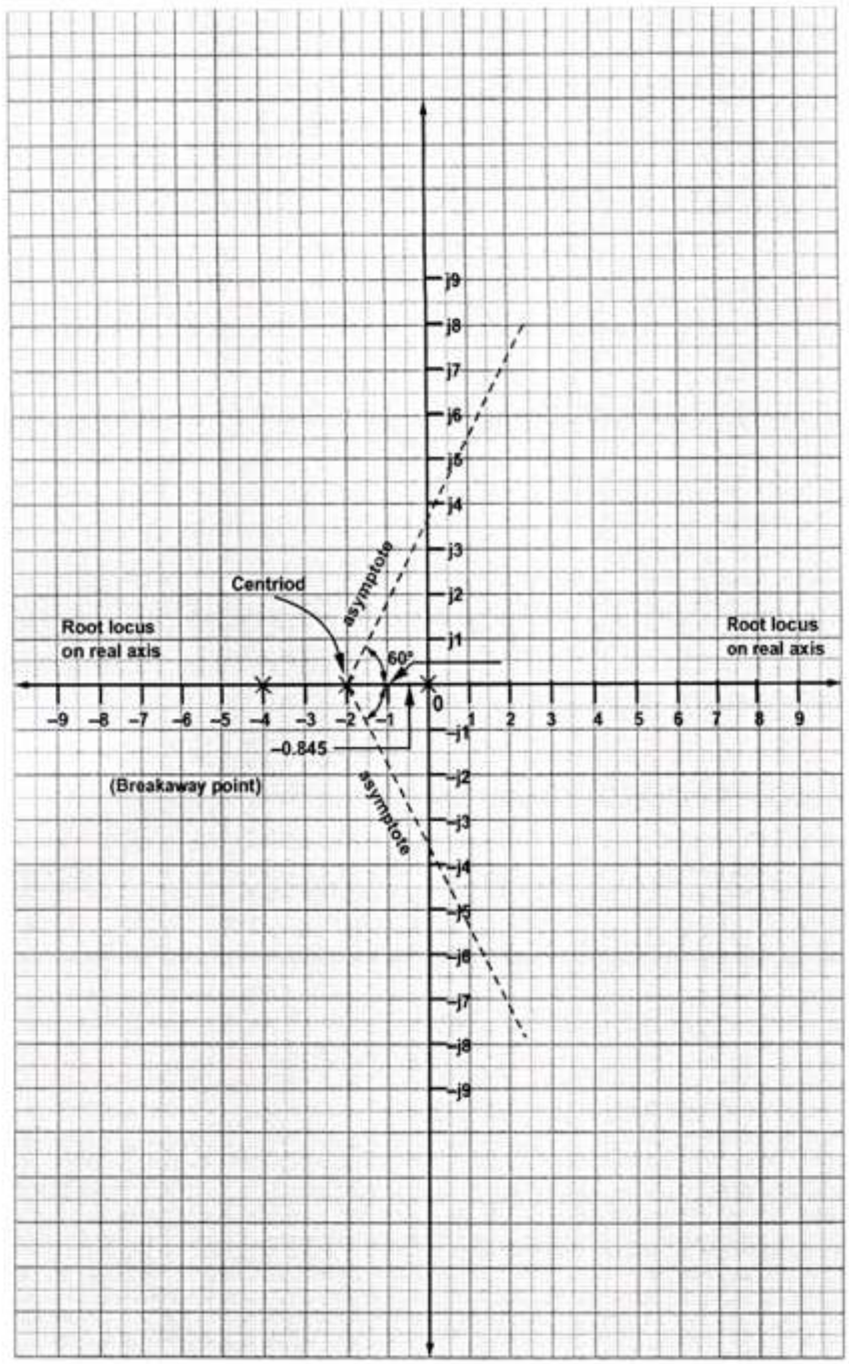
$$\frac{dK}{ds} = -(3s^2 + 12s + 8)$$

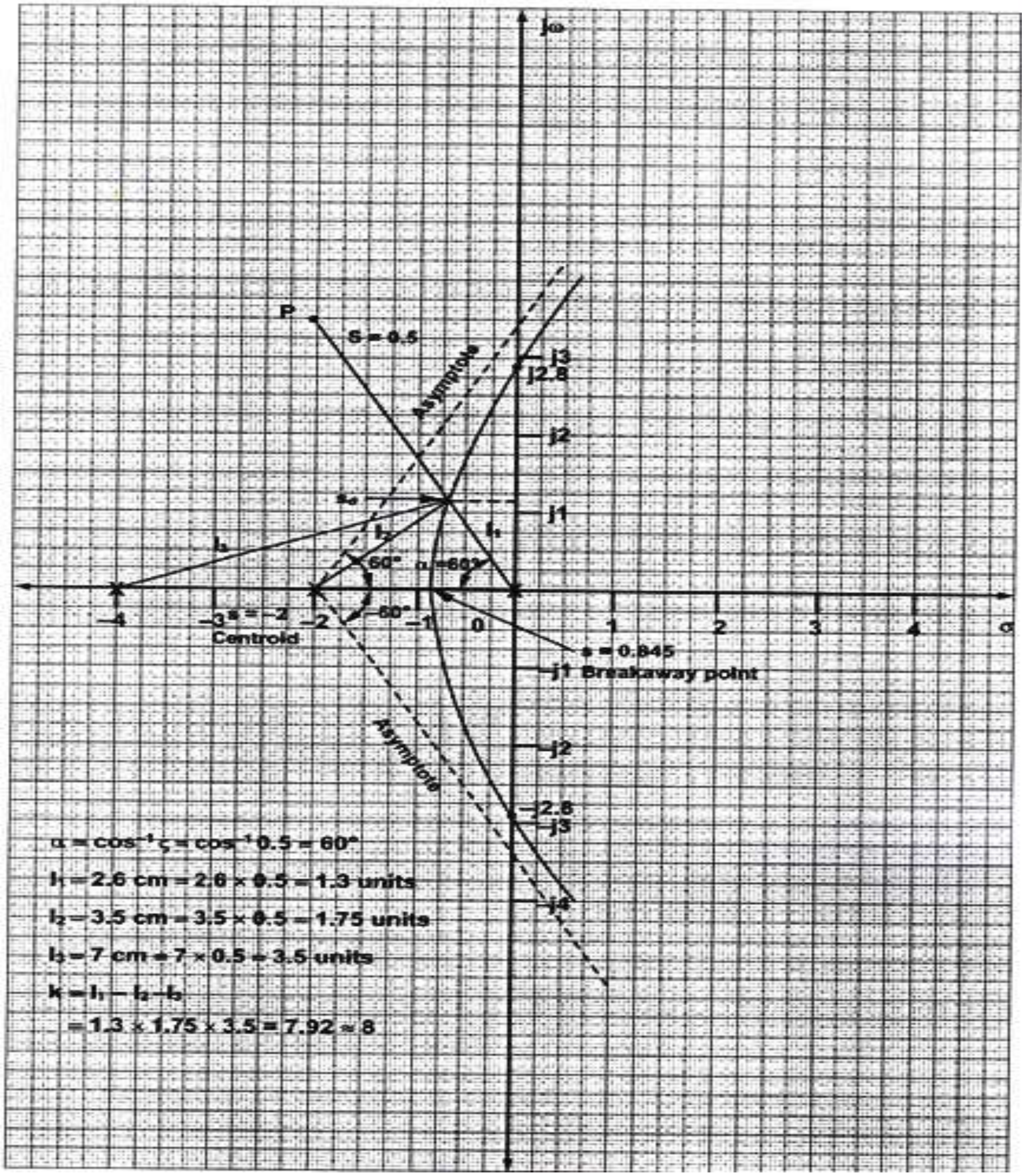
Put $\frac{dK}{ds} = 0$

$\therefore\ -(3s^2 + 12s + 8) = 0$

$(3s^2 + 12s + 8) = 0$

$$s = \frac{-12 \pm \sqrt{12^2 - 4 \times 3 \times 8}}{2 \times 3} = -0.845 \text{ or } -3.154$$





(b) Root locus sketch of $G(s) = \dfrac{K}{s(s+2)(s+4)}$

Check that K is positive for the given value of s.

When s = − 0.845, the value of K = 3.08

Since K is positive and real, this point is the actual breakaway point.

When s = − 3.155, the value of K = − 3.08

Since K is negative for s = − 3.155, this is not the actual breakaway point. The breakaway point is marked on the negative real axis as shown in Figure(b).

Step 7: Determination of angle of departure

Since no complex pole or zero is present, the necessity to determine the angle of departure or arrival does not arise.

Step 8 : Determination of crossing point of imaginary axis

The characteristic equation is given by,

$$s^3 + 6s^2 + 8s + K = 0$$

Substituting $s = j\omega$ in above equation we will get,

$$(j\omega)^3 + 6(j\omega)^2 + 8(j\omega) + K = 0$$

$$-j\omega^3 - 6\omega^2 + j8\omega + K = 0$$

Separating real and imaginary part and equating them to zero,

$$-j\omega^3 + j8\omega = 0$$

$$-j\omega^3 = -j8\omega$$

$$\omega^2 = 8$$

$$\omega = \pm\sqrt{8} = \pm 2.8$$

$$-6\omega^2 + K = 0$$

$$K = 6\omega^2 = 6\times 8 = 48$$

The crossing point of root locus on the imaginary axis is $\pm j2.8$. The value of K corresponding to this point is 48, which is the limiting value of K for stability of the system.

The complete root locus sketch is shown in Figure (b). The root locus has three branches. One branch starts at the pole $s = -4$ and travels away from the origin along the negative real axis to meet the zero at infinity. The other two root locus branches starts at $s = 0$ and $s = -2$ and travel through negative real axis, break away from real axis at $s = -0.845$, then crosses the imaginary axis at $s = \pm j2.8$ and travels parallel to asymptotes to meet the zeros at infinity.

Determination of K corresponding to $\zeta = 0.5$

Given that $\zeta = 0.5$ $(\cos\theta = \zeta)$

Let $\theta = \cos^{-1}\zeta = \cos^{-1} 0.5 = 60°$

A damping line is drawn for the specified damping which makes an angle 60° with the negative real axis as shown in Figure 2.17. The meeting point of the line and root locus gives the dominant pole, s_d.

The value of K corresponding to the point,

$$s = s_d = \frac{\text{Product of length of vector from all poles to the point, } s = s_d}{\text{Product of length of vectors from all zeros to the point, } s = s_d} = 8$$

The poles are s = 0, –4, –2 + j4 and –2 – j4

The zeros are located at infinity. The poles are shown in Figure (a).

Step 2 : Number of root locus branches, starting and ending points

Here n = 4 and m = 0. Number of root locus branches = 4. The root locus branches start from s = 0, –4, –2 + j4 and –2 – j4 and end at infinity.

Step 3 : Determination of root locus on real axis

There are two poles and zeros on the real axis. A test point on real axis is chosen between s = 0 and s = –4. It will be found that to the right of this point, the total number of real poles is an odd number. Hence it will be a part of the root locus. The same procedure is repeated after every pole on the real axis.

Step 4 : Determination of angles of asymptotes

Since there are four poles and no zeros, all the four root locus branches ends at zeros at infinity, the number of asymptotes required are,

$$n - m = 4 - 0 = 4$$

where, n - number of poles

m - number of zeros

Angle of asymptotes $\phi_A = \pm\dfrac{180°(2\theta+1)}{n-m}$ where $\theta = 0, 1, 2, 3$

If $\theta = 0$, angles $= \dfrac{\pm 180°}{4} = \pm 45°$

If $\theta = 1$, angles $= \dfrac{\pm 180° \times 3}{4} = \pm 135°$

If $\theta = 2$, angles $= \dfrac{\pm 180° \times 5}{4} = \pm\ 225° = \pm 135°$

If $\theta = 3$, angles $= \dfrac{\pm 180° \times 7}{4} = \pm 315° = \pm 45°$

Step 5 : Determination of centroid

The point of intersection of asymptotes on real axis is given by,

$$\text{Centroid} = \frac{\text{sum of poles} - \text{sum of zeros}}{n - m}$$

$$= \frac{0 - 4 - 2 + j4 - 2 - j4 - 0}{4 - 0} = \frac{-8}{4} = -2$$

The asymptotes are marked on real axis as shown in Figure (a).

Step 6 : Determination of breakaway and breakin points

The closed loop transfer function,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

$$\therefore\ \frac{C(s)}{R(s)} = \frac{\dfrac{K}{s(s+4)\,(s^2+4s+20)}}{1 + \dfrac{K}{s(s+4)\,(s^2+4s+20)}}$$

$$= \frac{K}{s(s+4)(s^2+4s+20)+K}$$

The characteristic equation is, $s(s+4)(s^2+4s+20)+K=0$

where K is given as,

$\therefore\ K = -s(s+4)(s^2+4s+20) = -(s^2+4s)(s^2+4s+20)$

$K = -(s^4+8s^3+36s^2+80s)$

On differentiating the equation of K with respect to s and equating it to zero we get,

$$\frac{dK}{ds} = -(4s^3+24s^2+72s+80)$$

$\therefore\ -(4s^3+24s^2+72s+80) = 0$

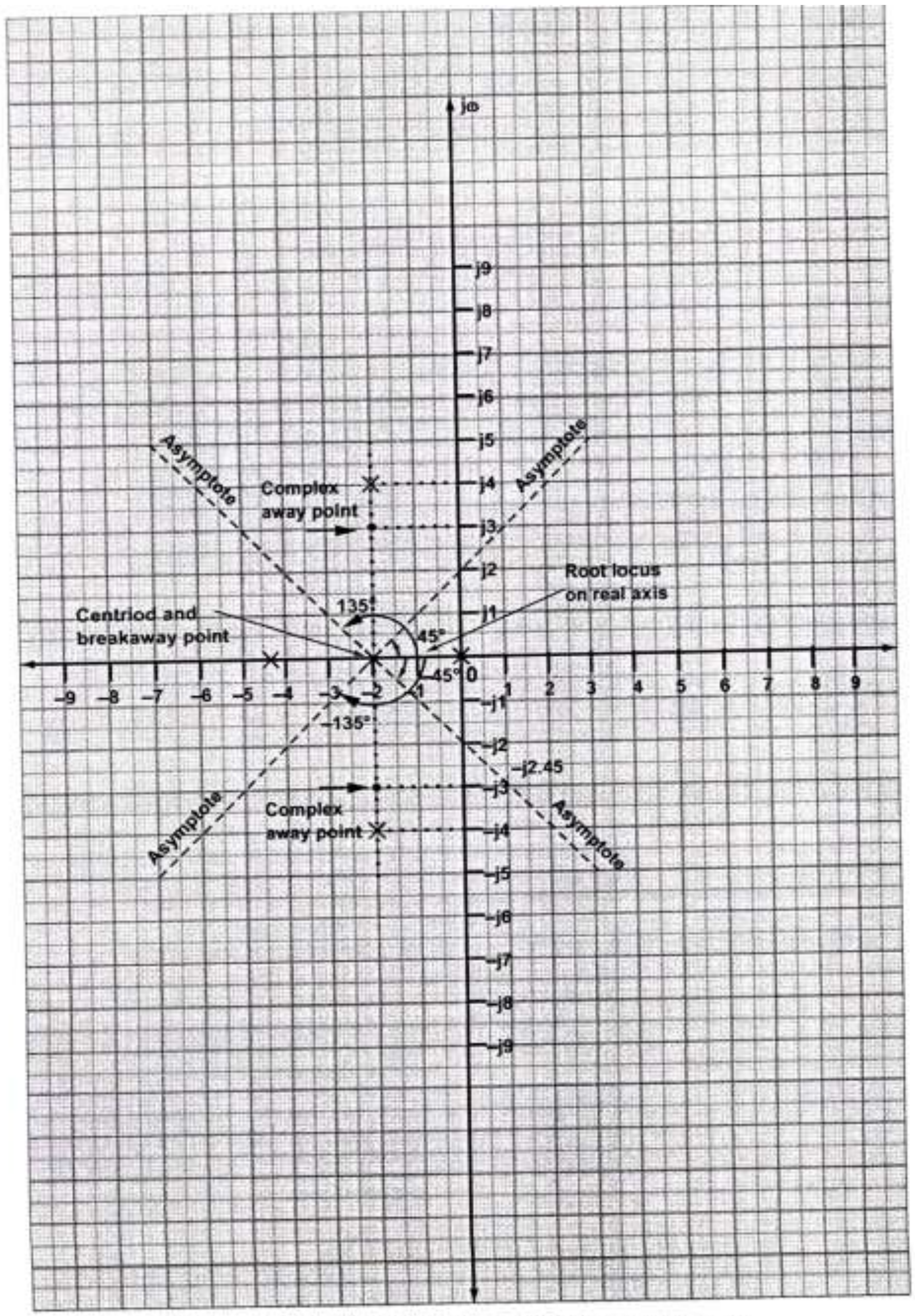


Figure (a) Figure showing the asymptotes, root locus on real axis and location of poles, centroid and breakaway points for $G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+20)}$

$$4s^3+24s^2+72s+80=0$$

On dividing by 4 we get,

$$s^3+6s^2+18s+20=0$$

One root of the equation is at $s=-2$.

Other roots are determined by Lin's method

The last two terms of the polynomial are chosen to be the first divisor.

1. I divisor = $18s + 20$

$$= s+\frac{20}{18} = s + 1.11$$

$$\begin{array}{r|l} & s^2 + 4.89s + 12.57 \\ \hline s + 1.11 & s^3 + 6s^2 + 18s + 20 \\ & s^3 + 1.11s^2 \\ \hline & \quad 4.89s^2 + 18s \\ & \quad 4.89s^2 + 5.43s \\ \hline \text{second divisor} \rightarrow & \qquad 12.57s + 20 \\ & \qquad 12.57s + 13.95 \\ \hline & \qquad\qquad 6.05 \\ \hline \end{array}$$

2. II divisor = $12.57s + 20$

$$= s+\frac{20}{12.57} = s + 1.59$$

$$\begin{array}{r|l} & s^2 + 4.41s + 11 \\ \hline s + 1.59 & s^3 + 6s^2 + 18s + 20 \\ & s^3 + 1.59s^2 \\ \hline & \quad 4.41s^2 + 18s \\ & \quad 4.41s^2 + 7s \\ \hline \text{third divisor} \rightarrow & \qquad 11s + 20 \\ & \qquad 11s + 17.49 \\ \hline & \qquad\qquad 2.51 \\ \hline \end{array}$$

3. III divisor = $11s + 20$

$$= s+\frac{20}{11} = s + 1.82$$

$$\begin{array}{r|l} & s^2 + 4.18s + 10.4 \\ \hline s + 1.82 & s^3 + 6s^2 + 18s + 20 \\ & s^3 + 1.82s^2 \\ \hline & \quad 4.18s^2 + 18s + 20 \\ & \quad 4.18s^2 + 7.6s \\ \hline & \qquad 10.4s + 20 \\ & \qquad 10.4s + 18.9 \\ \hline & \qquad\qquad 1.1 \\ \hline \end{array}$$

$$\begin{array}{r|l} & s^2 + 4s + 10 \\ \hline s+2 & s^3 + 6s^2 + 18s + 20 \\ & s^3 + 2s^2 \\ \hline & \quad 4s^2 + 18s \\ & \quad 4s^2 + 8s \\ \hline & \qquad 10s + 20 \\ & \qquad 10s + 20 \\ \hline & \qquad\quad 0 \\ \hline \end{array}$$

The polynomial $s^3 + 6s^2 + 18s + 20 = 0$ can be expressed as,

$$(s + 2)\,(s^2 + 4s + 10) = 0$$

The roots of the quadratic equation $s^2 + 4s + 10$ is given by,

$$s = \frac{-4 \pm \sqrt{4^2 - 4\times 10}}{2} = -2 \pm j2.45$$

The value of K is checked to be positive.

When $s = -2$

$$K = -[-64] = 64$$

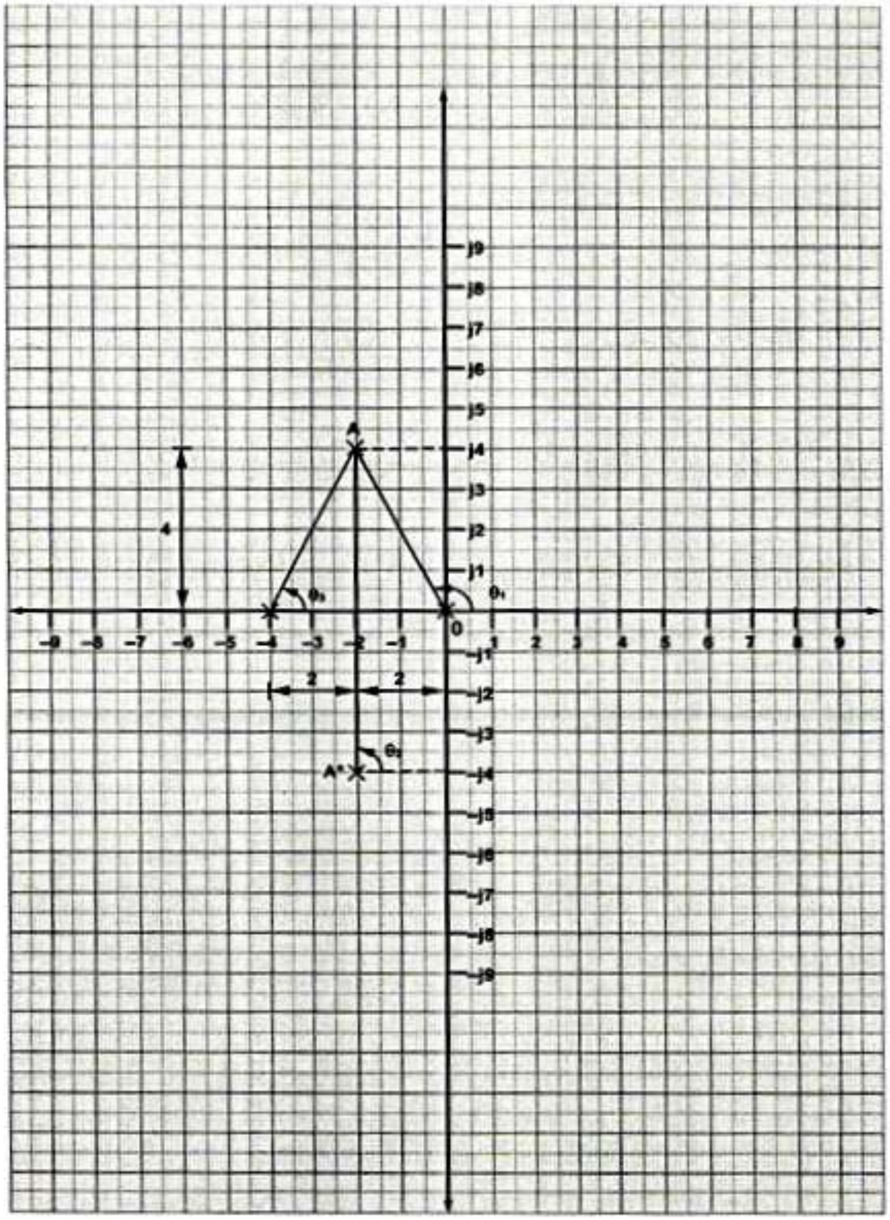


Figure (b)

When $s = -2 + j2.45$

$K = -[-100] = 100$

When $s = -2 - j2.45$,

$K = -[-100] = 100$

For all the roots, the value of K is positive and real. Hence all the three roots are actual breakaway points and are indicated in Figure (a).

Step 7 : Determination of angle of departure

The angle of departure is determined by drawing vectors from all other poles to the complex pole A as shown in Figure (b).

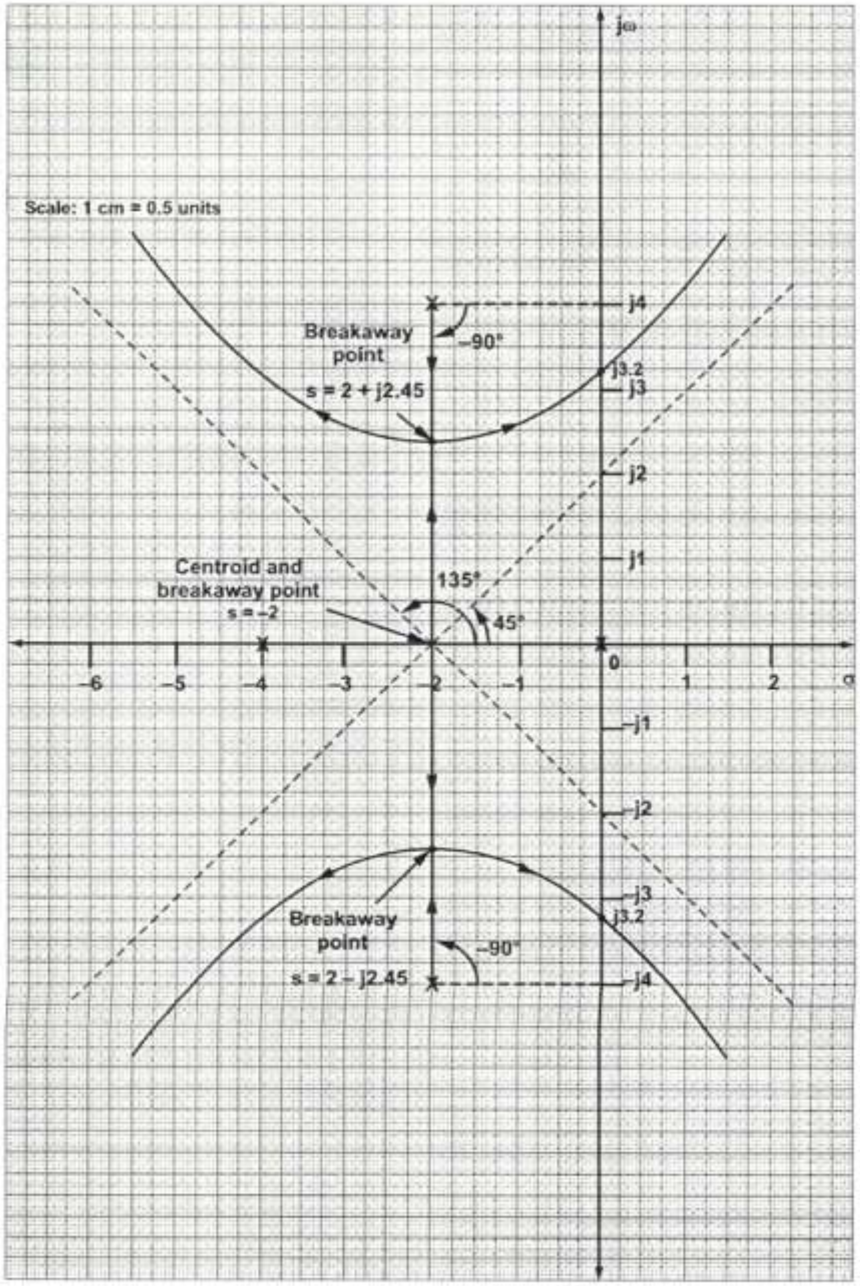


Figure (c) Root locus for $G(s)H(s) = \dfrac{K}{s(s+4)(s^2+4s+20)}$

The angles are,

$$\theta_1 = 180° - \tan^{-1}\frac{4}{2} = 117°$$

$$\theta_2 = 90°$$

$$\theta_3 = \tan^{-1}\frac{4}{2} = 63°$$

Angle of departure from complex pole A is given as,

$$\phi_p = 180° - (\theta_1 + \theta_2 + \theta_3)$$

$$= 180° - (117° + 90° + 63°)$$

$= -90°$

Thus the angle of departure from complex pole A = +90°.

The angles are marked at the complex poles.

Step 8 : Determination of crossing point on imaginary axis

The characteristic equation is given by,

$$s^4 + 8s^3 + 36s^2 + 80s + K = 0$$

Substituting $s = j\omega$ in the above equation we get,

$$(j\omega)^4 + 8(j\omega)^3 + 36(j\omega)^2 + 80(j\omega) + K = 0$$

$$\omega^4 - j8\omega^3 - 36\omega^2 + j80\omega + K = 0$$

Separating real and imaginary part and equating to zero,

$$-j8\omega^3 + j80\omega = 0 \qquad \omega^4 - 36\omega^2 + K = 0$$

$$-j8\omega^3 = -j80\omega \qquad K = -\omega^4 + 36\omega^2$$

$$\omega^2 = 10 \qquad \text{Put } \omega^2 = 10$$

$$\omega = \pm\sqrt{10} = \pm 3.2 \qquad \therefore\ K = -(10)^2 + 36\times 10 = 260$$

The crossing point of root locus on imaginary axis is $\pm j3.2$. The value of K at this crossing point is K = 260, which is the limiting value of K for stability of the system. The complete root locus is sketched as shown in Figure (c).

23. Sketch the Root locus for

$$G(s).H(s) = \frac{K(s+2)(s+3)}{(s+1)(s-1)}$$

(Nov/Dec 2012),(May/June 2013)

Solution

Step 1: Locate poles and zeros on the complex plane

The open loop poles are s = 1 and s = –1 and the zero is at s = –2, s = –3

Step 2: Number of root locus branches, starting and ending points

Here m = 2

The number of root locus branches = 2

The root locus branches start from s = 1 and –1 and end at s = –2 and –3

Step 3 and Step 4:

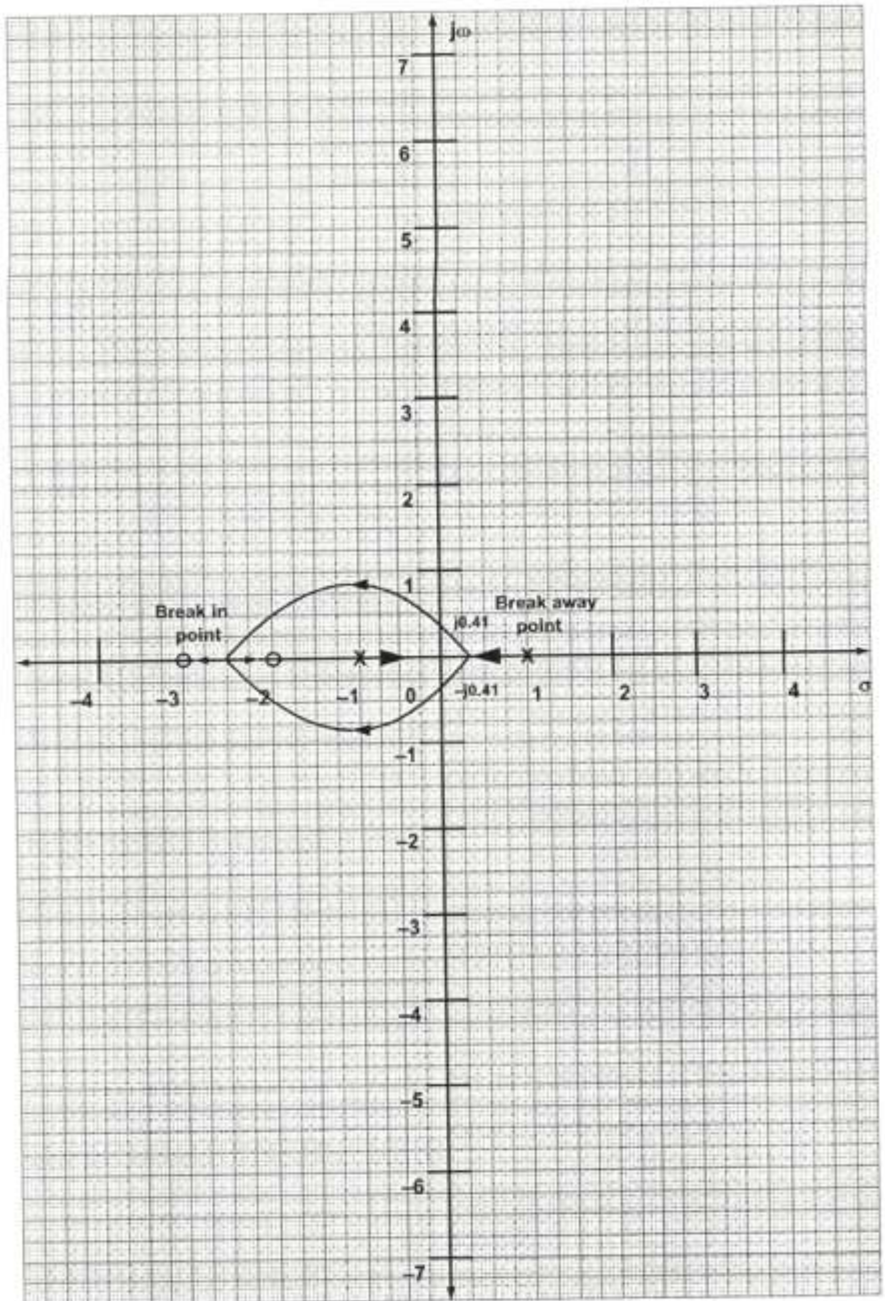
There is no asymptotes and centroid since the root locus ends at finite zeros.

Step 5: Determination of breakaway and breakin points

Breakaway point is obtained by using the condition

$$\frac{dK}{ds} = 0$$

$$1 + G(s).H(s) = 0$$



Root locus for $G(s).H(s) = \frac{K(s+2)(s+3)}{(s+1)(s-1)}$

$$1+\frac{K(s+2)(s+3)}{(s+1)(s-1)} = 0$$

$$\frac{dK}{ds} = \frac{(s^2+5s+6)(2s)-(s^2-1)\,(2s+5)}{[s^2+5s+6]^2}$$

$$s^2+2s-1=0$$

Therefore , $\sigma_b = 0.414, -2.414$

Step 6 : Determination of crossing point of imaginary axis

$$1+G(s).H(s)=0$$

$$1+\frac{K(s+2)(s+3)}{(s+1)(s-1)}=0$$

$$(s+1)(s-1)+K(s+2)(s+3)=0$$

$$(K+1)\,s^2+(5K-1)s+(6K-1)=0$$

Applying Routh's criterion to the above equation

s^2	$K+1$	$6K-1$
s^1	$5K-1$	0
s^0	$6K-1$	

For stability, $5K-1>0$

$$K>0.2$$

The Auxialry equation is $1.2s^2+0.2=0$

Therefore, $s^2=\frac{-2}{12}$

$$s=\pm j0.41$$

These are the crossing points in the imaginary axis. It is shown in Figure.

24. Sketch the root locus for the system with open-loop transfer function.

$$G(s)\,.\,H(s)\;=\frac{K}{s(s+3)(s^2+2s+2)}$$

(Nov/Dec 2011)

Solution

Step 1: Locate poles and zeros on the complex plane

The open loop poles are at $s=0$, $s=-3$, $s=-1+j1$, $s=-1-j1$

Step 2: Number of root locus branches, starting and ending points

Here $n=4$ and $m=0$

No.of root locus branches $=4$

The four root locus branches start at the open loop poles $0, -3, -1+j1$ and end at infinity.

Step 3: Determination of the root locus on real axis

No.of Asymptotes $=n-m=4$

The root locus from -3 to 0 on the real axis

Step 4: Determination of angle of asymptotes

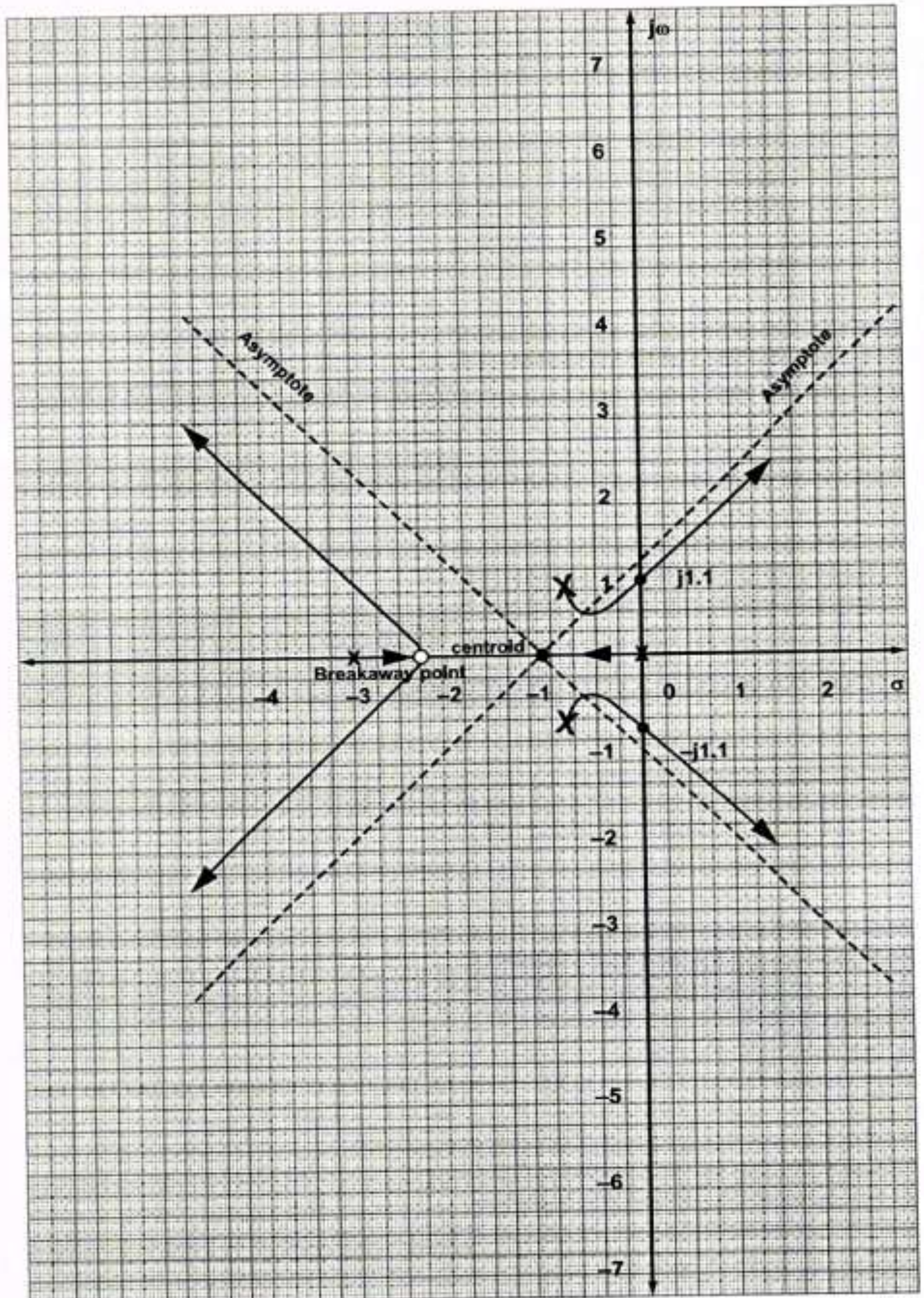
Asymptotic angle $\phi_A=\frac{\pm180(2\theta+1)}{n-m}$, $\theta=0,1,......$

$\theta=0$, angles $=\pm\frac{180^\circ}{4}=45^\circ$

$\theta=1$, angles $=\pm\frac{180^\circ\times3}{4}=135^\circ$

$\theta=2$, angles $=\pm\frac{180^\circ\times5}{4}=225^\circ$

$\theta=3$, angles $=\pm\frac{180^\circ\times7}{4}=315^\circ$



Root locus for $G(s)H(s) = \frac{K}{s(s+3)(s^2+2s+2)}$

Step 5 : Determination of centroid

$$\sigma_c = \frac{\sum P - \sum z}{n-m}$$

$$= \frac{(0-3-1+j1-1-j1)-0}{4} \quad = -1.25$$

Step 6: Determination of the breakaway points

The characteristic equation is

$$1+G(s).H(s)=0$$

$$1+\frac{K}{s(s+3)(s^2+2s+2)}=0$$

$$K=-s(s+3)(s^2+2s+2)$$

$$\frac{dK}{ds}=-4\,[s^3+3.75s^2+4s+1.5]=0$$

Thus $s=-2.3;\ -0.725\pm j0.365$

A breakaway point must occur at s = – 2.3 as this part of the real axis is on the root locus and the two locus branches which start from s = 0 and s = – 3 are approaching each other.

Step 7 : Determination of crossing points of imaginary axis

To find the points where the root loci cross the imaginary axis, we apply Routh's stability criterion. The system characteristic equation is

$$s(s+3)(s^2+2s+2)+K=0$$

$$s^4+5s^3+8s^2+6s+K=0$$

Routh Array:

s^4	1	8	K
s^3	5	6	
s^2	34/5	K	
s^1	$\dfrac{204/5-5K}{34/5}$	0	
s^0	K	0	

For stability of the system, $\frac{204}{5}-5K=0 \Rightarrow K=8.16$

The auxilary equation, formed from the coefficients of s^2 row when K=8, is

$(34/5)s^2+8.16=0$ from which $s=\pm j\,1.1$

The root locus is shown in the Figure.

25. A unity feedback control system has an open loop transfer function(s)=K /s(s^2+8s+32). Sketch Root Locus

(April/May18)

Solution

Step1: Locate poles and zeros on complex plane

The poles of the open loop transfer function are the roots of the denominator

$$s(s^2+8s+32)=0$$

$$\Rightarrow \quad s_2=0$$

$$s_{2,3}=\frac{-8\pm\sqrt{64-4(32)}}{2}$$

$$=\frac{-8\pm\sqrt{-64}}{2} \quad =-4\pm j4$$

Step 2: Number of root locus branches, starting and ending points

Here n = 3 and m = 0. The number of root locus branches = 3.

The three branches starts at $s_1 = 0$, $s_2 = -4 + j4$ and $s_3 = -4 - j4$ when K = 0 and terminate at infinity when K = ∞.

Step 3: Determination of root locus on real axis

All the points on the real axis between $-\infty$ to 0 lie on the root locus, since there is one pole right of these points.

Step 4: Determination of angle of asymptote

To find the angle a asymptote. The three branches that terminates at infinity do so along the asymp totes with angles.

$$\phi_A=\frac{\pm(2\theta+1)180^\circ}{n-m} \quad \theta=0,1,2,....(n-m-1)$$

$$=\frac{\pm(2\theta+1)180^\circ}{3} \quad \theta=0,1,2$$

$$\text{For } \theta=0 \quad \text{angles}=\pm\frac{180^\circ}{3}=60^\circ$$

$$\text{For } \theta=1 \quad \text{angles}=\pm\frac{3(180^\circ)}{3}=180^\circ$$

$$\text{For } \theta=2 \quad \text{angles}=\pm\frac{5(180^\circ)}{3}=300^\circ$$

Step 5: Determination of centroid

The asymptotes meet at a point known as centroid

$$\text{centroid } \sigma_c = \frac{\text{Sum of real parts of poles} - \text{Sum of real parts of seros}}{\text{Number of poles} - \text{Number of zeros}}$$

$$= \frac{-4-4-0}{3} = -\frac{8}{3} = -2.667$$

Mark the centroid on the real axis and draw the asymptotes with angles calculated in step 4 using protractor.

Step 6: Determination of breakaway points

To find break away points of root locus are the solution of $\frac{dK}{ds} = 0$

$$G(s)H(s) = \frac{K}{s(s^2+8s+32)}, \quad H(s) = 1$$

we know

$$1 + G(s)H(s) = 0$$

$$\Rightarrow \ 1 + \frac{K}{s(s^2+8s+32)} = 0$$

$$K = -s(s^2+8s+32)$$

$$\frac{dK}{ds} = 0$$

$$\Rightarrow \quad 3s^2 + 16s + 32 = 0$$

The roots are $\frac{-8 \pm j4\sqrt{2}}{3}$

The points are not on the root locus. Therefore there is no breakaway point.

Step 7: Determination of angle of departure

To find angle of departure ϕ_p of a root locus from a complex open loop pole is $\phi_p = 180 + \phi$

when ϕ is the net angle contribution at this pole by all other open loop poles and zero as shown in Figure (a).

The angle of departure at pole p_2 is

$$\phi_{p2} = 180 + \phi$$

$$\phi = -135^\circ - 90^\circ$$
$$= -225^\circ$$
$$\phi_{p2} = 180^\circ - 225^\circ = --45^\circ$$

...ilarly

$$\phi_{p3} = -\phi p^2 = -(-45^\circ) = 45^\circ$$

$$\tan^{-1}\left(\frac{4}{4}\right) = 45^\circ$$
$$\phi_{p2} = 180^\circ - 45^\circ = 135^\circ$$
$$\phi_{p3} = 90^\circ$$

...ing protractor mark the angle of departure of complex pole

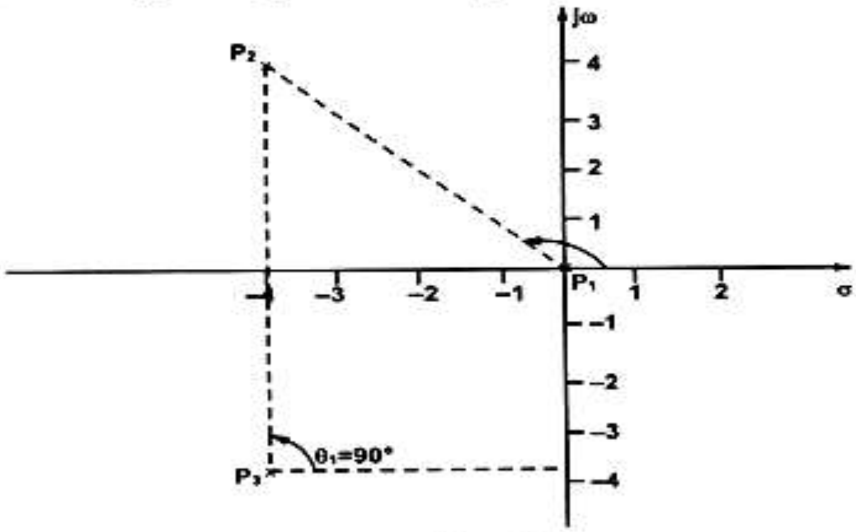


Figure (a)

Step 8: Determination of crossing points of imaginary axis

To find the crossing point on the imaginary axis can be found using Routh criterion. The characteristic equation is given by

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s^2 + 8s + 32)} = 0$$

$$s^3 + 8s^2 + 32s + K = 0$$

$$\begin{array}{c|ccc} s^3 & 1 & 32 & 0 \\ s^2 & 8 & K & 0 \\ s^1 & 256 - K/8 & 0 & \\ s^0 & K & & \end{array}$$

For stability

$$\frac{256 - K}{8} > 0 \quad \text{and} \quad K > 0$$

$$\Rightarrow \qquad 0 < K < 256$$

When = 256, the root locus crosses the imaginary axis.

The auxiliary equation is $8s^2 + K = 0 \Rightarrow 8s^2 + 256 = 0$. $\therefore\ s = j\sqrt{32}$

The complete root locus plot is shown in Figure (b).

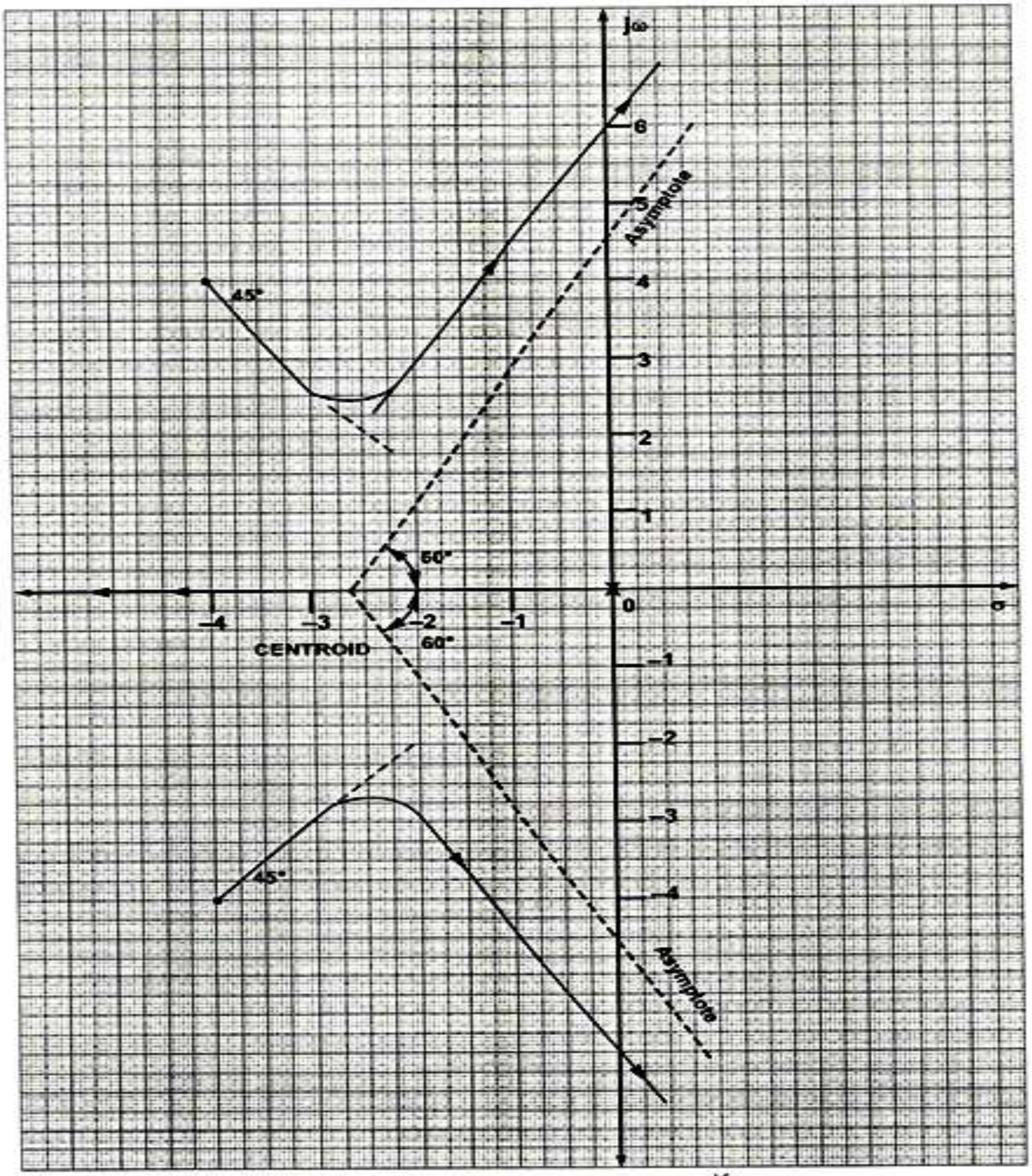


Figure (b) Root locus for $G(s)H(s) = \dfrac{K}{s(s^2+8s+32)}$

26. Plot the root locus for a unity feedback closed loop system whose open loop transfer function is

$$G(s) = \frac{K}{s(s+4)(s^2+2s+2)}$$

Solution

Step 1: Locate poles and zeros on the complex plane

The open loop poles are roots of the denominator of G(s)

$$s(s+4)(s^2+2s-2) = 0$$

$$\Rightarrow \quad s_1 = 0; \quad s_2 = -4; \quad s_{3,4} = \frac{-2 \pm \sqrt{4-8}}{2}; \quad -1 \pm j1$$

Step 2: Number of root locus branches, starting and ending points

Here n =4 and m = 0.

The number of root locus branches = 4.

The root locus branches start at open loop poles and end at infinity.

Step 3: Determination of root locus on real axis

All the four branches in the root locus start from open loop poles $s_1 = 0$; $s_2 = -4$; $s_3 = -1 + j1$ and $s_4 = -1 - j1$ when K = 0 and terminate at infinity when $K = \infty$.

All the point between –4 and 0 lie on the root locus

Step 4: Determination of angle of asymptote

The four branches that terminate at infinity do so along the asymptote with the angles

$$\phi_A = \frac{(2\theta+1)180^\circ}{n-m} \quad \theta = 0,1,2...(n-m-1)$$

$$= \frac{(2\theta+1)180^\circ}{3} \theta = 0,1$$

For $\theta = 0$, angles $= 45^\circ$

For $\theta = 1$, angles $= 135^\circ$

For $\theta = 2$, angles $= 225^\circ$

For $\theta = 3$, angles $= 315^\circ$

Step 5: Determination of centroid

The asymptotes meet at a point known as centroid σ_C

$$\sigma_C = \frac{\text{Sum of real parts of poles} - \text{Sum of real parts of zero}}{\text{Number of poles} - \text{Number of zeros}}$$

$$= \frac{0-4-1-1}{4} = -1.5$$

Mark the centroid on the real axis and draw the asymptotes with angles calculated in step 4.

Step 6: Determination of breakaway point

The break away points of the root locus are the solution of $\frac{dK}{ds} = 0$

$$K = -s(s+4)(s^2+2s+2)$$

$$= -(s^2+4s)(s^2+2s+2)$$

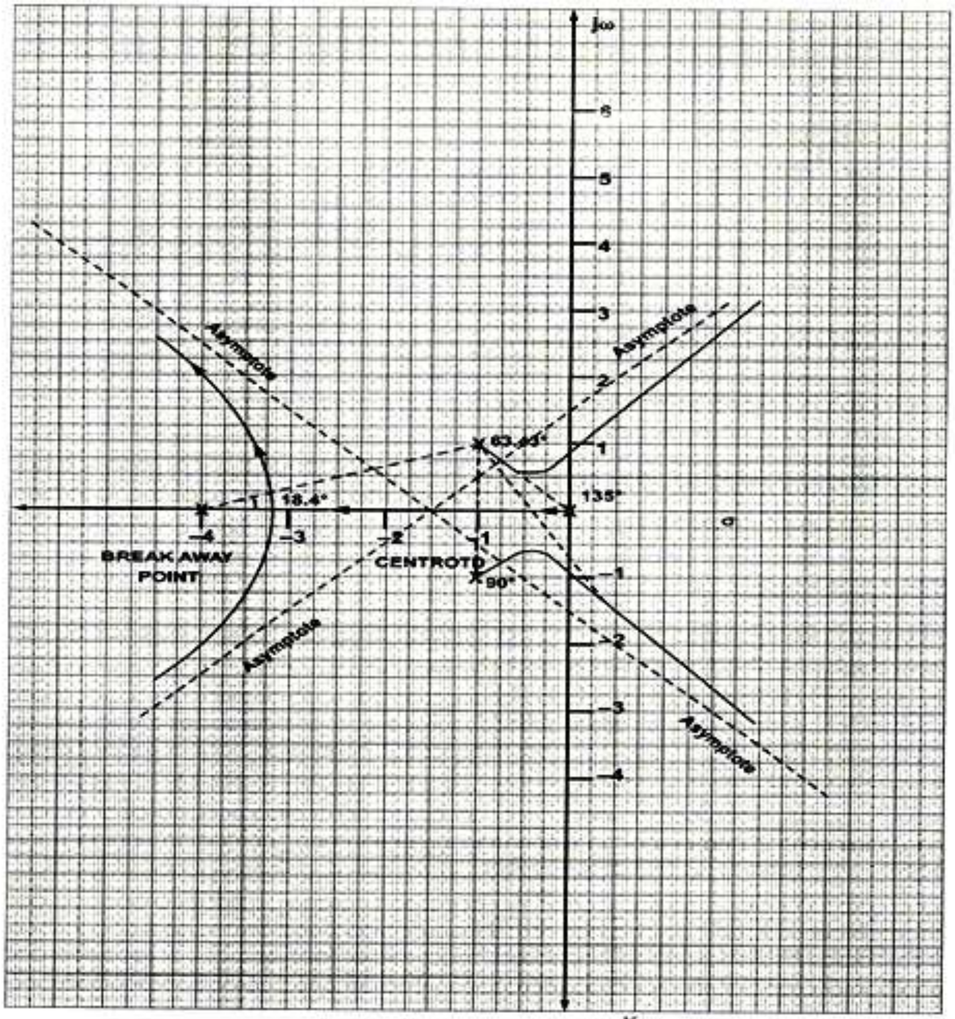
$$= -(s^4+6s^3+10s^2+8s)$$

$$\frac{dK}{ds} = 4s^3 + 18s^2 + 20s + 8 = 0$$

$$\Rightarrow 2s^3 + 9s^2 + 10s + 4 = 0$$

$$\Rightarrow s^3 + 4.5s^2 + 5s + 2 = 0$$

By trial and error procedure, we find the break away point at $s = -3.09$.



Root locus for $G(s)\,H(s) = \dfrac{K}{s(s+4)(s^2+2s+2)}$

Step 7: Determination of crossing point of imaginary axis

The intersection of root locus with imaginary axis can be obtained using Routh's criterion.

The characteristic equation is given by

$$s^4 + 6s^3 + 10s^2 + 8s + K = 0$$

s^4	1	10	K
s^3	6	8	0
s^2	26/3	K	
s^1	$\dfrac{208/3 - 6K}{26/3}$	0	
s^0	K		

For stability

$$K > 0$$

$$\text{and } \frac{208}{3} - 6K > 0 \Rightarrow 0 < K < \frac{107}{9}$$

The two branches of root locus cross the imaginary axis when $K = \frac{107}{9}$. To find the point of intersections with imaginary axis, consider the auxiliary equation is $\frac{26}{3}s^2 + \frac{104}{9} = 0$

$\therefore$ Points of intersection are $s = \pm\, j1.154$

The complete root locus is shown in Figure.

27. **Derive an expression for the response of critically damped second order system for**

unit step input.May-17

$$\frac{C(s)}{R(s)} = \frac{\omega^2{}_n}{s^2 + 2\varsigma\omega_n s + \omega^2{}_n}$$

$$\varsigma = 1$$

$$\frac{C(s)}{R(s)} = \frac{\omega^2{}_n}{s^2 + 2\omega_n s + \omega^2{}_n} = \frac{\omega^2{}_n}{(s+\omega_n)^2}$$

$$r(t) = 1;\ R(s) = \frac{1}{s}$$

$$C(s) = R(s)\frac{\omega^2{}_n}{(s+\omega_n)^2} = \frac{1}{s}\frac{\omega^2{}_n}{(s+\omega_n)^2} = \frac{\omega^2{}_n}{s(s+\omega_n)^2}$$

$$C(s) = \frac{\omega^2{}_n}{s(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{(s+\omega_n)^2} + \frac{C}{s+\omega_n}$$

$$A = s \times C(s)\Big|_{s=0} = \frac{\omega^2{}_n}{(s+\omega_n)^2}\Bigg|_{s=0} = \frac{\omega^2{}_n}{\omega^2{}_n} = 1$$

$$B = (s+\omega_n)^2 \times C(s)\Big|_{s=-\omega_n} = \frac{\omega^2{}_n}{s}\Bigg|_{s=-\omega_n} = -\omega_n$$

$$C = \frac{d}{ds}\left[s+\omega_n)^2 \times C(s)\right]\Big|_{s=-\omega_n} = \frac{d}{ds}\frac{\omega^2{}_n}{s}\Bigg|_{s=-\omega_n} = \frac{-\omega^2{}_n}{s^2}\Bigg|_{s=-\omega_n} = -1$$

$$\therefore\ C(s) = \frac{A}{s} + \frac{B}{s+\omega_n{}^2} + \frac{C}{s+\omega_n} = \frac{1}{s} - \frac{\omega_n}{s+\omega_n{}^2} - \frac{1}{s+\omega_n}$$

$$c(t) = L^{-1}\big(C(s)\big) = L^{-1}\left\{\frac{1}{s} - \frac{\omega_n}{s+\omega_n{}^2} - \frac{1}{s+\omega_n}\right\}$$

$$c(t) = 1 - \omega_n t e^{-\omega_n t} - e^{-\omega_n t}$$

$$c(t) = 1 - e^{-\omega_n t}(1+\omega_n t)$$

28. Outline the time response analysis of a first order system when it is subjected to a unit step input.
(April/May18)

The closed loop order system with unity feedback is shown in fig 2.6.

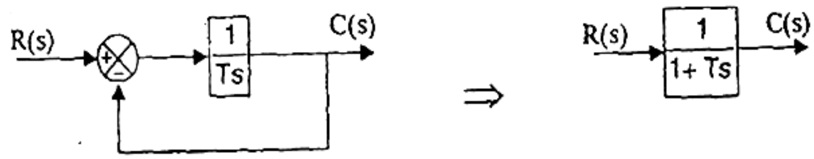


Fig 2.6 : Closed loop for first order system.

The closed loop transfer function of first order system, $\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$

If the input is unit step then, $r(t) = 1$ and $R(s) = \frac{1}{s}$.

$$\therefore \text{The response in s-domain, } C(s) = R(s)\frac{1}{(1+Ts)} = \frac{1}{s}\frac{1}{(1+Ts)} = \frac{1}{sT\left(\frac{1}{T}+s\right)} = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)}$$

By partial fraction expansion,

$$C(s) = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)} = \frac{A}{s} + \frac{B}{\left(s+\frac{1}{T}\right)}$$

A is obtained by multiplying C(s) by s and letting s = 0.

$$A = C(s) \times s\Big|_{s=0} = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)} \times s\Bigg|_{s=0} = \frac{\frac{1}{T}}{s+\frac{1}{T}}\Bigg|_{s=0} = \frac{\frac{1}{T}}{\frac{1}{T}} = 1$$

B is obtained by multiplying C(s) by (s+1/T) and letting s = −1/T.

$$B = C(s) \times \left(s+\frac{1}{T}\right)\Bigg|_{s=-\frac{1}{T}} = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)} \times \left(s+\frac{1}{T}\right)\Bigg|_{s=-\frac{1}{T}} = \frac{\frac{1}{T}}{s}\Bigg|_{s=-\frac{1}{T}} = \frac{\frac{1}{T}}{\frac{-1}{T}} = -1$$

$$\therefore C(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{T}}$$

$$\boxed{\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}}$$

The response in time domain is given by,

$$c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+\frac{1}{T}}\right\} = 1 - e^{-\frac{t}{T}}$$

The equation (2.13) is the response of the closed loop first order system for unit step input. For step input of step value, A, the equation (2.13) is multiplied by A.

$$\boxed{\begin{aligned} &\therefore \text{For closed loop first order system, Unit step response } = 1 - e^{-\frac{t}{T}} \\ &\text{Step response } = A\left(1 - e^{-\frac{t}{T}}\right) \end{aligned}}$$

When, $t = 0$, $c(t) = 1 - e^{0} = 0$
When, $t = 1T$, $c(t) = 1 - e^{-1} = 0.632$
When, $t = 2T$, $c(t) = 1 - e^{-2} = 0.865$
When, $t = 3T$, $c(t) = 1 - e^{-3} = 0.95$
When, $t = 4T$, $c(t) = 1 - e^{-4} = 0.9817$
When, $t = 5T$, $c(t) = 1 - e^{-5} = 0.993$
When, $t = \infty$, $c(t) = 1 - e^{-\infty} = 1$

Here T is called Time constant of the system. In a time of 5T, the system is assumed to have attained steady state. The input and output signal of the first order system is shown in fig 2.7.

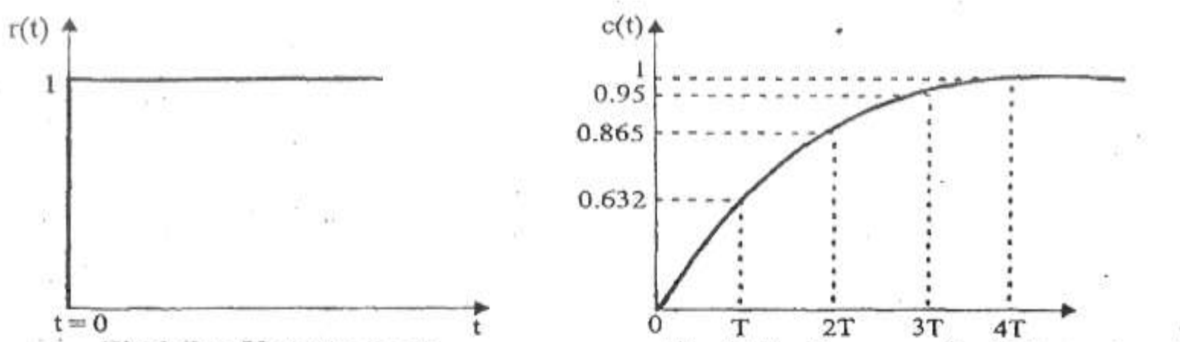


Fig 2.7a : Unit step input.   *Fig 2.7b : Response for Unit step input*

29.A unity feedback control system has an open loop transfer function G(s) =K(s+9) / s (s^2+4s+11)

(Nov/Dec 2017)

SOLUTION

Step 1 : To locate poles and zeros

The poles of open loop transfer function are the roots of the equations, $(s^2+4s+11)=0$.

The roots of the quadratic are, $s = \frac{-4 \pm \sqrt{4^2 - 4\times 11}}{2} = -2 \pm j2.64$

$\therefore$ The poles are lying at, s = 0, –2 + j2.64, –2 – j2.64

The zeros are lying at, s = –9 and infinity.

Let us denote the poles as p_1, p_2, p_3 finite zero by z_1.

Here, $p_1 = 0$, $p_2 = -2+j2.64$, $p_3 = -2-j2.64$ and $z_1 = -9$.

The poles are marked by X(cross) and zeros by "o" (circle) as shown in fig 4.24.1.

Step 2 : To find the root locus on real axis.

One pole and one zero lie on real axis.

Choose a test point to the left of s = 0, then to the right of this point, the total number of poles and zeros is one which is an odd number. Hence the portion of real axis from s = 0 to s = –9 will be a part of root locus.

If we choose a test point to the left of s = –9 then to the right of this point, the total number of poles and zeros is two, which is an even number. Hence the real axis from s = –9 to –∞ will not be a part of root locus.

The root locus on real axis is shown as a bold line in fig 4.24.1.

Step 3 : To find angles of asymptotes and centroid

Since there are 3 poles, the number of root locus branches are three. One root locus branch starts at the pole at origin and travel along negative real axis to meet the zero at s = –9. The other two root locus branches meet the zeros at infinity. The number of asymptotes required are two.

$$\text{Angles of asymptotes} = \frac{\pm 180^\circ (2q+1)}{n-m}; \qquad q = 0, 1, 2, \ldots\ n-m$$

Here, n = 3 and m = 0. $\therefore$ q = 0, 1, 2, 3.

$$\text{When } q = 0, \quad \text{Angles} = \pm\frac{180^\circ}{3} = \pm 60^\circ$$

$$\text{When } q = 1, \quad \text{Angles} = \pm\frac{180^\circ \times 3}{2} = \pm 270^\circ = \mp 90^\circ$$

$$\text{When } q = 2, \quad \text{Angles} = \pm\frac{180^\circ \times 5}{2} = \pm 450^\circ = \pm 90^\circ$$

Note *: It is enough if you calculate the required number of angles. Here it is given by first two values of angles. The remaining values will be repetitions of the previous values.*

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m} = \frac{0 - 2 + j2.64 - 2 - j2.64 - (-9)}{2} = 2.5$$

The centroid is marked and from the centroid, the angles of asymptotes are marked using a protractor. The asymptotes are drawn as dotted lines as shown 4.24.1.

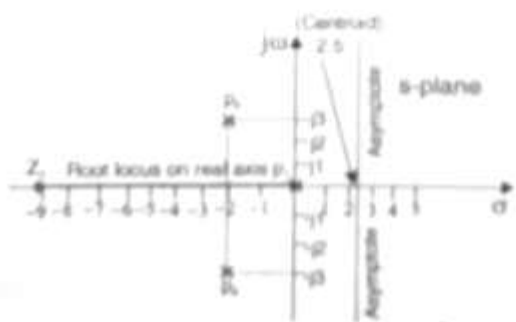


Fig 4.24.1 *: Figure showing the asymptotes, root locus on real axis and location of poles, zero and centroid*

Step 4 : To find the breakaway and breakin points

From the location of poles and zero and from the knowledge of typical sketches of root locus, it can be concluded that there is no possibility of breakaway or breakin points.

Step 5 : To find the angle of departure

Let us consider the complex pole p_2 as shown in fig 4.24.2. Draw vectors from all other poles and zero to the pole p_2 as shown in fig 4.24.2. Let the angles of these vectors be θ_1, θ_2 and θ_3.

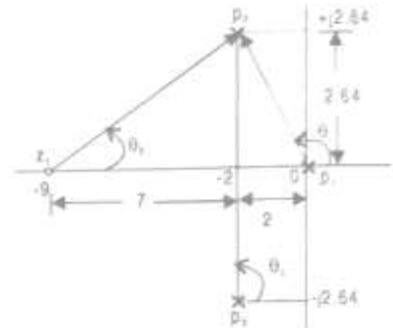


Fig 4.24.2

$$\text{Here, } \theta_1 = 180° - \tan^{-1}\frac{2.64}{2} = 127.1°$$

$$\theta_2 = 90°$$

$$\theta_3 = \tan^{-1}\frac{2.64}{7} = 20.7°$$

$$\left.\begin{array}{l}\text{Angle of departure from}\\ \text{the complex pole } p_2\end{array}\right\} = 180° - (\theta_1 + \theta_2) + \theta_3$$

$$= 180° - (127.1° + 90°) + 20.7° = -16.4°$$

The angle of departure at the complex pole p_3 is negative of the angle of departure at complex pole p_2.

$\therefore$ Angle of departure at pole $p_3 = -(-16.4) = +16.4°$

Mark the angles of departure at complex poles using protractor.

Step 6 : To find the crossing point of imaginary axis

$$\left.\begin{array}{l}\text{The closed loop}\\ \text{transfer function}\end{array}\right\} \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\dfrac{K(s+9)}{s\,(s^2+4s+11)}}{1+\dfrac{K\,(s+9)}{s\,(s^2+4s+11)}} = \frac{K(s+9)}{s\,(s^2+4s+11)+K(s+9)}$$

The characteristic equation is the denominator polynomial of C(s)/R(s).

$\therefore$ The characteristic equation is,

$$s\,(s^2+4s+11)+K\,(s+9)=0 \quad\Rightarrow\quad (s^3+4s^2+11s)+Ks+9K=0$$

put $s = j\omega$

$$(j\omega)^3+4(j\omega)^2+11(j\omega)+K(j\omega)+9K=0 \quad\Rightarrow\quad -j\omega^3-4\omega^2+j11\omega+jK\omega+9K=0$$

On equating imaginary part to zero,

$$-j\omega^3+j11\omega+jK\omega=0 \Rightarrow -j\omega^3=-j11\omega-jK\omega$$

$$\therefore\ \omega^2=11+K$$

$$\text{Put } K=8.8,\quad \therefore \omega^2=11+8.8=19.8$$

$$\omega=\pm\sqrt{19.8}=\pm4.4$$

On equating real part to zero,

$$-4\omega^2+9K=0 \quad\Rightarrow\quad 9K=4\omega^2$$

$$\text{Put, } \omega^2=11+K. \quad \therefore 9K=4(11+K)=44+4K$$

$$\therefore 9K-4K=44$$

$$\therefore 5K=44 \quad\Rightarrow\quad K=\frac{44}{5}=8.8$$

The crossing point of root locus is $\pm j4.4$. The value of K at this crossing point is K = 8.8 (This is the limiting value of K for the stability of the system).

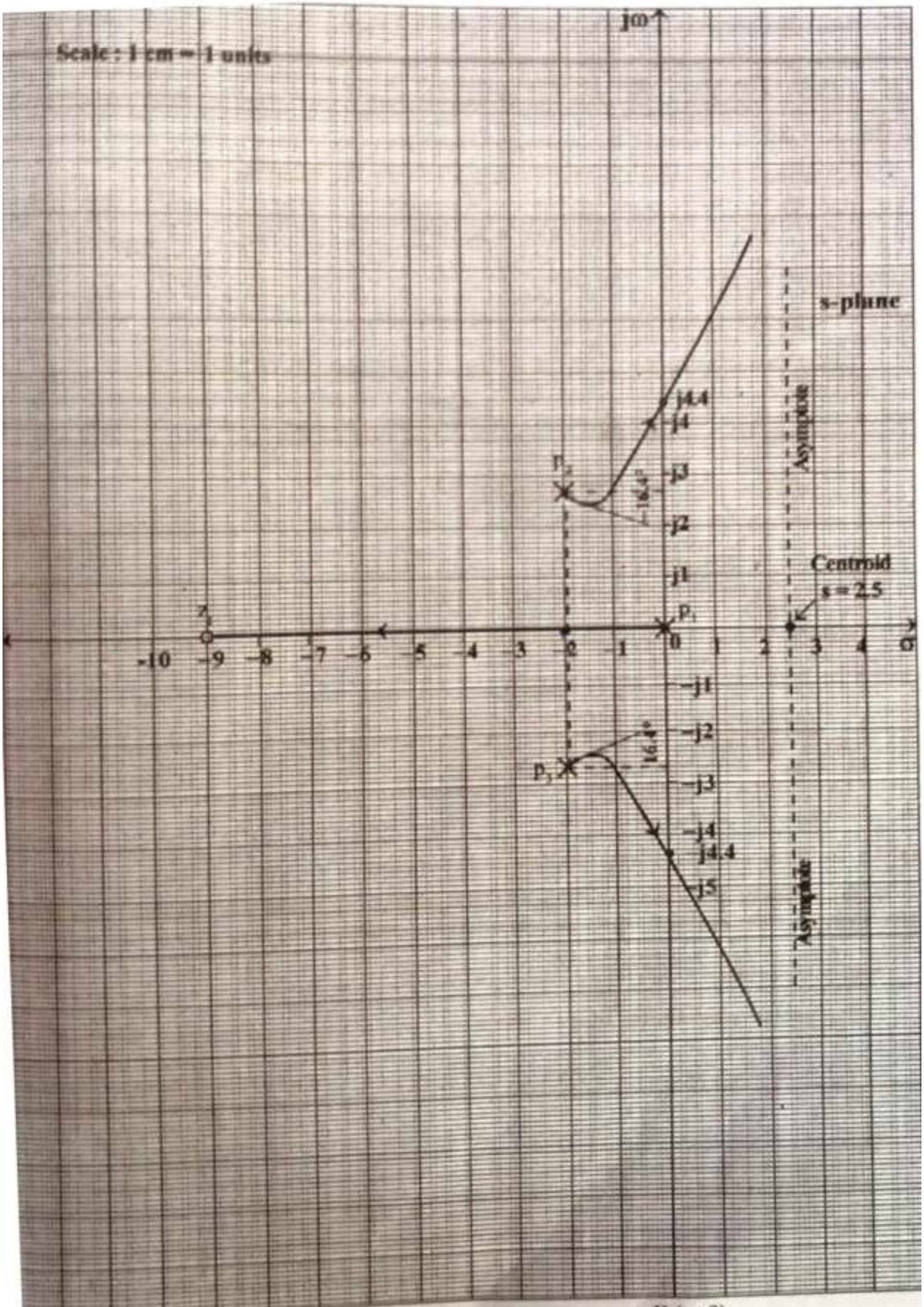


Fig 4.24.3. : Root locus sketch for, $1+G(s)=1+\dfrac{K(s+9)}{s(s^2+4s+11)}$.

30. Determine the response of the unity feedback system whose open loop transfer function is G(s) =4/s(s+5) (April/May 2018)

SOLUTION

Fig 1 : Closed loop system.

The closed loop system is shown in fig 1

The closed loop transfer function, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$

$$\frac{C(s)}{R(s)} = \frac{\frac{4}{s(s+5)}}{1+\frac{4}{s(s+5)}} = \frac{\frac{4}{s(s+5)}}{\frac{s(s+5)+4}{s(s+5)}} = \frac{4}{s(s+5)+4} = \frac{4}{s^2+5s+4} = \frac{4}{(s+4)(s+1)}$$

The response in s-domain, $C(s) = R(s)\,\frac{4}{(s+1)(s+4)}$

Since the input is unit step, $R(s) = \frac{1}{s}$; $\therefore C(s) = \frac{4}{s(s+1)(s+4)}$

By partial fraction expansion, we can write,

$$C(s) = \frac{4}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}$$

$$A = C(s) \times s\big|_{s=0} = \left.\frac{4}{(s+1)(s+4)}\right|_{s=0} = \frac{4}{1\times 4} = 1$$

$$B = C(s) \times (s+1)\big|_{s=-1} = \left.\frac{4}{s(s+4)}\right|_{s=-1} = \frac{4}{-1(-1+4)} = \frac{-4}{3}$$

$$C = C(s) \times (s+4)\big|_{s=-4} = \left.\frac{4}{s(s+1)}\right|_{s=-4} = \frac{4}{-4(-4+1)} = \frac{1}{3}$$

The time domain response c(t) is obtained by taking inverse Laplace transform of C(s).

Response in time domain, $c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{4}{3}\,\frac{1}{s+1} + \frac{1}{3}\,\frac{1}{s+4}\right\}$

$$= 1 - \frac{4}{3}e^{-t} + \frac{1}{3}e^{-4t} = 1 - \frac{1}{3}\left[4e^{-t} - e^{-4t}\right]$$

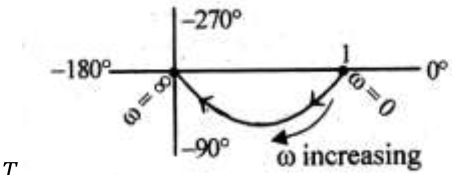
RESULT

Response of unity feedback system, $c(t) = 1 - \frac{1}{3}\left[4e^{-t} - e^{-4t}\right]$

Part-A

1.Draw the polar plot of G(s) =1/ (1+sT). (May/June 2012)(April/May 2015)

Let s=jω, $G(j\omega) = \frac{1}{1+j\omega T} = \frac{1}{\sqrt{1+\omega^2T^2}}\frac{1}{\tan^{-1}\omega T}$



$=\frac{1}{\sqrt{1+\omega^2T^2}}\tan^{-1}\omega T$

As $\omega \to 0$ G(jω)→ $1 < 0°$

As $\omega \to \infty$G(jω)→0< $-90°$

2.State the uses of Nicholas chart.(May/June2012)

Theclosedloopfrequencyresponsecanbedeterminedgraphicallyfromthelocusofopenloop frequencyresponse

3. Name the parameters which constitute frequency domain specifications. (Nov/Dec2011)

List out the different frequency domain specifications. (Nov/Dec2017)

Thefrequencydomainspecificationindicatestheperformanceofthesysteminfrequencydomain and theyare

- Resonantpeak,M,
- Resonantfrequency,ω,
- Bandwidth,

4. Give the advantages of frequency response analysis. (April/May2018)

1.Theabsoluteandrelativestabilityoftheclosedloopsystemcanbeestimated

2.Fromtheknowledgeoftheopenloopfrequencyresponse.

3.The correctivemeasure for noise disturbance andparametervariation canbe easily carried

4.Itcanbeextendedtocertainnon-linearsystem

5. Define phase margin. (Nov/Dec2013)(April/May2015)

Thephasemargin is thatamountofadditionalphase lagatgaincross-overfrequency, requiredtobringthesystemtothevergeofinstability.Itisgivenby,180 isthe phaseofG(jω)atthegaincrossoverfrequency.

6.What are constant Mand Ncircles?(Nov/Dec2013)(Nov/Dec2014)

Themagnitude,Moftheclosedlooptransferfunctionwithunityfeedbackwillbeintheformof circleandcomplexplaneforeachconstantvaluesofM.thefamilyofthesecirclesarecalledM- circles.;

Letn=tan whereaisthephase ofclosedlooptransferfunctionwithunityfeedback.Foreach constantvaluesofNcirclescanbedrawninthecomplexplane.Thefamiliesofthesecirclesare calledN-circles.

7.What is meant by‘cornerfrequency’?(Nov/Dec2012) (April/May2018)

The magnitudeplot can be approximated by asymptotic straight lines. The frequenciescorrespondingtothe meetingpointofasymptotesarecalledascornerfrequency.Theslopeofthe magnitudeplotchangesateverycornerfrequency.

8.What is Nicholas chart?(Nov/Dec2012)(April/May2011)

TheNicholaschartconsistsofMandNcontourssuperimposedonordinarygraph.AlongeachM-contourthemagnitudesoftheclosedloop system,Mwillbeaconstant.AlongeachN-contour,the phase oftheclosedloopsystemwillbeconstant.Theordinarygraphsconsistsofmagnitudein db,markedonthey-axisandthephaseindegreesmarkedonx-axis.TheNicholschartisusedto find theclosedloopfrequencyresponsefromtheopenloopfrequencyresponse.

9.Whati s polar plot?

ThepolarplotofasinusoidaltransferfunctionG(jω)isaplotofthemagnitudeofG(jω)versusthe phaseangle/argument G(jω)onpolarorrectangularco=ordinatesasωisvariedfromZeroto infinite.

10.What is frequency response?

Themagnitudeandphasefunctionofsinusoidaltransferfunctionofasystemarerealfunctionof frequencyω,andsotheyarecalledfrequencyresponse.

11.Define resonant peak?(May/June2014)

The maximumvalueofthemagnitudeofclosedlooptransferfunctioniscalledasresonantpeak.

12.Whati s cut-offregion?

Theslopeofthelog magnitudecurvenearthecut-offfrequencyiscalledcut-offrate.

13.Define bandwidth

Thebandwidthistherangeoffrequenciesforwhichthesystemgainismore then-3db

14.What are the advantage sof Nicholas chart?

1. Itisusedtofindclosedloopfrequencyresponsefromopenloopfrequencyresponse.
2. ThefrequencydomainspecificationcanbedeterminedfromNicholaschart.
3. Thegainofthesystemcanbeadjustedtosatisfythegivenspecification.

15.Write the necessary frequency domain specification for design of acontrol system?

Thefrequencydomainspecificationrequiredtodesignacontrolsystemare,

- Phasemargin
- Gain margin
- Resonantpeak

•Bandwidth

16.What is the advantage in design using root locus?

Theadvantage indesign usingrootlocustechnique isthatthe informationaboutclosedloop transientresponseandfrequencyresponsearedirectlyobtainedfromthepole-zeroconfigurationof thesysteminthes-plane.

17.How root loci are modified when a zero is added to openloop transfer function.

Theadditionofapoletotheopenlooptransferfunctionhastheeffectofpulingtherootlocusto theright,whichreducestherelativestabilityofthesystemandincreasethesettlingtime.

18.What is PI-controller and what are the effects on system performance?

The PI-controller is a device that produces an output signal consisting of two terms-one proportionaltotheinputsignal.

The proportional controller improves the steady state tracking accuracy, disturbance signal rejection andrelative stability. It alsodecreases the sensitivity ofthe system to parameters variation.

19.Write the MATLAB statement to draw the bode plot of the given system

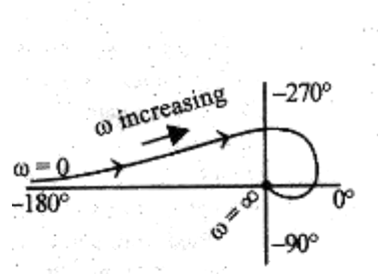
(May/June2013)

$G(s)=s^2/(1+0.2s)(1+0.02s)$

```
%programtoplotBodeplot
Clearall
Clc
S=tf('s')
disp('thegiventransferfunctionis,');
Gs=(s^2)/((1+0.2*s)*(1+0.02*s))
W=logspace(-1,2,200);
Bode(Gs,w)
Grid
```

20.Draw the polar plot of an integral term of transfer function.

(May/June2013)



21.Define gain margin.(April/May2015)(Nov/Dec2014)

Thegainmarginisdefinedasthevalueby whichgainofthesystemhastobe increasedtodrive systemtobevergeofinstability. Itisgivenreciprocalofthemagnitudeofopenlooptransfer function,atphasecrossoverfrequency.

22.What is resonance requency?(May/June2014,17)

Thefrequencyat whichtheresonantpeakoccursis calledresonantfrequency.theresonantpeakis themaximumvalueofthemagnitudeofclosedlooptransferfunction.

23. Write the expression for resonant frequency and peak in terms of time domain specifications.(Nov/Dec 2014)

Resonant frequency, $\omega r = \omega_n \sqrt{1-2\zeta^2}$

Resonant peak= $M_r = \frac{1}{2\zeta\sqrt{1-2\zeta}}$

24. How is gain margin and phase margin are determined from Bodeplot?

According todefinitionofG.M

G.M. =-20Log$|G(j\omega)H(j\omega)|G(j\omega)H(j\omega)|_{at\omega=\omega_{pc}} db$

$=0db-20Log|G(j\omega)H(j\omega)|_{at\omega=\omega_{pc}} db$

Now20Log$|G(j\omega)H(j\omega)|_{at\omega=\omega_{pc}}$ *indbcanbedirectlyreadoutfrommagnitudeplot.Extend*$\omega=\omega_{pc}$*line upwardstillit*intersect*sresul*tant*magnitudeplotsaypo*int*A.Themagnitudecorresponding topo*int*Ais*$|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$

Thedifferencebetween 0dbandmagnitudecorresponding topointAisGainmargin.IfpointAis below0 db ,GMispositiveandifpointAisabove0 db ,G.Misnegative.

SimilarlyP.M.$[\angle|G(j\omega)H(j\omega)|_{\omega=\omega gc}]-(-180^0)$

Now$\angle|G(j\omega)H(j\omega)|_{\omega=\omega gc}$*canbereadoutfromphseangleplot*.ThispointofintersectionispointC.

25. What should be the values of G.M. and P.M. for a good system?

Itisobviousthatasystemshouldhavegain which islowerthanthe criticalvalueso that thesystem isfarremovedfromunstable conditions

Thisisnecessarybecauseformostofthesystemsthetransferfunctionofcomponentsand Systemschangeswith variation in temperatureand pressuresofsurrounding environment Moreoverthegainsarealso dependentonsupply frequency,supply voltage,loading conditions,variation incontrolenergysourcessuchaspneumaticairpressure.

26. What are the different methods of compensation?

- seriescompensation
- parallelcompensation
- series-parallelcompensation

Aredesign oralteration ofsystemusinganadditionalsuitabledeviceiscalled compensationofa controlsystem.

27. Mention the need for lead compensation and lag compensation.

The phase lead compensator is required to achieve large bandwidth, shorter Rise time and less settling time. While the phase lag compensation is required to provide major damping and sufficient phase margin by providing attenuation in the higher frequency range.

28. What are the effects of lag-lead compensator? (May/June2016, 17)

Lag-leadcompensatorisusedwhen bothfastresponse andgoodstaticaccuracyare desired.useoflagleadcompensatorincreasesthelowfrequencygainwhichimproves thesteadystate.whileatthesametimeitincreasesbandwidthofthesystem,making the systemresponseveryfast.itisusedtoachievelargerbandwidth andhenceshorter rise time.

29. Howlag-leadcompensatoraredesignedusingrootlocus?Annauniv.Dec-11

It is known that the lead compensator increases thespeed ofthe responseand improvesstability.thelagcompensatorimprovesthesteady stateresponse.ifimprovementisbothtransientaswellasthesteadystateresponseisdesired thenthe lag-lead compensatorisused

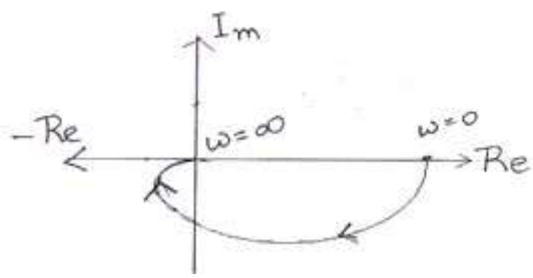
31. What type of compensator is suitable for high frequency noisy environment?

Thebandwidth ofleadcompensatorandlag-leadcompensatorishigh.Thisallowshigh frequencynoisesignalstoenterintothesystemwhichishighlyobjectionable.Thegain of such compensatorsishigh athighfrequenciesmakingnoisemoretroublesome.Thelag compensatoractsaslowpassfilteranddoesnotallowhighfrequency noisesignal toenter into thesystem. Hencelagcompensatorissuitableforhighfrequency noisy environment.

32. Why lag compensatorisnotusefulforimproving theperformanceofhigherorder systems?

lagcompensatorisbasicallylowpassfilter.The attenuation characteristicsoflag compensatorisusedtoimprovetheperformanceofthesystem thanitsphaselag characteristics. Itreducesthebandwidth makingsystemslower. It makessystemmore sensitivetoparametervariationsandlessstable.Asstabilityismajorproblem relatedto higherordersystems, lagcompensatorwhichmakessystemlessstable isnot used for higherordersystems.

33. Drawtheapproximatepolarplot foratype0 secondordersystem.(Nov/Dec2015)



34. What is the basis for the selection of a particular compensator for a system?(Nov/Dec2015)

Whenmainlytransientresponseisto beimproved,alead compensatorischosen. When steadystate responseistobeimproved whilenearlypreservingthetransientresponse,alagcompensatoris chosen. When both the transient and steady state response aretobe improved, a lag-lead compensatorischosen.

35. Define gain and phase crossover frequencies.(May/June2016)

(i)Gaincrossoverfrequency(ω_{gc}) :Thefrequencyatwhichthemagnitude $G(j\omega)H(j\omega)$ Isunity i.e.1iscalledgain crossoverfrequency.

Thefrequency at which phase angle of $G(j\omega)H(j\omega)$is-180^0iscalled phase crossoverfrequency,

36. Differrtiate between gain margin and phase margin.(April/May 2018)

Gain margin:

Thegainmarginisdefinedasthevalueby whichgainofthesystemhastobe increasedtodrive systemtobevergeofinstability. Itisgivenreciprocalofthemagnitudeofopenlooptransfer function,atphasecrossoverfrequency.

Phase margin:

Thephasemargin is thatamountofadditionalphase lagatgaincross-overfrequency, requiredtobringthesystemtothevergeofinstability.Itisgivenby,180 isthe phaseofG(jω)atthegaincrossoverfrequency

PARTB

1.ExplainthedetailaboutFrequencydomainspecifications

The basic objective of control system design is to meet the performance specifications. These specifications are the constraints or limitations put on the mathematical functions describing the system characteristics .Such frequency response specifications are described below.

Consider a general frequency response of a system

(i) **Bandwidth**: It is defined as the range of frequencies over which the system will respond satisfactorily .It can also be defined as range of frequencies in which the magnitude response is almost flat in nature. So it is defined as the range of frequencies over the magnitude of closed loop response i.e. $\left|\frac{C(j\omega)}{R(j\omega)}\right|$ does not drop by more than 3db from its zero frequency value.

(ii) **Cut-off frequency**: It is denoted by ω_b .The frequency at which the frequency of the closed loop response is 3db down from its frequency value is called cut-off frequency. The range 0 to ω_b is nothing but the bandwidth of the system whose frequency response is shown.

Bandwidth indicates the speed of the response .It indicates the ability to reproduce the input signal. It is inversely proportional to the rise time. Large bandwidth means small rise time means fast response.

(iii) **Cut-off rate** : The slope of the resultant magnitude curve near the cut-off frequency is called cut-off rate.

(iv) **Resonant peak** (M_r): It is maximum value of magnitude of the closed loop frequency as shown. Larger the value of resonant peak more is the value of peak overshoot of system for step input. It is a measure of relative stability of the system.

(v) **Resonant frequency** (ω_r): The frequency at which resonant peak M_r occurs in closed loop frequency response is called resonant frequency. It is inversely proportional to the rise time. These are the general specifications which are most important from the stability and relative stability analysis..

(vi) **Gain crossover frequency(ω_{gc}) :**The frequency at which magnitude of

$G(j\omega)H(j\omega)$ is unity i.e. 1 is called gain crossover frequency.Generally magnitude of $G(j\omega)H(j\omega)$ is expressed in db. And db value of one is $20\log_{10} 1=0$ db.

It can be defined as the frequency at which magnitude of $G(j\omega)H(j\omega)$ is 0 db at ω_{gc}

(vii) **Phase cross-over frequency (ω_{pc}):** The frequency at which phase angle of $G(j\omega)H(j\omega)$ is -180^0 is called phase cross-over frequency, ω_{pc} .

(viii) Gain Margin G.M.: As seen earlier in the root locus as gain 'K' is increased the system stability reduces and for a certain value of 'K' it becomes marginally stable. So gain margin is defined as the margin in gain allowable by which gain can be increased till system reaches on the verge of instability.

The positive gain margin means such increase in 'k' is possible before system becomes unstable, hence system is stable and negative gain margin means k is greater than K_{mar} and system is unstable so 'K' is required to be reduced to make the system stable. Mathematically it can be defined as reciprocal of the magnitude of the $G(j\omega)H(j\omega)$ measured at phase crossover frequency.

$$G.M = \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}$$

In decibels

$$G.M = 20 Log \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}$$

$$G.M = -20 \, Log_{10} |G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$$

More positive the G.M, more stable is the system.

Phase Margin (γ)

The phase marin γ , is defined as the additional phase lag to be added at the gain cross over frequency in order to bring the system to the verge of instability. The gain cross over frequency ω_{gc} is the frequency in order to bring the system to the open loop transfer function is unity (or it is the frequency at which the db magnitude is zero)
The phase margin γ, is obtained by adding 180^0 to the phase angle $\emptyset$of the 0pen loop transfer function at the gain cross over frequency

Phase margin, $\gamma = 180^0 + \emptyset_{gc}$

Where, $\emptyset_{gc} = \angle G(j\omega_{gc})$

Note : $|G(j\omega_{pc})|$ ***is the magnitude of*** $G(j\omega)$ **at** $\omega = \omega_{gc}$

The phase margin indicates the additional phase lag that can be provided to the system without affecting stability.

2.Discussaboutthe CorrelationBetweenTimeAndFrequencyResponseofsecondorder system.(May/June2014)(Nov/Dec2015)

The correlation between time and frequency response has an explicit form only for first and second order systems. The correlation for second-order system is discussed here.

Consider the magnitude and phase of a closed loop second order system as a function of normalized frequency, as given by equations (4.9) and (4.10).

$$\text{Magnitude of closed loop system, } M = |M(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2+(2\zeta u)^2}}$$

$$\text{Phase of closed loop system, } \alpha = \angle M(j\omega) = -\tan^{-1}\frac{2\zeta u}{1-u^2}$$

The magnitude and phase angle characteristics for normalized frequency u, for certain values of ζ are shown in fig 4.2 and 4.3. The frequency at which M has a peak value is known as the resonant frequency. The peak value of the magnitude is the resonant peak M_r. At this frequency the slope of the magnitude curve is zero. The frequency corresponding to M_r is u_r, which is the normalized resonant frequency.

From equations (4.14) and (4.15) we get,

$$\text{Resonant peak, } M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$\text{Resonant frequency, } \omega_r = \omega_n\sqrt{1-2\zeta^2}$$

$$\text{When } \zeta = 0, \quad \omega_r = \omega_n\sqrt{1-2\zeta^2} = \omega_n \qquad .....(4.27)$$

$$\text{When } \zeta = 0, \quad M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \infty \qquad .....(4.28)$$

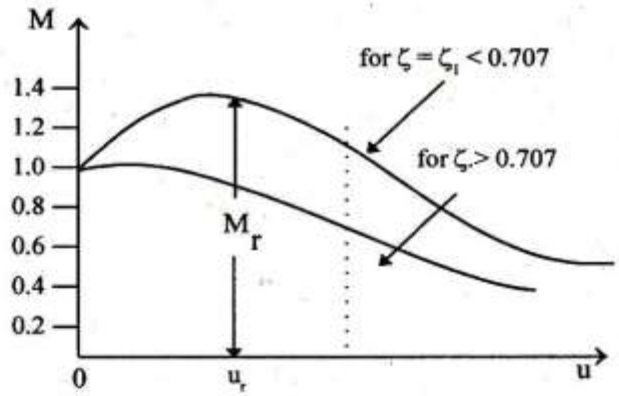


Fig 4.2 : Magnitude, M as a function of u.

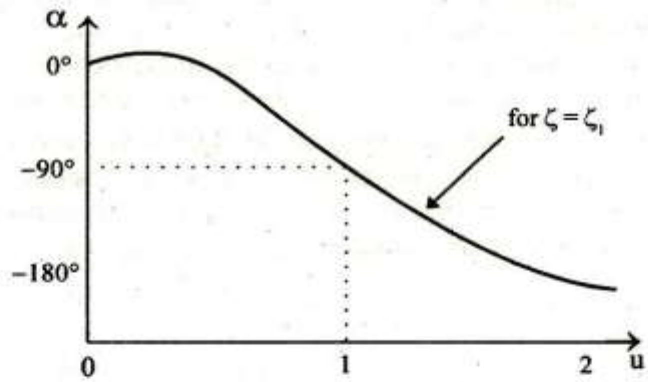


Fig 4.3 : Phase, α as a function of u.

From equations (4.27) and (4.28), it is clear that as ζ tends to zero, ω_r approaches ω_n, and M_r approaches infinity.

When $1-2\zeta^2 = 0$, $\omega_r = 0$, which means there is no resonant peak at this condition.

Let, $1-2\zeta^2 = 0$; $\therefore \zeta^2 = \frac{1}{2} \quad \Rightarrow \quad \zeta = \frac{1}{\sqrt{2}}$

For $0 < \zeta \le 1/\sqrt{2}$, the resonant frequency always has a value less than ω_n, and the resonant peak has a value greater than one.

For $\zeta > 1/\sqrt{2}$, the condition (dM/du) = 0, will not be satisfied for any real value of ω.

Hence when $\zeta > 1/\sqrt{2}$ the magnitude M decreases monotonically from M = 1 at u = 0 with increasing u. It follows that for $\zeta > 1/\sqrt{2}$ there is no resonant peak and the greatest value of M equals one.

The frequency at which M has a value of $1/\sqrt{2}$ is of special significane and is called the cut-off frequency ω_c. The signal frequencies above cut-off are greatly attenuated on passing through a system.

For feedback control system, the range of frequencies over which $M \ge 1/\sqrt{2}$ is defined as bandwidth ω_b. Control system being low-pass filters (at zero frequency M = 1), the bandwidth ω_b is equal to cut-off frequency ω_c.

In general the bandwidth of a control system indicates the noise-filtering characteristics of the system. Also, bandwidth gives a measure of the transient response.

The normalized bandwidth, $u_b = \frac{\omega_b}{\omega_n} = \left[1-2\zeta^2 + \sqrt{2-4\zeta^2+\zeta^4}\right]^{\frac{1}{2}}$

From the equation of u_b it is clear that u_b is a function of ζ alone. The graph between u_b and ζ is shown in fig 4.4.

The expression for the damped frequency of oscillation ω_d and peak overshoot M_p of the step response, for $0 \le \zeta \le 1$ are,

Damped frequency, $\omega_d = \omega_n\sqrt{1-\zeta^2}$ and Peak overshoot, $M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$

Comparison of the equation of M_r and M_p reveals that both are functions of only ζ.

The sketch of M_r and M_p for various value of ζ are shown in fig 4.5. The sketches reveals that a system with a given value of M_r must exhibit a corresponding value of M_p if subjected to a step input. For $\zeta > 1/\sqrt{2}$, the resonant peak M_r does not exist and the correlation breaks down. This is not a serious problem as for this range of ζ, the step response oscillations are well damped and M_p is negligible.

The comparison of the equation of ω_r and ω_d reveals that there exists a definite correlation between them. The sketch of ω_r/ω_d with respect to ζ is shown in fig 4.6.

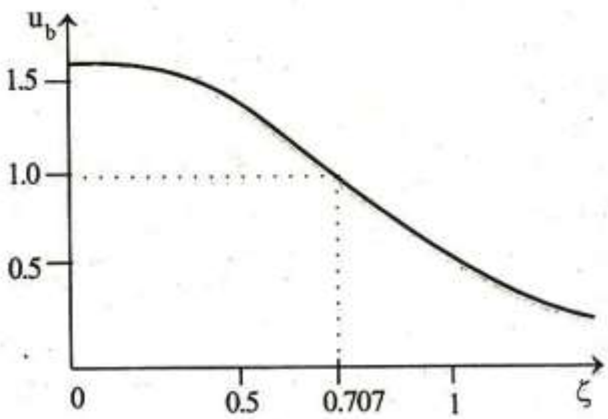
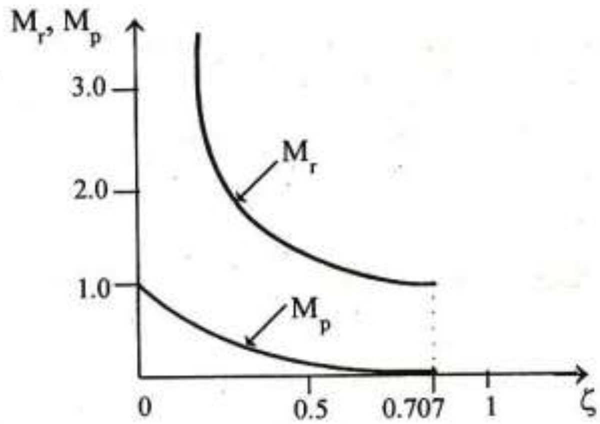
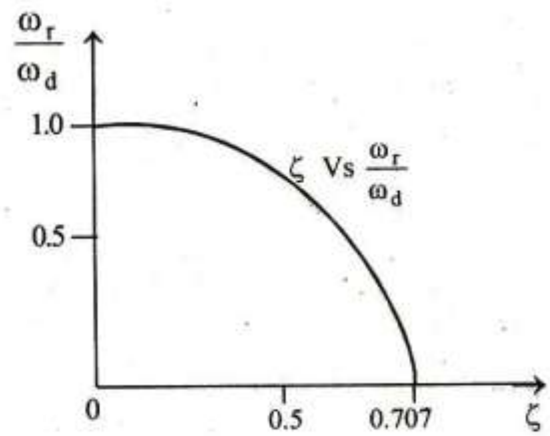


Fig 4.4 : Normalized bandwidth as a function of ζ.





3.Sketch the Bode Plot for the following transfer function and determine Phase Margin And Gain Margin(Nov/Dec2011) (May/June2015) (Nov/Dec 2017) (April/May18)

$$\boldsymbol{G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)}}$$

$On comparing the quadratic factor in the denominator of G(s) with standard form of quadratic factor we can estimate \zeta and \omega_n.$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 16s + 100$$

$On\ comparing\ we\ get, \omega_n^2 = 100$ $\qquad \therefore \boxed{\boldsymbol{\omega_n = 10\ rad/sec}}$

$$2\zeta\omega_n = 16 \Rightarrow \zeta = \frac{16}{2\omega_n} = \frac{16}{2\times 10} = 0.8 \qquad \therefore \boxed{\boldsymbol{\zeta = 0.8}}$$

$$\therefore G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)} = \frac{75(1+0.2s)}{100\, s\left(\frac{s^2}{100}+\frac{16s}{100}+1\right)} = \frac{0.75(1+0.2s)}{s(1+0.01s^2+0.16s)}$$

The sinusoidal transfer function G(jω)is obtained by replacing s by jω in G(s)

$$\therefore\ G(s) = \frac{0.75(1+j0.2\omega)}{j\omega(1+0.01(j\omega)^2+j0.16\omega)} = \frac{0.75(1+j0.2\omega)}{j\omega(1-0.01\omega^2+j0.16\omega)}$$

$$The\ corner\ frequencies\ are, \omega_{c1} = \frac{1}{0.2} = 5\, rad/sec\ \ and\ \omega_{c2} = \omega_n = 10\, rad/sec$$

$$\boxed{\boldsymbol{Note\text{: } For\ quadratic\ factor\ the\ corner\ frequency\ is\ \omega_n}}$$

<u>MAGNITUDE PLOT</u>

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{0.75}{j\omega}$	-	−20	
$1 + j0.2\omega$	$\omega_{c1} = \frac{1}{0.2} = 5$	20	$-20 + 20 = 0$
$\frac{1}{1 - 0.01\omega^2 + j0.16\omega}$	$\omega_{c2} = \omega_n = 10$	−40	$0 - 40 = -40$

Choose alow frequency ω_l *such that* $\omega_l < \omega_{c1}$ *and*
choose a high frequency ω_h *such that* $\omega_h > \omega_{c2}$
Let $\omega_l = 0.5\, rad/sec$ and $\omega_h = 20\, rad/sec$

$$At\ \omega = \omega_l, A = 20\ log\frac{0.75}{0.5} = \mathbf{3.5\ db}$$

$$At\ \omega = \omega_{c1}, A = 20\ log\frac{0.75}{5} = \mathbf{-16.5\ db}$$

$$At\ \omega = \omega_{c2}, A = \left[change\ in\ slopefrom\ \omega_{c1}\ to\ \omega_{c2} \times log\left(\frac{\omega_{c2}}{\omega_{c1}}\right)\right] + A_{(at\ \omega = \omega_{c1})} = 0 \times log\left(\frac{10}{5}\right) - 16.5 = \mathbf{-16.5\ db}$$

$$At\ \omega = \omega_{ch}, A = \left[change\ inslopefrom\ \omega_{c2}\ to\ \omega_{ch} \times log\left(\frac{\omega_{ch}}{\omega_{c2}}\right)\right] + A_{(at\ \omega = \omega_{c2})} = -40 \times log\left(\frac{20}{10}\right) - 16.5 = \mathbf{-56.2\ db}$$

PHASE PLOT

The phaseangleof $G(j\omega)$ *as a function of* ω *is given by,*

$$\phi = \angle G(j\omega) = \tan^{-1} 0.2\omega - 90 - \tan^{-1}\frac{0.16\omega}{1 - 0.01\omega^2}\ for\ \omega \le \omega_n$$

$$\phi = \angle G(j\omega) = \tan^{-1} 0.2\omega - 90 - \left(\tan^{-1}\frac{0.16\omega}{1 - 0.01\omega^2} + 180^\circ\right) for\ \ \omega > \omega_n$$

The phase angle of $G(j\omega)$ *are calculated for various values of* ω

TABLE-2

ω rad/sec	$\tan^{-1} 0.2\,\omega$ deg	$\tan^{-1}\frac{0.16\omega}{1 - 0.01\omega^2}$ deg	$\phi = \angle G(j\omega)$ deg	Points in phase plot
0.5	5.7	4.6	$-88.9 \approx -88$	e
1	11.3	9.2	$-87.9 \approx -88$	f
5	45	46.8	$-91.8 \approx -92$	g
10	63.4	90	$-116.6 \approx -116$	h
20	75.9	−46.8+180=133.2	$-147.3 \approx -148$	i
50	84.3	−18.4+180=161.6	$-167.3 \approx -168$	j
100	87.1	−92+180=170.8	$-173.7 \approx -174$	k

Let ϕ_{gc} *be the phase of* $G(j\omega)$ *at gain cross − over frequency,* $\omega_{gc.}$

$$We\ get, \phi_{gc} = 88^\circ\ \therefore Phase\ margin, \gamma = 180^\circ + \phi_{gc} = 180^\circ - 88^\circ = 92^\circ \Rightarrow \boxed{\boldsymbol{\gamma = 92^\circ}}$$

The phase plot crosses $180°$ *only at infinity. The* $|G(j\omega)|$ *at infinity is* $-\infty\ db$*. Hence gain margin is* $+\infty$*.*

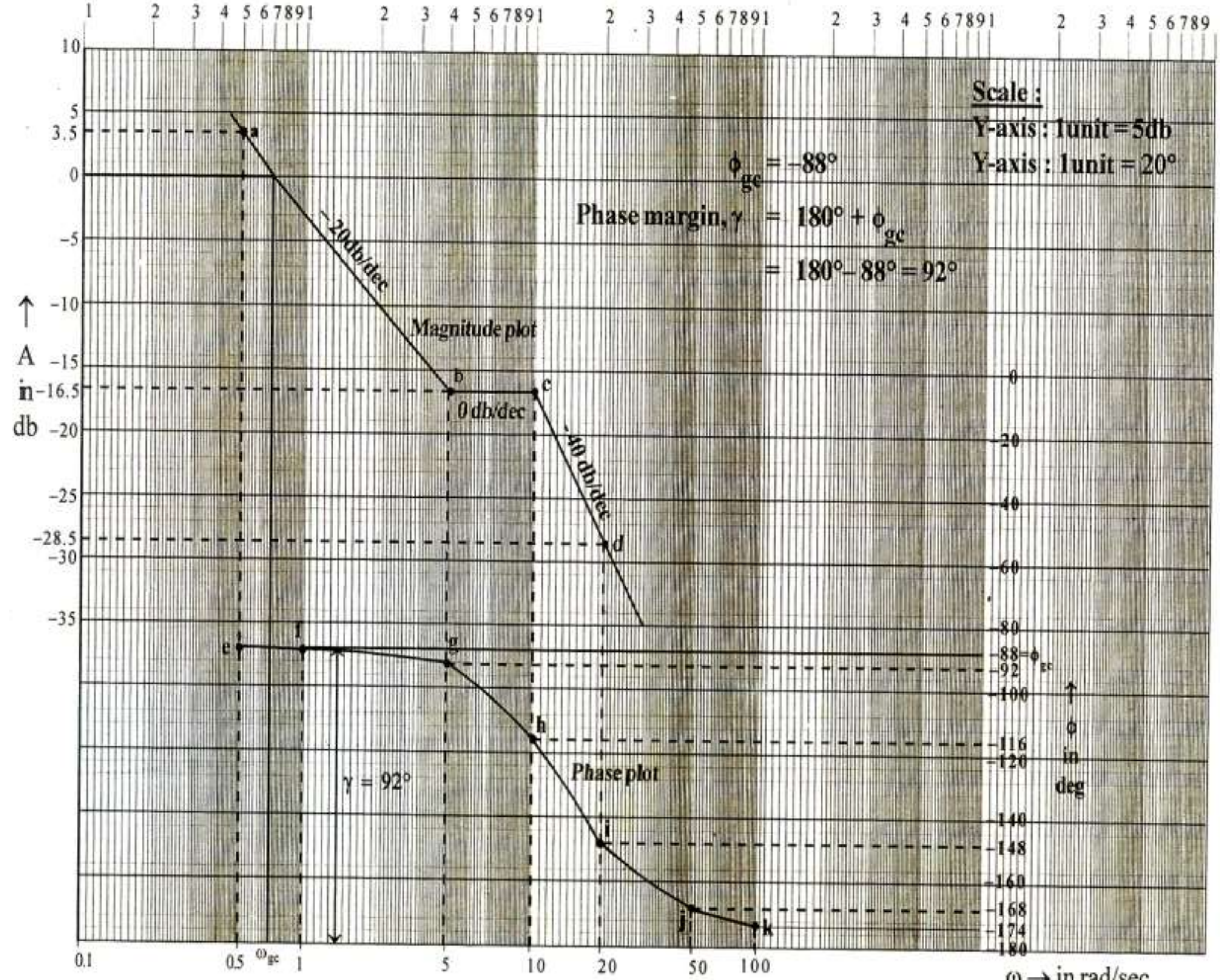


Fig 4.2.1: *Bode plot of transfer function,* $G(j\omega) = \dfrac{0.75\,(1 + j0.2\omega)}{j\omega\,(1 - 0.01\omega^2 + j0.16\omega)}$.

4. Plot the bode diagram for the following transfer function and obtain the gain and phase cross over frequencies. $G(s) = \frac{\mathbf{10}}{s(\mathbf{1+0.4}s)(\mathbf{1+0.1}s)}$ **(May /June 2013)(Nov/Dec 2014)(May/June2016,17)**

$$The\ corner\ frequencies\ are, \omega_{c1} = \frac{1}{0.4} = 2.5\, rad/sec\ , \omega_{c2} = \frac{1}{0.1} = 10\, rad/sec$$

<u>***Magnitude plot***</u>

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{10}{j\omega}$	-	– 20	
$\frac{1}{1+j0.4\omega}$	$\omega_{c1} = \frac{1}{0.4} = 2.5$	– 20	$-20-20 = -40$
$\frac{1}{1+j0.1\omega}$	$\omega_{c2} = \frac{1}{0.1} = 10$	– 20	$-40-20 = -60$

Choose alow frequency ω_l *such that* $\omega_l < \omega_{c1}$ *and*
choose a high frequency ω_h *such that* $\omega_h > \omega_{c2}$
Let $\omega_l = 0.1\, rad/sec$ and $\omega_h = 50\, rad/sec$

$$At\ \omega = \omega_l, A = 20\ log \frac{10}{0.1} \quad = \mathbf{40\ db}$$

$At\ \omega = \omega_{c1}, A = 20\ log \frac{10}{2.5} = \mathbf{12\ db}$

$At\ \omega = \omega_{c2}, A = \left[change\ in\ slope from\ \omega_{c1}\ to\ \omega_{c2} \times log\left(\frac{\omega_{c2}}{\omega_{c1}}\right)\right] + A_{(at\ \omega = \omega_{c1})} = -40 \times log\left(\frac{10}{2.5}\right) + 12 = \mathbf{-12\ db}$

$At\ \omega = \omega_{ch}, A = \left[change\ in\ slope from\ \omega_{c2}\ to\ \omega_{ch} \times log\left(\frac{\omega_{ch}}{\omega_{c2}}\right)\right] + A_{(at\ \omega = \omega_{c2})} = -60 \times log\left(\frac{50}{10}\right) - 12 = \mathbf{-54\ db}$

Draw the magnitude plot for $(0.1\, rad/sec\ , 40\ db)(2.5\, rad/sec, 12\ db)(1\ 0 rad/sec, 12\ db)$
$(50\, rad/sec, -54\ db)$

<u>PHASE PLOT</u>

The phase angle of $G(j\omega)$ *as a function of* ω *is given by,*

$$\phi = \angle G(j\omega) = -90 - \tan^{-1} 0.4\omega - \tan^{-1} 0.1\omega$$

TABLE-2

ω rad/sec	$\tan^{-1} 0.4\,\omega$ deg	$\tan^{-1} 0.1\,\omega$ deg	$\phi = \angle G(j\omega)$ deg	Points in phase plot
0.1	2.29	0.57	–92.86 ≈ –92	e
1	21.80	5.71	–117.5 ≈ –118	f
2.5	45.0	14.0	–149 ≈ –150	g
4	57.99	21.8	–169.79 ≈ –170	h
10	75.96	45.0	–210.96 ≈ –210	i
20	82.87	63.43	–236.3 ≈ –236	j

From the graph, the gain and phase crossover frequencies are found to be $5\, rad/sec$

$$\boxed{\boldsymbol{Gain\ \ crossover\ frequency =\ \ 5\, rad/sec}}$$

$$\boxed{\boldsymbol{Phase\ crossover\ frequency = 5\, rad/sec}}$$

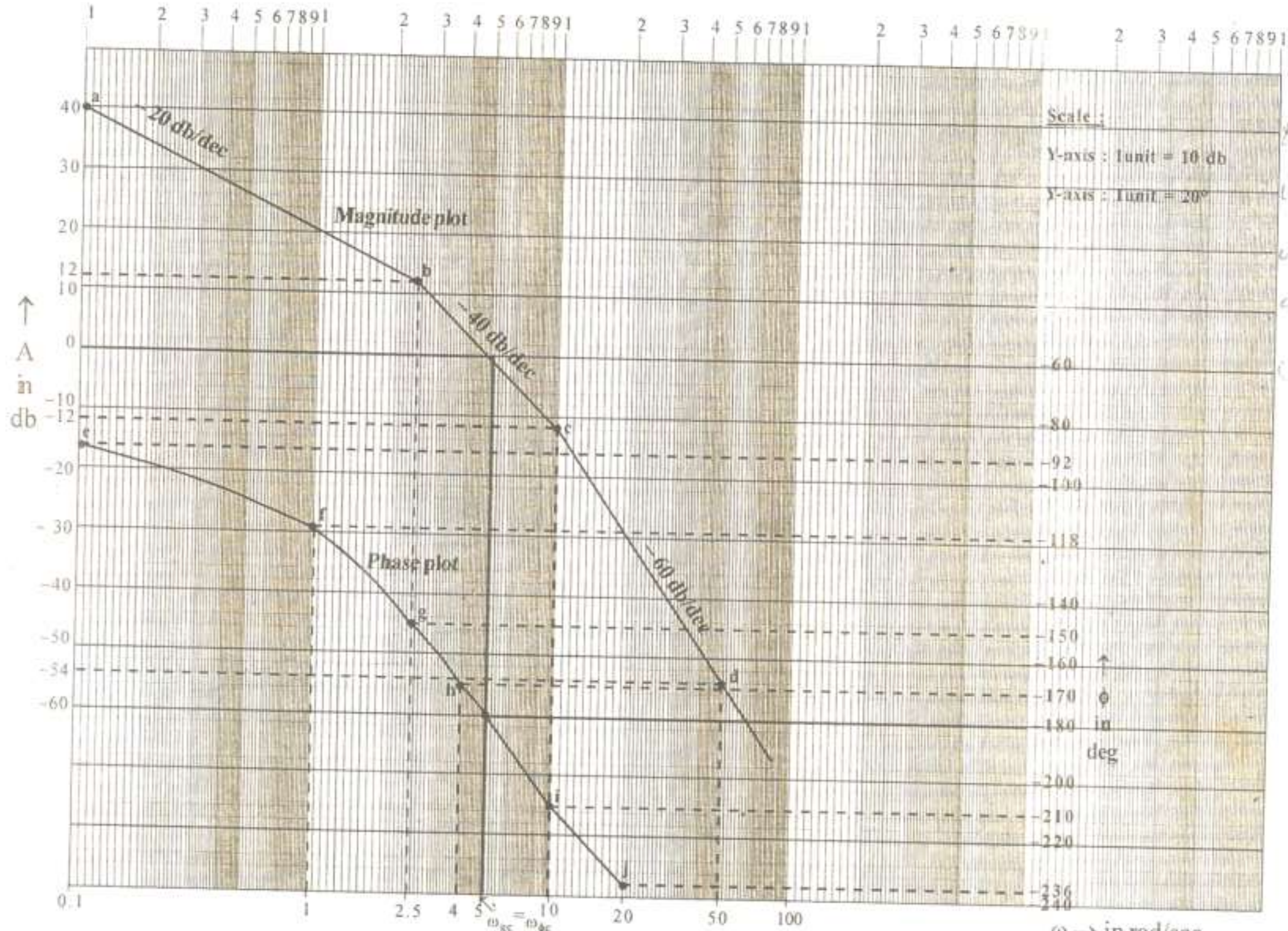


Fig 4.4.1: *Bode plot of transfer function,* $G(j\omega) = \dfrac{10}{j\omega\,(1 + j0.4\omega)\,(1 + j0.1\omega)}$

5. **Theopenlooptransferfunctionofunity feedbacksystemisgivenby $G(s)=1/s^2(1+s)(1+2s)$.sketchthepolarplotanddeterminethegainmarginand phase margin.(April/May 2011)(May2015)**

Given that, $G(s) = 1/s^2 (1+s)(1+2s)$

Put $s = j\omega$.

$$\therefore G(j\omega) = \frac{1}{(j\omega)2(1+j\omega)(1+j2\omega)}$$

$$The\ corner\ frequencies\ are, \omega_{c1} = \frac{1}{2} = 0.5\, rad/sec\,, \omega_{c2} = \frac{1}{1} = 1\, rad/sec$$

$$Magnitude\ equation, |G(j\omega)| = \frac{1}{\sqrt{\omega^4(1+\omega^2)(1+4\omega^2)}} = \frac{1}{\omega^2\sqrt{(1+\omega^2)(1+4\omega^2)}}$$

$$Phase\ angle\ equation, \angle G(j\omega) = -180 - \tan^{-1}\omega - \tan^{-1}2\omega$$

TABLE-1 : Magnitude and phase plot of G(jω) at various frequencies

ω rad/sec	0.45	0.5	0.55	0.6	0.65	0.7	0.75	1.0
|G(jω)|	3.3	2.5	1.9	1.5	1.2	0.97 ≈ 1	0.8	0.3
∠G(jω) deg	–246	–251	–256	–261	–265	–269	–273	–288

TABLE-2 : Real and imaginary part of G(jω)

ω rad/sec	0.45	0.5	0.55	0.6	0.65	0.7	0.75	1.0
$G_R(j\omega)$	–1.34	–0.81	–0.46	–0.23	–0.1	–0.02	0.04	0.09
$G_I(j\omega)$	3.01	2.36	1.84	1.48	1.2	1.0	0.8	0.29

From the graph,

$The\ -180° line\ cuts\ the\ plot\ at\ \infty, so\ |G(j\omega_{pc})| = \infty$

$$\therefore Gain\ margin\ , K_g = \frac{1}{|G(j\omega_{pc})|} = \frac{1}{\infty} = 0\ \boxed{\boldsymbol{Gain\ margin, K_g = 0}}$$

$The\ plot\ cuts\ the\ unity\ circle, draw\ the\ line\ from\ center\ through\ the\ meeting\ point\ of\ plot\ and\ unity\ circle, corresponding\ angle\ \phi_{gc} measured.\ \phi_{gc} = -270°$

$\therefore Phase\ margin, \gamma = 180° + \phi_{gc} \quad = 180° - 270° - 90°$

$$\boxed{\boldsymbol{Phase\ margin, \gamma = -90°}}$$

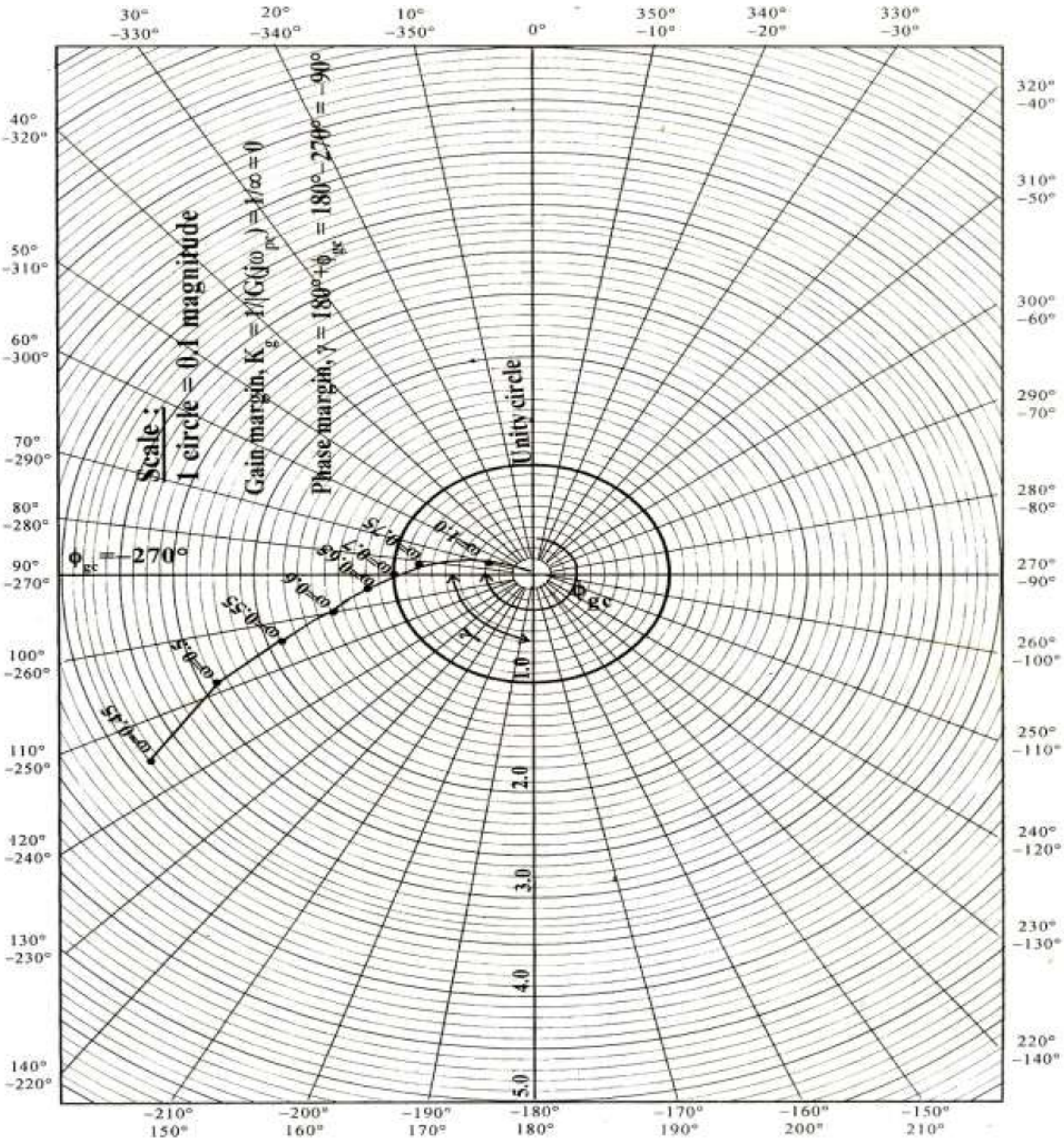


Fig 4.8.1: Polar plot of $G(j\omega) = 1/(j\omega)^2 (1+j\omega)(1+j2\omega)$, (using polar coordinates)

6. **The open loop transfer function of unity feedback system is given by $G(s) = \frac{1}{s(1+s)^2}$ Sketch the polar plot and determines the gainmargin and phase margin. (May/June 2016)**

$$Given that, G(s) = \frac{1}{s(1+s)^2} \; put \; s = j\omega, G(j\omega) = \frac{\mathbf{1}}{\boldsymbol{j\omega(1+j\omega)^2}} = \frac{\mathbf{1}}{\boldsymbol{j\omega(1+j\omega)(1+j\omega)}}$$

The corner frequency is $\omega_{c1} = 1 \; rad/sec.$

$$Magnitude\ equation, |G(j\omega)| = \frac{1}{\sqrt{\omega^2(1+\omega^2)(1+\omega^2)}} = \frac{1}{\omega(1+\omega^2)} = \frac{1}{\omega+\omega^3}$$

$$Phase\ angle\ equation, \angle G(j\omega) = -90° - 2\tan^{-1}\omega$$

TABLE-1: Magnitude and phase of G(jw) at various frequencies

ω rad/sec	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1
$\lvert G(j\omega)\rvert$	2.2	1.6	1.2	1	0.8	0.6	0.5	0.4
$\angle G(j\omega)$ deg	−134	−143	−151	−159	−167	−174	−180	−185

TABLE-2 : Real and imaginary part of G(jω) at various frequencies

ω rad/sec	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1
$G_R(j\omega)$	−1.53	−1.28	−1.05	−0.93	−0.78	−0.6	−0.5	−0.4
$G_I(j\omega)$	−1.58	−0.96	−0.58	−0.36	−0.18	0.06	0	0.03

From the graph,

$$Gain\ margin, K_g = \frac{1}{\left|G(j\omega_{pc})\right|} = \frac{1}{0.5} = 2 \qquad \boxed{\boldsymbol{Gain\ margin, K_g = 2}}$$

$$Phase\ margin, \gamma = 180° + \phi_{gc} = 180° - 159° \boxed{\boldsymbol{Phase\ margin, \gamma = 21°}}$$

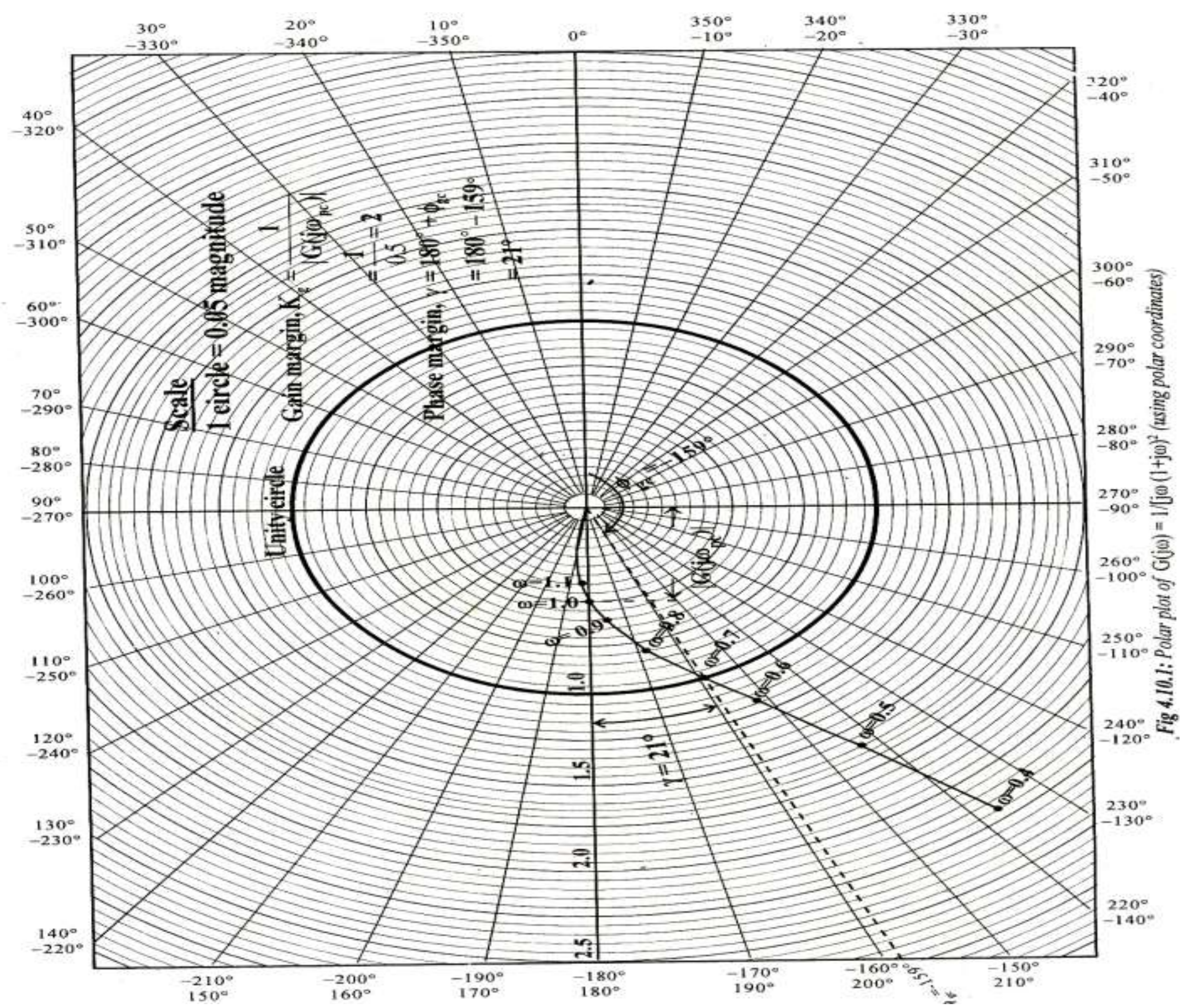


Fig 4.10.1: Polar plot of $G(j\omega) = 1/[j\omega(1+j\omega)^2]$ (using polar coordinates)

7. Consider a unity feedback system open loop transfer function $G(s) = \frac{K}{s(1+0.2s)(1+0.05s)}$ Sketch the polar plot and determine the value of K so that (i) Gain margin is 18db (ii) Phase margin is 60^0. (Nov/Dec 2014)

$$Given\ that, G(s) = \frac{\mathrm{K}}{\mathrm{s}(1+\mathrm{j}0.2\mathrm{s})(1+0.05\mathrm{s})}, \text{let K} = 1. \text{Put s} = \mathrm{j}\omega.$$

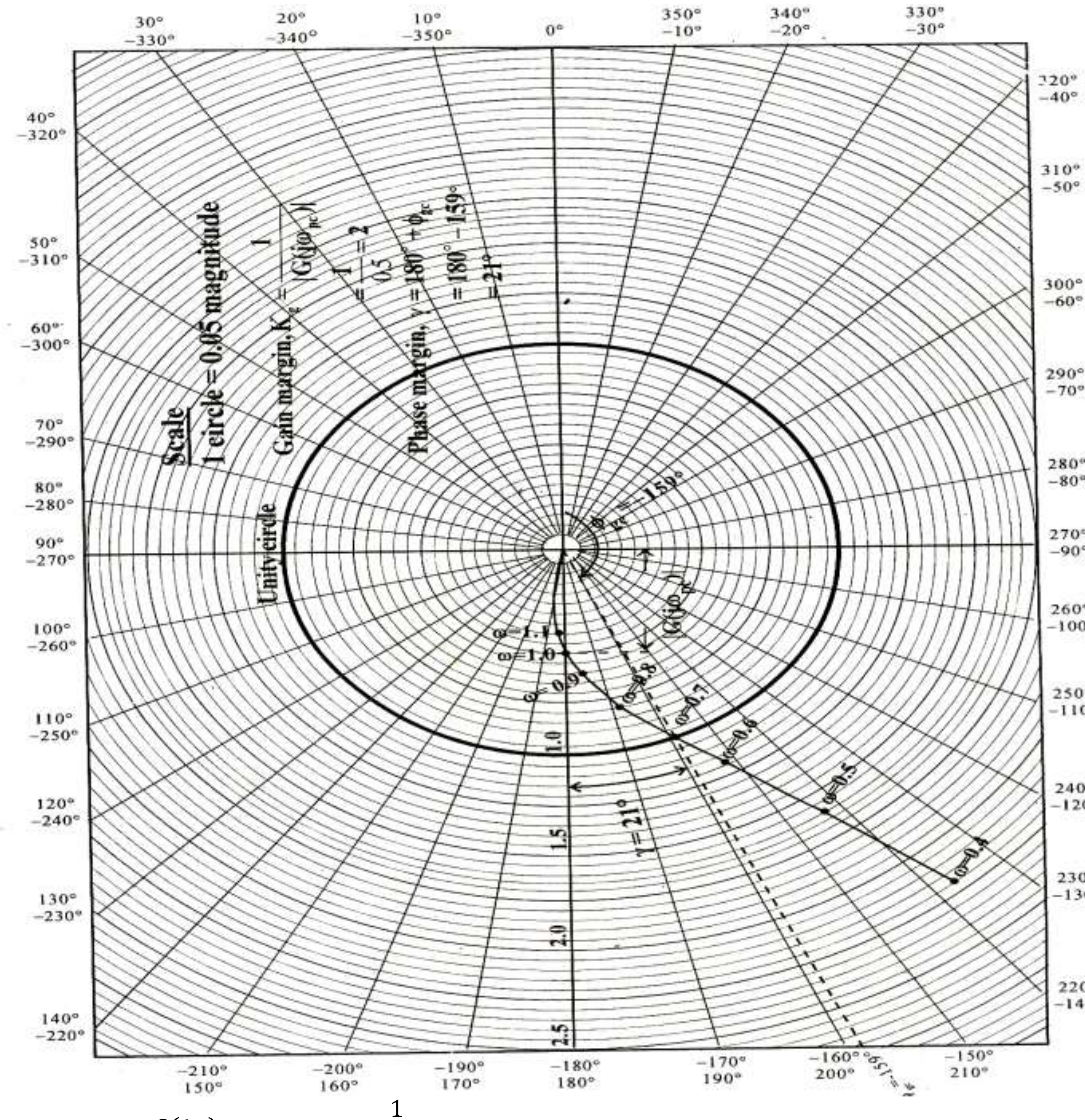


Fig 4.10.1: Polar plot of $G(j\omega) = 1/[j\omega(1+j\omega)^2$ (using polar coordinates)

$$\therefore G(j\omega) = \frac{1}{\mathrm{j}\omega(1+\mathrm{j}0.2\omega)(1+\mathrm{j}0.05\omega)}$$

$$\text{Corner frquencies are, } \omega_{c1} = \frac{1}{0.2} = 5\ \text{rad/sec}, \omega_{c2} = \frac{1}{0.05} = 20\ \text{rad/sec}$$

$$Magnitude equation, |G(j\omega)| = \frac{1}{\sqrt{\omega^2(1+0.4\omega^2)(1+0.025\omega^2)}}$$
$$= \frac{1}{\omega\sqrt{(1+0.4\omega^2)(1+0.025\omega^2)}}$$
$$Phase\ angle\ equation, \angle G(j\omega) = -90° - \tan^{-1} 0.2\omega - \tan^{-1} 0.05\omega$$

TABLE-1 : Magnitude and phase of G(jω) at various frequencies

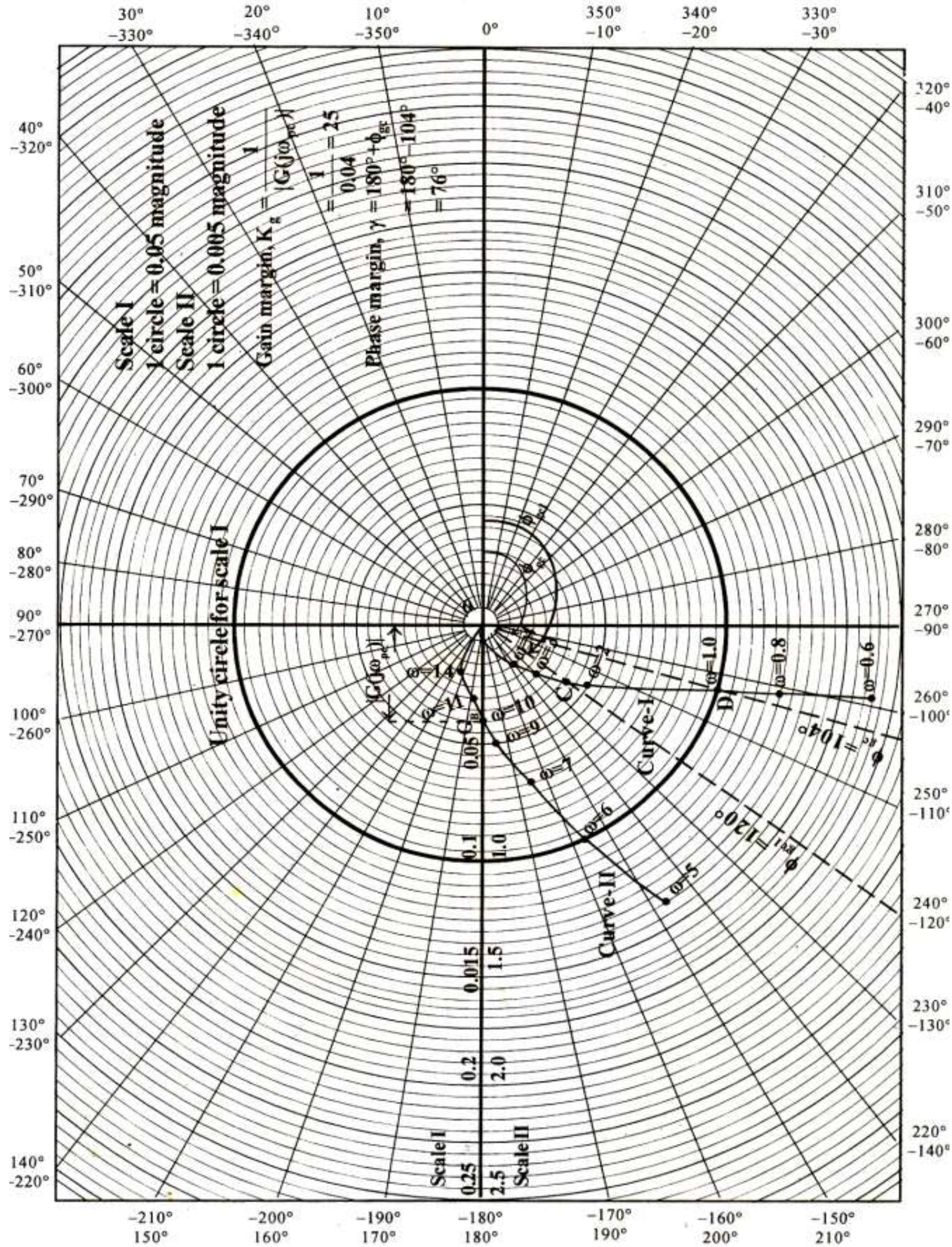
ω rad/sec	0.6	0.8	1	2	3	4
$\lvert G(j\omega)\rvert$	1.65	1.23	1.0	0.5	0.3	0.2
$\angle G(j\omega)$ deg	−98	−101	−104	−117.5	−129.4	−140

ω rad/sec	5	6	7	9	10	11	14
$\lvert G(j\omega)\rvert$	0.14	0.1	0.07	0.05	0.04	0.03	0.02
$\angle G(j\omega)$ deg	−149	−157	−164	−176	−180	−184	−195

TABLE-2 : Real and imaginary part of G(jω) at various frequencies

ω rad/sec	0.6	0.8	1	2	3	4
$G_R(j\omega)$	−0.23	−0.23	−0.24	−0.23	−0.19	−0.15
$G_I(j\omega)$	−1.63	−1.21	−0.97	−0.44	−0.23	−0.13

ω rad/sec	5	6	7	9	10	11	14
$G_R(j\omega)$	−0.120	−0.092	−0.067	−0.050	−0.04	−0.030	−0.019
$G_I(j\omega)$	−0.072	−0.039	−0.019	−0.0034	0	0.002	0.005



From the graph, eith $K = 1$,

Gain margin, $K_g\ 1/0.04 = 25$. *Gainmargin in db* $= 20\log 25 = 28\ db$. *Phase margin,* $\gamma = 76°$.

Case(i)

With $K = 1$, *let* $G(j\omega)$ *cut the* $-180°$ *line at Band gain corresponding to that point be* G_B. *From the polar plot* $G_B = 0.04$. *The gain margin of* 28*db with* $K = 1$ *has tobe reduced to* 18 *dband so* K *has to be increased to a value greater than one.*

$Let\ G_A be\ the\ gain\ at -180°\ for\ a\ gain\ margin\ of\ 18\ db.$

$$Now, 20\ log\frac{1}{G_A} = 18\ \Rightarrow log\frac{1}{G_A} = \frac{18}{20}\ \Rightarrow \frac{1}{G_A} = 10^{18/20} \Rightarrow G_A = \frac{1}{10^{18/20}} = 0.125$$

$$The\ value\ of\ K\ is\ given\ by, K = \frac{G_A}{G_B} = \frac{0.125}{0.04} = 3.125. \qquad \therefore \boxed{\boldsymbol{K = 3.125}}$$

<u>Case(ii)</u>

$With\ K = 1, the\ gain\ margin\ is\ 76°. This\ has\ to\ be\ reduced\ to\ 60°.$

$Hence\ gain\ has\ to\ be\ increased.$

$Let\ \phi_{gc2} be\ the\ phase of G(j\omega) for\ a\ phase\ margin\ of\ 60°.$

$\therefore 60° = 180° +\ \phi_{gc2}\ \Rightarrow\ \phi_{gc2} = 60° - 180° = -120°.$

$In\ the\ graph -120° line\ cut\ the\ plot\ at\ point\ C\ and\ cut\ the\ unity\ circle\ at\ point\ D.$

$Let\ G_c = Magnitude\ of\ G(j\omega)\ at\ point$ C.

$G_D = Magnitude\ of\ G(j\omega)\ at\ point\ D. From\ the\ graph,$

$$G_c = 0.425\ and\ \ G_D = 1. Now, K = \frac{G_D}{G_c} = \frac{1}{0.425} = 2.353 \qquad \therefore \boxed{\boldsymbol{K = 2.353}}$$

8. Given $G(s)=Ke^{-0.2s}/s(s+2)(s+8)$. Find K so that the system is stable with, (a) Gain margin equal to 2db, (b) Phase margin=45°. (April/May 2011)(Nov/Dec2012)

SOLUTION

Let us take K = 1, and convert the given transfer function to time constant form or bode form.

$$\therefore G(s)=\frac{e^{-0.2s}}{s\,(s+2)(s+8)}=\frac{e^{-0.2s}}{s\times 2\left(1+\frac{s}{2}\right)\times 8\left(1+\frac{s}{8}\right)}=\frac{0.0625\;e^{-0.2s}}{s\,(1+0.5s)(1+0.125s)}$$

The sinusoidal transfer function G(jw) is obtained by replacing s by jw G(s).

$$\therefore\; G(j\omega)=\frac{0.0625\;e^{-j0.2\omega}}{j\omega\,(1+j0.5\omega)\,(1+j0.125\omega)}$$

Note: $\left|0.0625\,e^{-j0.2\omega}\right| = 0.0625$ *and* $\angle(0.0625e^{-j0.2\omega}) = -0.2\omega$ *radians.*

MAGNITUDE PLOT

The corner frequencies are, $\omega_{c1}=\frac{1}{0.5}=2$ rad / sec and $\omega_{c2}=\frac{1}{0.125}=8$ rad / sec

The various terms of $G(j\omega)$ are listed in table-1 in the increasing order of their corner frequencies. Also the table shows the slope contributed by each term and the change in slope at the corner frequency.

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{0.0625}{j\omega}$	-	-20	
$\frac{1}{1+j0.5\omega}$	$\omega_{c1}=\frac{1}{0.5}=2$	-20	$-20-20=-40$
$\frac{1}{1+j0.125\omega}$	$\omega_{c2}=\frac{1}{0.125}=8$	-20	$-40-20=-60$

Let us calculate A at $\omega_l, \omega_{c1}, \omega_{c2}$ and ω_h.

$$\text{At } \omega = \omega_l,\ A = 20\log\left|\frac{0.0625}{j\omega}\right| = 20\log\frac{0.0625}{0.5} = -18\text{db}$$

$$\text{At } \omega = \omega_{c1},\ A = 20\log\left|\frac{0.0625}{j\omega}\right| = 20\log\frac{0.0625}{2} = -30\text{db}$$

$$\text{At } \omega = \omega_{c2},\ A = \left[\text{Slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(\text{at } \omega=\omega_{c1})} = -40\times\log\frac{8}{2} + (-30) = -54\text{db}$$

$$\text{At } \omega = \omega_h,\ A = \left[\text{Slope from } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A_{(\text{at } \omega=\omega_{c2})} = -60\times\log\frac{50}{8} + (-54) = -102\text{db}$$

Let the points a,b,c,d be the points corresponding to frequencies ω_l, ω_{cl}, ω_{c2} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose a scale of 1unit = 10db on y-axis. The frequencies are marked in decades from 0.01 to 100 rad/sec on logarithmic scale in x-axis. Fix the points a, b, c, and d on the graph. Join the points by straight line and mark the slope on the respective region.

PHASE PLOT

The phase angle of G(jω) as a function of ω is given by,

$$\phi = -0.2\omega \times \frac{180^\circ}{\pi} - 90^\circ - \tan^{-1}0.5\omega - \tan^{-1}0.125\omega$$

The phase angle of G(jω) are calculated for various values of ω and listed in table-2.

TABLE-2

ω rad/sec	$-0.2\ \omega\ (180^0/\pi)$ deg	$\tan^{-1} 0.5\ \omega$ deg	$\tan^{-1} 0.125\ \omega$ deg	$\phi = \angle G(j\omega)$ deg	Point in phase plot
0.01	−0.1145	0.2864	0.0716	−90.4 ≈ −90	e
0.1	−1.145	2.862	0.716	−94.7 ≈ −94	f
0.5	−5.7	14	3.6	−113.3 ≈ −114	g
1	−11.4	26	7.12	−134.4 ≈ −134	h
2	−22.9	45	14	−171.9 ≈ −172	i
3	−34.37	56.30	20.56	−201.2 ≈ −202	j
4	−45.84	63.43	26.57	−225.8 ≈ −226	k

On the same semilog graph sheet choose a scale of 1unit = 20° on the y-axis on the right side of the semilog graph sheet. Mark the calculated phase angle on the graph sheet. Join the points by smooth curve.

The magnitude and phase plot are shown in fig 4.3.1.

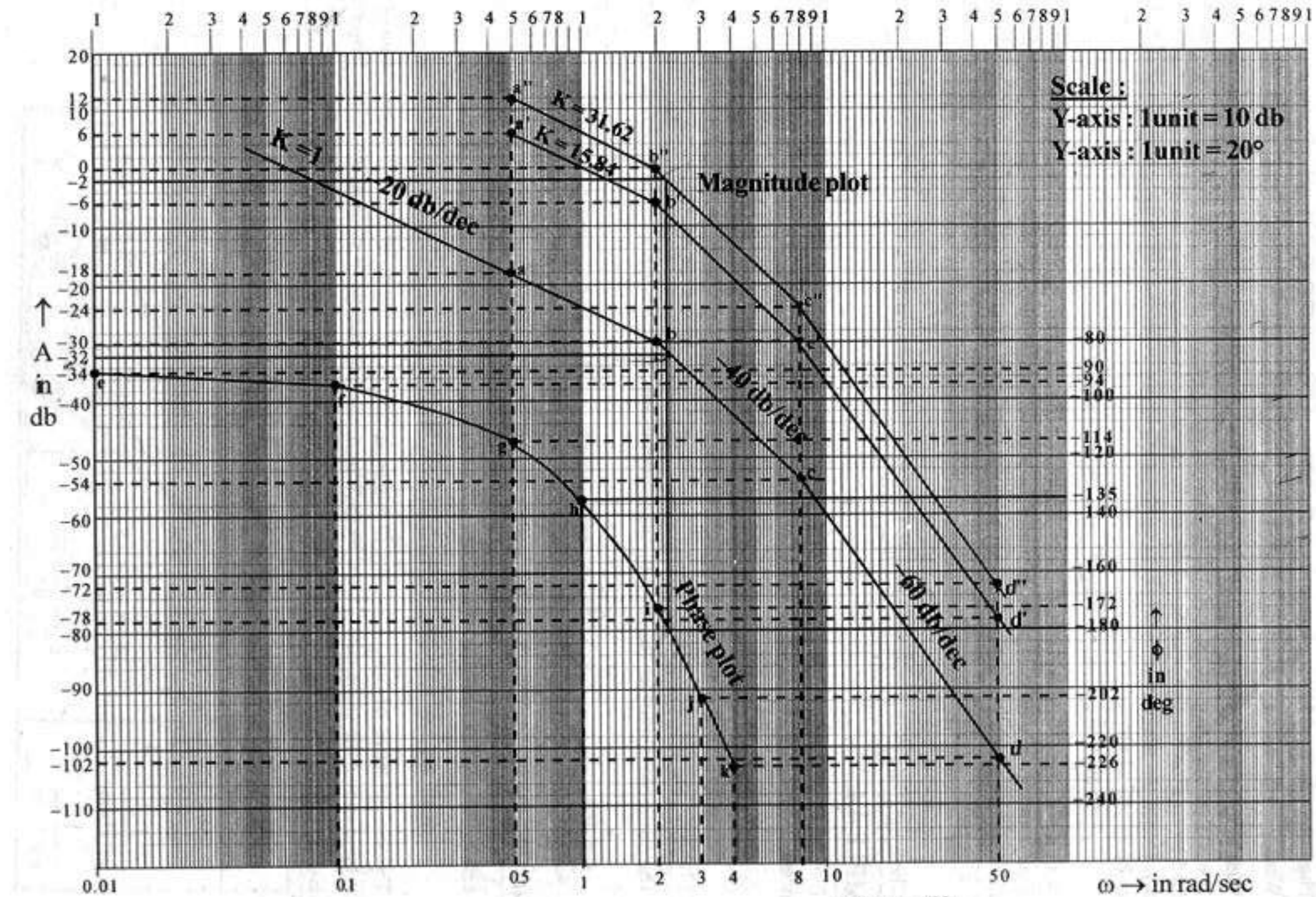


***Fig 4.3.1:** Bode plot of transfer function,* $G(j\omega) = \dfrac{0.0625\,K\,e^{-j0.2\omega}}{j\omega(1+j0.5\omega)(1+j0.125\omega)}$

CALCULATION OF K

Phase margin, $\gamma = 180^0 + \phi_{gc}$, where ϕ_{gc} is the phase of $G(j\omega)$ at $\omega = \omega_{gc}$.

When $\gamma = 45°$, $\phi_{gc} = \gamma - 180^0 = 45° - 180^0 = -135°$.

With K = 1, the db gain at $\phi = -135°$ is –24 db. This gain should be made zero to have to PM of 45°. Hence to every point of magnitude plot a db gain of 24 db should be added. The corrected magnitude plot is obtained by shifting the plot with K = 1 by 24 db upwards. The magnitude correction is independent of frequency. Hence the magnitude of 24 db is contributed by the term K. The value of K is calculated by equating 20logK to 24 db.

$$\therefore 20 \log K = 24 \quad ; \quad K = 10^{24/20} \quad ; \quad K = 15.84$$

With K = 1, the gain margin = –(–32) = 32 db. But the required gain margin is 2 db. Hence to every point of magnitude plot a db gain of 30 db should be added. This addition of 30 db shifts the plot upwards. The magnitude correction is independent of frequency. Hence the magnitude of 30 db is contributed by the term K. The value of K is calculated by equating 20logK to 30 db.

$$\therefore 20 \log K = 30 \quad ; \quad K = 10^{30/20} \quad ; \quad K = 31.62$$

The magnitude plot with K = 15.84 and 31.62 are shown in fig 4.3.1.

9. Forthefunction,G(s)=5(1+2s)/(1+4s)(1+0.25s),sketchthebodemagnitudeplot.

(May/June2012)

For the function, $G(s) = \frac{5(1+2s)}{(1+4s)(1+0.25s)}$, draw the bode plot.

SOLUTION

The sinusoidal transfer function $G(j\omega)$ is obtained by replacing s by $j\omega$ in G(s).

$$\therefore G(j\omega) = \frac{5(1+j2\omega)}{(1+j4\omega)(1+j0.25\omega)}$$

MAGNITUDE PLOT

The corner frequencies are, $\omega_{c1} = \frac{1}{4} = 0.25$ rad / sec, $\omega_{c2} = \frac{1}{2} = 0.5$rad / sec, $\omega_{c3} = \frac{1}{0.25} = 4$ rad / sec

The various terms of $G(j\omega)$ are listed in table-1 in the increasing order of their corner frequencies. Also the table shows the slope contributed by the each term and the change in slope at the corner frequency.

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$. Let $\omega_l = 0.1$ rad/sec and $\omega_h = 10$ rad/sec.

Let $A = |G(j\omega)|$ in db and let us calculate A at ω_l, ω_{c1}, ω_{c2}, ω_{c3} and ω_h.

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/deg
5	-	0	-
$\frac{1}{1+j4\omega}$	$\omega_{c1} = \frac{1}{4} = 0.25$	-20	$0-20=-20$
$1+j2\omega$	$\omega_{c2} = \frac{1}{2} = 0.5$	20	$-20+20=0$
$\frac{1}{1+j0.25\omega}$	$\omega_{c3} = \frac{1}{0.25} = 4$	-20	$0-20=-20$

At $\omega = \omega_l$, $A = |G(j\omega)| = 20 \log 5 = +14$ db

At $\omega = \omega_{c1}$, $A = |G(j\omega)| = 20 \log 5 = +14$ db

$$\text{At } \omega = \omega_{c2},\ A = \left[\text{Slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(\text{at } \omega=\omega_{c1})} = -20 \times \log\frac{0.5}{0.25} + 14 = +8\text{db}$$

$$\text{At } \omega = \omega_{c3},\ A = \left[\text{Slope from } \omega_{c2} \text{ to } \omega_{c3} \times \log\frac{\omega_{c3}}{\omega_{c2}}\right] + A_{(\text{at } \omega=\omega_{c2})} = 0 \times \log\frac{4}{0.5} + 8 = +8\text{db}$$

$$\text{At } \omega = \omega_h,\ A = \left[\text{Slope from } \omega_{c3} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c3}}\right] + A_{(\text{at } \omega=\omega_{c3})} = -20 \log\frac{10}{4} + 8 = 0 \text{ db}$$

Let the points a, b, c, d and e be the points correponding to frequencies ω_l, ω_{c1}, ω_{c2}, ω_{c3} and ω_h espectively on the magnitude plot. In a semilog graph sheet choose a scale of 1unit = 5 db on y axis. The frequencies are marked in decades from 0.1 to 100 rad/sec on logarithmic scales on x-axis. Fix the points a, b, c, d and e on the graph. Join the points by a straight line and mark the slope in the respective region.

PHASE PLOT

The phase angle of $G(j\omega)$, $\phi = \tan^{-1}(2\omega) - \tan^{-1}(4\omega) - \tan^{-1}(0.25\omega)$

The phase angle of $G(j\omega)$ are calculated for various values of ω and listed the table-2.

TABLE-2

ω	$\tan^{-1} 2\omega$ deg	$\tan^{-1} 4\omega$ deg	$\tan^{-1} 0.25\omega$ deg	$\phi = \angle G(j\omega)$	Points in phase plot
0.1	11.3	21.8	1.43	$-11.93 \approx -12$	f
0.25	26.56	45.0	3.5	$-21.94 \approx -22$	g
0.5	45.0	63.43	7.1	$-25.53 \approx -26$	h
2	75.96	82.87	26.56	$-33.47 \approx -33$	i
4	82.87	86.42	45.0	$-48.55 \approx -49$	j
10	87.13	88.56	68.19	$-69.62 \approx -70$	k
50	89.42	89.71	85.42	$-85.71 \approx -86$	l

On the same semilog graph sheet choose a scale of 1unit = 10° on y-axis on the right side of the semilog graph sheet. Mark the calculated phase angle on the graph sheet, Join the points by a smooth curve. The magnitude and phase plots are shown in fig 4.6.1.

10. Draw the polar plot for the following transfer function G(s)=1/s(1+s)(1+2s).

(Nov/Dec2011, 2016,May-17)(Nov/Dec 2017)

Given that, $G(s) = 1/s(1+s)(1+2s)$

Put $s = j\omega$.

$$\therefore G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+j2\omega)}$$

The corner frequencies are $\omega_{c1} = 1/2 = 0.5$ rad/sec and $\omega_{c2} = 1$ rad/sec. The magnitude and phase angle of $G(j\omega)$ are calculated for the corner frequencies and for frequencies around corner frequencies and tabulated in table-1. Using polar to rectangular conversion, the polar coordinates listed in table-1 are converted to rectangular coordinates and tabulated in table-2. The polar plot using polar coordinates is sketched on a polar graph sheet as shown in fig 4.7.1. The polar plot using rectangular coordinates is sketched on an ordinary graph sheet as shown in fig 4.7.2.

$$G(j\omega) = \frac{1}{(j\omega)(1+j\omega)(1+j2\omega)} = \frac{1}{\omega\angle 90^\circ \sqrt{1+\omega^2}\angle\tan^{-1}\omega\sqrt{1+4\omega^2}\angle\tan^{-1}2\omega}$$

$$= \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}}\angle -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

$$\therefore |G(j\omega)| = \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}} = \frac{1}{\omega\sqrt{1+4\omega^2+\omega^2+4\omega^4}} = \frac{1}{\omega\sqrt{1+5\omega^2+4\omega^4}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

TABLE-1 : Magnitude and phase of G(jω) at various frequencies

ω rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1.0
$\lvert G(j\omega)\rvert$	2.2	1.8	1.5	1.2	0.9	0.7	0.3
$\angle G(j\omega)$ deg	−144	−150	−156	−162	−171	−179.5 ≈ −180	−198

TABLE-2 : Real and imaginary part of G(jω) at various frequencies

ω rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1.0
$G_R(j\omega)$	−1.78	−1.56	−1.37	−1.14	−0.89	−0.7	−0.29
$G_I(j\omega)$	−1.29	−0.9	−0.61	−0.37	−0.14	0	0.09

RESULT

Gain margin, $K_g = 1.4286$

Phase margin, $\gamma = +12^\circ$

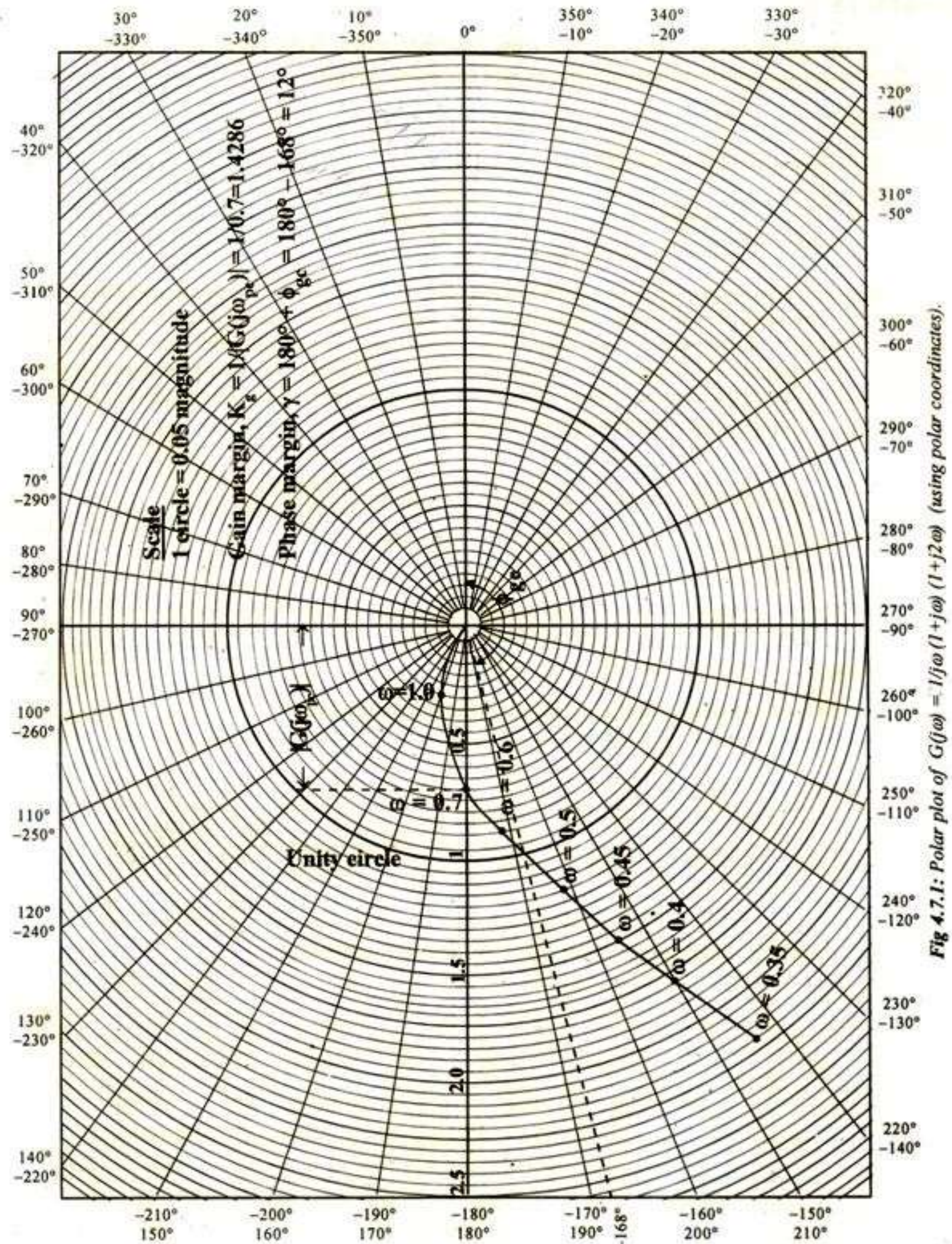


Fig 4.7.1: Polar plot of $G(j\omega) = 1/j\omega\,(1+j\omega)\,(1+j2\omega)$ (using polar coordinates).

11.For the following T.F draw the Bode plot and obtain Gain cross over frequency (wgc) , Phase cross over frequency , Gain Margin and Phase Margin.(Nov/Dec-2016)

G(s) = 20 / [s (1+3s) (1+4s)]

Solution:

The sinusoidal T.F of G(s) is obtained by replacing s by jw in the given T.F
G(jw) = 20 / [jw (1+j3w) (1+j4w)]

Corner frequencies:
wc1= 1/4 = 0.25 rad /sec ;
wc2 = 1/3 = 0.33 rad /sec

Choose a lower corner frequency and a higher Corner frequency
wl= 0.025 rad/sec ;
wh = 3.3 rad / sec

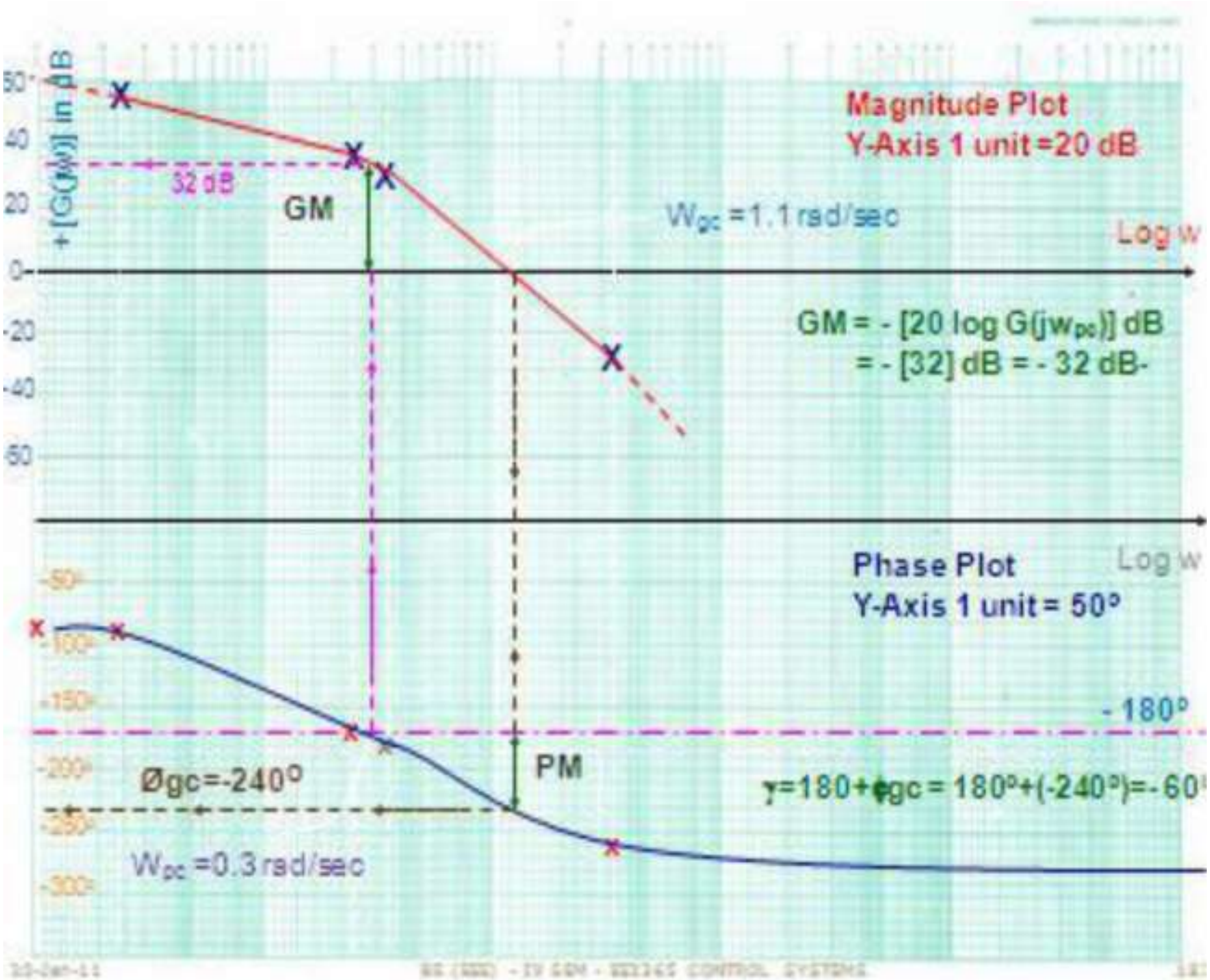
Calculation of Gain (A) (MAGNITUDE PLOT)
A @ wl ; A= 20 log [20 / 0.025] = 58 .06 dB
A @ wc1 ; A = [Slope from wl to wc1 x log (wc1 / wl] + Gain (A)@wl
= - 20 log [0.25 / 0.025] + 58.06
= 38.06 dB
A @ wc2 ; A = [Slope from wc1 to wc2 x log (wc2 / wc1] + Gain (A)@ wc1
= - 40 log [0.33 / 0.25] + 38
= 33 dB
A @ wh ; A = [Slope from wc2 to wh x log (wh / wc2] + Gain (A) @ wc2
= - 60 log [3.3 / 0.33] + 33
=-27 dB

Calculation of Phase angle for different values of frequencies [PHASE PLOT]

$Ø = -90^{O} - \tan^{-1} 3w - \tan^{-1} 4w$

When

Frequency in rad / sec	**Phase angles in Degree**
w=0	$Ø= -90^{0}$
w = 0.025	$Ø= -99^{0}$
w = 0.25	$Ø= -172^{0}$
w = 0.33	$Ø= -188^{0}$
w =3.3	$Ø= -259^{0}$
w =∞	$Ø= -270^{0}$



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Calculations of Gain cross over frequency

The frequency at which the dB magnitude is Zero

wgc = 1.1 rad / sec

- **Calculations of Phase cross over frequency**

The frequency at which the Phase of the system is - 180o
wpc = 0.3 rad / sec

- Gain Margin

  The gain margin in dB is given by the negative of dB magnitude of G(jw) at phase cross over frequency

  GM = - { 20 log [G(jwpc)] = - { 32 } = -32 dB

- **Phase Margin**

  Ґ = 180^0+ Øgc= 180^0 + (- 2400^o) = -60^0

- **Conclusion**

  For this system GM and PM are negative in values. Therefore the system is unstable in nature.

12. Explain theuseofnichol'schart toobtainclosed loopfrequencyresponsefromopenloop frequencyresponseofaunityfeedbacksystem.(Nov/Dec2015)

TheG(jω)H(jω)locus,i.e., thepolarplotissketchedonthestandardnichol'schart.Themeetingpointof G(jω)H(jω)locuswithconstantMcirclegivesthemagnitudeoftheclosedloopsystemandthemeeting pointwithNcirclegivesthephaseofthe closedloopsystem.

Constant magnitude locithat is M -circles and constant phase angle locithat is N-circles are the fundamentalcomponentsin designingtheNicholschart.TheconstantMandconstantNcirclesinG(jω) planecanbeusedfortheanalysisanddesignofcontrolsystems.Howeverthe constantMandconstantN circlesingainphaseplanearepreparedforsystemdesignandanalysisastheseplotssupplyinformationwithlessmanip ulations.Gainphaseplaneisthegraphhavinggainindecibelalongtheordinate(vertical axis)andphaseanglealongtheabscissa(horizontalaxis).TheMandNcirclesofG(jω)in thegainphase planearetransformedintoMandNcontoursin rectangularco-ordinates.

ApointontheconstantMlociinG(jω)planeistransferredtogainphaseplanebydrawingthevector directedfromtheoriginofG(jω)planeto aparticularpointonMcircleandthenmeasuringthelengthin dbandangleindegree.ThecriticalpointinG(jω),planecorrespondstothepointofzerodecibeland-180°in thegain phaseplane.PlotofMandNcirclesin gain phaseplaneisknownasNicholschart/plot. The Nichols plot is named after the American engineer N.B Nichols who formulated this plot. Compensators canbe designed usingNichols plot. Nicholsplottechnique ishowever alsoused in designingofdcmotor.Thisisusedin signalprocessingandcontroldesign.Nyquistplotin complexplane showshowphaseoftransferfunctionandfrequencyvariationofmagnitudearerelated.Wecanfindout thegainandphaseforagivenfrequency.Angleofpositiverealaxisdeterminesthephaseanddistance fromoriginofcomplexplanedeterminesthegain.There are someadvantagesofNicholsplotin control systemengineering.Theyare:Gainandphasemargincanbedeterminedeasilyandalsographically. Closedloopfrequencyresponseisobtainedfromopenloopfrequencyresponse.Gainofthesystemcan beadjustedtosuitablevalues.Nicholschartprovidesfrequencydomainspecifications.Therearesome drawbacksofNicholsplotalso.

UsingNicholsplotsmallchangesingaincannotbeencounteredeasily..ConstantMandNcirclesin the Nicholschartaredeformedintosquashedcircles.ThecompleteNicholschartextendsforthephaseangle ofG(jω)from0to-360°.

Theregionof G(jω)usedforanalysisofsystemsinbetween-90°to-270°.Thesecurvesrepeatafter every180°interval.IftheopenloopT.FofunityfeedbacksystemG(s)isexpressed as

$G(s)= G(s)e^{j\theta}= G(s)[\cos\theta+ j\sin\theta]$

Closedloop T.F is $M(s)= G(s)/1+G(s)$

Substituting $s=j\omega$ in the above eq. frequency functions are,

$G(j\omega)= G(j\omega)[\cos\theta+ j\sin\theta]$ and $M(j\omega)= Me^{j\varphi}= G(j\omega)/1+ G(j\omega)$

Eliminating $G(j\omega)$ from above two eq. $M=G(j\omega)/\sqrt{G(j\omega)^2 + 2G(j\omega)\cos\theta+1}$ and $\varphi= \tan^{-1}\sin\theta/G(j\omega)+\cos\theta$.

22. Construct bode plot for the system whose open loop transfer function is given below and determine (i) the gain margin (ii) the phase margin and (iii) closed loop system stability. (Nov/Dec2015)

$$G(j\omega)H(j\omega) = \frac{4}{j\omega(1+j0.5\omega)(1+j0.08\omega)}.$$

The corner frequencies are $\omega_{c1} = \frac{1}{0.5} = 2$ rad/sec. $\omega_{c2} = \frac{1}{0.08} = 12.5$ rad/sec.

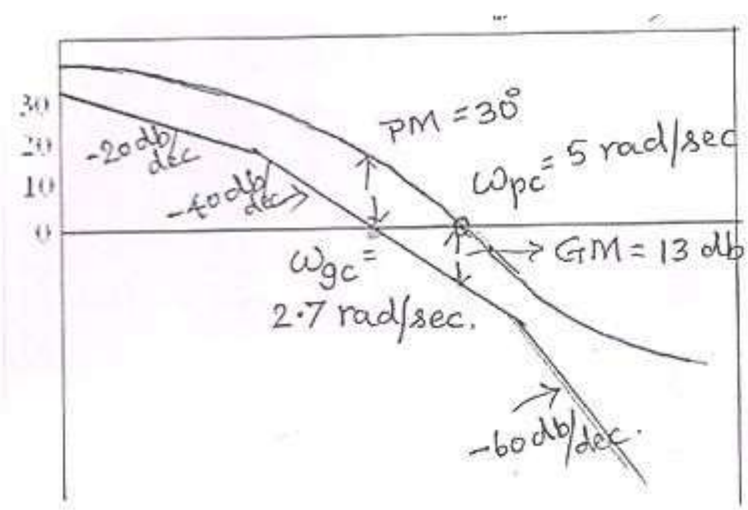
$$20\log_{10}|G(j\omega)H(j\omega)| = 20\log_{10}4 - 20\log_{10}\omega - 20\log_{10}|(1+j0.5\omega| - 20\log_{10}|(1+j0.08\omega|$$

Draw the Bode plots of individual term in the gain equation. Then add all the gains to get the following Bode plot.

To draw the Phase angle plot:

$$\angle G(j\omega)H(j\omega) = 0^\circ - 90^\circ - \tan^{-1}0.5\omega - \tan^{-1}0.08\omega$$

ω (rad/sec)	1	2	8	10	20	50
$\angle G(j\omega)H(j\omega)$	-121°	-144°	-198°	-207°	-234°	-252°



$\omega_{gc} = 2.7$ rad/sec.; $\omega_{pc} = 5$ rad/sec.

Gain Margin (GM) = -(Gain) at ω_{pc} = 13 db ;

Phase Margin (PM) = $180^\circ + \angle G(j\omega)H(j\omega)$ at ω_{gc} = $180^\circ + (-150^\circ) = 30^\circ$

The GM and PM both are positive; therefore, the closed-loop system is stable.

23. Construct the polar plot and determine the gain and phase margin of a unity feedback control system whose open loop transfer function is, $G(s)= (1+0.2s)(1+0.025s) / s^3(1+0.005s)(1+0.001s)$.

(April/May 2018)

SOLUTION

Given that $G(s) = \dfrac{(1+0.2s)(1+0.025s)}{s^3(1+0.005s)(1+0.001s)}$

$$\therefore G(j\omega) = \frac{(1+j0.2\omega)(1+j0.025\omega)}{(j\omega)^3(1+j0.005\omega)(1+j0.001\omega)}$$

$$= \frac{\sqrt{1+(0.2\omega)^2}\angle\tan^{-1}0.2\omega\sqrt{1+(0.025\omega)^2}\angle\tan^{-1}0.025\omega}{\omega^3\angle 270^\circ\sqrt{1+(0.005\omega)^2}\angle\tan^{-1}0.005\omega\sqrt{1+(0.001\omega)^2}\angle\tan^{-1}0.001\omega}$$

$$|G(j\omega)| = \frac{\sqrt{1+(0.2\omega)^2}\sqrt{1+(0.025\omega)^2}}{\omega^3\sqrt{1+(0.005\omega)^2}\sqrt{1+(0.001\omega)^2}}$$

$$\angle G(j\omega) = \tan^{-1}0.2\omega + \tan^{-1}0.025\omega - 270^\circ - \tan^{-1}0.005\omega - \tan^{-1}0.001\omega$$

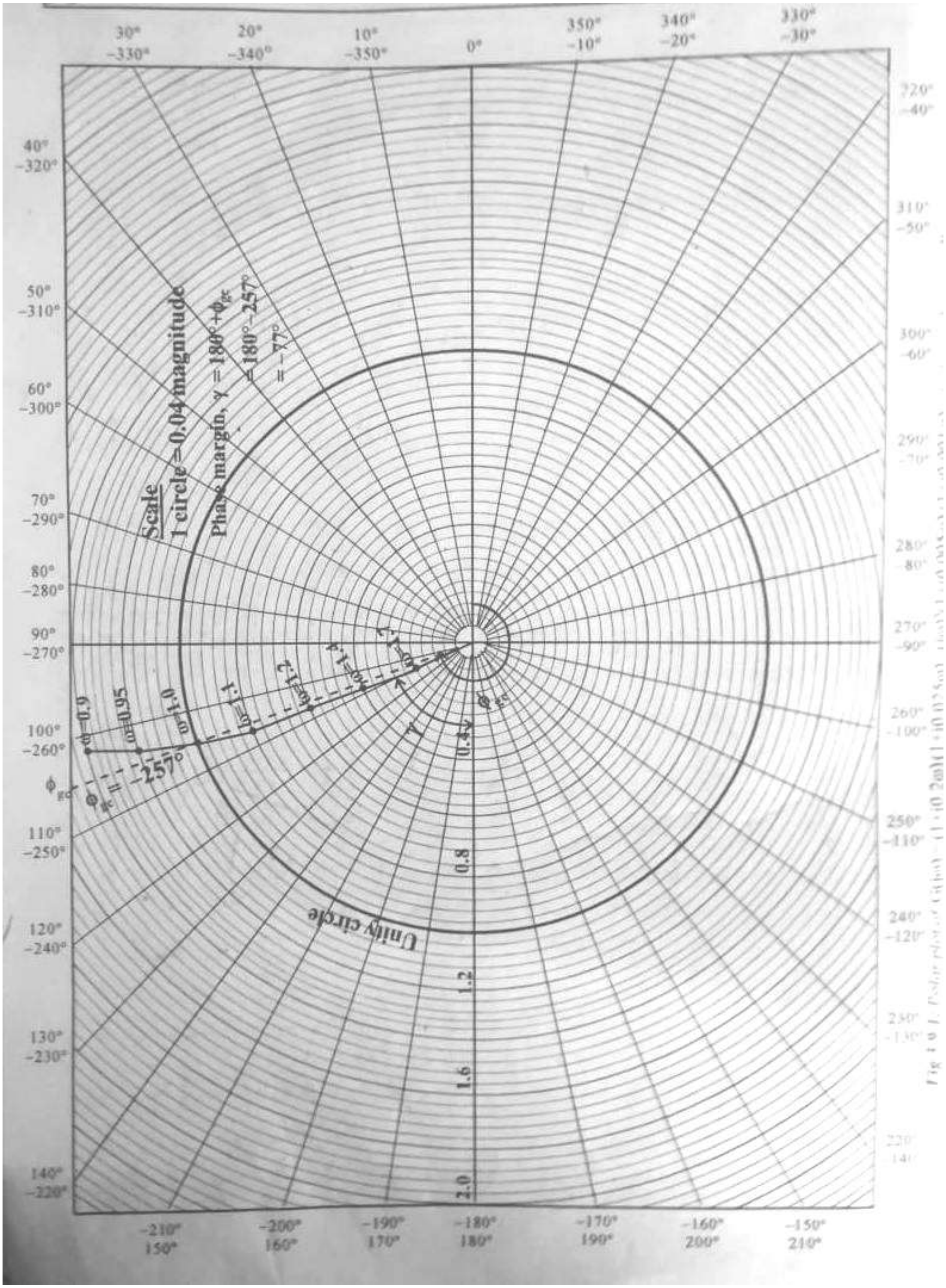
The magnitude and phase angle of $G(j\omega)$ are calculated for various frequencies and listed in table-1. Using the polar to rectangular conversion, the polar coordinates listed in table-1 are converted to rectangular coordinates and tabulated in table-2. The polar plot using polar coordinates is sketched on a polar graph sheet as shown in fig 3.9.1. The polar plot using rectangular coordinates is sketched on an ordinary graph sheet as shown in fig 3.9.2.

TABLE-1 : Magnitude and phase of $G(j\omega)$

ω, rad/sec	0.9	0.95	1.0	1.1	1.2	1.4	1.7
$\lvert G(j\omega)\rvert$	1.4	1.2	1.0	0.8	0.6	0.4	0.2
$\angle G(j\omega)$, deg	−259	−258	−257	−256	−255	−253	−249

TABLE-2 : Real and imaginary part of $G(j\omega)$

ω, rad/sec	0.9	0.95	1.0	1.1	1.2	1.4	1.7
$G_R(j\omega)$	−0.27	−0.25	−0.22	−0.19	−0.16	−0.12	−0.07
$G_I(j\omega)$	1.37	1.17	0.97	0.78	0.58	0.38	0.19


Scale
1 circle = 0.04 magnitude
Phase margin, γ = 180° + φgc
= 180° − 257°
= −77°
Unity circle
ω=0.9
ω=0.95
ω=1.0
ω=1.1
ω=1.2
ω=1.4
ω=1.7
−257°
φgc
0.4
0.8
1.2
1.6
2.0

PART A

1. Write the necessary and sufficient condition for stability in routh stability criterion.

(May/June 2013) What are the necessary conditions for stability? (Nov/Dec 2017)

The necessary and sufficient condition for system to be stable is all the terms in the first column of Routh array must have same sign. There should not be any sign change in the first column of Routh array.

2. What are the advantages of Nyquist plot? (Nov/Dec 2012)

- It gives same information about absolute stability as provided by Routhcriterion.
- It also indicates relative stability giving the values of GM and PM.
- Information regarding frequency response can be obtained.
- Very useful for analyzing conditionally stable systems.

3. Give any two limitations of Routh criterion. (Nov/Dec 2011),(Nov/Dec 2012)

- It is valid only for real coeffients of the characteristic equation.
- It does not provide exact locations of the closed loop poles in left half of the s-plane.
- It does not suggest methods of stabilizing an unstable system.
- Applicable only to linear systems.

4. Define BIBO stability.

A linear relaxed system is said to have BIBO stability if every bounded(finite) input results in a bounded(finite) output.

5. Define phase margin. (May/June 2014)

The phase marginγ is that amount of additional phase lag at gain cross-over frequency, ω_{gc} required to bring the system to the verge of instability. It is given by, $180° + \emptyset gc, where \emptyset gc$ is the phase of G(jω) at the gain cross over frequency.

6. What is meant by relative stability? (May/June 2014)

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It indicates the closeness of the system to stable region.it is an indication of the strength or degree of stability.

7. What is characteristics equation? (May/June 2016,17)

Thedenominator polynomial of C(S)/R(S)is the characteristics equation of the system.

8. Define Nyquist stability criterion.(May/June 2016,17)

If G(s)H(s)-contour in the G(s)H(s)-plane corresponding to Nyquistcontourin s-plane encircles the point -1+j0 in the anti-clockwise direction as many times as the number of right half s-plane poles of G(s)H(s). Then the closed loop system is stable.

9. What is principle of argument?

The principle of argument states that let F(s) be an analytic function and if an arbitrary closed contour in the clockwise direction is chosen in the s-plane so that F(s) is analytic at every pointof the contour. Then corresponding F(s)-plane mapped in the F(s) plane will encircle.

10. What is impulse response?

The impulse response of a system is the inverse Laplace transforms of the system transfer function.

11. How are the roots of characteristics equation are related to stability? (Nov/Dec 2015)

If the roots of characteristics equation has positive real part then the impulse response of the system is not bounded (the impulse response will be infinite as $t\rightarrow\infty$). Hence the system will be unstable. If the roots have negative real parts then the impulse response is bounded (the impulse response becomes 0 as $t\rightarrow\infty$). Hence the system will be stable.

12. What is quadrantal symmetry?

The symmetry of roots with respect to both real and imaginary axis is called quadrantalsymmetry.

13. What will be the nature of impulse response if the roots of characteristic equation are lying on left half of s-plane?

When the roots are lying on the real axis on the right half of s-plane. The response is exponentially increasing. When the roots are complex conjugate and lying on the right half of s-plane, the response is oscillatory with exponentially increasing amplitude.

14. What is relation between stability and coefficient of characteristic polynomial?

If the coefficients of characteristic polynomial are negative or zero, then some of roots lies on right half of s-plane. Hence the system is unstable. If the coefficients of characteristic polynomial are positive and if no coefficient is zero then there is a possibility of the system to be stable provided all the roots are lying on left half of s-plane.

15. What is auxillary polynomial?

In the construction of routh array a row of all zero indicates the existence of an even polynomial as a factor of the given characteristic equation. In an even polynomial the exponents of s are even integers or zero only. This even polynomial factor is called auxillarypolynomial.the coefficients of auxillary polynomial are given by the elements of the row just above the row of all zeros.

16. What is the basis for the selection of aparticular compensator for a system?

(Nov/Dec 2015)

When mainly transient response is to be improved, a lead compensator is chosen. When steady state response is to be improved, while nearly preserving the transient response, a lag compensator is chosen. When both the transient and steady state response are to be improved, a lag-lead compensator is chosen.

17. What are the two notations of the system stability to be satisfied for a linear time invariant system to be stable? (Dec-16)

A linear time- invariant system is stable if the following two notions of system stability are satisfied.

i) When the system is by a bounded input, the output is bounded.

ii)In the absence of the input, the output tends towards zero irrespective of initial conditions.

18. Why frequency domain compensation is normally carried out using the bode plots? Dec-16

Because they are easier to draw and modify. Also the gain adjustment can be conveniently carried out and also the error constants are always clearly in evident in the bode plots.

19.State Routh stability criterion.(April/May2018)

Routh criterionstates that the necessary aand sufficient condition for stability is that all of the elelments in the first column of routh array be positive.if this condition is not met, the system is unstable and the number of sign changes in the elements of first coulumn of routh array corresponds to the number of roots of characteristic equation in the right half of the s-plane.

20. Give the need for lag, lead and lag-lead compensation. (Nov/Dec 2017)

Lag compensation:

When a system is stable and does not satisfy the steady state performance specifications then lag compensation can be employed .

Lead compensation:

When the system is stable /unstable and requires imorovement in transient state response then the lead compensation is employed.

Lag-lead compensation:

When improvements in both steady state and transient response are rrequired.

PART B

1) Using Routh criterion, determine the stability of the system represented by the characteristic equation, $S^4 + 8s^3 + 18s^2 + 16s + 5 = 0$. Comment on the location of the roots of characteristic equation?(Nov/Dec-16,May-17)

SOLUTION

The characteristic equation of the system is, $s^4+8s^3+18s^2+16s+5=0$.

The given characteristic equation is 4^{th} order equation and so it has 4 roots. Since the highest power of s is even number, form the first row of routh array using the coefficients of even powers of s and form the second row using the coefficients of odd powers of s.

s^4	:	1	18	5	.... Row-1
s^3	:	8	16		.... Row-2

The elements of s^3 row can be divided by 8 to simplify the computations.

s^4	:	1	18	5	.... Row-1
s^3	:	1	2		.... Row-2
s^2	:	16	5		.... Row-3
s^1	:	1.7			.... Row-4
s^0	:	5			.... Row-5

Column-1

$$s^2 : \frac{1\times 18-2\times 1}{1} \quad \frac{1\times 5-0\times 1}{1}$$

$$s^2 : \; 16 \quad 5$$

$$s^1 : \frac{16\times 2-5\times 1}{16}$$

$$s^1 : \; 1.6875 \approx 1.7$$

$$s^0 : \frac{1.7\times 5-0\times 16}{1.7}$$

$$s^0 : \; 5$$

Onexamining the elements of first column of routh array it is observed that all the elements are positive and there is no sign change. Hence all the roots are lying on the left half of s-plane and the system is stable.

RESULT;

1. Stable system

2. All the four roots are lying on the left half of s-plane.

2). Construct routh array and determine the stability of the system whose characteristic equation is $S^6+2s^5+8s^4+12s^3+20s^2 16s+16=0$. All so determine the number of rootslying on right half of s-plane,lefthalf of s-plane and on imaginaryaxis.(May/June 2013),(May/June 2012),

SOLUTION

The characteristic equation of the system is, $s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$.

The given characteristic polynomial is 6^{th} order equation and so it has 6 roots. Since the highest power of s is even number, form the first row of routh array using the coefficients of even powers of s and form the second row using the coefficients of odd powers of s.

s^6	:	1	8	20	16	.... Row-1
s^5	:	2	12	16		.... Row-2

The elements of s^5 row can be divided by 2 to simplify the calculations.

s^6	:	1	8	20	16	....Row-1
s^5	:	1	6	8		....Row-2
s^4	:	1	6	8		....Row-3
s^3	:	0	0			....Row-4
s^3	:	1	3			....Row-4
s^2	:	3	8			....Row-5
s^1	:	0.33				....Row-6
s^0	:	8				....Row-7

Column-1

On examining the elements of 1st column of routh array it is observed that there is no sign change. The row with all zeros indicate the possibility of roots on imaginary axis. Hence the system is limitedly or marginally stable.

The auxiliary polynomial is,

$$s^4+6s^2+8=0$$

Let, $s^2=x$

$$\therefore x^2+6x+8=0$$

The roots of quadratic are, $x=\dfrac{-6\pm\sqrt{6^2-4\times 8}}{2}$

$$=-3\pm 1=-2 \text{ or } -4$$

The roots of auxiliary polynomial is,

$$s=\pm\sqrt{x}=\pm\sqrt{-2} \text{ and } \pm\sqrt{-4}$$
$$=+j\sqrt{2},-j\sqrt{2},+j2 \text{ and } -j2$$

$$s^4 : \frac{1\times 8-6\times 1}{1} \quad \frac{1\times 20-8\times 1}{1} \quad \frac{1\times 16-0\times 1}{1}$$

s^4 : 2 12 16

divide by 2

s^4 : 1 6 8

$$s^3 : \frac{1\times 6-6\times 1}{1} \quad \frac{1\times 8-8\times 1}{1}$$

s^3 : 0 0

The auxiliary equation is, $A = s^4+6s^2+8$. On differentiating A with respect to s we get,

$$\frac{dA}{ds}=4s^3+12s$$

The coefficients of $\frac{dA}{ds}$ are used to form s^3 row.

s^3 : 4 12

divide by 4

s^3 : 1 3

$$s^2 : \frac{1\times 6-3\times 1}{1} \quad \frac{1\times 8-0\times 1}{1}$$

s^2 : 3 8

$$s^1 : \frac{3\times 3-8\times 1}{3}$$

s^1 : 0.33

$$s^0 : \frac{0.33\times 8-0\times 3}{0.33}$$

s^0 : 8

The roots of auxiliary polynomial are also roots of characteristic equation. Hence 4 roots are lying on imaginary axis and the remaining two roots are lying on the left half of s-plane.

RESULT

1. The system is limitedly or marginally stable.
2. Four roots are lying on imaginary axis and remaining two roots are lying on left half of s-plane.

3). Construct routh array and determine the stability of the system represented by the characteristic equation $s^5+s^4+2s^3+2s^2+3s+5=0$. Comment on the location of the roots of characteristicequation?

From the second row using the coefficients of even powers of S.

$$
\begin{array}{llccl}
s^5 & : & 1 & 2 \quad 3 & \text{.... Row-1} \\
s^4 & : & 1 & 2 \quad 5 & \text{.... Row-2} \\
s^3 & : & \epsilon & -2 & \text{.... Row-3} \\
s^2 & : & \dfrac{2\epsilon+2}{\epsilon} & 5 & \text{.... Row-4} \\
s^1 & : & \dfrac{-(5\epsilon^2+4\epsilon+4)}{2\epsilon+2} & & \text{.... Row-5} \\
s^0 & : & 5 & & \text{.... Row-6}
\end{array}
$$

$$s^3 : \frac{1\times 2-2\times 1}{1} \quad \frac{1\times 3-5\times 1}{1}$$

$$s^3 : \quad 0 \qquad -2$$

Replace 0 by ϵ

$$s^3 : \quad \epsilon \qquad -2$$

$$s^2 : \frac{\epsilon\times 2-(-2\times 1)}{\epsilon} \quad \frac{\epsilon\times 5-0\times 1}{\epsilon}$$

$$s^2 : \frac{2\epsilon+2}{\epsilon} \qquad 5$$

$$s^1 : \frac{\frac{2\epsilon+2}{\epsilon}\times(-2)-(5\times\epsilon)}{\frac{2\epsilon+2}{\epsilon}}$$

$$s^1 : \frac{-(5\epsilon^2+4\epsilon+4)}{2\epsilon+2}$$

$$s^0 : \frac{\frac{-(5\epsilon^2+4\epsilon+4)}{2\epsilon+2}\times 5-0\times\frac{2\epsilon+2}{\epsilon}}{\frac{-(5\epsilon^2+4\epsilon+4)}{2\epsilon+2}}$$

$$s^0 : 5$$

On letting $\epsilon \to 0$, we get

$$
\begin{array}{lllcl}
 & s^5 & : & \boxed{1} \quad 2 \quad 3 & \text{.... Row-1} \\
 & s^4 & : & 1 \quad 2 \quad 5 & \text{.... Row-2} \\
 & s^3 & : & 0 \quad -2 & \text{.... Row-3} \\
 & s^2 & : & \infty \quad 5 & \text{.... Row-4} \\
+\text{ to }- \;/\; -\text{ to }+ & s^1 & : & -2 & \text{.... Row-5} \\
 & s^0 & : & 5 & \text{.... Row-6}
\end{array}
$$

Column-1

On observing the elements of first column of routh array, it is found that there are two sign changes. Hence two roots are lying on the right half of s-plane and the system is unstable. The remaining three roots are lying on the left half of s-plane.

RESULT

1. The system is unstable.
2. Two roots are lying on right half of s-plane and three roots are lying on left half of s-plane.

4). Byrouth stability criterion determine the stability of the system represented by the characteristic equation $9s^5-20s^4+10s^3-s^2-9s-10=0$ .Comment on the location of the roots of characteristicequation?

SOLUT ION

The characteristic polynomial of the system is, $9s^5-20s^4+10s^3-s^2-9s-10=0$

On examining the coefficients of the characteristic polynomial, it is found that some of the coefficients are negative and so some roots will lie on the right half of s-plane. Hence the system is unstable. The routh array can be constructed to find the number of roots lying on right half of s-plane.

The given characteristic polynomial is 5th order equation and so it has 5 roots. Since the highest power of s is odd number, form the first row of routh array using the coefficients of odd powers of s and form the second row using the coefficients of even powers of s.

s^5	:	9	10	−9	.... Row-1
s^4	:	−20	−1	−10	.... Row-2
s^3	:	9.55	−13.5		.... Row-3
s^2	:	−29.3	−10		.... Row-4
s^1	:	−16.8			.... Row-5
s^0	:	−10			.... Row-6

Column-1

$$s^3: \frac{-20\times10-(-1)\times9}{-20} \quad \frac{-20\times(-9)-(-10)\times9}{-20}$$

s^3: 9.55 −13.5

$$s^2: \frac{9.55\times(-1)-(-13.5)\times(-20)}{9.55} \quad \frac{9.55\times(-10)}{9.55}$$

s^2: −29.3 −10

$$s^1: \frac{-29.3\times(-13.5)-(-10)\times9.55}{-29.3}$$

s^1: −16.8

$$s^0: \frac{-16.8\times(-10)}{-16.8}$$

s^0: −10

By examining the elements of Ist column of routh array it is observed that there are three sign changes and so three roots are lying on the right half of s-plane and the remaining two roots are lying on the left half of s-plane.

RESULT

1. The system is unstable.
2. Three roots are lying on right half of s-plane and two roots are lying on left half of s-plane.

5). The characteristic polynomial of a system is,$s^7+9s^6+24s^5+24s^4+24s^3+24s^2+23s+15=0$. Determine the locationof roots on s-plane and stability of the system?

SOLUTION

METHOD-I

The characteristic equation is, $s^7+9s^6+24s^5+24s^4+24s^3+24s^2+23s+15=0$.

The given characteristic polynomial is 7th order equation and so it has 7 roots. Since the highest power of s is odd number, form the first row of array using the coefficients of odd powers of s and form the second row using the coefficients of even powers of s as shown below.

s^7	:	1	24	24	23	.... Row-1
s^6	:	9	24	24	15	.... Row-2

Divide s^6 row by 3 to simplify the computations.

s^7	:	1	24	24	23	.... Row-1
s^6	:	3	8	8	5	.... Row-2
s^5	:	1	1	1		.... Row-3
s^4	:	1	1	1		.... Row-4
s^3	:	0	0			.... Row-5
s^3	:	2	1			.... Row-5
s^2	:	0.5	1			.... Row-6
s^1	:	−3				.... Row-7
s^0	:	1				.... Row-8

Column-I

$$s^5: \frac{3\times24-8\times1}{3} \quad \frac{3\times24-8\times1}{3} \quad \frac{3\times23-5\times1}{3}$$

s^5: 21.33 21.33 21.33

Divide by 21.33

s^5: 1 1 1

$$s^4: \frac{1\times8-1\times3}{1} \quad \frac{1\times8-1\times3}{1} \quad \frac{1\times5-0\times3}{1}$$

s^4: 5 5 5

Divide by 5

s^5: 1 1 1

$$s^3: \frac{1\times1-1\times1}{1} \quad \frac{1\times1-1\times1}{1}$$

s^3: 0 0

Since we get a row of zeros, there exists an even polynomial, the even polynomial is nothing but, the auxiliary polynomial.

The auxiliary polynomial is,

$$s^4 + s^2 + 1 = 0$$

Divide the characteristic equation by auxiliary polynomial to get the quotient polynomial.

$$\begin{array}{l|l}
 & s^3 + 9s^2 + 23s + 15 \quad \text{(Quotient Polynomial)} \\
\hline
s^4 + s^2 + 1 & s^7 + 9s^6 + 24s^5 + 24s^4 + 24s^3 + 24s^2 + 23s + 15 \\
\text{Even} & \underline{s^7_{(-)} \quad {}_{(-)}+ s^5 \quad {}_{(-)}+ s^3} \\
\text{Polynomial)} & 9s^6 + 23s^5 + 24s^4 + 23s^3 + 24s^2 + 23s + 15 \\
 & \underline{{}_{(-)}9s^6 + \quad {}_{(-)}+ 9s^4 \quad {}_{(-)}+ 9s^2} \\
 & 23s^5 + 15s^4 + 23s^3 + 15s^2 + 23s + 15 \\
 & \underline{{}_{(-)}23s^5 \quad {}_{(-)}+ 23s^3 \quad {}_{(-)}+ 23s} \\
 & 15s^4 \quad + 15s^2 \quad + 15 \\
 & \underline{{}_{(-)}15s^4 \quad {}_{(-)}+ 15s^2 \quad {}_{(-)}+ 15} \\
 & 0
\end{array}$$

The characteristic polynomial can be expressed as a product of quotient polynomial and auxiliary polynomial.

$$\therefore s^7 + 9s^6 + 24s^5 + 24s^4 + 24s^3 + 24s^2 + 23s + 15 = 0$$

$$\Downarrow$$

$$\underbrace{(s^4 + s^2 + 1)}_{\text{Even polynomial}} \underbrace{(s^3 + 9s^2 + 23s + 15)}_{\text{Quotient polynomial}} = 0$$

The routh array is constructed for quotient polynomial as shown below.

$$\begin{array}{lcll}
s^3 & : & 1 & 23 \\
s^2 & : & 9 & 15
\end{array}$$

Divide s^2 row by 3,

$$\begin{array}{lcll}
s^3 & : & 1 & 23 \\
s^2 & : & 3 & 5 \\
s^1 & : & 21.33 & \\
s^0 & : & 5 &
\end{array}$$

Column-1

$$s^1 : \frac{3 \times 23 - 5 \times 1}{3}$$

$$s^1 : \ 21.33$$

$$s^0 : \frac{21.33 \times 5 - 0 \times 3}{21.33}$$

$$s^0 : \ 5$$

The elements of column-1 of quotient polynomial are all positive and there is no sign change. Hence all the roots of quotient polynomial are lying on the left half of s-plane. To determine the stability the roots of auxiliary polynomial should be evaluated.

The auxiliary equation is, $s^4 + s^2 + 1 = 0$.

Put, $s^2 = x$ in the auxiliary equation. $\quad \therefore s^4 + s^2 + 1 = x^2 + x + 1 = 0$

The roots of quadradic are, $x = \dfrac{-1 \pm \sqrt{1-4}}{2} = -\dfrac{1}{2} \pm j\dfrac{\sqrt{3}}{2} = 1\angle 120° \text{ or } 1\angle -120°$

$$\begin{aligned}
\text{But } s^2 = x, \quad \therefore s = \pm\sqrt{x} &= \pm\sqrt{1\angle 120°} && \text{or } \pm\sqrt{1\angle -120°} \\
&= \pm\sqrt{1}\angle 120°/2 && \text{or } \pm\sqrt{1}\angle -120°/2 \\
&= \pm 1\angle 60° && \text{or } \pm 1\angle -60° \\
&= \pm(0.5 + j0.866) && \text{or } \pm(0.5 - j0.866)
\end{aligned}$$

The roots of auxiliary equation are complex and has quadrantal symmetry. Two roots of auxiliary equation are lying on the right half of s-plane and the other two on the left half of s-plane.

The roots of characteristic equation are given by the roots of auxiliary polynomial and the roots of quotient polynomial. Hence we can conclude that two roots of characteristic equation are lying on the right half of s-plane and so system is unstable. The remaining five roots are lying on left half of s-plane.

6) The characteristic polynomial of a system is,$s^7 + 9s^5 + 9s^4 + 4s^3 + 20s^2 + 36s + 36 = 0$. Determine the locationof roots on the s-plane and hence the stability of the system?

SOLUTION

The characteristic equation is, $s^7 + 5s^6 + 9s^5 + 9s^4 + 4s^3 + 20s^2 + 36s + 36 = 0$.

The given characteristic polynomial is 7th order equation and so it has 7 roots. Since the highest power of s is odd number, form the first row of array using the coefficients of odd powers of s and form the second row of array using the coefficients of even powers of s as shown below.

s^7	:	1	9	4	36	.... Row-1
s^6	:	5	9	20	36	.... Row-2

Divide s^6 row by 5 to simplify the computations.

s^7	:	1	9	4	36	.... Row-1
s^6	:	1	1.8	4	7.2	.... Row-2
s^5	:	1	0	4		.... Row-3
s^4	:	1	0	4		.... Row-4
s^3	:	0	0			.... Row-5

$$s^5 : \frac{1\times 9 - 1.8\times 1}{1} \quad \frac{1\times 4 - 4\times 1}{1} \quad \frac{1\times 36 - 7.2\times 1}{1}$$

$s^5 : 7.2 \quad 0 \quad 28.8$

Divide by 7.2

$s^5 : 1 \quad 0 \quad 4$

$$s^4 : \frac{1\times 1.8 - 0\times 1}{1} \quad \frac{1\times 4 - 4\times 1}{1} \quad \frac{1\times 7.2 - 0\times 1}{1}$$

$s^4 : 1.8 \quad 0 \quad 7.2$

Divide by 1.8

$s^4 : 1 \quad 0 \quad 4$

$$s^3 : \frac{1\times 0 - 0\times 1}{1} \quad \frac{1\times 4 - 4\times 1}{1}$$

$s^3 : 0 \quad 0$

The row of all zeros indicate the existence of even polynomial, which is also the auxiliary polynomial. The auxiliary polynomial is, $s^4 + 4 = 0$. Divide the characteristic equation by auxiliary equation to get the quotient polynomial.

The characteristic equation can be expressed as a product of quotient polynomial and auxiliary equation.

$$\therefore s^7 + 5s^6 + 9s^5 + 9s^4 + 4s^3 + 20s^2 + 36s + 36 = 0$$

$$\underbrace{(s^4 + 4)}_{\text{Even polynomial}} \quad \underbrace{(s^3 + 5s^2 + 9s + 9)}_{\text{Quotient polynomial}} = 0$$

The routh array is constructed for quotient polynomial as shown below.

s^3	:	1	9
s^2	:	5	9
s^1	:	7.2	
s^0	:	9	

Column-1

$$s^1 : \frac{5\times 9 - 9\times 1}{5}$$

$s^1 : 7.2$

$$s^0 : \frac{7.2\times 9 - 0\times 5}{7.2}$$

$s^0 : 9$

There is no sign change in the elements of first column of routh array of quotient polynomial. Hence all the roots of quotient polynomial are lying on the left half of s-plane.

To determine the stability, the roots of auxiliary polynomial should be evaluated.

The auxiliary polynomial is, $s^4 + 4 = 0$.

$$
\begin{array}{r|l}
 & s^3 + 5s^2 + 9s + 9 \\
\hline
s^4 + 4 & s^7 + 5s^6 + 9s^5 + 9s^4 + 4s^3 + 20s^2 + 36s + 36 \\
 & s^7 \qquad\qquad\qquad + 4s^3 \\
\hline
 & \quad 5s^6 + 9s^5 + 9s^4 \qquad + 20s^2 + 36s + 36 \\
 & \quad 5s^6 \qquad\qquad\qquad + 20s^2 \\
\hline
 & \qquad 9s^5 + 9s^4 \qquad\qquad + 36s + 36 \\
 & \qquad 9s^5 \qquad\qquad\qquad + 36s \\
\hline
 & \qquad\quad 9s^4 \qquad\qquad\qquad + 36 \\
 & \qquad\quad 9s^4 \qquad\qquad\qquad + 36 \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

There is no sign change in the elements of first column of routh array of quotient polynomial. Hence all the roots of quotient polynomial are lying on the left half of s-plane.

To determine the stability, the roots of auxiliary polynomial should be evaluated.

The auxiliary polynomial is, $s^4 + 4 = 0$.

Put, $s^2 = x$ in the auxiliary equation, $\therefore s^4 + 4 = x^2 + 4 = 0$

$\therefore x^2 = -4 \quad \Rightarrow \quad x = \pm\sqrt{-4} = \pm j2 = 2\angle 90^\circ$ or $2\angle -90^\circ$

But, $s = \pm\sqrt{x} = \pm\sqrt{2\angle 90^\circ}$ or $\pm\sqrt{2\angle -90^\circ} = \pm\sqrt{2}\angle 90^\circ/2$ or $\pm\sqrt{2}\angle -90^\circ/2$

$= \pm\sqrt{2}\angle 45^\circ$ or $\pm\sqrt{2}\angle -45 = \pm(1+j1)$ or $\pm(1-j1)$

The roots of auxiliary equation are complex and has quadrantal symmetry. Two roots of auxiliary equation are lying on the right half of s-plane and the other two on the left half of s-plane.

The roots of characteristic equation are given by roots of quotient polynomial and auxiliary polynomial. Hence we can conclude that two roots of characteristic equation are lying on the right half of s-plane and so the system is unstable. The remaining five roots are lying on the left half of s-plane.

RESULT

1. The system is unstable.
2. Two roots are lying on the right half of s-plane and five roots are lying on the left half of s-plane.

7).Use the routh stability criterion to determine the location of roots on the s-plane and hence the stability for the system represented by the characteristic equation (May/June 2015)(May/June 2016)

$$s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0.$$

SOLUTION

The characteristic equation of the system is, $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$.

The given characteristic polynomial is 5^{th} order equation and so it has 5 roots. Since the highest power of s is odd number, form the first row of routh array using the coefficients of odd powers of s and form the second row using the coefficients of even powers of s.

s^5 : 1 8 7 .... Row-1

s^4 : 4 8 4 .... Row-2

Divide s^4 row by 4 to simplify the calculations.

s^5	:	1	8	7	.... Row-1
s^4	:	1	2	1	.... Row-2
s^3	:	1	1		.... Row-3
s^2	:	1	1		.... Row-4
s^1	:	$\in$			.... Row-5
s^0	:	1			.... Row-6

Column-1

$s^3 : \frac{1\times8-2\times1}{1} \quad \frac{1\times7-1\times1}{1}$

$s^3 : 6 \quad 6$

Divide by 6

$s^3 : 1 \quad 1$

$s^2 : \frac{1\times2-1\times1}{1} \quad \frac{1\times1-0\times1}{1}$

$s^2 : 1 \quad 1$

$s^1 : \frac{1\times1-1\times1}{1}$

$s^1 : 0$

Let $0 \rightarrow \in$

$s^1 : \in$

$s^0 : \frac{\in\times1-0\times1}{\in}$

$s^0 : 1$

When $\in \rightarrow 0$, there is no sign change in the first column of routh array. But we have a row of all zeros (s^1 row or row-5) and so there is a possibility of roots on imaginary axis. This can be found from the roots of auxiliary polynomial. Here the auxiliary polynomial is given by s^2 row.

The auxiliary polynomial is, $s^2 + 1 = 0$; $\quad \therefore s^2 = -1$ or $s = \pm\sqrt{-1} = \pm j1$

The roots of auxiliary polynomial are $+j1$, and $-j1$, lying on imaginary axis. The roots of auxiliary polynomial are also roots of characteristic equation. Hence two roots of characteristic equation are lying on imaginary axis and so the system is limitedly or marginally stable. The remaining three roots of characteristic equation are lying on the left half of s-plane.

RESULT

1. The system is limitedly or marginally stable.
2. Two roots are lying on imaginary axis and three roots are lying on left half of s-plane.

8). Use the routh stability criterion to determine the location of roots on the s-plane and hence the stability for the system represented by the characteristic equation$s^6+s^5+3s^4+3s^3+3s^2+2s+1=0$

(May 2015)

SOLUTION

The characteristic polynomial of the system is, $s^6+s^5+3s^4+3s^3+3s^2+2s+1=0$.

The given characteristic polynomial is 6^{th} order equation and so it has 6 roots. Since the highest power of s is even number, form the first row of routh array using the coefficients of even powers of s and form the second row using the coefficients of odd powers of s as shown below.

$$
\begin{array}{llllll}
s^6 & : & 1 & 3 & 3 & 1 \quad \text{.... Row-1} \\
s^5 & : & 1 & 3 & 2 & \quad \text{.... Row-2} \\
s^4 & : & \epsilon & 1 & 1 & \quad \text{.... Row-3} \\
s^3 & : & \frac{3\epsilon-1}{\epsilon} & \frac{2\epsilon-1}{\epsilon} & & \quad \text{.... Row-4} \\
s^2 & : & \frac{-2\epsilon^2+4\epsilon-1}{3\epsilon-1} & 1 & & \quad \text{.... Row-5} \\
s^1 & : & \frac{4\epsilon^2-\epsilon}{2\epsilon^2-4\epsilon+1} & & & \quad \text{.... Row-6} \\
s^0 & : & 1 & & & \quad \text{.... Row-7}
\end{array}
$$

$$s^4 : \frac{1\times3-3\times1}{1} \quad \frac{1\times3-2\times1}{1} \quad \frac{1\times1-0\times1}{1}$$

$$s^4 : 0 \quad 1 \quad 1$$

let $0 \rightarrow \epsilon$

$$s^4 : \epsilon \quad 1 \quad 1$$

$$s^3 : \frac{\epsilon\times3-1\times1}{\epsilon} \quad \frac{\epsilon\times2-1\times1}{\epsilon}$$

$$s^3 : \frac{3\epsilon-1}{\epsilon} \quad \frac{2\epsilon-1}{\epsilon}$$

$$s^2 : \frac{\frac{3\epsilon-1}{\epsilon}-\frac{2\epsilon-1}{\epsilon}\times\epsilon}{\frac{3\epsilon-1}{\epsilon}} \quad \frac{\frac{3\epsilon-1}{\epsilon}\times1-0\times\epsilon}{\frac{3\epsilon-1}{\epsilon}}$$

$$s^2 : \frac{-2\epsilon^2+4\epsilon-1}{3\epsilon-1} \quad 1$$

On letting $\epsilon \rightarrow 0$, we get,

$$
\begin{array}{llllll}
s^6 & : & 1 & 3 & 3 & 1 \quad \text{.... Row-1} \\
s^5 & : & 1 & 3 & 2 & \quad \text{.... Row-2} \\
s^4 & : & 0 & 1 & 1 & \quad \text{.... Row-3} \\
s^3 & : & -\infty & -\infty & & \quad \text{.... Row-4} \\
s^2 & : & 1 & 1 & & \quad \text{.... Row-5} \\
s^1 & : & 0 & & & \quad \text{.... Row-6} \\
s^0 & : & 1 & & & \quad \text{.... Row-7}
\end{array}
$$

+ to −

− to +

$$s^1 : \frac{\frac{-2\epsilon^2+4\epsilon-1}{3\epsilon-1}\times\frac{2\epsilon-1}{\epsilon}-\frac{3\epsilon-1}{\epsilon}\times1}{\frac{-2\epsilon^2+4\epsilon-1}{3\epsilon-1}}$$

$$s^1 : \frac{(-2\epsilon^2+4\epsilon-1)(2\epsilon-1)-(3\epsilon-1)(3\epsilon-1)}{\epsilon(-2\epsilon^2+4\epsilon-1)}$$

$$s^1 : \frac{-4\epsilon^3+\epsilon^2}{\epsilon(-2\epsilon^2+4\epsilon-1)} = \frac{4\epsilon^2-\epsilon}{2\epsilon^2-4\epsilon+1}$$

$$s^0 : \frac{\frac{4\epsilon^2-\epsilon}{4\epsilon^2-4\epsilon+1}\times1-0\times\frac{-2\epsilon^2+4\epsilon-1}{3\epsilon-1}}{(4\epsilon^2-\epsilon)/(4\epsilon^2-4\epsilon+1)}$$

$$s^0 : 1$$

Since there is a row of all zeros (s^1 row) there is a possibility of roots on imaginary axis. The auxiliary polynomial is $s^2+1=0$.

The roots of auxiliarypolynomial are, $s = \pm\sqrt{-1} = \pm j1$

The roots of auxiliary polynomial are also roots of characteristic equation. Hence two roots are lying on imaginary axis. Therefore divide the characteristic polynomial by auxiliary equation and construct the routh array for quotient polynomial to find the roots lying on right half of s-plane.

The characteristic polynomial can be expressed as a product of auxiliary polynomial and quotient polynomial.

$$\therefore s^6 + s^5 + 3s^4 + 3s^3 + 3s^2 + 2s + 1 = 0 \quad \Rightarrow \quad \underbrace{(s^2+1)}_{\text{Even polynomial}} \quad \underbrace{(s^4 + s^3 + 2s^2 + 2s + 1)}_{\text{Quotient polynomial}} = 0$$

The routh array for quotient polynomial is constructed as shown below.

$$\begin{array}{lllll} s^4 & : & 1 & 2 & 1 & \text{.... Row-1} \\ s^3 & : & 1 & 2 & & \text{.... Row-2} \\ s^2 & : & \epsilon & 1 & & \text{.... Row-3} \\ s^1 & : & \frac{2\epsilon - 1}{\epsilon} & & & \text{.... Row-4} \\ s^0 & : & 1 & & & \text{.... Row-5} \end{array}$$

$$\begin{array}{r|l} & \quad\;\; s^4 + s^3 + 2s^2 + 2s + 1 \text{ (Quotient polynomial)} \\ \hline s^2 + 1 \text{ (Even polynomial)} & s^6 + s^5 + 3s^4 + 3s^3 + 3s^2 + 2s + 1 \\ & \underline{\overset{(-)}{s^6} \quad \overset{(-)}{+} s^4} \\ & s^5 + 2s^4 + 3s^3 + 3s^2 + 2s + 1 \\ & \underline{\overset{(-)}{s^5} \quad \overset{(-)}{+} s^3} \\ & 2s^4 + 2s^3 + 3s^2 + 2s + 1 \\ & \underline{\overset{(-)}{2s^4} \quad \overset{(-)}{+} 2s^2} \\ & 2s^3 + s^2 + 2s + 1 \\ & \underline{\overset{(-)}{2s^3} \quad \overset{(-)}{+} 2s} \\ & s^2 \quad + 1 \\ & \underline{\overset{(-)}{s^2} \quad \overset{(-)}{+} 1} \\ & 0 \end{array}$$

$s^2 : \frac{1 \times 2 - 2 \times 1}{1} \quad \frac{1 \times 1 - 0 \times 1}{1}$

$s^2 : 0 \quad 1$

let $0 \to \epsilon$

$s^2 : \epsilon \quad 1$

$s^1 : \frac{\epsilon \times 2 - 1 \times 1}{\epsilon}$

$s^1 : \frac{2\epsilon - 1}{\epsilon}$

$s^0 : \frac{\frac{2\epsilon-1}{\epsilon} \times 1 - 0 \times \epsilon}{(2\epsilon - 1)/\epsilon}$

$s^0 : 1$

On letting $\epsilon \to 0$, we get

$$\begin{array}{lllll} s^4 & : & 1 & 2 & 1 & \text{.... Row-1} \\ s^3 & : & 1 & 2 & & \text{.... Row-2} \\ s^2 & : & 0 & 1 & & \text{.... Row-3} \\ s^1 & : & -\infty & & & \text{.... Row-4} \\ s^0 & : & 1 & & & \text{.... Row-5} \end{array}$$

Column-1

+ to −

− to +

On examining the first column of the routh array of quotient polynomial, we found that there are two sign changes. Hence two roots are lying on the right half of s-plane and other two roots of quotient polynomial are lying on the left half of s-plane.

The roots of characteristic equation are given by roots of auxiliary polynomial and quotient polynomial. Hence two roots are lying on imaginary axis, two roots are lying on right half of s-plane and the remaining two roots are lying on left half of s-plane. Hence the system is unstable.

RESULT

1. The system is unstable.
2. Two roots are lying on imaginary axis, two roots are lying on right half of s-plane and two roots are lying on left half of s-plane.

9).Determine the range of K for stability of unity feedback system whose open loop transfer function is $(s) = \frac{K}{s(s+1)(s+2)}$.using Routh stability criterion.(Nov/Dec 2012) (April/May18)

SOLUTION

The closed loop transfer function, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K}{s(s+1)(s+2)}}{1+\frac{K}{s(s+1)(s+2)}} = \frac{K}{s(s+1)(s+2)+K}$

The characteristic equation is, $s(s+1)(s+2)+K=0$

$\therefore s(s^2+3s+2)+K=0 \quad \Rightarrow \quad s^3+3s^2+2s+K=0$

The routh array is constructed as shown below.

The highest power of s in the characteristic polynomial is odd number. Hence form the first row using the coefficients of odd powers of s and form the second row using the coefficients of even powers of s.

$$
\begin{array}{lcll}
s^3 & : & \boxed{1} & 2 \\
s^2 & : & 3 & K \\
s^1 & : & \frac{6-K}{3} & \\
s^0 & : & K &
\end{array}
$$

Column-1

$s^1 : \frac{3\times 2 - K\times 1}{3}$

$s^1 : \frac{6-K}{3}$

$s^0 : \frac{\frac{6-K}{3}\times K - 0\times 3}{(6-K)/3}$

$s^0 : K$

For the system to be stable there should not be any sign change in the elements of first column. Hence choose the value of K so that the first column elements are positive.

From s^0 row, for the system to be stable, $K > 0$

From s^1 row, for the system to be stable, $\frac{6-K}{3} > 0$

For $\frac{6-K}{3} > 0$, the value of K should be less than 6.

$\therefore$ The range of K for the system to be stable is $0<K<6$.

<u>RESULT;</u>

The value of K is in the range $0 < K < 6$ for the system to be stable.

10). The open loop transfer function of a unity feedback control system is given by

$$\boldsymbol{G(s) = \frac{K}{(s+2)(s+4)(s^2+6s+25)}}$$

By applying the routh criterion discuss the stability of the closed-loop system as a function of K.determine the value of K which will cause sustained oscillations in the closed-loop system. What are the corresponding oscillating frequencies? (Nov/Dec 2013), (May/June 2014)(Nov/Dec 2015)

SOLUTION

$$\left.\begin{array}{l}\text{The closed loop}\\ \text{transfer function}\end{array}\right\} \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\dfrac{K}{(s+2)(s+4)(s^2+6s+25)}}{1+\dfrac{K}{(s+2)(s+4)(s^2+6s+25)}} = \frac{K}{(s+2)(s+4)(s^2+6s+25)+K}$$

The characteristic equation is given by the denominator polynomial of closed loop transfer function.

The characteristic equation is, $(s+2)(s+4)(s^2+6s+25)+K=0$.

$$\therefore (s^2+6s+8)(s^2+6s+25)+K=0 \quad \Rightarrow \quad s^4+12s^3+69s^2+198s+200+K=0$$

The routh array is constructed as shown below. The highest power of s in the characteristic equation is even number. Hence form the first row using the coefficients of even powers of s and form the second row using the coefficients of odd powers of s.

s^4	:	1	69	200+K	..... Row-1
s^3	:	12	198		..... Row-2

Divide s^3 row by 12 to simplify the calculations

		Column-1			
s^4	:	1	69	200+K	..... Row-1
s^3	:	1	16.5		..... Row-2
s^2	:	52.5	200+K		..... Row-3
s^1	:	$\frac{666.25-K}{52.5}$			..... Row-4
s^0	:	200+K			..... Row-5

$$s^2 : \frac{1\times 69-16.5\times 1}{1} \quad \frac{1\times(200+K)}{1}$$

$$s^2 : 52.5 \qquad 200+K$$

$$s^1 : \frac{52.5\times 16.5-(200+K)\times 1}{52.5}$$

$$s^1 : \frac{666.25-K}{52.5}$$

$$s^0 : \frac{\dfrac{666.25-K}{52.5}\times(200+K)}{(666.25-K)/52.5}$$

$$s^0 : 200+K$$

For the system to be stable there should not be any sign change in the elements of first column. Hence choose the value of K so that the first column elements are positive.

From s^1 row, for the system to be stable, $(666.25-K) > 0$.

Since $(666.25-K) > 0$, should be less than 666.25.

From s^0 row, for the system to be stable, $(200+K) > 0$

Since $(200+K) > 0$, K should be greaterthan −200, but practical values of K starts from 0. Hence K should be greater than 0.

$\therefore$ The range of K for the system to be stable is $0<K<666.25$.

When K= 666.25 the s^1 row becomes zero, which indicates the possibility of roots on imaginary axis. A system will oscillate if it has roots on imaginary axis and no roots on right half of s-plane.

When K=666.25, the coefficients of auxiliary equation are given by the s^2 row.

$\therefore$ The auxiliary equation is, $52.5s^2+200+K=0$

$$52.5s^2+200+666.25=0$$

$$s^2 = \frac{-200-666.25}{52.5} = -16.5$$

$$s = \pm\sqrt{-16.5} = \pm j\sqrt{16.5} = \pm j4.06$$

When K = 666.25, the system has roots on imaginary axis and so it oscillates. The frequency of oscillation is given by the value of root on imaginary axis.

$\therefore$ The frequency of oscillation, ω = 4.06 rad/sec.

RESULT

1. The range of K for stability is 0<K<666.25
2. The system oscillates when K = 666.25
3. The frequency of oscillation, ω = 4.06 rad/sec. (When K = 666.25).

11). The open loop transfer function of a unity feedback system is given by, $G(s) = \frac{K(s+1)}{s^3+as^2+2s+1}$ Determine the value of K and a so that the system oscillates at a frequency of 2 rad/sec. (April/May 2011)

SOLUTION

$$\left.\begin{matrix}\text{The closed loop}\\ \text{transfer function}\end{matrix}\right\} \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K(s+1)}{s^3+as^2+2s+1}}{1+\frac{K(s+1)}{s^3+as^2+2s+1}} = \frac{K(s+1)}{s^3+as^2+2s+1+K(s+1)}$$

The characteristic equation is, $s^3+as^2+2s+1+K(s+1)=0$.

$$s^3+as^2+2s+1+Ks+K=0 \quad \Rightarrow \quad s^3+as^2+(2+K)s+1+K=0$$

The routh array of characteristic polynomial is constructed as shown below. The maximum power of s is odd, hence the first row of routh array is formed using coefficients of odd powers of s and the second row of routh array is formed using coefficients of even powers of s.

If the elements of s^1 row are all zeros then there exist an even polynomial (or auxiliary polynomial). If the roots of the auxiliary polynomial are purely imaginary then the roots are lying on imaginary axis and the system oscillates. The frequency of oscillation is the root of auxiliary polynomial.

Routh array

s^3 :	1	2+K
s^2 :	a	1+K
s^1 :	$\frac{a(2+K)-(1+K)}{a}$	
s^0 :	1+K	

From s^2 row, the auxiliary polynomial is,

$$as^2+(1+K)=0 \quad \Rightarrow \quad as^2=-(1+K) \quad \Rightarrow \quad s=\pm j\sqrt{\frac{1+K}{a}}$$

Given that, $s=\pm j2$, $\quad \therefore \sqrt{\frac{1+K}{a}}=2 \quad \Rightarrow \quad \frac{1+K}{a}=4 \quad \Rightarrow \quad K=4a-1$

From s^1 row, $\frac{a(2+K)-(1+K)}{a}=0 \Rightarrow a(2+K)-(1+K)=0 \Rightarrow 2a+Ka-1-K=0$

$$\therefore \quad 2a-1+K(a-1)=0$$

Put, K = 4a –1

$\therefore 2a - 1 + (4a-1)(a-1) = 0 \Rightarrow 2a - 1 + 4a^2 - 4a - a + 1 = 0 \Rightarrow 4a^2 - 3a = 0$ (or) $a(4a-3) = 0$

Since $a \neq 0$, $4a - 3 = 0$, $\therefore a = 3/4$

When $a = (3/4)$, $K = 4a - 1 = 4 \times (3/4) - 1 = 2$

RESULT

When the system oscillates at a frequency of 2 rad/sec, $K = 2$ and $a = 3/4$.

12). A feedback system has open loop transfer function of $G(s) = \dfrac{Ke^{-s}}{s(s^2+5s+9)}$. Determine the maximum value of K for stability of closed loop system.

SOLUTION

Generally control systems have very low bandwidth which implies that it has very low frequency range of operation. Hence for low frequency ranges the term e^{-s} can be replaced by, $1-s$, (i.e., $e^{-sT} \approx 1-sT$).

$$\therefore G(s) = \frac{K e^{-s}}{s(s^2+5s+9)} \approx \frac{K(1-s)}{s(s^2+5s+9)}$$

$$\left.\begin{matrix}\text{The closed loop}\\ \text{transfer function}\end{matrix}\right\} \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\dfrac{K(1-s)}{s(s^2+5s+9)}}{1+\dfrac{K(1-s)}{s(s^2+5s+9)}} = \frac{K(1-s)}{s(s^2+5s+9)+K(1-s)}$$

The characteristic equation is given by the denominator polynomial of closed loop transfer function.

$\therefore$ The characteristic equation is, $s(s^2+5s+9) + K(1-s) = 0$

$$\therefore s(s^2+5s+9) + K(1-s) = s^3 + 5s^2 + 9s + K - Ks = 0 \Rightarrow s^3 + 5s^2 + (9-K)s + K = 0$$

The routh array of characteristic polynomial is constructed as shown below.

The maximum power of s in the characteristic polynomial is odd, hence form the first row of routh array using coefficients of odd powers of s and second row of routh array using coefficients of even powers of s.

s^3	:	1	9−K
s^2	:	5	K
s^1	:	9−1.2K	
s^0	:	K	

$$s^1 : \frac{5\times(9-K) - K\times 1}{5}$$

$$s^1 : \frac{45-5K-K}{5}$$

$$s^1 : \frac{45-6K}{5} \approx 9 - 1.2K$$

$$s^0 : \frac{(9-1.2K)\times K}{(9-1.2K)}$$

$$s^0 : K$$

From s^1 row, for stability of the system, $(9-1.2K)>0$

If $(9-1.2K)>0$ then $1.2K<9$; $\therefore K < \dfrac{9}{1.2} = 7.5$

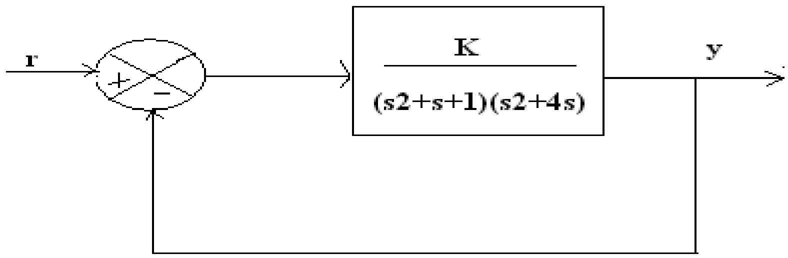
From s^0 row, for stability of the system, $K>0$

Finally we can conclude that for stability of the system K should be in the range of $0<K<7.5$

RESULT

For stability of the system K should be in the range of, $0 < K < 7.5$.

13. Consider the closed loop system shown in fig. determine the range of K for which the system is stable. (Nov/Dec 2014)



The Characteristics equation is

$1+G(s)\,H(s) = 0$

$1+K/\ (s^2+s+1)\ (s^2+4s) = 0, (s^2+s+1)\ (s^2+4s) + K/\ (s^2+s+1)\ (s^2+4s) = 0$

$S^4+9s^3+s^2+4s+k=0$

<u>Routh array:</u>

S^4	1	1	K
S^3	9	4	0
S^2	0.56	K	0
S^1	2.24-9K/0.56	0	0
S^0	K	0	0

When the system is stable then coefficients of s^1 and s^0(in routh array not in equation) should be greater than zero.(Since coefficients of s^4,s^3,s^2 are positive values)

$$2.24-9K/0.56 > 0$$

$$2.24-9K > 0$$

$$2.24 > 9K$$

$$K < 2.24/9$$

$$K < 0.254$$

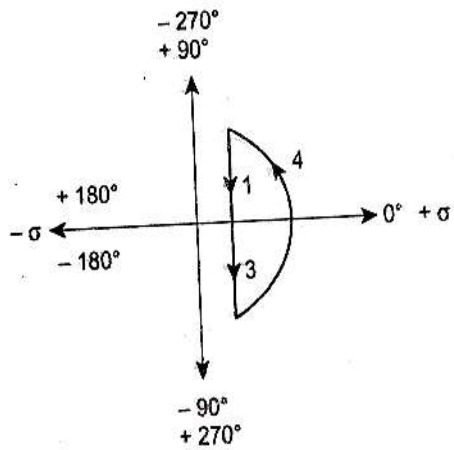
And K > 0

Therefore the range of K is $\quad 0 < K < 0.254$

1

14. ***Sketch the Nyquist plot for a system with open loop transfer function $G(S)\,H(S) = \dfrac{K\,(1+0.4\,s)\,(s+1)}{(1+8\,s)\,(s-1)}$ and determine the range of K for which the system is stable.***

Solution : ***Step 1 :*** From given G(S) H(S), first find type of system by identifying how many poles lie on origin of *s*-plane. Since there is no pole at origin, section 2 does not come into picture. So only section 4 is to be considered. So the path for type 0 system is shown below.



Step 2 : Analysis of section (4) : Any point on semicircle can be represented by a phasor $s = R\,e^{j\phi}$ where $R \to \infty$. ϕ varies from $-90°$ to $+90°$.

$$G(R\,e^{j\phi}) = \underset{R\to\infty}{\mathrm{Lt}} \frac{K\,(1+0.4\,R\,e^{j\phi})\,(R\,e^{j\phi}+1)}{(1+8\,R\,e^{j\phi})\,(R\,e^{j\phi}-1)} \quad \text{as } R >> 1$$

$$= \frac{K0.4\ \ R\,e^{j2\phi}}{8\,R\,e^{j2\phi}}$$

$$G(R\,e^{j\phi}) = \frac{K0.4}{8} = 0.05\,K$$

$\therefore$ G(S) varies from $-90°$ to $+90°$ in counter clockwise direction with 0.05 K radius in $+\sigma$ axis. $\phi_2 - \phi_1 = 90° + 90° = +180°$.

Step 3 : To determine intersection : Rationalise the function.

$$G(S) = \frac{K\,(1+0.4\,s)\,(s+1)}{(1+8\,s)\,(-1+s)}\ \frac{(1-8\,s)\,(-1-s)}{(1-8\,s)\,(-1-s)}$$

Substitute $s = j\omega$.

$$G(j\omega) = \frac{K\,[(1+0.4\,j\omega)(1+j\omega)(1-j8\omega)(1-j\omega)]}{(1+j8\omega)(j\omega-1)(1-j8\omega)(1-j\omega)}$$

$$= \frac{K\,[(1+1.4\,j\omega-0.4\,\omega^2)(-1+j7\omega-8\,\omega^2)]}{(1+64\,\omega^2)(1+\omega^2)}$$

$$= \frac{K\,[-1-17.4\,\omega^2+3.2\,\omega^4+j5.6\omega-j14\,\omega^3]}{(1+64\,\omega^2)(1+\omega^2)}$$

$$G(j\omega) = \underbrace{\frac{K\,[-1-17.4\,\omega^2+3.2\,\omega^4]}{(1+64\,\omega^2)(1+\omega^2)}}_{\text{Real part}} + \underbrace{\frac{jK\,[5.6\,\omega-14\,\omega^3]}{(1+64\,\omega^2)(1+\omega^2)}}_{\text{Imaginary part}}$$

Equating imaginary part to zero, we get,

$$\frac{K\,[5.6\,\omega-14\,\omega^3]}{(1+64\,\omega^2)(1+\omega^2)} = 0$$

$$5.6\,\omega - 14\,\omega^3 = 0$$

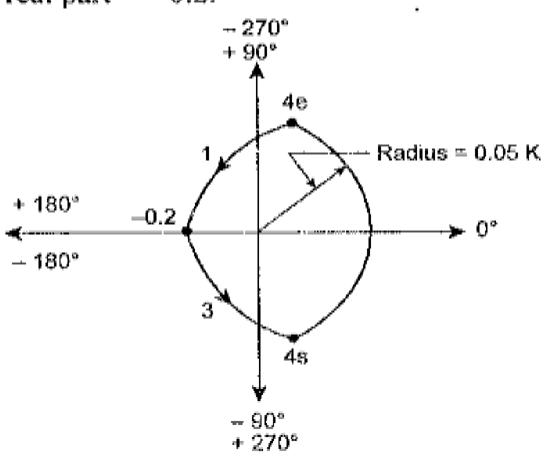
$$\omega^2 = \frac{5.6}{14} = 0.4$$

$$\therefore \boxed{\omega = 0.632}$$

Substitute $\omega = 0.632$ in real part, we get,

$$\text{Real part} = \frac{K\,[-1-17.4\,\omega^2+3.2\,\omega^4]}{(1+64\,\omega^2)(1+\omega^2)} = K\,(-0.2)$$

Assume K = 1, now, real part = – 0.2.



While drawing the path, the angle of section 4 is from – 90° to + 90°. So it is drawn by a semicircle. Section 1 joins 4 *e* and – 0.2 (Because absence of Section 2. So Section 1 is directly connected with intersection on real part *i.e.*, – 0.2) and similarly Section 3 joins 4 *s* and – 0.2.

Case (i) : Step 4 : If K = 1 , then point of intersection lie before $-1+j0$. Therefore number of encirclements $N_{-1} = 0$. The number of open loop poles in RHS is one,

$$i.e., \; P_{-1} = 1 \text{ at } s = +1$$

Condition for stability : $\quad z_{-1} = 1$

So the given system is unstable.

Case (ii) : If K = 5, then point of intersection lie $[-0.2\,K = -0.2 \times 5 = -1]$ exactly at $-1+j0$. Therefore number of encirclements $N_{-1} = 0$. $P_{-1} = 1$ at $s = +1$.

Condition for stability : $\quad z_{-1} = 0 + 1 = 1$

So the given system is unstable.

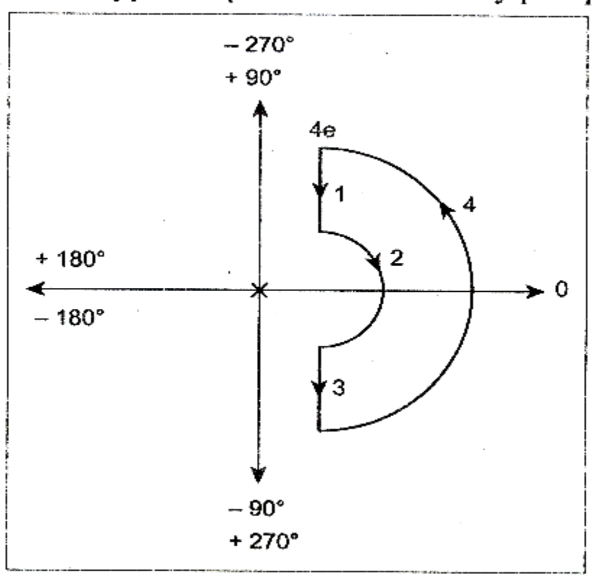
Case (iii) : If K > 5, for example here we take K = 6, then the point of intersection lie $[-0.2\,K = -0.2 \times 6 = -1.2]$ at $-1.2+j0$. Therefore number of encirclements $N_{-1} = 1$, $P_{-1} = 1$ at $s = +1$.

Condition for stability : $\quad z_{-1} = 1 + 1 = 2$

So the given system is unstable.

15. ***Draw the Nyquist plot for the system whose open loop transfer function*** $G(s)H(s) = \dfrac{K}{s(s+1)(s+10)}$ ***Determine the range of K for which closed loop system is stable.*** ***(May/June*** **2016)**

☺ ***Solution: Step 1:*** It is a type one problem. So draw Nyquist path in s-plane.



Step 2: Analysis of Section 2: Any point on semicircle can be represented by phasor $s = \varepsilon\, e^{j\theta}$ as $\varepsilon \to 0$ and θ varies from $+90°$ to $-90°$.

Substitute the value of s in problem

$$G(\varepsilon\, e^{j\theta}) = \underset{\varepsilon \to 0}{\text{Lt}} \frac{K}{\varepsilon\, e^{j\theta}(\varepsilon\, e^{j\theta}+2)(\varepsilon\, e^{j\theta}+10)} \quad \text{as } \varepsilon \to 0,\ \varepsilon << 1$$

$$= \underset{\varepsilon \to 0}{\text{Lt}} \frac{K}{\varepsilon\,(2)\,(10)}\, e^{-j\theta}$$

$$= \frac{K}{0}\, e^{-j\theta} = \infty\, e^{-j\theta}$$

Compare with 2, $\varepsilon\, e^{j\theta}$ is negligible.

Now substitute value of θ from $90°$ to $-90°$.

$\therefore$ G(S) varies from $-90°$ to $+90°$ with ∞ radius in counterclockwise direction.

Check: $\theta_2 - \theta_1 = +90° - (-90°) = +180°$ counter clockwise

Step 3: Analysis of Section 4: Any point on semicircle can be represented by a phasor $s = R\, e^{j\phi}$ where $R \to \infty$ and ϕ varies $-90°$ to $+90°$.

Substitute the value of 's' in the given problem.

$$G(R\, e^{j\phi}) = \underset{R \to \infty}{\text{Lt}} \frac{4}{R\, e^{j\phi}(R\, e^{j\phi}+2)(R\, e^{j\phi}+10)}, \quad \text{Since } R >> 1 \text{ so neglect}$$

$$= \underset{R \to \infty}{\text{Lt}} \frac{K}{R^3\, e^{j3\phi}} = \frac{K}{\infty}\, e^{-j3\phi} = 0\, e^{-j3\phi}$$

Substitute $\phi = -90°, +90°$.

$\therefore$ G(S) varies from $+270°$ to $-270°$ with zero radius in clockwise direction.

$$\phi_2 - \phi_1 = -270° - 270° = -540°$$

$$\phi_2 - \phi_1 = -540° \text{ clockwise direction}$$

Step 4: To find intersection of Nyquist plot on real axis:

Now, rationalise the given G(S).

$$G(s) = \frac{k}{s\,(s+2)\,(s+10)} \times \frac{s\,(2-s)\,(10-s)}{s\,(2-s)\,(10-s)}$$

$$G(s) = \frac{k\,(s^3 - 12\,s^2 + 20\,s)}{s^2\,(4-s^2)\,(100-s^2)}$$

Substitute $s = j\omega$.

$$G(j\omega) = \frac{k\,(-j\,\omega^3 + 12\,\omega^2 + j\,20\,\omega)}{-\omega^2\,(4+\omega^2)\,(100+\omega^2)}$$

Separate real and imaginary parts,

$$\frac{12\,k\,\omega^2}{-\omega^2(4+\omega^2)(100+\omega^2)} - \frac{k\,j\,\omega(\omega^2-20)}{\omega^2(4+\omega^2)(100+\omega^2)}$$

$$\frac{12}{(4+\omega^2)(100+\omega^2)} - \frac{k(\omega^2-20)}{\omega(4+\omega^2)(100+\omega^2)}$$

Equating imaginary part to zero, we get ω value.

$$\frac{k(\omega^2-20)}{\omega(4+\omega^2)(100+\omega^2)} = 0$$

$$k(\omega^2-20) = 0$$

$$\omega^2 = 20$$

$$\omega = \sqrt{20}$$

$$\omega = \infty$$

Substitute 'ω' value in either real part (or) in given function $G(s)$ to get intersection.

$$G(s) = \frac{k}{s(s+2)(s+10)} = \frac{k}{20\,s+12\,s^2+s^3}$$

Substitute $s = j\omega$

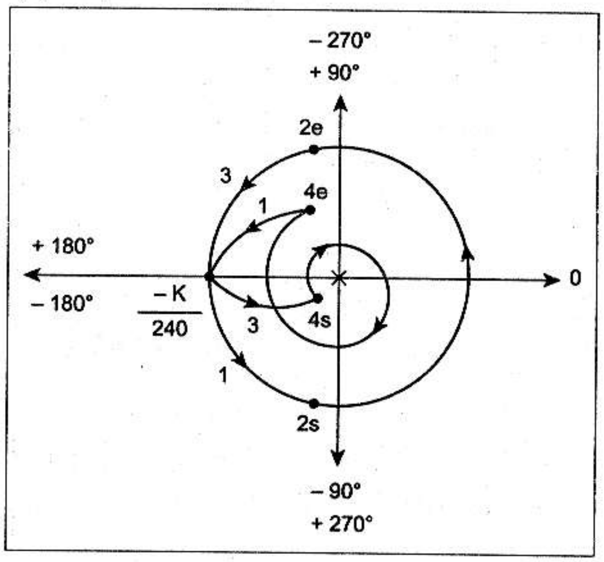
$$G(j\omega) = \frac{k}{20\,j\omega - 12\,\omega^2 - j\,\omega^3}$$

$$G(j\sqrt{20}\,) = \frac{k}{20\,j\sqrt{20} - 12\,(20) - j(\sqrt{20}\,)\,(20)}$$

$$G(j\sqrt{20}\,) = \frac{-k}{240}$$

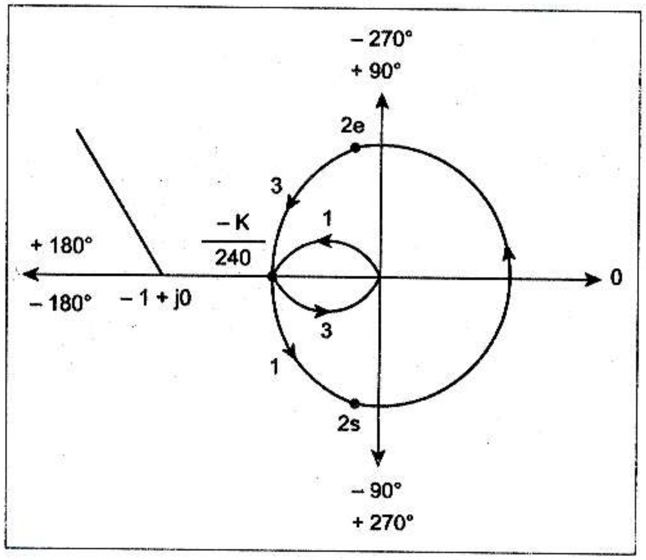
So, plot cuts the real axis as $\frac{-k}{240}$.

Step 5: Complete Nyquist path:



2 e is connected to 4 s by section 3 through $\frac{-k}{240}$. 4 e is connected to 2 s by section 1 through $\frac{-k}{240}$.

As section 4 is of zero radius, the plot simplifies as shown below.



To find Stability: Draw line from $(-1 + j0)$ point as shown. The plot does not tou $(-1 + j0)$ point. So absence of encirclement. *i.e.*, $N_{-1} = 0$.

From the given problem, poles at $s = 0, s = -0.1, s = -0.2$ which lies on LHS of s-plan But p_{-1} is the number of poles lie on RHS of s-plane.

$$\text{So } p_{-1} = 0$$

Condition for stability: $\quad z_{-1} = N_{-1} + p_{-1} = 0$

So the given system in closed loop is stable.

Range of values of K for stability:

$$\left| \frac{-K}{240} \right| < 1$$

$$K < 240$$

So range of values of K for stability is $\boxed{0 < K < 240}$

16. Explain in detail the design procedure of lead compensator using Bode plot.

(May/June 2013)(Nov/Dec 2015)

Step-1 : The open loop gain K of the given system is determined to satisfy the requirement of the error constant.

Step-2 : The bode plot is drawn for the uncompensated system using the value of K, determined from the previous step. *[Refer Chapter-4 for the procedure to sketch bode plot].*

Step-3 : The phase margin of the uncompensated system is determined from the bode plot.

Step-4 : Determine the amount of phase angle to be contributed by the lead network by using the formula given below,

$$\phi_m = \gamma_d - \gamma + \epsilon$$

where,

ϕ_m = Maximum phase lead angle of the lead compensator

γ_d = Desired phase margin

γ = Phase margin of the uncompensated system

ϵ = Additional phase lead to compensate for shift in gain crossover frequency

Choose an initial choice of ϵ as 5^0

Note : *If ϕ_m is more than 60° then realize the compensator as cascade of two lead compensator with each compensator contributing half of the required angle).*

Step-5 : Determine the transfer function of lead compensator

Calculate α using the equation, $\alpha = \dfrac{1 - \sin\phi_m}{1 + \sin\phi_m}$

From the bode plot, determine the frequency at which the magnitude of $G(j\omega)$ is $-20 \log 1/\sqrt{\alpha}$ db. This frequency is ω_m.

Calculate T from the relation, $\omega_m = \dfrac{1}{T\sqrt{\alpha}}$ $\quad \therefore T = \dfrac{1}{\omega_m\sqrt{\alpha}}$

$$\left.\begin{array}{l}\text{Transfer function of}\\ \text{lead compensator}\end{array}\right\} G_c(s) = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} = \frac{\alpha(1 + sT)}{(1 + \alpha sT)}$$

Step-6 : Determine the open loop transfer function of compensated system.

The lag compensator is connected in series with G(s) as shown in fig 6.12. When the lead network is inserted in series with the plant, the open loop gain of the system is attenuated by the factor α ($\because \alpha<1$), so an amplifier with the gain of $1/\alpha$ has to be introduced in series with the compensator to nullify the attenuation caused by the lead compensator.

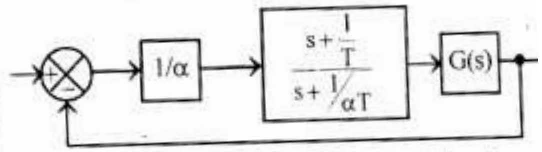


Fig 6.12 : Block diagram of lead compensated system.

Open loop transfer function of the overall system $\Big\}$ $G_0(s) = \dfrac{1}{\alpha} \times \dfrac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \times G(s)$

$$= \frac{1}{\alpha} \times \frac{\alpha(1 + sT)}{(1 + s\alpha T)} \times G(s) = \frac{(1 + sT)}{(1 + s\alpha T)} \times G(s)$$

Step-7 : Verify the design.

Finally the Bode plot of the compensated system is drawn and verify whether it satisfies the given specifications. If the phase margin of the compensated system is less than the required phase margin then repeat step 4 to 10 by taking ϵ as 5^0 more than the previous design.

17. Design a lead compensator for a unity feedback system with open loop transfer function,G(s)=K/S(S+1)(S+5) to satisfy the following specifications (i)velocity error constant,K_v>= 50 (ii)phase margin is >= 20˚. (Nov/Dec 2011,16)(April/May18)

$$\text{Let } s = j\omega, \therefore G(j\omega) = \frac{50}{j\omega(1 + j\omega)(1 + j0.2\omega)}$$

MAGNITUDE PLOT

The corner frequency are, ω_{c1} = 1 rad/sec and ω_{c2} = 1/0.2 = 5 rad/sec.

The various terms of G(jω) are listed in table-6. Also the table shows the slope contributed by each term and the change in slope at the corner frequencies.

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{50}{j\omega}$	–	–20	–
$\frac{1}{1+j\omega}$	$\omega_{c1} = 1$	–20	–20 + (–20) = –40
$\frac{1}{1+j0.2\omega}$	$\omega_{c2} = \frac{1}{0.2} = 5$	–20	–40 – 20 = –60

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$.

Let ω_l = 0.5 rad/sec and ω_h=10 rad/sec

Let $A = |G(j\omega)|$ in db

$$\text{At } \omega = \omega_l, \quad A = 20\log\left|\frac{50}{j\omega}\right| = 20\log\frac{50}{0.5} = 40 \text{ db}$$

$$\text{At } \omega = \omega_{c1}, \quad A = 20\log\left|\frac{50}{j\omega}\right| = 20\log\frac{50}{1} = 34 \text{ db}$$

$$\text{At } \omega = \omega_{c2}, \quad A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(\text{at } \omega = \omega_{c1})}$$

$$= -40 \times \log\frac{5}{1} + 34 = 6 \text{ db}$$

$$\text{At } \omega = \omega_h, \quad A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A_{(\text{at } \omega = \omega_{c2})}$$

$$= -60 \times \log\frac{10}{5} + 6 = -12 \text{ db}$$

Let the points a, b, c and d be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose appropriate scales and fix the points a, b, c and d. Join the points by straight lines and mark the slope on the respective region. The magnitude plot is shown in fig 6.6.2.

PHASE PLOT

The phase angle of $G(j\omega)$ as a function of ω is given by,

$$\phi = \angle G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}0.2\omega$$

The phase angle of $G(j\omega)$ are calculated for various values of ω and listed in table-2.

TABLE-2

ω rad/sec	0.1	0.5	1.0	5	10
ϕ deg	-96°	-122	-146	-214	-238

On the same semilog sheet take another y-axis, choose appropriate scale and draw phase plot as shown in fig 6.6.2.

Step-3 : Determine the phase margin

Let, ϕ_{gc} = Phase of $G(j\omega)$ at gain crossover frequency.

γ = Phase margin of uncompensated system.

From the bode plot of uncompensated system we get, $\phi_{gc} = -224^\circ$.

Now, $\gamma = 180^\circ + \phi_{gc} = 180^\circ - 224^\circ = -44^\circ$

The phase margin of the system is negative and so the system is unstable Hence lead compensation is required to make the system stable and to have a phase margin of 20°.

Step-4 : Find ϕ_m

The desired phase margin, $\gamma_d \geq 20°$

Let additional phase lead required, $\in = 5°$

Maximum lead angle, $\phi_m = \gamma_d - \gamma + \in = 20° - (-44°) + 5° = 69°$

Since the lead angle required is greater than 60°, we have to realise the lead compensator as cascade of two lead compensators with each compensator providing half of the required phase lead angle.

$$\therefore \phi_m = \frac{69°}{2} = 34.5°$$

Step-5 : Determine the transfer function of lead compensator

$$\alpha = \frac{1 - \sin\phi_m}{1 + \sin\phi_m} = \frac{1 - \sin 34.5°}{1 + \sin 34.5°} = 0.28$$

$$\left.\begin{array}{l}\text{The db magnitude}\\ \text{corresponding to}\end{array}\right\} \omega_m = -20 \log \frac{1}{\sqrt{a}} = -20 \log \frac{1}{\sqrt{0.28}} = -5.5 \text{ db.}$$

From the bode plot of uncompensated system the frequency, ω_m corresponding to a db gain of –5.5 db is found to be 7.8 rad/sec.

$$\therefore \omega_m = 7.8 \text{ rad/sec.}$$

$$\text{Now, } T = \frac{1}{\omega_m \sqrt{\alpha}} = \frac{1}{7.8\sqrt{0.28}} = 0.24$$

$$\left.\begin{array}{l}\text{Transfer function of}\\ \text{the lead compensator}\end{array}\right\} G_c(s) = \frac{\left(s + \frac{1}{T}\right)^2}{\left(s + \frac{1}{\alpha T}\right)^2} = \alpha^2 \frac{(1 + sT)^2}{(1 + s\alpha T)^2}$$

$$= (0.28)^2 \frac{(1 + 0.24s)^2}{(1 + 0.28 \times 0.24s)^2} = 0.0784 \frac{(1+0.24s)^2}{(1+0.067s)^2}$$

Step-6 : Open loop transfer function of compensated system.

The block diagram of the compensated system is shown in fig 6.6.1.

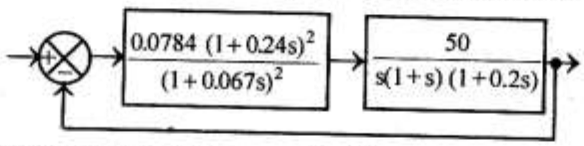


Fig 6.6.1 : Block diagram of lead compensated system.

The attenuation provided by the compensator can be retained to reduce the large value of op loop gain, so that the unstable system can be easily brought to stable region.

Let $G_0(s)$ be open loop transfer function of compensated system.

$$G_0(s) = \frac{0.0784\,(1+0.24s)^2}{(1+0.067s)^2} \times \frac{50}{s\,(1+s)\,(1+0.2s)} = \frac{4\,(1+0.24s)^2}{s\,(1+s)\,(1+0.2s)\,(1+0.067s)^2}$$

Step-7 : Draw the bode plot of compensated system to verify the design.

$$\text{Put, } s = j\omega \text{ in } G_0(s), \ \therefore G_0(j\omega) = \frac{4\,(1 + j0.24\omega)^2}{j\omega\,(1 + j\omega)\,(1 + j0.2\omega)\,(1 + j0.067\omega)^2}$$

At $\omega = \omega_{c1}$, $A_0 = 20 \log\left|\frac{4}{j\omega}\right| = 20 \log\frac{4}{1} = 12$ db

At $\omega = \omega_{c2}$, $A_0 = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A_0 \text{ at } (\omega = \omega_{c1})$

$$= -40 \times \log\frac{4.2}{1} + 12 = -13 \text{ db}$$

At $\omega = \omega_{c3}$, $A_0 = \left[\text{slope from } \omega_{c2} \text{ to } \omega_{c3} \times \log\frac{\omega_{c3}}{\omega_{c2}}\right] + A_0 \text{ at } (\omega = \omega_{c2})$

$$= 0 \times \log\frac{5}{4.2} + (-13) = -13 \text{ db}$$

At $\omega = \omega_{c4}$, $A_0 = \left[\text{slope from } \omega_{c3} \text{ to } \omega_{c4} \times \log\frac{\omega_{c4}}{\omega_{c3}}\right] + A_0 \text{ at } (\omega = \omega_{c3})$

$$= -20 \times \log\frac{15}{5} + (-13) = -22.5 \text{ db} \approx -23 \text{ db}$$

At $\omega = \omega_h$, $A_0 = \left[\text{slope from } \omega_{c4} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c4}}\right] + A_0 \text{ at } (\omega = \omega_{c4})$

$$= -60 \times \log\frac{30}{15} + (-23) = -41 \text{ db}$$

TABLE-3

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$4/j\omega$		-20	–
$\frac{1}{1+j\omega}$	$\omega_{c1} = 1$	-20	$-20 - 20 = -40$
$(1+j0.24\omega)^2$	$\omega_{c2} = \frac{1}{0.24} = 4.2$	$+40$	$-40 + 40 = 0$
$\frac{1}{1+j0.2\omega}$	$\omega_{c3} = \frac{1}{0.2} = 5$	-20	$0 - 20 = -20$
$\frac{1}{(1+j0.067\omega)^2}$	$\omega_{c4} = \frac{1}{0.067} = 15$	-40	$-20 - 40 = -60$

Let the points e, f, g, h, i and j be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2}, ω_{c3}, ω_{c4} and ω_h respectively on the magnitude plot of compensated system. The magnitude plot of compensated system is drawn on the same semilog graph sheet by using the same scales as shown in fig 6.6.2.

PHASE PLOT

The phase angle of $G_0(j\omega)$ as a function of ω is given by,

$$\phi_0 = \angle G_0(j\omega) = 2\tan^{-1}0.24\omega - 90^\circ - \tan^{-1}\omega - \tan^{-1}0.2\omega - 2\tan^{-1}0.067\omega.$$

The phase angle of $G_0(j\omega)$ are calculated for various values of ω and listed in table - 4.

TABLE-4

ω rad/sec	0.1	0.5	6.0	2.0	5	10	15
$\angle G_0(j\omega)$ deg	−94	−112	−127°	−139 ≈ −140	−150	−171°	−189 ≈ −188

In the same semilog sheet and by using the same scales, the phase plot of compensated system is sketched as shown in fig 6.6.2.

Let, ϕ_{gc0} = Phase of $G_0(j\omega)$ at new gain crossover frequency (ω_{gc0}).

and γ_0 = Phase margin of compensated system.

From the bode plot of compensated system we get, $\phi_{gc0} = -140°$.

Now, $\gamma_0 = 180° + \phi_{gc0} = 180° - 140° = 40°$

CONCLUSION

The phase margin of the compensated system is satisfactory. Hence the design is acceptable.

RESULT

$$\left.\begin{matrix}\text{The transfer function}\\ \text{of lead compensator}\end{matrix}\right\} G_c(s) = \frac{0.0784\,(1 + 0.24s)^2}{(1 + 0.067s)^2} = \frac{(s + 4.17)^2}{(s + 14.92)^2}.$$

$$\left.\begin{matrix}\text{Open loop transfer function}\\ \text{of lead compensated system}\end{matrix}\right\} G_0(s) = \frac{4\,(1 + 0.24s)^2}{s\,(1 + s)\,(1 + 0.2s)\,(1 + 0.067s)^2}$$

18. Design a lag-lead compensator so as to meet the following specifications: static velocity error constant $K_v = 10\ sec^{-1}$. Phase margin= 50° and gain margin >= 10 dB.

(April/May 2011)(May/June 2012) (Nov/Dec 2017)

SOLUTION

Step-1 : Determine K

For unity feedback system,

Velocity error constant, $K_v = \underset{s\to 0}{Lt}\ sG(s)$

Given that, $K_v = 80$.

$$\therefore \underset{s\to 0}{Lt}\ s\,G(s) = \underset{s\to 0}{Lt}\ s\,\frac{K}{s(s+3)(s+6)} = 80 \quad \Rightarrow \quad \frac{K}{3\times 6} = 80 \quad \Rightarrow \quad K = 80\times 3\times 6 = 1440$$

$$\therefore G(s) = \frac{1440}{s(s+3)(s+6)} = \frac{1440}{s\times 3(1+s/3)\times 6(1+s/6)} = \frac{80}{s(1+0.33s)(1+0.167s)}$$

Step-2: Bode plot of uncompensated system.

In G(s), put $s = j\omega$

$$\therefore G(j\omega) = \frac{80}{j\omega(1+j0.33\omega)(1+j0.167\omega)}$$

MAGNITUDE PLOT

The corner frequencies are ω_{c1} and ω_{c2}.

Here ω_{c1} = 1/0.33 = 3 rad/sec and ω_{c2} = 1/0.167 = 6 rad/sec.

The various terms of G(jω) are listed in table-1. Also the table shows the slope contributed by each term and the change in slope at the corner frequency.

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{80}{j\omega}$	–	–20	–
$\frac{1}{1+j0.33\omega}$	$\omega_{c1}=\frac{1}{0.33}=3$	–20	–20 –20 = –40
$\frac{1}{1+j0.167\omega}$	$\omega_{c2}=\frac{1}{0.167}=6$	–20	–40 –20 = –60

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$

Let ω_l = 0.5 rad/sec and ω_h = 20 rad/sec.

Let A = |G(jω)| in db

$$\text{At } \omega = \omega_l, \quad A = 20\log\frac{80}{\omega} = 20\log\frac{80}{0.5} = 44 \text{ db}$$

$$\text{At } \omega = \omega_{c1}, \quad A = 20\log\frac{80}{\omega} = 20\log\frac{80}{3} = 28.5 \text{ db} \approx 28 \text{ db}$$

$$\text{At } \omega = \omega_{c2}, \quad A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A \text{ at } (\omega = \omega_{c1})$$

$$= -40 \times \log\frac{6}{3} + 28 = 16 \text{ db}$$

$$\text{At } \omega = \omega_h, \quad A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A \text{ at } (\omega = \omega_{c2})$$

$$= -60 \times \log\frac{20}{6} + 16 = -15 \text{ db}$$

Let the points a, b, c and d be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose appropriate scales and fix the points a, b, c and d. Join the points by straight lines and mark the slope on the respective region. The magnitude plot is shown in fig 6.9.2.

PHASE PLOT

The phase angle of G(jω) as a function of ω is given by

$$\phi = \angle G(j\omega) = -90° - \tan^{-1}0.33\omega - \tan^{-1}0.167\omega.$$

The phase angle of G(jω) are calculated for various values of ω and listed in table-2.

TABLE-2

ω rad/sec	0.5	1.0	3.0	6	10	20
∠G(jω) deg	−104	−118	−161 ≈ −160	−198	−222	−244.7 ≈ −244

On the same semilog sheet take another y-axis, choose appropriate scale and draw phase plot as shown in fig 6.9.2.

Step-3 : Find phase margin of uncompensated system.

Let, ϕ_{gc} = Phase of G(jω) at gain crossover frequency

γ = Phase margin of uncompensated system.

From the bode plot of uncompensated system we get, $\phi_{gc} = -226°$.

Now, $\gamma = 180° + \phi_{gc} = 180° - 226° = -46°$

Step-4 : Choose a new phase margin

The desired phase margin, $\gamma_d = 35°$

The phase margin of compensated system, $\gamma_n = \gamma_d + \epsilon$

Let initial choice of $\epsilon = 5°$

$\therefore \gamma_n = \gamma_d + \epsilon = 35° + 5° = 40°$.

Step-5 : Determine new gain crossover frequency

Let, ω_{gcn} = New gain crossover frequency and ϕ_{gcn} = Phase of G(jω) at ω_{gcn}

Now, $\gamma_n = 180° + \phi_{gcn}$, $\therefore \phi_{gcn} = \gamma_n - 180° = 40° - 180° = -140°$

From the bode plot we found that the frequency corresponding to a phase of −140° is 1.8 rad/sec.

Let, ω_{gcl} = Gain crossover frequency of lag compensator.

Choose ω_{gcl} such that, $\omega_{gcl} > \omega_{gcn}$. Let ω_{gcl} = 4 rad/sec.

Step-6 : Calculate β of lag compensator

From the bode plot we found that the db magnitude at ω_{gcl} is 23 db.

$\therefore |G(j\omega)|$ in db at $(\omega = \omega_{gcl}) = A_{gcl} = 23$ db.

Also, $A_{gcl} = 20\log\beta$; $\therefore \beta = 10^{A_{gcl}/20} = 10^{23/20} = 14$

Step-7 : Determine the transfer function of lag section.

The zero of the lag compensator is placed at a frequency one-tenth of ω_{gcl}.

$\therefore$ Zero of lag compensator, $z_{c1} = \frac{-1}{T_1} = \frac{\omega_{gcl}}{10}$

$$\text{Now, } T_1 = \frac{10}{\omega_{gcl}} = \frac{10}{4} = 2.5$$

Pole of lag compensator, $p_{c1} = \frac{1}{\beta T_1} = \frac{1}{14 \times 2.5} = \frac{1}{35}$

$$\left.\begin{array}{l}\text{Transfer function of}\\ \text{lag section}\end{array}\right\} G_1(s) = \beta\frac{(1+sT_1)}{(1+s\beta T_1)} = 14\frac{(1+2.5s)}{(1+35s)}$$

Step-8 : Determine the transfer function of lead section.

Let $\alpha = 1/\beta$; $\therefore \alpha = 1/14 = 0.07$

$$\left.\begin{array}{l}\text{The db gain (magnitude)}\\ \text{corresponding to } \omega_m\end{array}\right\} = -20\log\frac{1}{\sqrt{\alpha}} = -20\log\frac{1}{\sqrt{0.07}} = -11.5\text{ db} \approx -12\text{ db}$$

From the bode plot of uncompensated system the frequency ω_m corresponding to a db pair of -12 db is found to be 17 rad/sec.

$$\therefore \omega_m = 17\text{ rad/sec}$$

$$\therefore T_2 = \frac{1}{\omega_m\sqrt{\alpha}} = \frac{1}{17\sqrt{0.07}} = 0.22$$

$$\left.\begin{array}{l}\text{Transfer function}\\ \text{of lead section}\end{array}\right\} G_2(s) = \alpha\frac{(1+sT_2)}{(1+s\alpha T_2)} = 0.07\frac{(1+0.22s)}{(1+0.0154s)}$$

Step-10 : Determine the transfer function of lag-lead compensator.

$$\left.\begin{array}{l}\text{Transfer function}\\ \text{of lag - lead compensator}\end{array}\right\} G_c(s) = G_1(s) \times G_2(s) = 14\frac{(1+2.5s)}{(1+35s)} \times 0.07\frac{(1+0.22s)}{(1+0.0154s)}$$

$$= \frac{(1+2.5s)(1+0.22s)}{(1+35s)(1+0.0154s)}$$

Step-11 : Determine open loop transfer function of compensated system.

The lag-lead compensator is connected in series with G(s) as shown in fig 6.9.1.

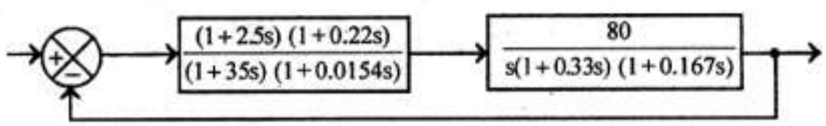


Fig 6.9.1 : Block diagram of lag-lead compensated system.

$$\left.\begin{array}{l}\text{Open loop transfer}\\ \text{function of}\\ \text{compensated system}\end{array}\right\} G_0(s) = \frac{80(1+2.5s)(1+0.22s)}{s(1+35s)(1+0.0154s)(1+0.33s)(1+0.167s)}$$

Step-12 : Bode plot of compensated system.

Put $s = j\omega$ in $G_0(s)$

$$\therefore G_0(j\omega) = \frac{80(1+j2.5\omega)(1+j0.22\omega)}{j\omega(1+j35\omega)(1+j0.0154\omega)(1+j0.33\omega)(1+j0.167\omega)}$$

MAGNITUDE PLOT

There are six corner frequencies, which are given below.

$$\omega_{c1} = \frac{1}{35} = 0.03\,\text{rad/sec} \; ; \; \omega_{c2} = \frac{1}{2.5} = 0.4\,\text{rad/sec} \; ; \; \omega_{c3} = \frac{1}{0.33} = 3\,\text{rad/sec} \; ;$$

$$\omega_{c4} = \frac{1}{0.22} = 4.5\,\text{rad/sec}; \; \omega_{c5} = \frac{1}{0.167} = 6\,\text{rad/sec} \; ; \; \omega_{c6} = \frac{1}{0.0154} = 65\,\text{rad/sec}.$$

The various terms of $G_0(j\omega)$ are listed in table-3. Also the table shows the slope contributed b each term and the change in slope at the corner frequency.

TABLE-3

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{80}{j\omega}$	–	–20	–
$\frac{1}{1+j35\omega}$	$\omega_{c1} = \frac{1}{35} = 0.03$	–20	–20 –20 = –40
$1+j2.5\omega$	$\omega_{c2} = \frac{1}{2.5} = 0.4$	+20	–40 +20 = –20
$\frac{1}{1+j0.33\omega}$	$\omega_{c3} = \frac{1}{0.33} = 3$	–20	–20 –20 = –40
$1+j0.22\omega$	$\omega_{c4} = \frac{1}{0.22} = 4.5$	+20	–40 +20 = –20
$\frac{1}{1+j0.167\omega}$	$\omega_{c5} = \frac{1}{0.167} = 6$	–20	–20 –20 = –40
$\frac{1}{1+j0.0154\omega}$	$\omega_{c6} = \frac{1}{0.0154} = 65$	–20	–40 –20 = –60

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c6}$

Let ω_l = 0.01 rad/sec and ω_h = 80 rad/sec

Let $A_0 = |G_0(j\omega)|$ in db.

$$\text{At } \omega = \omega_l, \quad A_0 = 20\log\frac{80}{0.01} = 78 \text{ db}$$

$$\text{At } \omega = \omega_{c1}, \quad A_0 = 20\log\frac{80}{0.03} = 68.5 \text{ db} \approx 68 \text{ db}$$

$$\text{At } \omega = \omega_{c2}, \quad A_0 = -40\times\log\frac{0.4}{0.03} + 68 = 23 \text{ db}$$

$$\text{At } \omega = \omega_{c3}, \quad A_0 = -20\times\log\frac{3}{0.4} + 23 = 5 \text{ db}$$

19. Discuss detailed about lag compensator. (Nov/Dec 2013,May-17)

A compensator having the characteristics of a lag network is called a lag compensator. If a sinusoidal signal is applied to a lag network, then in steady state the output will have a phase lag with respect to input.

Lag compensation results in a large improvement in steady state performance but results in slower response due to reduced bandwidth. The attenuation due to the lag compensator will shift the gain crossover frequency to a lower frequency point where the phase margin is acceptable. Thus, the lag compensator will reduce the bandwidth of the system and will result in slower transient response.

Lag compensator is essentially a low pass filter and so high frequency noise signals are attenuated. If the pole introduced by the compensator is not cancelled by a zero in the system, then lag compensator increases the order of the system by one.

S-PLANE REPRESENTATION OF LAG COMPENSATOR

The lag compensator has a pole at $s = -1/\beta T$ and a zero at $s = -1/T$. The pole-zero plot of lag compensator is shown in fig 6.3. Here, $\beta>1$, so the zero is located to the left of the pole on the negative real axis. The general form of lag compensator transfer function is given by equation (6.1).

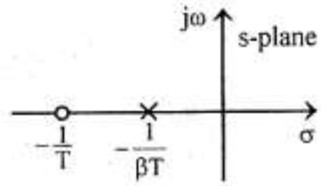


Fig 6.3 : Pole-zero plot of lag compensator.

$$\text{Transfer function of lag compensator, } G_c(s) = \frac{s+z_c}{s+p_c} = \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \quad .....(6.1)$$

where, $T > 0$ and $\beta > 1$

$$\text{The zero of lag compensator, } z_c = \frac{1}{T} \quad .....(6.2)$$

$$\text{The pole of lag compensator, } p_c = \frac{1}{\beta T} = \quad .....(6.3)$$

$$\text{From equation(6.2) we get, } T = \frac{1}{z_c} \quad .....(6.4)$$

$$\text{From equation(6.3) we get, } \beta = \frac{z_c}{p_c} \quad .....(6.5)$$

REALISATION OF LAG COMPENSATOR USING ELECTRICAL NETWORK

The lag compensator can be realised by the R-C network shown in fig 6.4.

Let, $E_i(s)$ = Input voltage

$E_o(s)$ = Output voltage

In the network shown in fig 6.4, the input voltage is applied to the series combination of R_1, R_2 and C. The output voltage is obtained across series combination of R_2 and C.

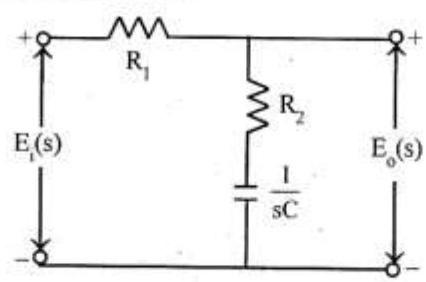


Fig 6.4 : Electrical lag compensator.

By voltage division rule,

$$E_o(s) = E_i(s)\ \frac{(R_2 + 1/sC)}{(R_1 + R_2 + 1/sC)} = E_i(s)\ \frac{(sCR_2+1)/sC}{[sC(R_1+R_2)+1]/sC} = E_i(s)\ \frac{(sCR_2+1)}{[sC(R_1+R_2)+1]}$$

The transfer function of the electrical network is the ratio of output voltage to input voltage,

$$\left.\begin{matrix}\text{Transfer function}\\ \text{of electrical network}\end{matrix}\right\} \frac{E_o(s)}{E_i(s)} = \frac{CR_2(s + 1/CR_2)}{C(R_1+R_2)\left[s + 1/C(R_1+R_2)\right]}$$

$$= \frac{\left(s+\frac{1}{R_2C}\right)}{\left(\frac{R_1+R_2}{R_2}\right)\left[s+\frac{1}{((R_1+R_2)/R_2)\,R_2C}\right]} \qquad .....(6.6)$$

But the transfer function of lag compensator is given by,

$$G_c(s) = \frac{(s+\frac{1}{T})}{(s+\frac{1}{\beta T})} \qquad .....(6.7)$$

On comparing equations (6.6) and (6.7) we get,

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{\beta}\,\frac{(s+\frac{1}{T})}{(s+\frac{1}{\beta T})} \qquad .....(6.8)$$

where, $T = R_2C$ and $\beta = (R_1+R_2)/R_2$

The transfer function of RC network as given by equation(6.8) is similar to the general form with an attenuation of $1/\beta$ (since $\beta > 1$, $(1/\beta) < 1$). If the attenuation is not required then an amplifier with gain β can be connected in cascade with RC network to nullify the attenuation.

FREQUENCY RESPONSE OF LAG COMPENSATOR

Consider the general form of lag compensator,

$$G_c(s) = \frac{\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\beta T}\right)} = \frac{(sT+1)/T}{(s\beta T+1)/\beta T} = \beta\frac{(1+sT)}{(1+s\beta T)} \qquad .....(6.9)$$

Sinusoidal transfer function of lag compensator is obtained by letting $s = j\omega$ in equation (6.9).

$$\therefore G_c(j\omega) = \frac{\beta(1+j\omega T)}{(1+j\omega\beta T)} \qquad .....(6.10)$$

$$\text{When } \omega = 0,\ G_c(j\omega) = \beta \qquad .....(6.11)$$

From equation(6.11) we can say that the lag compensator provides a dc gain of β (here $\beta > 1$). If the dc gain of the compensator is not desirable then it can be eliminated by a suitable attenuation.

Let us assume that the gain β is eliminated by a suitable attenuation network. Now, $G_c(j\omega)$ is given by,

$$G_c(j\omega) = \frac{1+j\omega T}{1+j\omega\beta T} = \frac{\sqrt{1+(\omega T)^2}\angle\tan^{-1}\omega T}{\sqrt{1+(\omega\beta T)^2}\angle\tan^{-1}\omega\beta T} \qquad .....(6.12)$$

The sinusoidal transfer function shown in equation(6.12) has two corner frequencies and they are denoted as ω_{c1} and ω_{c2}.

Here, $\omega_{c1} = \frac{1}{\beta T}$ and $\omega_{c2} = \frac{1}{T}$.

Since, $\beta T > T$, $\omega_{c1} < \omega_{c2}$.

Let, $A = |G_c(j\omega)|$ in db $= 20 \log \dfrac{\sqrt{1+(\omega T)^2}}{\sqrt{1+(\omega\beta T)^2}}$ .....(6.13)

At very low frequencies i.e., upto ω_{c1}, $\omega T << 1$ and $\omega\beta T << 1$.

$$\therefore A \approx 20 \log 1 = 0$$

In the frequency range from ω_{C1} to ω_{C2}, $\omega T << 1$ and $\omega\beta T >> 1$.

$$\therefore A \approx 20 \log \frac{1}{\sqrt{(\omega\beta T)^2}} = 20 \log \frac{1}{\omega\beta T}$$

At very high frequencies i.e., after ω_{c2}, $\omega T >> 1$ and $\omega\beta T >> 1$.

$$\therefore A \approx 20 \log \frac{\sqrt{(\omega T)^2}}{\sqrt{(\omega\beta T)^2}} = 20 \log \frac{1}{\beta}$$

The approximate magnitude plot of lag compensator is shown in fig 6.5. The magnitude plot of Bode plot of $G_c(j\omega)$ is a straight line through 0 db upto ω_{c1}, then it has a slope of –20 db/dec upto ω_{c2} and after ω_{c2} it is a straight line with a constant gain of 20 log (1/β).

20. Draw the pole zero diagram of a lead compensator. Propose lead compensation using electrical network. Derive the transfer function. Draw the Bode plot.(Nov/Dec 2012).

Discuss detailed about lead compensator. (Nov/Dec 2013)

A compensator having the characteristics of a lead network is called a lead compensator. If a sinusoidal signal is applied to the lead network, then in steady state the output will have a phase lead with respect to the input.

The lead compensation increases the bandwidth, which improves the speed of response and also reduces the amount of overshoot. Lead compensation appreciably improves the transient response, whereas there is a small change in steady state accuracy. Generally, lead compensation is provided to make an unstable system as a stable system.

A lead compensator is basically a high pass filter and so it amplifies high frequency noise signals. If the pole introduced by the compensator is not cancelled by a zero in the system, then lead compensation increases the order of the system by one.

S-PLANE REPRESENTATION OF LEAD COMPENSATOR

The lead compensator has a zero at $s = -1/T$ and a pole at $s = -1/\alpha T$. The pole-zero plot of lead compensator is shown in fig 6.9. Here, $\alpha < 1$, so the zero is closer to the origin than the pole. The general form of lead compensator transfer function is given by equation (6.19),

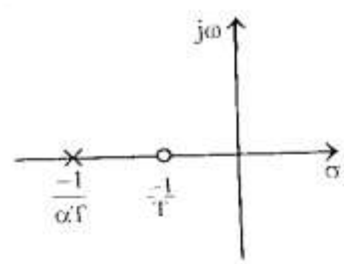


Fig 6.9 : Pole-zero plot of lead compensator.

$$G_c(s) = \frac{s + z_c}{s + p_c} = \frac{\left(s + \frac{1}{T}\right)}{\left(s + \frac{1}{\alpha T}\right)}$$ .....(6.

where, $T > 0$ and $\alpha < 1$

Also we can express ϕ_m in terms of α and α in terms of ϕ_m as shown below.

$$\phi_m = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right) \qquad .....(6.34)$$

$$\alpha = \frac{1 - \sin\phi_m}{1 + \sin\phi_m} \qquad .....(6.35)$$

Note : The equations (6.33), (6.34) and (6.35) can be derived by a similar analysis shown in section 6.2 for lag compensator after replacing β by α.

21. Derive the transferfunction of lag-lead compensator.(Nov/Dec 2015)

Realize the basic compensators using electrical network and obtain the transfer function. (Nov/Dec 2017)

REALISATION OF LAG COMPENSATOR USING ELECTRICAL NETWORK

The lag compensator can be realised by the R-C network shown in fig 6.4.

Let, $E_i(s)$ = Input voltage

$E_o(s)$ = Output voltage

In the network shown in fig 6.4, the input voltage is applied to the series combination of R_1, R_2 and C. The output voltage is obtained across series combination of R_2 and C.

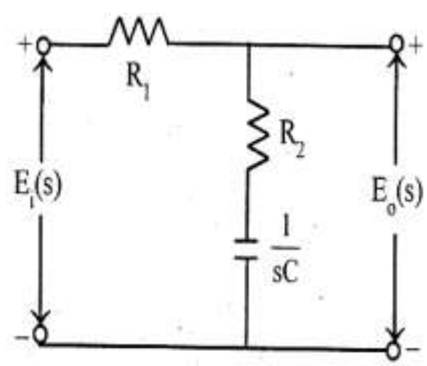


Fig 6.4 : Electrical lag compensator.

By voltage division rule,

$$E_0(s) = E_i(s)\,\frac{(R_2 + 1/sC)}{(R_1 + R_2 + 1/sC)} = E_i(s)\,\frac{(sCR_2+1)/sC}{[sC(R_1+R_2)+1]/sC} = E_i(s)\,\frac{(sCR_2+1)}{[sC(R_1+R_2)+1]}$$

The transfer function of the electrical network is the ratio of output voltage to input voltage,

$$\left.\begin{array}{l}\text{Transfer function}\\ \text{of electrical network}\end{array}\right\}\ \frac{E_0(s)}{E_i(s)} = \frac{CR_2(s + 1/CR_2)}{C(R_1+R_2)\left[s + 1/C(R_1+R_2)\right]}$$

$$= \frac{\left(s + \frac{1}{R_2 C}\right)}{\left(\frac{R_1 + R_2}{R_2}\right)\left[s + \frac{1}{((R_1 + R_2)/R_2)\, R_2 C}\right]} \qquad .....(6.6)$$

But the transfer function of lag compensator is given by,

$$G_c(s) = \frac{(s + \frac{1}{T})}{(s + \frac{1}{\beta T})} \qquad .....(6.7)$$

On comparing equations (6.6) and (6.7) we get,

$$\frac{E_0(s)}{E_i(s)} = \frac{1}{\beta} \frac{(s + \frac{1}{T})}{(s + \frac{1}{\beta T})} \qquad .....(6.8)$$

where, $T = R_2 C$ and $\beta = (R_1 + R_2)/R_2$

The transfer function of RC network as given by equation(6.8) is similar to the general form with an attenuation of $1/\beta$ (since $\beta > 1$, $(1/\beta) < 1$). If the attenuation is not required then an amplifier with gain β can be connected in cascade with RC network to nullify the attenuation.

REALISATION OF LEAD COMPENSATOR USING ELECTRICAL NETWORK

The lead compensator can be realised by the RC network shown in fig 6.10.

Let, $E_i(s)$ = Input voltage, and $E_0(s)$ = Output voltage

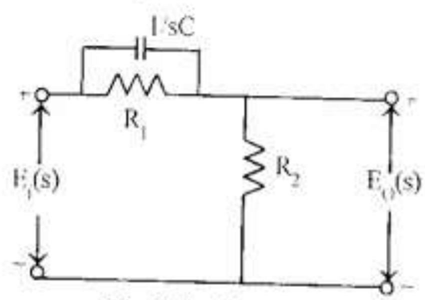


Fig 6.10 : Electrical lead compensator.

In the network shown in fig 6.10, the input voltage is applied to the series combination of ($R_1 \| C$) and R_2. The output voltage is obtained across R_2.

By voltage division rule,

$$\text{Output voltage, } E_0(s) = E_i(s) \frac{R_2}{R_2 + \frac{\left(R_1 \times \frac{1}{sC}\right)}{\left(R_1 + \frac{1}{sC}\right)}}$$

$$E_0(s) = E_i(s) \cdot \frac{R_2}{R_2 + \frac{R_1}{(R_1 Cs + 1)}} = E_i(s) \frac{R_2}{\frac{R_2(R_1 Cs + 1) + R_1}{R_1 Cs + 1}}$$

The transfer function of the electrical network is the ratio of output voltage to input voltage.

$$\left.\begin{array}{l}\text{Transfer function of}\\ \text{electrical network}\end{array}\right\} \frac{E_0(s)}{E_i(s)} = \frac{R_2(R_1Cs + 1)}{[R_1R_2Cs + R_2 + R_1]} = \frac{R_1 C R_2\left[s + \frac{1}{R_1C}\right]}{R_1 C R_2\left[s + \frac{(R_1 + R_2)}{R_1 C R_2}\right]}$$

$$= \frac{\left[s + \frac{1}{R_1 C}\right]}{\left[s + \left(\frac{1}{R_2/(R_1 + R_2)}\right)\frac{1}{R_1 C}\right]} \quad .....(6.24)$$

The general form of lead compensator transfer function is,

$$G_c(s) = \frac{\left(s + \frac{1}{T}\right)}{\left(s + \frac{1}{\alpha T}\right)} \quad .....(6.25)$$

On comparing equations (6.24) and (6.25) we get,

$$\frac{E_0(s)}{E_i(s)} = \frac{s + \frac{1}{T}}{\left(s + \frac{1}{\alpha T}\right)} \quad .....(6.26)$$

where, $T = R_1C$ and $\alpha = \frac{R_2}{R_1 + R_2}$

The transfer function of the RC network is similar to the general form of transfer function of lead compensator.

Lag-Lead Compensator

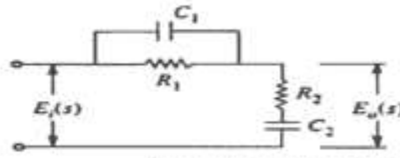
The lag-lead compensator is the combination of a lag compensator and a lead compensator. The lag-section is provided with one real pole and one real zero, the pole being to the right of zero, whereas the lead section has one real pole and one real came with the zero being to the right of the pole.

The transfer function of the lag-lead compensator will be

$$G(s) = \left(\frac{s + \frac{1}{\tau_1}}{s + \frac{1}{\alpha\tau_1}}\right)\left(\frac{s + \frac{1}{\tau_2}}{s + \frac{1}{\beta\tau_2}}\right)$$

The figure shows lag lead compensator

$$E_o(s) = \frac{E_i(s)}{\dfrac{R_1 \times \dfrac{1}{sC_1}}{R_1 + \dfrac{1}{sC_1}} + R_2 + \dfrac{1}{sC_2}}\left(R_2 + \frac{1}{sC_2}\right)$$



Lag-lead compensator

$$\frac{E_o(s)}{E_i(s)} = \frac{\left(R_1 + \dfrac{1}{sC_1}\right)\left(R_2 + \dfrac{1}{sC_2}\right)}{R_1\dfrac{1}{sC_1} + \left(R_2 + \dfrac{1}{sC_2}\right)\left(R_1 + \dfrac{1}{sC_1}\right)}$$

$$= \frac{\dfrac{(sC_1R_1 + 1)}{sC_1}\,\dfrac{(sC_2R_2 + 1)}{sC_2}}{\dfrac{R_1}{sC_1} + \dfrac{(R_2sC_2 + 1)}{sC_2}\,\dfrac{(R_1sC_1 + 1)}{sC_1}}$$

$$= \frac{\dfrac{(1 + sC_1R_1)(1 + sC_2R_2)}{s^2C_1C_2}}{\dfrac{R_1sC_2 + R_2sC_2 + 1 + R_1R_2s^2C_1C_2 + R_1sC_1}{s^2C_1C_2}}$$

$$= \frac{(1 + sC_1R_1)(1 + sC_2R_2)}{s^2R_1R_2C_1C_2 + s(R_1C_1 + R_2C_2) + 1 + R_1sC_2}$$

$$= \frac{C_1R_1\,C_2R_2\left(s + \dfrac{1}{C_1R_1}\right)\left(s + \dfrac{1}{C_2R_2}\right)}{R_1R_2C_1C_2\left[s^2 + \left\{\dfrac{1}{R_2C_2} + \dfrac{1}{R_1C_1} + \dfrac{1}{R_2C_1}\right\}s + \dfrac{1}{R_1R_2C_1C_2}\right]}$$

$$= \frac{\left(s + \dfrac{1}{C_1R_1}\right)\left(s + \dfrac{1}{C_2R_2}\right)}{s^2 + \left(\dfrac{1}{R_1C_1} + \dfrac{1}{R_2C_1} + \dfrac{1}{R_2C_2}\right)s + \dfrac{1}{R_1R_2C_1C_2}}$$

The above transfer functions are comparing with

$$G(s) = \frac{\left(s + \dfrac{1}{\tau_1}\right)\left(s + \dfrac{1}{\tau_2}\right)}{\left(s + \dfrac{1}{\alpha\tau_1}\right)\left(s + \dfrac{1}{\beta\tau_2}\right)}$$

Then

$$\frac{1}{\tau_1} = \frac{1}{C_1R_1}, \qquad \frac{1}{\tau_2} = \frac{1}{C_2R_2}$$

$$\frac{1}{\alpha\tau_1} + \frac{1}{\beta\tau_2} = \frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2}$$

$$\frac{1}{\alpha\beta\tau_1\tau_2} = \frac{1}{R_1R_2C_1C_2}$$

$$\tau_1 = C_1R_1$$

$$\tau_2 = C_2R_2$$

Therefore

$$\alpha\beta\tau_1\tau_2 = R_1R_2C_1C_2$$

$$\alpha\beta = 1 \qquad \text{or} \qquad \beta = \frac{1}{\alpha}$$

$$G(s) = \frac{\left(s + \frac{1}{\tau_1}\right)\left(s + \frac{1}{\tau_2}\right)}{\left(s + \frac{1}{\alpha\tau_1}\right)\left(s + \frac{\alpha}{\tau_2}\right)} \qquad \text{where } \alpha > 1$$

$$\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} = \frac{1}{\alpha\tau_1} + \frac{\alpha}{\tau_2}$$

22. Write the procedure for the design of Lag compensator using Bode plot.(May/June 2016,17)

1. Determine the open loop gain necessary to satisfy the specified error constant. If the phase margin of the uncompensated system with this gain is unsatisfactory, then design the lag network as per succeeding steps.
2. Find the frequency $wc2$ where the uncompensated system makes a phase margin contribution of $\phi_2 = \phi_s + \varepsilon$ where ϕ_2 is measured above -180 line. Allow for $\varepsilon = 5\ to\ 15$ for phase – lag contributed by the network at $wc2$.
3. Measure the gain of the uncompensated system at $wc2$ and equate it to the required high frequnecfy network attenuation$(20 \log \beta)$. Calculate these from the b parameter of the network. This procedure ensures that the compensated cross over frequency will lie at $wc2$.
4. Choose the upper corner frequency $(\omega_2) = 1/\tau$ of the network one octave to one decade below $wc2$ i.e

$$\omega_2 = 1/\tau = wc_2/2\ to wc_2/10$$

   A larger value of τ than that calculated from this rule is undesirable from the point of view of realization as it leads to excessively large size of capacitor for the network.
5. With $\beta and \tau$ determined, the lag compensator design is complete. Draw the frequency response of the compensated system to check the resulting phase margin.
6. If this is any additional specification, check if it is satisfied. Redesign the compensator by choosing another valve of τ, if found necessary.

For the system discussed above, the examination of the bode plots of the lag compensator system revails that

(i) Cross over frequency is reduced.

(ii) The high frequency end of the lag magnitude plot is lowered by 120 lag b db.

Thus we find the lag compensator reduces the system bandwidth and the additional attenuation of high frequency improves the signal/noise ratio of the system.

23. Draw the pole zero diagram of a lead compensator. Propose lead compensation using electrical network. Derive the transfer function. Draw the Bode plot.(Nov/Dec 2012).

Discuss detailed about lead compensator. (Nov/Dec 2013,May-17)

A compensator having the characteristics of a lead network is called a lead compensator. If a sinusoidal signal is applied to the lead network, then in steady state the output will have a phase lead with respect to the input.

The lead compensation increases the bandwidth, which improves the speed of response and also reduces the amount of overshoot. Lead compensation appreciably improves the transient response, whereas there is a small change in steady state accuracy. Generally, lead compensation is provided to make a unstable system as a stable system.

A lead compensator is basically a high pass filter and so it amplifies high frequency noise signals. If the pole introduced by the compensator is not cancelled by a zero in the system, then lead compensation increases the order of the system by one.

S-PLANE REPRESENTATION OF LEAD COMPENSATOR

The lead compensator has a zero at $s = -1/T$ and a pole at $s = -1/\alpha T$. The pole-zero plot of lead compensator is shown in fig 6.9. Here, $\alpha < 1$, so the zero is closer to the origin than the pole. The general form of lead compensator transfer function is given by equation (6.19),

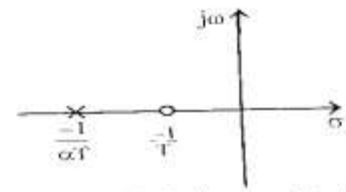


Fig 6.9 : Pole-zero plot of lead compensator.

$$G_c(s) = \frac{s + z_c}{s + p_c} = \frac{\left(s + \frac{1}{T}\right)}{\left(s + \frac{1}{\alpha T}\right)} \quad .....(6.$$

where, $T > 0$ and $\alpha < 1$

The zero of lead compensator. $z_c = \frac{1}{T}$ .....(6.20)

The pole of lead compensator. $p_c = \frac{1}{\alpha T}$ .....(6.21)

From equation (6.20) we get, $T = \frac{1}{z_c}$ .....(6.22)

From equation (6.21) we get. $\alpha = \frac{z_c}{p_c}$ .....(6.23)

REALISATION OF LEAD COMPENSATOR USING ELECTRICAL NETWORK

The lead compensator can be realised by the RC network shown in fig 6.10.

Let, $E_i(s)$ = Input voltage, and $E_0(s)$ = Output voltage

In the network shown in fig 6.10, the input voltage is applied to the series combination of $(R_1 \| C)$ and R_2. The output voltage is obtained across R_2.

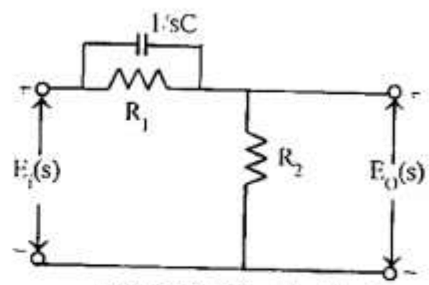


Fig 6.10 : Electrical lead compensator.

By voltage division rule,

$$\text{Output voltage, } E_0(s) = E_i(s)\,\frac{R_2}{R_2 + \dfrac{\left(R_1 \times \dfrac{1}{sC}\right)}{\left(R_1 + \dfrac{1}{sC}\right)}}$$

$$E_0(s) = E_i(s)\,.\,\frac{R_2}{R_2 + \dfrac{R_1}{(R_1Cs + 1)}} = E_i(s)\,\frac{R_2}{\dfrac{R_2(R_1Cs + 1) + R_1}{R_1Cs + 1}}$$

The transfer function of the electrical network is the ratio of output voltage to input voltage.

$$\left.\begin{array}{l}\text{Transfer function of}\\ \text{electrical network}\end{array}\right\}\ \frac{E_0(s)}{E_i(s)} = \frac{R_2(R_1Cs + 1)}{[R_1R_2Cs + R_2 + R_1]} = \frac{R_1\,C\,R_2\left[s + \dfrac{1}{R_1C}\right]}{R_1\,C\,R_2\left[s + \dfrac{(R_1 + R_2)}{R_1\,C\,R_2}\right]}$$

$$= \frac{\left[s + \dfrac{1}{R_1\,C}\right]}{\left[s + \left(\dfrac{1}{R_2/(R_1 + R_2)}\right)\dfrac{1}{R_1\,C}\right]} \quad .....(6.24)$$

The general form of lead compensator transfer function is,

$$G_c(s) = \frac{\left(s + \dfrac{1}{T}\right)}{\left(s + \dfrac{1}{\alpha T}\right)} \quad .....(6.25)$$

On comparing equations (6.24) and (6.25) we get,

$$\frac{E_0(s)}{E_i(s)} = \frac{s + \frac{1}{T}}{\left(s + \frac{1}{\alpha T}\right)} \quad .....(6.26)$$

where, $T = R_1C$ and $\alpha = \frac{R_2}{R_1 + R_2}$

The transfer function of the RC network is similar to the general form of transfer function of lead compensator.

FREQUENCY RESPONSE OF LEAD COMPENSATOR

Consider the general form of lead compensator,

$$G_c(s) = \frac{s + \frac{1}{T}}{\left(s + \frac{1}{\alpha T}\right)} = \frac{(1 + sT)/T}{(s\alpha T + 1)/\alpha T} = \alpha \frac{(1 + sT)}{(1 + \alpha sT)} \quad .....(6.27)$$

The sinusoidal transfer function of lead compensator is obtained by letting $s = j\omega$ in equation (6.27).

$$\therefore G_c(j\omega) = \alpha \frac{(1 + j\omega T)}{(1 + j\omega\alpha T)} \quad .....(6.28)$$

$$\text{When } \omega = 0,\ G_c(j\omega) = \alpha \quad .....(6.29)$$

From equation (6.29) we can say that the lead compensator provides an attenuation of α (Here $\alpha < 1$). If the attenuation of the compensator is not desirable then it can be eliminated by a suitable amplifier.

Let us assume that the attenuation α is eliminated by a suitable amplifier network. Now, $G_c(j\omega)$ is given by,

$$G_c(j\omega) = \frac{(1 + j\omega T)}{(1 + j\omega\alpha T)} = \frac{\sqrt{1 + (\omega T)^2} \angle \tan^{-1} \omega T}{\sqrt{1 + (\omega\alpha T)^2} \angle \tan^{-1} \omega\alpha T} \quad .....(6.30)$$

Sinusoidal transfer function shown in equation (6.30) has two corner frequencies ω_{c1} and ω_{c2}.

Here, $\omega_{c1} = \frac{1}{T}$ and $\omega_{c2} = \frac{1}{\alpha T}$. Since, $T > \alpha T$, $\omega_{c1} < \omega_{c2}$

$$\text{Let } A = |G_c(j\omega)| \text{ in db} = 20 \log \frac{\sqrt{1 + (\omega T)^2}}{\sqrt{1 + (\omega\alpha T)^2}} \quad .....(6.31)$$

At very low frequencies i.e., upto ω_{c1}, $\omega T << 1$ and $\omega\alpha T << 1$

$$\therefore A \approx 20 \log 1 = 0$$

Also we can express ϕ_m in terms of α and α in terms of ϕ_m as shown below.

$$\phi_m = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right) \qquad .....(6.34)$$

$$\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m} \qquad .....(6.35)$$

Note : The equations (6.33), (6.34) and (6.35) can be derived by a similar analysis shown in section 6.2 for lag compensator after replacing β by α.

24. Determine the stability of closed loop system by Nyquist stability criterion, whose open loop transfer system is given by, G(s).H(s)=(S+2)/(S+1)(S-1). (Nov/Dec 2017)

$(s+1)(s-1)$

SOLUTION

Given that, $G(s)H(s) = \dfrac{(s+2)}{(s+1)(s-1)}$

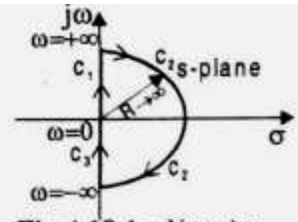


Fig 4.18.1 : *Nyquist Contour in s-plane*

The open loop transfer function does not have a pole at origin. Hence choose the Nyquist contour on s-plane enclosing the entire right half plane as shown in fig 4.18.1

The Nyquist contour has three sections C_1, C_2 and C_3. The mapping of each section is performed separately and the overall Nyquist plot is obtained by combining the individual sections.

MAPPING OF SECTION C_1

In section C_1, ω varies from 0 to $+\infty$. The mapping of section C_1 is given by the locus of $G(j\omega)H(j\omega)$ as ω is varied from 0 to ∞. This locus is the polar plot of $G(j\omega)H(j\omega)$.

Note : $(-1+j\omega)$ represents a point in second quadrant

$$G(s)H(s) = \frac{s+2}{(s+1)(s-1)} = \frac{2(1+0.5s)}{(1+s)(-1+s)}$$

Let $s = j\omega$. $\quad \therefore G(j\omega)H(j\omega) = \dfrac{2(1+j0.5\omega)}{(1+j\omega)(-1+j\omega)} = \dfrac{2\sqrt{1+0.25\omega^2}\ \angle \tan^{-1}0.5\omega}{\sqrt{1+\omega^2}\ \angle\tan^{-1}\omega\ \sqrt{1+\omega^2}\ \angle(180^\circ - \tan^{-1}\omega)}$

$$= \frac{2\sqrt{1+0.25\omega^2}}{1+\omega^2}\angle(-180+\tan^{-1}0.5\omega)$$

$$\therefore |G(j\omega)H(j\omega)| = \frac{2\sqrt{1+0.25\omega^2}}{1+\omega^2}$$

$$\angle G(j\omega)H(j\omega) = -180^\circ + \tan^{-1}0.5\omega$$

The exact shape of $G(j\omega)H(j\omega)$ locus is determined by calculating the magnitude and phase of $G(j\omega)H(j\omega)$ for various values of ω.

ω rad/sec	0	0.4	1.0	2.0	10.0	∞
$\lvert G(j\omega)H(j\omega)\rvert$	2	1.76	1.12	0.57	0.1	0
$\angle G(j\omega)H(j\omega)$ deg	−180	−168	−153	−135	−101	−90

From the above analysis, we can conclude that $G(j\omega)H(j\omega)$ locus starts at -180° axis at a magnitude of -2 for $\omega = 0$ and meets the origin along -90° axis when $\omega = +\infty$.

The section C_1 in s-plane and its corresponding mapping in G(s)H(s)-plane are shown in fig 4.18.2. and 4.18.3.

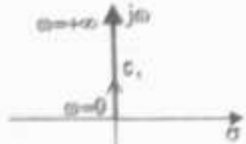


Fig 4.18.2 : Section C_1 in s-plane

$\Rightarrow$

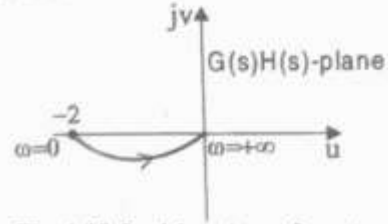


Fig 4.18.3 : Mapping of section C_1 in G(s)H(s)-plane

MAPPING OF SECTION C_2

The mapping of section C_2 from s-plane to G(s)H(s)-plane is obtained by letting $s = \underset{R\to\infty}{Lt} R\,e^{j\theta}$ in G(s)H(s) and varying θ from $+\pi/2$ to $-\pi/2$. Since $s \to R\,e^{j\theta}$ and $R \to \infty$, G(s) H(s) can be approximated as shown below, [i.e., $(1+sT) \approx sT$].

$$G(s)H(s) = \frac{2(1+0.5s)}{(1+s)\,(-1+s)} \approx \frac{2\times 0.5s}{s\times s} = \frac{1}{s}$$

Let, $s = \underset{R\to\infty}{Lt} R\,e^{j\theta}$.

$$\therefore G(s)H(s)\Big|_{s=\underset{R\to\infty}{Lt} R e^{j\theta}} = \frac{1}{\underset{R\to\infty}{Lt} R\,e^{j\theta}} = 0e^{-j\theta}$$

When $\theta = \frac{\pi}{2}$, $\quad G(s)H(s) = 0e^{-j\frac{\pi}{2}}$ .....(1)

When $\theta = -\frac{\pi}{2}$, $\quad G(s)H(s) = 0e^{j\frac{\pi}{2}}$ .....(2)

From the equations (1) and (2) we can say that section C_2 in s-plane (fig 4.18.4) is mapped as circular arc of zero radius around origin in G(s)H(s)-plane with argument varying from $-\pi/2$ to $+\pi/2$ as shown in fig 4.18.5.

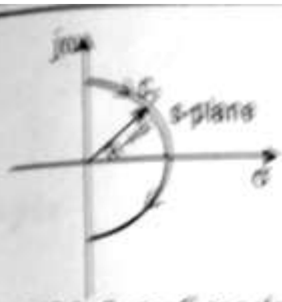


Fig 4.18.4 : Section C_2 in s-plane

$\Rightarrow$

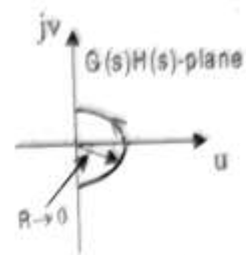


Fig 4.18.5 : Mapping of section C_2 in G(s)H(s)-plane

MAPPING OF SECTION C_3

In section C_3, ω varies from $-\infty$ to 0. The mapping of section C_3 is given by the locus of $G(j\omega)H(j\omega)$ as ω is varied from $-\infty$ to 0. This locus is the inverse polar plot of $G(j\omega)H(j\omega)$.

The inverse polar plot is given by the mirror image of polar plot with respect to real axis. The section C_3 in s-plane and its corresponding contour in G(s)H(s) plane are shown in fig 4.18.6 and fig 4.18.7.

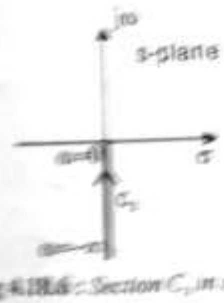


Fig 4.18.6 : Section C_2 in s-plane

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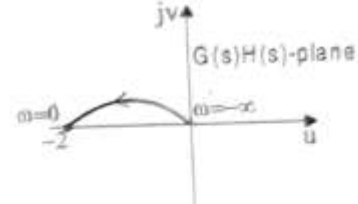
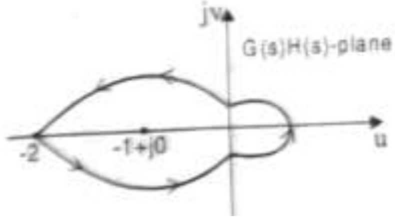


Fig 4.18.7 : *Mapping of section C_2 in $G(s)H(s)$-plane*



25. Construct the Nyquist plot for a sysytem whose open loop transfer function is given by $G(s)H(s)= K(1+s)^2/s^3$.(April/May 2018)

Given that, $G(s)H(s) = \frac{K(1+s)^2}{s^3}$

The open loop transfer function has three poles at origin. Hence choose the Nyquist contour on s-plane enclosing the entire right half plane except the origin as shown in fig 4.14.1.

The Nyquist contour has four sections C_1, C_2, C_3 and C_4. The mapping of each section is performed separately and the overall Nyquist plot is obtained by combining the individual sections.

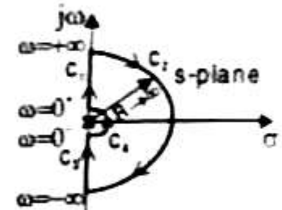


Fig 4.14.1 : *Nyquist Contour in*

MAPPING OF SECTION C_1

In section C_1, ω varies from 0 to +∞. The mapping of section C_1 is given by the locus of $G(j\omega)H(j\omega)$ as ω is varied from 0 to ∞. This locus is the polar plot of $G(j\omega)H(j\omega)$.

$$G(s)H(s) = \frac{K(1+s)^2}{s^3}$$

Let $s = j\omega$.

$$\therefore G(j\omega)H(j\omega) = \frac{K(1+j\omega)^2}{(j\omega)^3} = \frac{K(1-\omega^2+2j\omega)}{-j\omega^3} = \frac{K(1-\omega^2)}{-j\omega^3} + \frac{K\,2j\omega}{-j\omega^3} = -\frac{2K}{\omega^2} + j\frac{K(1-\omega^2)}{\omega^3}$$

When the $G(j\omega)H(j\omega)$ locus crosses real axis the imaginary term will be zero and the corresponding frequency is the phase crossover frequency, ω_{pc}.

$$\therefore \text{At } \omega = \omega_{pc}, \quad K(1-\omega_{pc}^2) = 0 \quad \Rightarrow \quad 1-\omega_{pc}^2 = 0 \quad \Rightarrow \quad \omega_{pc} = 1\,\text{rad/sec}$$

At $\omega = \omega_{pc} = 1$ rad / sec,

$$G(j\omega)H(j\omega) = -\frac{2K}{\omega^2} = -\frac{2K}{1^2} = -2K$$

$$G(j\omega)\,H(j\omega) = \frac{K(1+j\omega)^2}{(j\omega)^3} = \frac{K\sqrt{1+\omega^2}\,\angle\tan^{-1}\omega\,\sqrt{1+\omega^2}\,\angle\tan^{-1}\omega}{\omega^3\angle 270^\circ} = \frac{K(1+\omega^2)}{\omega^3}\angle(2\tan^{-1}\omega - 270^\circ)$$

As $\omega \to 0$, $\quad G(j\omega)H(j\omega) \to \infty\angle -270^\circ$

As $\omega \to \infty$, $\quad G(j\omega)H(j\omega) \to 0\angle -90^\circ$

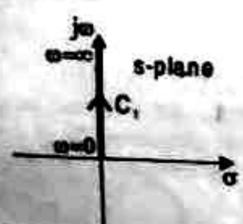


Fig 4.14.2 : *Section C_1 in s-plane*

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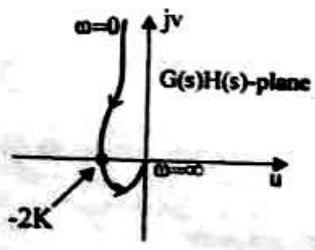


Fig 4.14.3 : *Mapping of section C_1 in G(s)H(s)-plane*

From equations (1), (2) and (3) we can say that the polar plot starts at −270° axis at infinity, crosses real axis at −2K and ends at origin in third quadrant. The section C_1 and its mapping are shown in fig 2 and 3.

MAPPING OF SECTION C_2

The mapping of section C_2 from s-plane to G(s) H(s)-plane is obtained by letting $s = \underset{R\to\infty}{Lt} R\,e^{j\theta}$ in G(s)H(s) and varying θ from +π/2 to −π/2. Since $s \to R\,e^{j\theta}$ and $R\to\infty$, the G(s) H(s) can be approximated as shown below, [i.e., $(1+sT) \approx sT$].

$$G(s)H(s) = \frac{K(1+s)^2}{s^3} \approx \frac{Ks^2}{s^3} = \frac{K}{s}$$

Let $s = \underset{R \to \infty}{Lt} Re^{j\theta}$

$$\therefore G(s)\,H(s)\Big|_{s=\underset{R\to\infty}{Lt} Re^{j\theta}} = \frac{K}{s}\Big|_{s=\underset{R\to\infty}{Lt} Re^{j\theta}} = \frac{K}{\underset{R\to\infty}{Lt} Re^{j\theta}} = 0e^{-j\theta}$$

When $\theta = \frac{\pi}{2}$, $\quad G(s)\,H(s) = 0e^{-j\frac{\pi}{2}}$ .....(4)

When $\theta = -\frac{\pi}{2}$, $\quad G(s)\,H(s) = 0e^{j\frac{\pi}{2}}$ .....(5)

From the equations (4) and (5) we can say that section C_2 in s-plane (fig 4.14.4.) is mapped as circular arc of zero radius around origin in G(s)H(s)-plane with argument (phase) varying from $-\pi/2$ to $+\pi/2$ as shown in fig 4.14.5

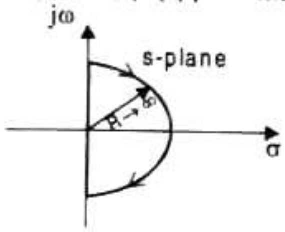


Fig 4.14.4 : *Section C_2 in s-plane*

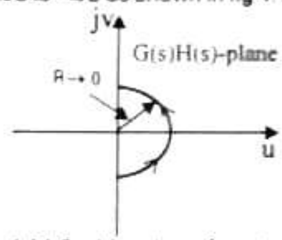


Fig 4.14.5 : *Mapping of section C_2 in G(s)H(s)-plane*

Mapping of section C_3

In section C_3, ω varies from $-\infty$ to 0. The mapping of section C_3 is given by locus of $G(j\omega)\,H(j\omega)$ as ω is varied from $-\infty$ to 0. This locus is the inverse polar plot of $G(j\omega)\,H(j\omega)$.

The inverse polar plot is given by the mirror image of polar plot with respect to real axis. The section C_3 in s-plane and its corresponding contour in G(s)H(s) plane are shown in fig 4.14.6 and fig 4.14.7.

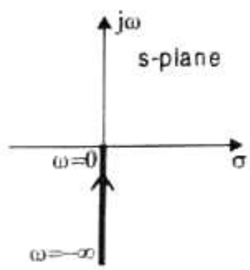


Fig 4.14.6 : *Section C_3 in s-plane*

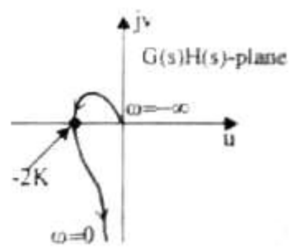


Fig 4.14.7 : *Mapping of section C_3 in G(s)H(s)-plane*

Mapping of section C_4

The mapping of section C_4 from s-plane to G(s) H(s)-plane is obtained by letting $s = \underset{R\to 0}{Lt} Re^{j\theta}$ in G(s)H(s) and varying θ from $-\pi/2$ to $+\pi/2$. Since $s \to Re^{j\theta}$ and $R \to 0$, the G(s)H(s) can be approximated as shown below, [i.e., $(1+sT) \approx 1$].

$$G(s)H(s) = \frac{K(1+s)^2}{s^3} \approx \frac{K \times 1}{s^3} = \frac{K}{s^3}$$

Let $s = \underset{R\to 0}{Lt} Re^{j\theta}$

$$\therefore G(s)\,H(s)\Big|_{s=\underset{R\to 0}{Lt} Re^{+j\theta}} = \frac{K}{s^3}\Big|_{s=\underset{R\to 0}{Lt} Re^{+j\theta}} = \frac{K}{\underset{R\to 0}{Lt} (Re^{+j\theta})^3} = \infty e^{-j3\theta}$$

When $\theta = -\frac{\pi}{2}$, $G(s)\,H(s) = \infty e^{+j\beta\frac{\pi}{2}}$

When $\theta = \frac{\pi}{2}$, $G(s)\,H(s) = \infty e^{-j\beta\frac{\pi}{2}}$

From the equations (6) and (7) we can say that section C_4 in s-plane (fig 4.14.8.) is mapped as a circular arc of in… radius with argument (phase) varying from $+3\pi/2$ to $-3\pi/2$ as shown in fig 4.14.9.

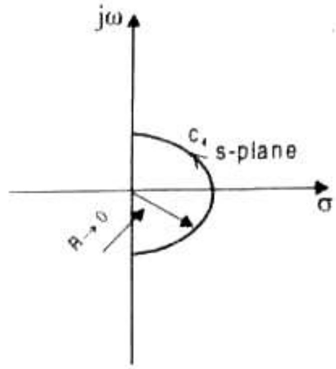


Fig 4.14.8 : *Section C_4 in s-plane*

⇒

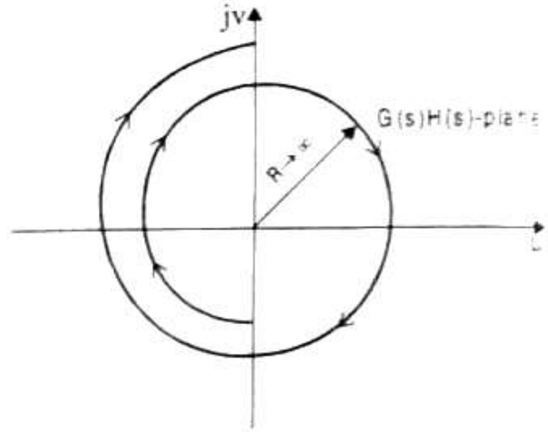


Fig 4.14.9 : *Mapping of section C_4 in G(s)H(s)-plane*

COMPLETE NYQUIST PLOT

The entire Nyquist plot in G(s)H(s)-plane can be obtained by combining the mappings of individual sections as shown in fig 4.14.10.

STABILITY ANALYSIS

When, $-2K = -1$, the contour passes through $-1+j0$ point and corresponding value of K is the limiting value of K for stability.

$$\therefore \text{Limiting value of } K = \frac{1}{2} = 0.5$$

When K < 0.5

When K is less than 0.5, the contour crosses real axis at a point between 0 and $-1+j0$. On travelling through Nyquist plot along the indicated direction it is observed that the $-1+j0$ point is encircled in clockwise direction two times. Therefore the system is unstable. [Since there are two clockwise encirclement and no right half open loop poles, the closed loop system will have two poles on right half of s-plane]

When K > 0.5

When K is greater than 0.5, the contour crosses real axis at a point between $-1+j0$ and $-\infty$. On travelling through Nyquist plot along the indicated direction it is observed that $(-1+j0)$ point is encircled in both clockwise and anticlockwise direction one time. Hence net encirclement is zero. Also the open loop system has no poles at the right half of s-plane. Therefore the closed loop system is stable.

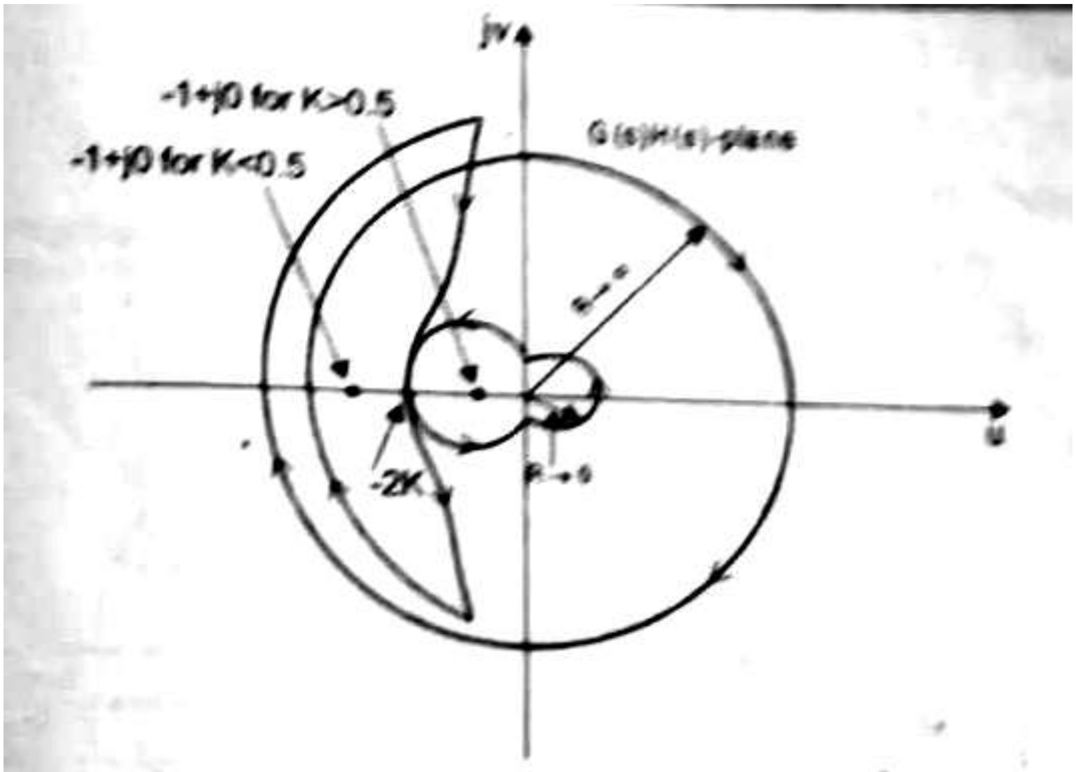


Fig 4.14.10 : *Nyquist plot of* $G(s)H(s) = \dfrac{K(1+s)^2}{s^3}$

Part-A

1. Define state variable.

The state of a dynamical system is a minimal set of variables(known as state variables) such that the knowledge of these variables at $t\text{-}t_0$ together with the knowledge of the inputs for $t > t_0$, completely determines the behavior of the system for $t > t_0$

2. Write the general form of state variable matrix.

The most general state-space representation of a linear system with m inputs, p outputs and n state variables is written in the following form:

= AX + BU

Y = CX + DU

Where = state vector of order n X 1.
U = input vector of order n X1.

A=System matrix of order n X n.

B=Input matrix of order n X m

C =output matrix of order p X n

D = transmission matrix of order p X m

3. Write the relationship between z-domain and s-domain.

All the poles lying in the left half of the S-plane, the system is stable in S-domain. Corresponding in Z-domain all poles lie within the unit circle. Type equation here.

4. What are the methods available for the stability analysis of sampled data control system?

The following three methods are available for the stability analysis of sampled data control system

1. Juri's stability test. 2. Bilinear transformation. 3. Root locus technique.

5. What is the necessary condition to be satisfied for design using state feedback?

The state feedback design requires arbitrary pole placements to achieve the desire performance. The necessary and sufficient condition to be satisfied for arbitrary pole placement is that the system is completely state controllable.

6. What is controllability?(or)
When do you say that a system is completely state controllable?(Nov/Dec 2015)

A system is said to be completely state controllable if it is possible to transfer the system state from any initial state $X(t_0)$ at any other desired state X(t), in specified finite time by a control vector U(t).

7. What is observability? (Or)
When a system is said to be completely observable? (May/June 2016, 17)
State the concept of observability. (April/May 2018)

A system is said to be completely observable if every state X(t) can be completely identified by measurements of the output Y(t) over a finite time interval.

8. Write the properties of state transition matrix.

The following are the properties of state transition matrix

1. $\Phi(0) = e^{Ax0} = I$ (unit matrix).
2. $\Phi(t) = e^{At} = (e^{-At})^{-1} = [\Phi(-t)]^{-1}$.
3. $\Phi(t_1+t_2) = e^{A(t1+t2)} = \Phi(t_1)\,\Phi(t_2) = \Phi(t_2)\,\Phi(t_1)$.

9. Define sampling theorem.

Sampling theorem states that a band limited continuous time signal with highest frequency f_m, hertz can be uniquely recovered from its samples provided that the sampling rate F_s is greater than or equal to $2f_m$ samples per second.

10. What is sampled data control system?

When the signal or information at any or some points in a system is in the form of discrete pulses, then the system is called discrete data system or sampled data system.

11. What is Nyquist rate?

The Sampling frequency equal to twice the highest frequency of the signal is called as Nyquist rate. $f_s=2f_m$

12. What is similarity transformation?

The process of transforming a square matrix **A** to another similar matrix **B** by a transformation $\mathbf{P^{-1}AP = B}$ is called similarity transformation. The matrix P is called transformation matrix.

13. What is meant by diagonalization?

The process of converting the system matrix **A** into a diagonal matrix by a similarity transformation using the modal matrix **M** is called diagonalization.

14. What is modal matrix?

The modal matrix is a matrix used to diagonalize the system matrix. It is also called diagonalization matrix.

If A = system matrix.

M = Modal matrix

And M^{-1}=inverse of modal matrix.

Then $M^{-1}AM$ will be a diagonalized system matrix.

15. How the modal matrix is determined?

The modal matrix M can be formed from eigenvectors. Let m_1, m_2, m_3 …. m_n be the eigenvectors of the n^{th} order system. Now the modal matrix M is obtained by arranging all the eigenvectors column wise as shown below.

Modal matrix , $M = [m_1, m_2, m_3 \ldots . m_n]$.

16. What is the need for controllability test?

The controllability test is necessary to find the usefulness of a state variable. If the state variables are controllable then by controlling (i.e. varying) the state variables the desired outputs of the system are achieved.

17. What is the need for observability test?

The observability test is necessary to find whether the state variables are measurable or not. If the state variables are measurable then the state of the system can be determined by practical measurements of the state variables.

18. State the condition for controllability by Gilbert's method.

Case (i) when the eigen values are distinct

Consider the canonical form of state model shown below which is obtained by using the transformation X=MZ.

$= \Lambda Z + U$

$Y=Z + DU$

Where, $\Lambda = M^{-1}AM$; = CM , = $M^{-1}B$ and M = Modal matrix.

In this case the necessary and sufficient condition for complete controllability is that, the matrix must have no row with all zeros. If any row of the matrix is zero then the corresponding state variable is uncontrollable.

Case(ii) when eigen values have multiplicity

In this case the state modal can be converted to Jordan canonical form shown below

= JZ + U

Y=Z + DU   Where, $J = M^{-1}AM$

In this case the system is completely controllable, if the elements of any row of that correspond to the last row of each Jordan block are not all zero.

19. State the condition for observability by Gilbert's method.

Consider the transformed canonical or Jordan canonical form of the state model shown below which is obtained by using the transformation, X =MZ

= ΛZ + U

Y=Z + DU   (Or)

= JZ + U

Y=Z + DU   where   = CM and M = modal matrix.

The necessary and sufficient condition for complete observability is that none of the columns of the matrix be zero. If any of the column is of has all zeros then the corresponding state variable is not observable.

20. State the duality between controllability and observability.

The concept of controllability and observability are dual concepts and it is proposed by kalman as principle of duality.The principle of duality states that a system is completely state controllable if and only if its dual system is completely state controllable if and only if its dual system is completely observable or viceversa.

21. What is the need for state observer?

In certain systems the state variables may not be available for measurement and feedback. In such situations we need to estimate the unmeasurable state variables from the knowledge of input and output. Hence a state observer is employed which estimates the state variables from the input and output of the system. The estimated state variable can be used for feedback to design the system by pole placement.

22. How will you find the transformation matrix, P_o to transform the state model to observable phase variable form?

- Compute the composite matrix for observability,Q_0
- Determine the characteristic equation of the system $|\lambda I - A| = 0$.

Using the coefficients $a_1, a_2, \ldots. a_{n-1}$ of characteristic equation form a matrix, W.
Now the transformation matrix, P_0 is given by $P_0 = W\ Q_0^T$.

23. Write the observable phase variable form of state model.

The observable phase variable form of state model is given by the following equations

$= A_0 Z + B_0 u.$

$Y \quad = C_0 Z + Du$

Where, $A_0 =$, $B_0 =$ and $C_0 = [\ 0\ 0\ \ldots..\ 0\ 1\ ]$

24. What is the pole placement by state feedback?

The pole placement by state feedback is a control system design technique, in which the state variables are used for feedback to achieve the desired closed loop poles.

25. How control system design is carried in state space?

In state space design of control system, any inner parameter or variable of a system are used for feedback to achieve the desired performance of the system. The performance of the system is related to the location of closed loop poles. Hence in state space design the closed loop poles are placed at the desired location by means of state feedback through an appropriate state feedback gain matrix, K.

26. What is meant by State of a dynamic system?(Nov/Dec 2015)

The state of a dynamic system is defined as a minimal set of variables such that the knowledge of these variables at $t=t_0$ together with the knowledge of inputs $t \geq 0$ completely determine the behavior of the system for $t > t_0$.

27. What are the various types of system the state space approach is used?

The state variable approach is a powerful tool/technique for the analysis and design of control systems. The analysis and design of the following systems can be carried using state space method.

1. Linear system.
2. Non-linear system.
3. Time invariant system.
4. Time varying system
5. Multiple input and multiple output system.

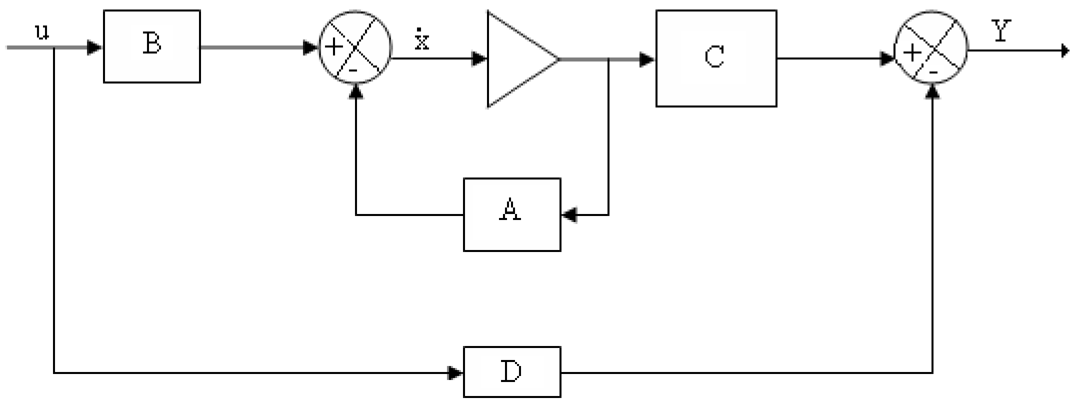
28. What are the draws backs in the transfer function model and analysis?

The state space analysis is a modern approach and also easier for analysis using digital computers. The conventional methods of analysis employ the transfer function of the system. The drawbacks in the transfer function model and analysis are,

1. Transfer function is defined under zero initial conditions
2. Transfer function is applicable to linear time invariant systems.

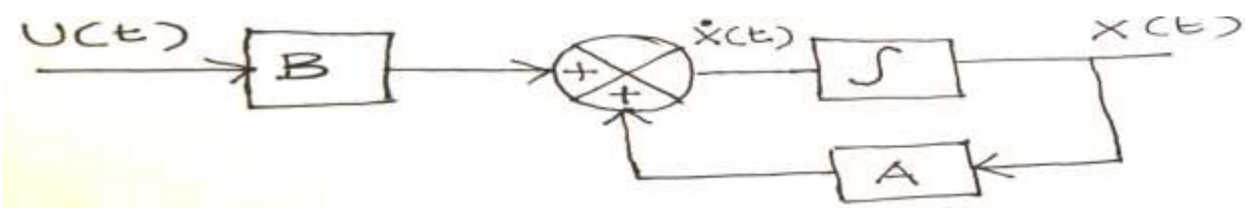
3. Transfer function analysis is restricted to single input and single output systems.
4. Does not provides information regarding the internal state of the system.

29. What is meant by state space? (May/June 2016)
The space whose coordinate axes are nothing but the ‘n’ state variables with time as the implicit variable is called state space.
30. Draw the block diagram representation of a state model.(May-17)



31. State the Limitations of state variables feedback. Nov/Dec-2016
controller design using classical methods, e.g., root locus or frequency domain method are limited to only LTI systems, particularly SISO (single input single output) systems since for MIMO (multi input multi output) systems controller design using classical approach becomes more complex.These limitations of classical approach led to the development of state variable approach of system modeling and control which formed a basis of modern control theory.

32.For a first order differential equation described by x(t)=ax(t)+bu(t), draw the block diagram form of state diagram.Nov/Dec-16



33. Write the homogeneous and non-homogeneous state equation. (Nov/Dec 2017)
The solution of homogeneous state equation is

$$X(t) = e^{At} X(0)$$

The solution of non-homogeneous state equation is

$$X(t) = e^{A(t-t_0)} X(t_0) + \int_{t_0}^{t} e^{A(t-\tau)} Bu(\tau)d\tau$$

34. Define state trajectory. (Nov/Dec 2017)

In dynamical systems, a trajectory is the set of points in state space that are the future states resulting from a given initial state.

35. Write the advantages of state space analysis. (April/May 2018)

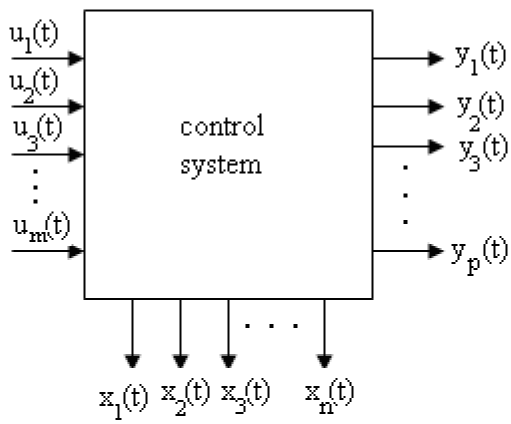
- It can be applied to non linear system.
- It can be applied to multiple input and multiple output systems.
- It gives idea about the internal state of the system.

Part B

1. Give the Procedure for state space formulation.(May-17)

The state of a dynamic system is a minimal set of variables(known as state variables) such that the knowledge of these variables at t=t0 together with the knowledge of he inputs for t ≥ t0,completely determines the behaviour of the system for t > t0.

In the state variable formulation of a system ,in general ,a system consists of m-inputs, p-output and n-state variables .The state representation of the system may be visualized as shown in figure.



The different variables may be represented by the vector as shown below.

$$\text{Input vector} = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}$$

$$\text{Output vector} = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_p(t) \end{bmatrix}$$

$$\text{state variable vector} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$$

The state variable representation can be arranged in the form of n number of first order differential equations as shown below

$$dx_1/dt = \dot{x}_1 = f(x_1, x_2, \ldots\ldots x_n;\ u_1, u_2 \ldots\ldots u_m)$$

$$dx_2/dt = \dot{x}_2 = f_2(x_1, x_2, \ldots\ldots x_n;\ u_1, u_2 \ldots\ldots u_m)$$

$$\vdots$$

$$dx_n/dt = \dot{x}_n = f(x_1, x_2, \ldots\ldots x_n;\ u_1, u_2 \ldots\ldots u_m)$$

The n numbers of differential equations may be written in vector notation as

$$\dot{X}(t) = f(X(t), U(t))$$

STATE MODEL:

The state model of a system consists of the state equation and output equation. The state equation of a system is a function of state variables and inputs

For linear time invariant systems the first derivatives of state variables can be expressed as a linear combination of state variables and inputs.

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + b_{11}u_1 + b_{12}u_2 + \ldots + b_{1m}u_m$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + b_{21}u_1 + b_{22}u_2 + \ldots + b_{2m}u_m$$

$$\vdots$$

$$\dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + b_{n1}u_1 + b_{n2}u_2 + \ldots + b_{nm}u_m$$

In the matrix form the above equations can also be expressed as,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nm} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

The matrix equation also be written as

$$\dot{X}(t) = AX(t) + BU(t)$$

Where ,
X(t) = State vector of order (n×1)
U(t) = Input vector of order (m×1)
A = System matrix of order (n×n)
B = Input matrix of order (n×m)

The equation $\dot{X}(t)=AX(t)+BU(t)$ is called as the state equation of linear time invariant system.

The output at any time are functions of state variables and inputs.

Output vector ,Y(t)=f(X(t),U(t))

Hence the output variables can be expressed as a linear combination of state variables and inputs.

$$y_1=c_{11}x_1+c_{12}x_2+\dots+c_{1n}x_n+d_{11}u_1+d_{12}u_2+\dots+d_{1m}u_m$$

$$y_2=c_{21}x_1+c_{22}x_2+\dots+c_{2n}x_n+d_{21}u_1+d_{22}u_2+\dots+d_{2m}u_m$$

$$\vdots$$

$$y_p=c_{p1}x_1+c_{p2}x_2+\dots+c_{pn}x_n+d_{p1}u_1+d_{p2}u_2+\dots+d_{pm}u_m$$

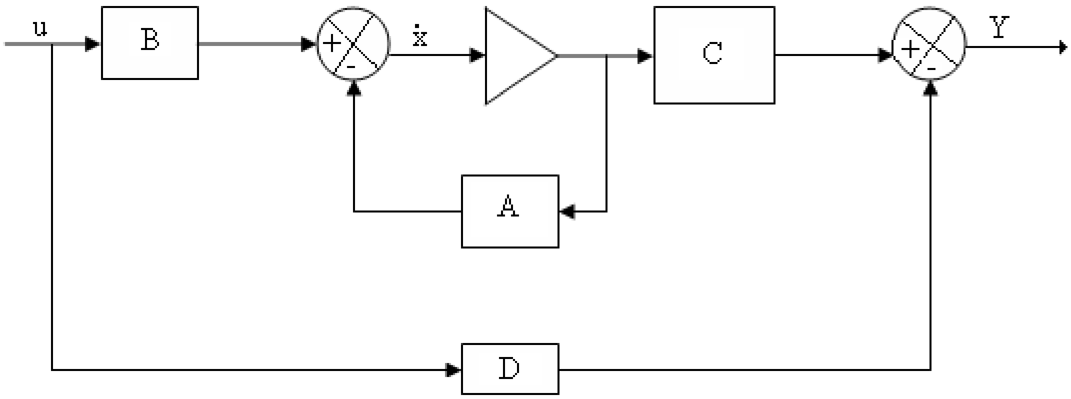
In the matrix form the above equations can be expressed as,

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & & \vdots \\ c_{p1} & c_{p2} & \cdots & c_{pn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1m} \\ d_{21} & d_{22} & \cdots & d_{2m} \\ \vdots & \vdots & & \vdots \\ d_{p1} & d_{p2} & \cdots & d_{pm} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

The matrix equation can also be written as, Y(t)=CX(t)+DU(t)
Where , X(t)=State vector of order (n×1)
U(t)=Input vector of order (m×1)
C=Output vector of order (p×1)
D=Transmission matrix of order (p×m)
The equation Y(t)=CX(t)+DU(t) is called the output equation of linear time invariant system.

$\dot{X}(t)=AX(t)+BU(t)$;state equation.

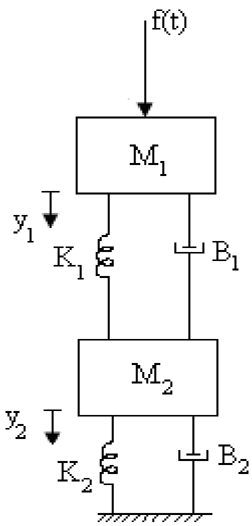
Y(t)=CX(t)+DU(t);output equation.
The block diagram representation of the state model is



THE SELECTON OF STATE VARIABLES FOR DIFFERENT SYSTEMS:

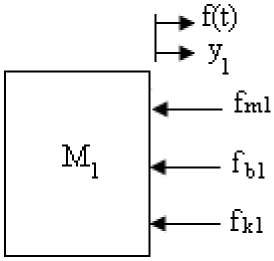
1. In the mechanical system, the potential and kinetic energy of a mass are functions of position and velocity of the mass. Therefore, position and velocity can be chosen as state variables.
2. In an electric circuit like RLC network, capacitors and inductors are energy storage elements. Therefore the rate of change of current in an inductor, and the rate of change of voltage across a capacitor can be chosen as state variables. The voltage across resistor and current through the resistor cannot be chosen as state variables, because resistance is not an energy storage element.
3. In chemical engineering systems, the rate of change of variables are often temperature, pressure and flow. Therefore rate of change of temperature, rate of change of pressure, rate of change of flow are often chosen as state variables.

2. Obtain the state model of the mechanical system shown in fig.in which f(t) is the input and y_2(t) is the output. (Nov/Dec 2015)



Solution:

Free body diagram of M_1 is



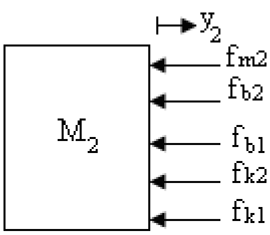
By Newton's second law, the force balance equation at node M_1 is ,

$f(t) = f_{m1} + f_{b1} + f_{k1}.$

$$f(t) = M_1\, d^2y_1/dt^2 + B_1\, d(y_1 - y_2)/dt + K_1(y_1 - y_2)$$

$$f(t) = M_1\, d^2y_1/dt^2 + B_1\, dy_1/dt - B_1\, dy_2/dt + K_1 y_1 - K_1 y_2 \longrightarrow (1)$$

Free body diagram of M_2 is,



By Newton's second law, the force balance equation at M_2 is,

$$f_{m2} + f_{b2} + f_{b1} + f_{k2} + f_{k1} = 0$$

$$M_2\, d^2y_2/dt^2 + B_2\, dy_2/dt + B_1\, d(y_2 - y_1)/dt + K_2 y_2 + K_1(y_2 - y_1) = 0$$

$$M_2\, d^2y_2/dt^2 + B_2\, dy_2/dt + B_1\, dy_2/dt - B_1\, dy_1/dt + K_2 y_2 + K_1 y_2 - K_1 y_1 = 0 \longrightarrow (2)$$

Let us choose four state variables x_1,x_2,x_3 and x_4.

$$x_1 = y_1$$

$$x_2 = y_2$$

$$x_3 = dy_1/dt$$

$$x_4 = dy_2/dt$$

On substituting above variables and f(t) = u

Equation (1) becomes

$$u = M_1 \dot{x}_3 + B_1 x_3 - B_1 x_4 + K_1 x_1 - K_1 x_2$$

$$\dot{x}_3 = -\frac{K_1}{M_1} x_1 + \frac{K_1}{M_1} x_2 - \frac{B_1}{M_1} x_3 + \frac{B_1}{M_1} x_4 + \frac{1}{M_1} u \longrightarrow (3)$$

Equation (2) becomes

$$M_2 \dot{x}_4 + B_2 x_4 + B_1 x_4 - B_1 x_3 + K_2 x_2 + K_1 x_2 - K_1 x_1 = 0$$

$$\dot{x}_4 = \frac{K_1}{M_2} x_1 - \frac{(K_1 + K_2)}{M_2} x_2 + \frac{B_1}{M_2} x_3 - \frac{(B_1 + B_2)}{M_2} x_4 \longrightarrow (4)$$

$$\dot{x}_1 = x_3 \longrightarrow (5)$$

$$\dot{x}_2 = x_4 \longrightarrow (6)$$

The state equations of the mechanical system are,

$$\dot{x}_1 = x_3$$

$$\dot{x}_2 = x_4$$

$$\dot{x}_3 = -\frac{K_1}{M_1}x_1 + \frac{K_1}{M_1}x_2 - \frac{B_1}{M_1}x_3 + \frac{B_1}{M_1}x_4 + \frac{1}{M_1}u$$

$$\dot{x}_4 = \frac{K_1}{M_2}x_1 - \frac{(K_1+K_2)}{M_2}x_2 + \frac{B_1}{M_2}x_3 - \frac{(B_1+B_2)}{M_2}x_4$$

On arranging the state equations in the matrix form,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_1}{M_1} & \frac{K_1}{M_1} & -\frac{B_1}{M_1} & \frac{B_1}{M_1} \\ \frac{K_1}{M_2} & -\frac{(K_1+K_2)}{M_2} & \frac{B_1}{M_2} & -\frac{(B_1+B_2)}{M_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M_1} \\ 0 \end{bmatrix} \begin{bmatrix} u \end{bmatrix}$$

Let the displacement y_1 and y_2 be the outputs of the system.

$$y_1 = x_1$$

$$y_2 = x_2$$

The output equation in matrix form is given by,

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

3. Construct a state model for a system characterized by the differential equation

$d^3y/dt^3 + 6\, d^2y/dt^2 + 11\, dy/dt + 6y + u = 0$

Give the block diagram representation of the state model.

Solution:

The state variables X_1, X_2, and X_3 are related to phase variables as follows

$X_1 = Y$

$X_2 = dy/dt = \dot{X}_1$

$X_3 = d^2y/dt^2 = \dot{X}_2$

Put $Y = X_1$, $d^2y/dt^2 = X_2$ and $d^3y/dt^3 = \dot{X}_3$ in the given equation.

$\dot{X}_3 + 6X_3 + 11X_2 + 6X_1 + u = 0$

$\dot{X}_3 = -6X_1 - 11X_2 - 6X_3 - u$

The state equations are

$\dot{X}_1 = X_2$

$\dot{X}_2 = X_3$

$\dot{X}_3 = -6X_1 - 11X_2 - 6X_3 - u$

On arranging the state equation in the matrix form we get,

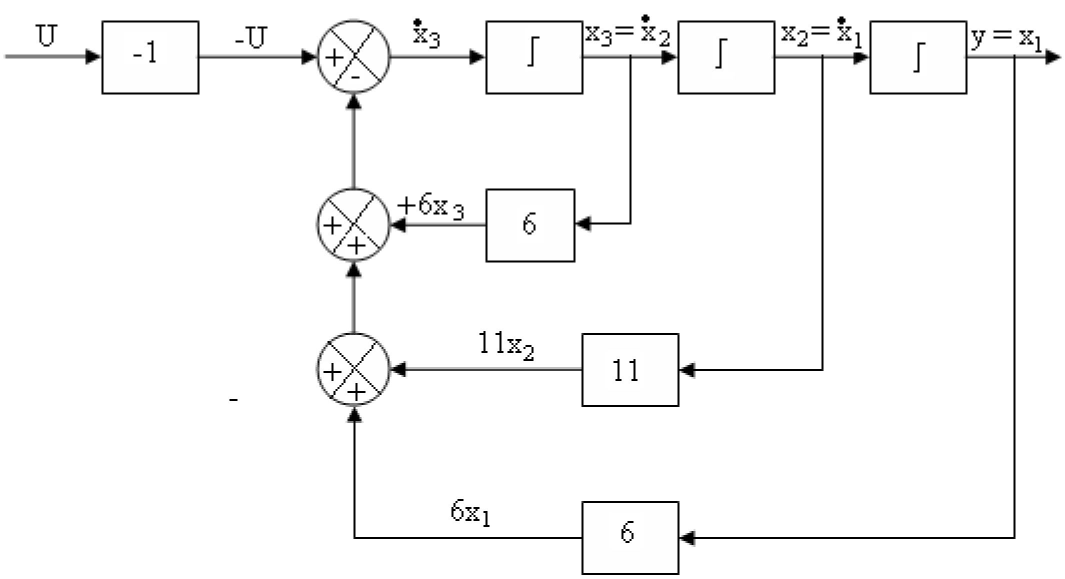
$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} [u]$$

Here ,Y=output.

But, Y=X1

The output equation is $y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$

The block diagram form of the state diagram of the system is shown in below figure.



4. Give the methods for state space model system realization:

A given system transfer function can be realized in many different ways. They are

1. Controllable canonical realization.
2. Observable canonical realization.
3. Cascade form realization
4. Parallel form realization.

CONTROLLABLE CANONICAL REALIZATION:

The controllable canonical form so named for its use in design of a controller. It is also known as controllable direct form realization.

$H(s) = Y(s)/X(s) = X_1(s)\ Y(s)/X(s)\ X_1(s)$.
$H(s) = H_1(s)\ H_2(s)$.

5. Find the controllable canonical realization of the given system H(s) =s+2/s+5.
(April/May 2018)

Solution:

Given $H(s) = s+2/s+5$

Let $H(s) = Y(s)/X(s)$

$X_1(s)\ Y(s)/X(s)\ X_1(s) = s+2/s+5$.

Where

$H_1(s) = X_1(s)/X(s) = 1/s+5$

$H_2(s) = Y(s)/X_1(s) = s+2$

$X_1(s)/X(s) = 1/s+5$

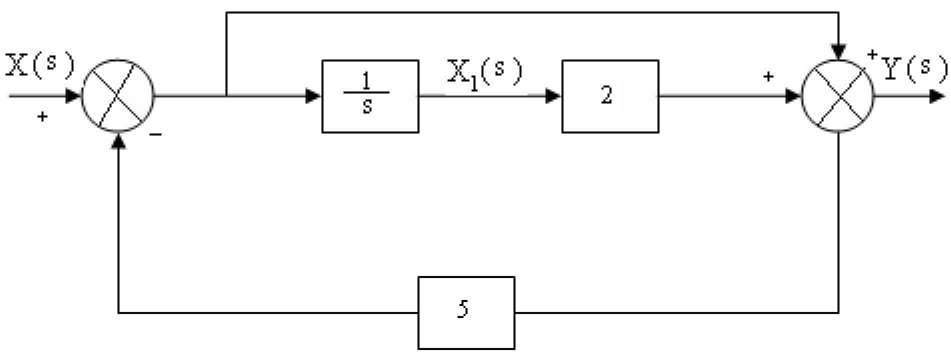
$(s+5)X_1(s) = X(s)$

$S\ X_1(s) = X(s) – 5X_1(s)$.

$Y(s)/X_1(s) = s+2$

$Y(s) = sX_1(s)+2X_1(s)$.

The complete system is shown in figure



6. Find the controllable canonical realization of the given systems^2 +2s+3/s^ 4 +3s^ 3+12s^ 2+9s+8.

Solution:

given

$H(s) = s^2+2s+3/s^4+3s^3+12s^2+9s+8$

$H(s) = Y(s)/X(s) = X_1(s)\ Y(s)/X(s)\ X_1(s)$

$Y(s)/X_1(s) = s^2+2s+3$

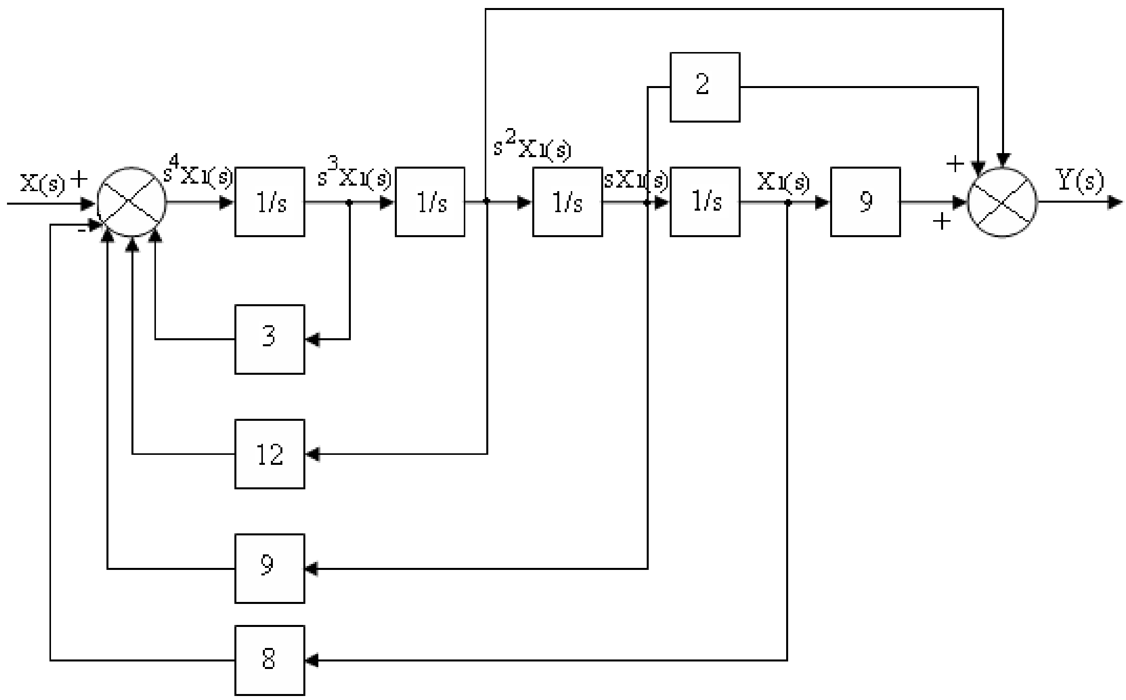
$Y(s) = (s^2+2s+3)X_1(s)$

$Y(s) = s^2 X_1(s) + 2sX_1(s)+3X_1(s) \longrightarrow (1)$

$X_1(s)/X(s) = 1/s^4+3s^3+12s^2+9s+8$

$s^4 X_1(s) = X(s)-3s^3X_1(s)-12s^2X_1(s)-9sX_1(s)-8X_1(s) \longrightarrow (2)$

The realization of equation (1) and (2) is shown in figure



7. Find the observable canonical realization of the system H(s)= s+2/ s^4 + 4 s^3 + 3 s^2 + 12s+5.

Solution:

Given

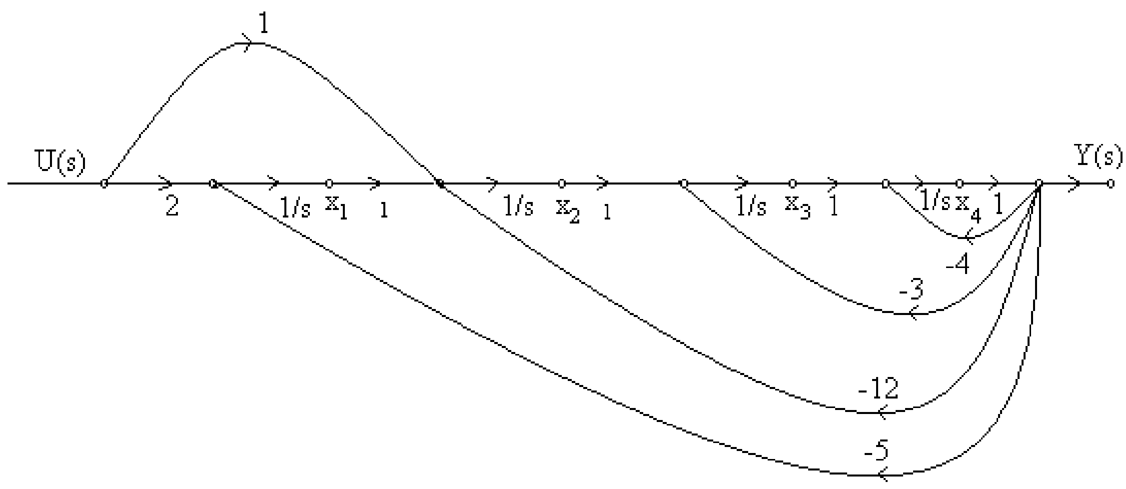
$H(s) = s+2/s^4 + 4s^3 + 3s^2 + 12s+5$

$H(s) = (1/s^3 + 2/s^4)/(1-(-4/s -3/s^2 -12/s^3 - 5/s^4)$

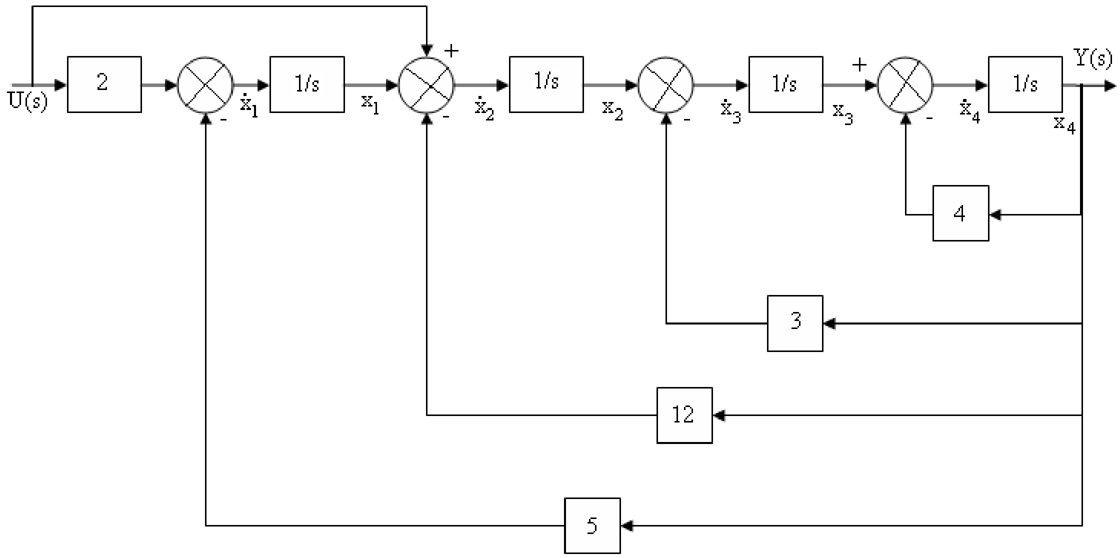
The gains of forward paths are $1/s^3$, $2/s^4$.

The feedback loop gains are $-4/s$, $-3/s^2$, $-12/s^3$, $-5/s^4$.

The signal flow graph that satisfies the above requirement is shown in figure



The observable canonical realization can be obtained by converting the signal flow graph into block diagram representation as shown in figure.



TIME DOMAIN SOLUTION OF STATE EQUATION:

Time domain equation of state equation is

$$X(t) = e^{A(t-t_0)} X(t_0) + \int_{t_0}^{t} e^{A(t-\tau)} Bu(\tau) d\tau$$

8. State and prove the properties of state transition matrix. (Nov/Dec 2015)

PORPERTIES OF THE TRANSITION MATRIX:

1. $\Phi(0)$ is an identity matrix

$$X(t) = e^{At} X(0)$$

$$= \Phi(t)\, X(0)$$

At t=0;

$$X(0) = \Phi(0)X(0)$$

$$\Phi(0) = I$$

2. $\Phi(t) = e^{At} = (e^{-At})^{-1} = \left[\Phi(-t)\right]^{-1}$

$$\Phi(t) = \left[\Phi(-t)\right]^{-1}$$

Taking inverse on both sides. We have

$$\Phi^{-1}(t) = \left[\Phi(-t)\right]$$

3. $\Phi(t_1+t_2) = e^{A(t_1+t_2)} = e^{At_1}\, e^{At_2} = \Phi(t_1)\ \Phi(t_2)$

i.e $\Phi(t_1+t_2) = \Phi(t_1)\Phi(t_2)$

4. $\left[\Phi(t)\right]^{n} = (e^{At})^{n} = e^{Ant} = \Phi(nt)$

i.e $\left[\Phi(t)\right]^n = \Phi(nt)$

LAPLACE TRANSFORM SOLUTION OF STATE EQUATIONS:

Consider the state equation

$$\dot{X} = AX + BU$$

And the output equation

Y= CX+Du

Taking laplace transform on both sides yields

sX(s) –X(0) = AX(s) + BU(s)

(sI-A)X(s) = X(0) + BU(s)

Where I is the identity matrix

$$X(s) = (sI-A)^{-1}[X(0)+BU(s)]$$

$$= \Phi(s)\,[X(0) + BU(s)]$$

Where

$$\Phi(s) = (sI-A)^{-1}$$

$$X(s) = \Phi(s)X(0) + \Phi(s)BU(s)$$

$$X(t) = L^{-1}\left[\Phi(s)\,X(0)\right] + L^{-1}\left[\Phi(s)BU(s)\right]$$

9. A linear time invariant system is characterized by the state equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Where u is a unit step function. Compute the solution of these equation assuming initial condition $X_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ **. Use inverse laplace transform technique**.(Nov/Dec 2016)

Solution:

Given

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix};\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$X(t) = L^{-1}\left[\Phi(s)\,X(0)\right] + L^{-1}\left[\Phi(s)BU(s)\right]$$

We know

$$\Phi(s) = (sI-A)^{-1}$$

$$sI-A = s\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$(sI-A)^{-1} = Adj(sI-A)/Det(sI-A) = 1/(s-1)^2 \begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/(s-1) & 0 \\ 1/(s-1)^2 & 1/(s-1) \end{bmatrix}$$

$$\Phi(s)x(0) = \begin{bmatrix} 1/(s-1) & 0 \\ 1/(s-1)^2 & 1/(s-1) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/(s-1) \\ 1/(s-1)^2 \end{bmatrix}$$

$$L^{-1}\left[\Phi(s)x(0)\right] = L^{-1} \begin{bmatrix} 1/(s-1) \\ 1/(s-1)^2 \end{bmatrix}$$

$$= \begin{bmatrix} L^{-1}(1/(s-1)) \\ L^{-1}(1/(s-1)^2) \end{bmatrix} = \begin{bmatrix} e^t \\ te^t \end{bmatrix}$$

$$B\,\Phi(s) = \begin{bmatrix} 1/(s-1) & 0 \\ 1/(s-1)^2 & 1/(s-1) \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1/(s-1) \end{bmatrix}$$

$$\Phi(s)Bu(s) = \begin{bmatrix} 0 \\ 1/(s-1) \end{bmatrix} \begin{bmatrix} 1/s \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1/s(s-1) \end{bmatrix}$$

$$L^{-1}\left[\Phi(s)Bu(s)\right] = \begin{bmatrix} 0 \\ -1 + e^t \end{bmatrix}$$

$$x(t) = \begin{bmatrix} e^t \\ te^t \end{bmatrix} + \begin{bmatrix} 0 \\ -1 + e^t \end{bmatrix}$$

$$x(t) = \begin{bmatrix} e^t \\ -1 + (t+1)e^t \end{bmatrix}$$

DERIVATION OF TRANSFER FUNCTION FROM STATE MODEL:

The derivation of transfer function from state model is

$$T(s) = Y(s)/U(s) = C(sI-A)^{-1}B+D$$

10.Find the transfer function for the system which is represented in state space representation as follows

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 1 \\ -3 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Solution:

We have

$$T(s) = Y(s)/U(s) = C(sI-A)^{-1}B+D$$

Given D = 0.Therefore

$T(s) = C(sI-A)^{-1}B$

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 1 \\ -3 & -4 & -5 \end{bmatrix}$$

$$sI-A = s\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 1 \\ -3 & -4 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} s+2 & -1 & 0 \\ 0 & s+3 & -1 \\ 3 & 4 & s+5 \end{bmatrix}$$

$(sI-A)^{-1} = \mathrm{adj}\,(sI-A)/\det(sI-A)$

$$\mathrm{adj}\,(sI-A) = \begin{bmatrix} s^2+8s+19 & s+5 & 1 \\ -3 & s^2+7s+10 & s+2 \\ -3(s+3) & -4-s & s^2+5s+6 \end{bmatrix}$$

$\det(sI-A) = s^3+10s^2+35s+41$

$$(sI-A)^{-1} = 1/s^3+10s^2+35s+41 \begin{bmatrix} s^2+8s+19 & s+5 & 1 \\ -3 & s^2+7s+10 & s+2 \\ -3(s+3) & -4-s & s^2+5s+6 \end{bmatrix}$$

$T(s) = C(sI-A)^{-1}B$

$$= 1/s^3+10s^2+35s+41 \begin{bmatrix} -3 \\ s^2+7s+10 \\ s+2 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$T(s) = \; s+2/s^3+10s^2+35s+41$

2. Given that $A_1 = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix}$; $A_2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$; $A = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$ compute e^{At} .

Solution:

Here

$A = A_1 + A_2$ (May-17)

$$e^{At} = e^{(A_1+A_2)t} = e^{A_1 t} \cdot e^{A_2 t}$$

$$sI - A_1 = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix}$$

$$= \begin{bmatrix} s-\sigma & 0 \\ 0 & s-\sigma \end{bmatrix}$$

$$\Delta_1 = |sI - A_1| = \begin{vmatrix} s-\sigma & 0 \\ 0 & s-\sigma \end{vmatrix} = (s-\sigma)^2$$

$$(sI-A_1)^{-1} = 1/\Delta_1 \begin{bmatrix} s-\sigma & 0 \\ 0 & s-\sigma \end{bmatrix} = \begin{bmatrix} 1/s-\sigma & 0 \\ 0 & 1/s-\sigma \end{bmatrix}$$

$$e^{A_1 t} = L^{-1}\left[(sI-A_1)^{-1}\right] = \begin{bmatrix} e^{-\sigma t} & 0 \\ 0 & e^{-\sigma t} \end{bmatrix}$$

$$sI - A_2 = s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$$

$$= \begin{bmatrix} s & -\omega \\ \omega & s \end{bmatrix}$$

$$\Delta_2 = |sI-A_2| = \begin{vmatrix} s & -\omega \\ \omega & s \end{vmatrix} = s^2+\omega^2$$

$$(sI-A_2)^{-1} = 1/\Delta_2 \begin{bmatrix} s & \omega \\ -\omega & s \end{bmatrix} = \begin{bmatrix} s/s^2+\omega^2 & \omega/s^2+\omega^2 \\ -\omega/s^2+\omega^2 & s/s^2+\omega^2 \end{bmatrix}$$

$$e^{At} = e^{A_1t} \cdot e^{A_2t} = \begin{bmatrix} e^{-\sigma t} & 0 \\ 0 & e^{-\sigma t} \end{bmatrix} \begin{bmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} e^{-\sigma t}\cos\omega t & e^{-\sigma t}\sin\omega t \\ -e^{-\sigma t}\sin\omega t & e^{-\sigma t}\cos\omega t \end{bmatrix}$$

11. A system is described by

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

Y = [10] x. check controllability and observability.

Solution:

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Controllable matrix is

$$Q_c = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

The determinant of $|Q_C| = -1 \neq 0$
Rank of $Q_c= 2=n$
Given system is controllable.
Observable matrix is

$$Q_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \begin{bmatrix} 1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The determinant of $Q_0 =1 \neq 0$.
Rank of $Q0=2 = n$
Given system is observable.

12. Check forcontrollability and observabilityof a system having following coefficient matrices. $\boldsymbol{A} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{-6} & \mathbf{-11} & \mathbf{-6} \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \\ \mathbf{1} \end{bmatrix};\ \boldsymbol{C^T} = \begin{bmatrix} \mathbf{10} \\ \mathbf{5} \\ \mathbf{1} \end{bmatrix}$ **(Nov/Dec 2015, 2016) (May/June 2016,17)**

$\underline{\boldsymbol{Controllability}}$:

$Q_C = [B \quad AB \quad A^2B]$;

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -12 \end{bmatrix}$$

$$A^2B = A(AB) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ -12 \end{bmatrix} = \begin{bmatrix} 0 \\ -12 \\ 55 \end{bmatrix}$$

$$Qi_C = [B \quad AB \quad A^2B] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -12 \\ 1 & -12 & 55 \end{bmatrix} = -59$$

$$Q_C \neq 0, The\ rank\ is\ 3$$

$\therefore The\ system\ is\ completely\ controllable.$

Observability:

$Q_o = [C^T \quad A^TC^T \quad (A^T)^2C^T];$

$$C^T = \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix};\ A^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}$$

$$A^TC^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}\begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix}; = \begin{bmatrix} -6 \\ -1 \\ -1 \end{bmatrix}$$

$$(A^T)^2C^T = A^T(A^TC^T) = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}\begin{bmatrix} -6 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 5 \end{bmatrix}$$

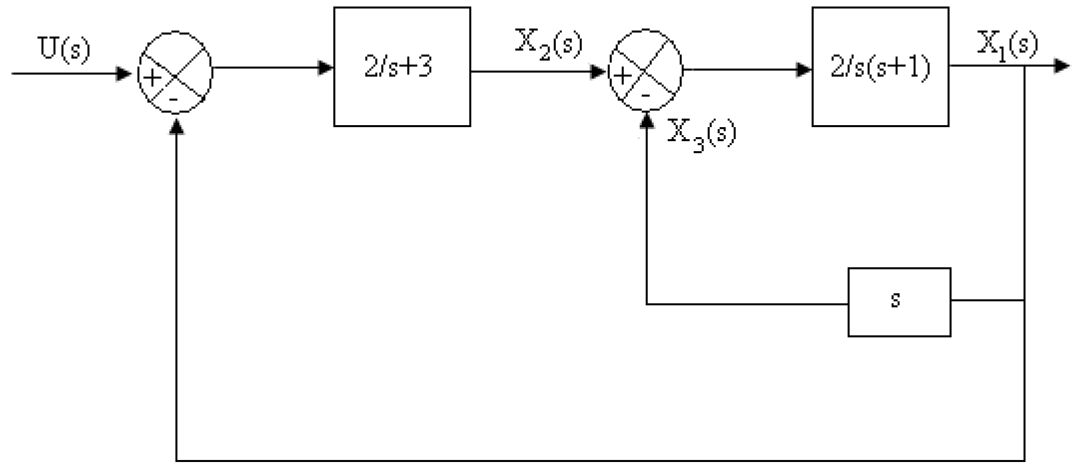
$$Q_o = [C^T \quad A^TC^T \quad (A^T)^2C^T] = \begin{bmatrix} 10 & -6 & 6 \\ 5 & -1 & 5 \\ 1 & -1 & 5 \end{bmatrix} = 96,$$

$$Q_o \neq 0\ The\ rank\ is\ 3$$

$\therefore The\ system\ is\ completely\ observable.$

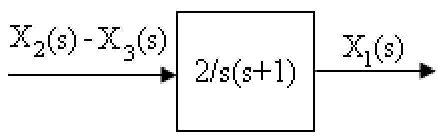
.

12. Write the state equations for the system shown in figure in which x_1,x_2 and x_3 constitute the state vector. Determine whether the system is completely controllable and observable.



Solution:

The state equation are obtained by writing equation for the output of each block and then taking inverse laplace transform.



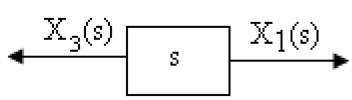
$$X_1(s) = [X_2(s) – X_3(s)]\ [2/s(s+1)]$$

$$s(s+1)X_1(s) = 2X_2(s) - 2X_3(s)$$

$$s^2 X_1(s) + sX_1(s) = 2X_2(s) - 2X_3(s)$$

On taking inverse laplace transform,

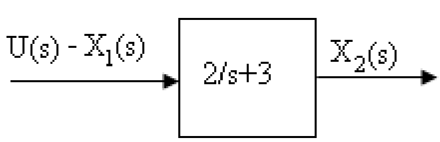
$$x_1 + x_1 = 2x_2 - 2x_3 \longrightarrow (1)$$



$$X_3(s) = sX_1(s)$$

On taking inverse laplace transform,

$$x_3 = \dot{x}_1 \longrightarrow (2)$$



$$X_2(s) = \left[U(s) - X_1(s)\right]\left[\ 2/s+3\right]$$

$$X_2(s)(s+3) = 2U(s) - 2X_1(s)$$

$$sX_2(s) + 3X_2(s) = 2U(s) - 2X_1(s)$$

On taking inverse Laplace transform,

$$\dot{x}_2 = -2x_1 - 3x_2 + 2u \longrightarrow (3)$$

From equation (2) we get,

$$\dot{x}_1 = x_3$$

$$\ddot{x}_1 = \dot{x}_3$$

Equation (1) becomes

$$\dot{x}_3 = 2x_2 - 3x_3 \longrightarrow (4)$$

The state equation are given by equation (2),(3) and (4)

$$\dot{x}_1 = x_3$$
$$\dot{x}_2 = -2x_1 - 3x_2 + 2u$$
$$\dot{x}_3 = 2x_2 - 3x_3$$

The output equation is Y= X_1.

The state model in the matrix form is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

COMPUTATION OF STATE TRANSITION MATRIX BY CANONICAL TRANSFORMATION:

The state transition matrix

$$e^{At} = M e^{\wedge t} M^{-1}$$

When the eigen values are distinct the above equation can be used to compute the state transition matrix.

When the eigen values have multiplicity the system matrix cannot be diagonalized but can be transformed to Jordanmatrix.When one of the eigan values λ_1 repeats q times the solution of state equation and state transition matrix

$$e^{At} = M\,Q(t)\, e^{\wedge t} M^{-1}$$

13. Convert the following system matrix to canonical form and hence calculate the state transition matrix

$$A = \begin{bmatrix} 4 & 1 & -2 \\ 1 & 0 & 2 \\ 1 & -1 & 3 \end{bmatrix}$$

Solution:

1. To find eigen values:

$$\lambda I - A = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 1 & -2 \\ 1 & 0 & 2 \\ 1 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda - 4 & -1 & 2 \\ -1 & \lambda & -2 \\ -1 & 1 & \lambda - 3 \end{bmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda - 4 & -1 & 2 \\ -1 & \lambda & -2 \\ -1 & 1 & \lambda - 3 \end{vmatrix} = (\lambda - 1)(\lambda - 3)^2$$

The eigen values are

$\lambda_1 = 1$; $\lambda_2 = 3$;$\lambda_3 = 3$.

14. To find eigen vectors:

$$\lambda_1 I - A = 1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 1 & -2 \\ 1 & 0 & 2 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} -3 & -1 & 2 \\ -1 & 1 & -2 \\ -1 & 1 & -2 \end{bmatrix}$$

Let the cofactors of $(\lambda_1 I - A)$ along 1st row be C_{11},C_{12} and C_{13}.

$$m_1 = \begin{bmatrix} C_{11} \\ C_{12} \\ C_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Let C_{21},C_{22} and C_{23} be cofactor of $(\lambda_1 I - A)$ along 2nd row

$$m_2 = \begin{bmatrix} C_{21} \\ C_{22} \\ C_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 4 \end{bmatrix}$$

$$\lambda_2 I - A = \lambda_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 1 & -2 \\ 1 & 0 & 2 \\ 1 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda - 4 & -1 & 2 \\ -1 & \lambda & -2 \\ -1 & 1 & \lambda - 3 \end{bmatrix}$$

$$m_2 = \begin{bmatrix} C_{11} \\ C_{12} \\ C_{13} \end{bmatrix} = \begin{bmatrix} \lambda_2^2 - 3\lambda_2 + 2 \\ \lambda_2 - 1 \\ \lambda_2 - 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

The eigenvector m_3 is given by

$$m_3 = \begin{bmatrix} \frac{d}{d\lambda_2} C_{11} \\ \frac{d}{d\lambda_2} C_{12} \\ \frac{d}{d\lambda_2} C_{13} \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

15. Find canonical form of system matrix:

$$M = \begin{bmatrix} m_1 & m_2 & m_3 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 3 \\ 8 & 2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$M^{-1} = M_{cof}^T / \Delta_M$$

$$\Delta_M = \begin{vmatrix} 0 & 2 & 3 \\ 8 & 2 & 1 \\ 4 & 2 & 1 \end{vmatrix} = 16$$

$$M_{cof}^T = \begin{bmatrix} 0 & -4 & 8 \\ 4 & -12 & 8 \\ -4 & 24 & -16 \end{bmatrix}^T = \begin{bmatrix} 0 & 4 & -4 \\ -4 & -12 & 24 \\ 8 & 8 & -16 \end{bmatrix}$$

$$M^{-1} = M_{cof}^T / \Delta_M = 1/16 \begin{bmatrix} 0 & 4 & -4 \\ -4 & -12 & 24 \\ 8 & 8 & -16 \end{bmatrix} = 1/4 \begin{bmatrix} 0 & 1 & -1 \\ -1 & -3 & 6 \\ 2 & 2 & -4 \end{bmatrix}$$

$$M^{-1} A M = 1/4 \begin{bmatrix} 0 & 1 & -1 \\ -1 & -3 & 6 \\ 2 & 2 & -4 \end{bmatrix} \begin{bmatrix} 4 & 1 & -2 \\ 1 & 0 & 2 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 2 & 3 \\ 8 & 2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

$$J = M^{-1} A M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

16. Compute state transition matrix

The state transition matrix

$$e^{At} = M\,Q(t)\,e^{\wedge t}\,M^{-1}$$

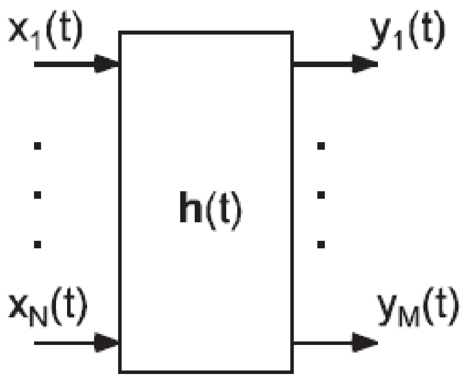
$$Q(t) = \begin{bmatrix} 1 & t & t^2/2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$$

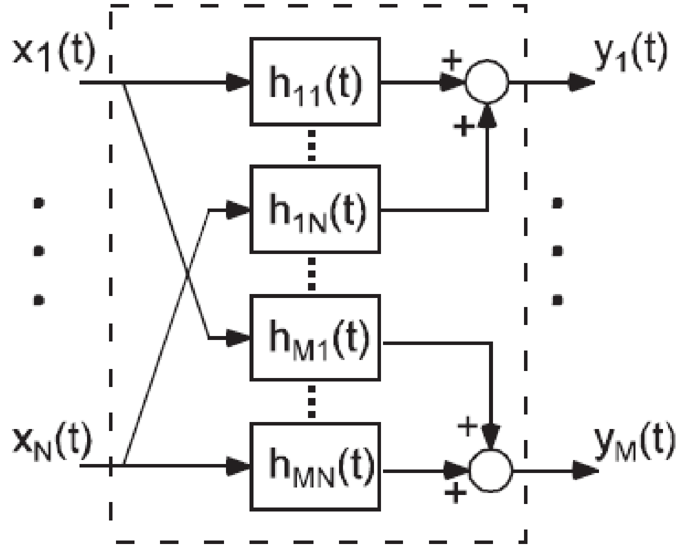
$$e^{\wedge t} = \begin{bmatrix} e^{\lambda_1 t} & 0 & 0 \\ 0 & e^{\lambda 2t} & 0 \\ 0 & 0 & e^{\lambda_3 t} \end{bmatrix} = \begin{bmatrix} e^{-t} & 0 & 0 \\ 0 & e^{3t} & 0 \\ 0 & 0 & e^{3t} \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 0 & 2 & 3 \\ 8 & 2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & t & t^2/2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{-t} & 0 & 0 \\ 0 & e^{3t} & 0 \\ 0 & 0 & e^{3t} \end{bmatrix} 1/4 \begin{bmatrix} 0 & 1 & -1 \\ -1 & -3 & 6 \\ 2 & 2 & -4 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} (t+1)e^{3t} & t\,e^{3t} & -2t\,e^{3t} \\ (2t^2-t)\,e^{3t} & 2e^{t} + (2t^2-5t-1)e^{3t} & -2e^{t} - (4t^2-10t-2)e^{3t} \\ t^2 e^{3t} & e^{t} + (t^2-2t-1)e^{3t} & -e^{t} - (2t^2-4t-2)e^{3t} \end{bmatrix}$$

17. With a neat block diagram, derive the state model and its equations of a linear multi-input-multi-output system. (May/June 2016)





Multi-input-multi output system:

Systems with more than one input and/or more than one output are known as Multi-Input Multi-Output systems, or they are frequently known by the abbreviation MIMO.

This is in contrast to systems that have only a single input and a single output (SISO), like we have been discussing previously.

Let's say that we have two outputs, y_1 and y_2, and two inputs, u_1 and u_2. These are related in our system through the following system of differential equations:

$$y_1'' + a_1 y_1' + a_0(y_1 + y_2) = u_1(t)$$

$$y_2' + a_2(y_2 - y_1) = u_2(t)$$

now, we can assign our state variables as such, and produce our first-order differential equations:

$$x_1 = y_1$$

$$x_4 = y_2$$

$$x_1' = y_1' = x_2$$

$$x_2' = -a_1 x_2 - a_0(x_1 + x_4) + u_1(t)$$

$$x_4' = -a_2(x_4 - x_1) + u_2(t)$$

And finally we can assemble our state space equations:

$$x' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -a_0 & -a_1 & 0 & -a_0 \\ 0 & 0 & 0 & 1 \\ a_2 & 0 & 0 & -a_2 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x(t)$$

18. Explain about the effect of state feedback. (May/June 2016)

- In the pole placement design process, it is assumed that the actual state is available for feedback. In practice, the actual state may not be measurable, so it is necessary to design a state observer.
- Therefore the design process involves a two stage process. First stage includes determination of the feedback gain matrix to yield the desired characteristic equation and the second stage involves the determination of the observer gain matrix to yield the desired observer characteristic equation.
- The closed loop poles of the observed-state feedback control system consist of the poles due to the pole placement design and the poles due to the observer design. If the order of the plant is n, then the observer is also nth order and the resulting characteristic equation for the entire closed-loop system becomes the order of 2n.
- The desired closed-loop poles to be generated by state feedback are chosen in such a way that the system satisfies the performance requirements. The poles of the observer are usually chosen so that the observer response is much faster than system response.
- A rule of thumb is to choose an observer response at least two to five times faster than system response. The maximum speed of the observer is limited only by the noise and sensitivity problem involved in the control system.
- Since the observer poles are placed left of the desired closedloop poles in the pole placement process, the closed loop poles will dominate the response.

19. Explain the concept of controllability and observablility. (Nov/Dec 2017)

CONTROLLABILITY:

The controllability verifies the usefulness of state variables. In the controllability test we can find, whether the state variable can be controlled to achieve the desired output. The choice of state variables is arbitrary while forming the state model. After determining the state model, the controllability of the state variable is verified.

Definition:

A system is said to be completely state controllable if it is possible to transfer the system state from any initial state $X(t_0)$ to any other desired state $X(t_0)$ in specified finite time by a control vector U(t).

Controllability test:

A general nth order multi-input multi-output time invariant system is completely controllable if and only if the rank of the composite matrix

$Q_c = [\ B : AB : A^2B \ldots A^{n-1}B\ ]$ is n

OBSERVABILITY:

A system is said to be completely observable if the knowledge of the outputs Y and input u over a finite interval $t_0 \leq t \leq t_f$ suffices to determine every state $X(t_0)$.

OBSERVABLILITY TEST:

A general nth order multi-input multi-output time invariant system is completely controllable if and if the rank of the composite matrix

$$Q_0 = \begin{bmatrix} C \\ CA \\ \cdot \\ \cdot \\ \cdot \\ C^{n-1}A \end{bmatrix}$$

20. Obtain the complete solution of nonhomogeneous state equation using time domain method.
(Nov/Dec 2017)

The state equation of n^{th} order system is given by

$$\dot{\mathbf{X}}(t) = \mathbf{A}\,\mathbf{X}(t) + \mathbf{B}\,\mathbf{U}(t) \quad ; \qquad \mathbf{X}(0) = \mathbf{X}_0$$

where $\mathbf{X}_0$ is initial condition vector.

The state equation of equation (6.60) can be rearranged as shown below.

$$\dot{\mathbf{X}}(t) - \mathbf{A}\,\mathbf{X}(t) = \mathbf{B}\,\mathbf{U}(t)$$

Premultiply both sides of equation (6.61) by $e^{-\mathbf{A}t}$

$$e^{-\mathbf{A}t}\left[\dot{\mathbf{X}}(t) - \mathbf{A}\,\mathbf{X}(t)\right] = e^{-\mathbf{A}t}\,\mathbf{B}\,\mathbf{U}(t)$$

$$e^{-\mathbf{A}t}\dot{\mathbf{X}}(t) + e^{-\mathbf{A}t}(-\mathbf{A})\,\mathbf{X}(t) = e^{-\mathbf{A}t}\,\mathbf{B}\,\mathbf{U}(t)$$

Consider the differential of $e^{-\mathbf{A}t}\,\mathbf{X}(t)$

$$\frac{d}{dt}\left(e^{-\mathbf{A}t}\,\mathbf{X}(t)\right) = e^{-\mathbf{A}t}\,\dot{\mathbf{X}}(t) + e^{-\mathbf{A}t}\,(-\mathbf{A})\,\mathbf{X}(t)$$

On comparing equations (6.62) and (6.63) we can write,

$$\frac{d}{dt}\left(e^{-\mathbf{A}t}\,\mathbf{X}(t)\right) = e^{-\mathbf{A}t}\,\mathbf{B}\,\mathbf{U}(t)$$

$$d\left(e^{-\mathbf{A}t}\,\mathbf{X}(t)\right) = e^{-\mathbf{A}t}\,\mathbf{B}\,\mathbf{U}(t)\,dt$$

On integrating the equation (6.64) between limits 0 to t we get,

$$e^{-\mathbf{A}t}\,\mathbf{X}(t) = \mathbf{X}_0 + \int_0^t e^{-\mathbf{A}\tau}\,\mathbf{B}\,\mathbf{U}(\tau)\,d\tau$$

where $\mathbf{X}_0$ = Initial condition vector = Integral constant

τ = Dummy variable substituted for t.

Premultiply both sides of equation (6.65) by e^{At},

$$e^{At}e^{-At}X(t) = e^{At}X_0 + e^{At}\int_0^t e^{-A\tau} B\,U(\tau)\,d\tau$$

$$X(t) = e^{At}X_0 + e^{At}\int_0^t e^{-A\tau} B\,U(\tau)\,d\tau \qquad (6.66)$$

The term e^{At} independent of the integral variable τ, and so e^{At} can be brought inside the integral function.

$$\therefore X(t) = e^{At}X_0 + \int_0^t e^{At}e^{-A\tau} B\,U(\tau)\,d\tau$$

$$X(t) = e^{At}X_0 + \int_0^t e^{A(t-\tau)} B\,U(\tau)\,d\tau \qquad (6.67)$$

The equation (6.67) is the solution of state equation, when the initial conditions are known at $t = 0$. If initial conditions are known at $t = t_0$ then the solution of state equation is given by equation (6.68).

$$X(t) = e^{A(t-t_0)}X(t_0) + \int_{t_0}^t e^{A(t-\tau)} B\,U(\tau)\,d\tau \qquad (6.68)$$

The state transition matrix e^{At} is denoted by the symbol $\phi(t)$, i.e., $\phi(t) = e^{At}$

Hence, $e^{A(t-t_0)}$ can be expressed as, $e^{A(t-t_0)} = \phi(t-t_0)$ (6.69)

and, $e^{A(t-\tau)}$ can be expressed as, $e^{A(t-\tau)} = \phi(t-\tau)$ (6.70)

The equations (6.67) and (6.68) can also be expressed as

$$X(t) = \phi(t)X(0) + \int_0^t \phi(t-\tau) B\,U(\tau)\,d\tau \quad \text{if the initial conditions are known at } t = 0 \qquad (6.71)$$

$$X(t) = \phi(t-t_0)X(t_0) + \int_{t_0}^t \phi(t-\tau) B\,U(\tau)\,d\tau \quad \text{if the initial conditions are known at } t = t_0 \qquad (6.72)$$

21.Determine the canonical state model of the sytem whose transfer function is T(s)=2(s+5)/(s+2)(s+3)(s+4).(April/May2018)

SOLUTION

By partial fraction expansion,

$$\frac{Y(s)}{U(s)} = \frac{2\,(s+5)}{(s+2)\,(s+3)\,(s+4)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s+4}$$

$$A = \left.\frac{2\,(s+5)}{(s+3)\,(s+4)}\right|_{s=-2} = \frac{2(-2+5)}{(-2+3)\,(-2+4)} = \frac{2\times 3}{1\times 2} = 3$$

$$B = \left.\frac{2\,(s+5)}{(s+2)\,(s+4)}\right|_{s=-3} = \frac{2(-3+5)}{(-3+2)(-3+4)} = \frac{2\times 2}{-1\times 1} = -4$$

$$C = \left.\frac{2\,(s+5)}{(s+2)\,(s+3)}\right|_{s=-4} = \frac{2(-4+5)}{(-4+2)\,(-4+3)} = \frac{2\times 1}{-2\times(-1)} = 1$$

$$\therefore \frac{Y(s)}{U(s)} = \frac{3}{s+2} - \frac{4}{s+3} + \frac{1}{s+4} \qquad (6.12.1)$$

The equation (6.12.1) can be rearranged as shown below

$$\frac{Y(s)}{U(s)} = \frac{3}{s(1+2/s)} - \frac{4}{s(1+3/s)} + \frac{1}{s(1+4/s)}$$

$$\therefore Y(s) = \left[\frac{\frac{1}{s}}{1+\frac{1}{s}\times 2}\times 3\right]U(s) - \left[\frac{\frac{1}{s}}{1+\frac{1}{s}\times 3}\times 4\right]U(s) + \left[\frac{\frac{1}{s}}{1+\frac{1}{s}\times 4}\right]U(s) \qquad (6.12.2)$$

The equation (6.12.2) can be represented by the block diagram in fig 1.

Assign state variables at the output of the integrators as shown in fig 1. At the input of the integrators we have first derivative of the state variables. The state equations are formed by adding all the incoming signals to the integrator and equating to the corresponding first derivative of state variable.

The state equations are

$$\dot{x}_1 = -2x_1 + u \quad ; \quad \dot{x}_2 = -3x_2 + u \quad ; \quad \dot{x}_3 = -4x_3 + u$$

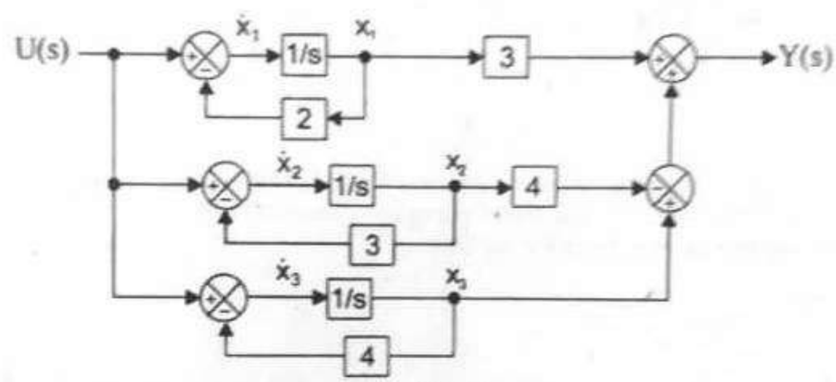


Fig 1.

The output equation is, $y = 3x_1 - 4x_2 + x_3$

The state model in matrix form is given below

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-2 & 0 & 0\\ 0 & -3 & 0\\ 0 & 0 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}1\\ 1\\ 1\end{bmatrix}[u] \quad ; \quad y = [3 \quad -4 \quad 1]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

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Question Paper Code : 50714

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

Fifth Semester

Electronics and Instrumentation Engineering

IC6501 – CONTROL SYSTEMS

(Common to Electrical and Electronics Engineering/Instrumentation and Control Engineering)

(Regulations 2013)

Time : Three Hours

Maximum : 100 Marks

Codes / Tables / Charts to be permitted, if any may be indicated

Answer ALL questions

PART – A

(10×2=20 Marks)

1. Define open loop and closed loop control system.
2. What are the basic elements used for modeling mechanical translational system ?
3. Distinguish between type and order of a system.
4. What is the effect on system performance when a proportional controller is introduced in a system ?
5. List out the different frequency domain specifications.
6. Give the need for lag/lag-lead compensation.
7. What are the necessary conditions for stability ?
8. What are the effects adding open loop poles and zero on the nature of the root locus and on system ?
9. Write the homogeneous and nonhomogeneous state equation.
10. Define state trajectory.

PART – B (5×13=65 Marks)

11. a) Find the transfer function $\frac{y_2(s)}{f(s)}$.

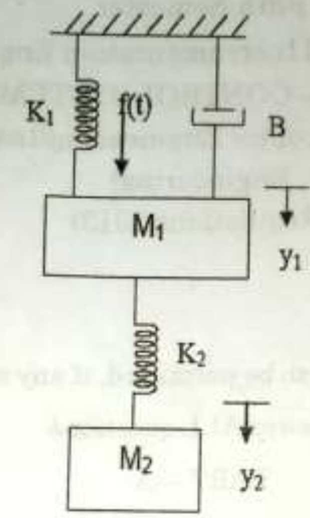

Fig. 11 a

(OR)

b) Find the overall gain C(S) / R(S) for the signal flow graph shown in Fig. 11 b.

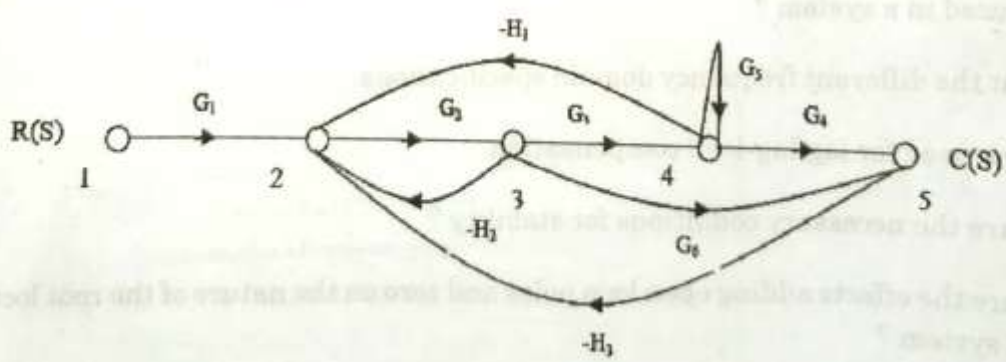

Fig. 11 b

12. a) Derive the expressions for second order system for under damped case and when the input is unit step.

(OR)

b) Find the static error coefficients for a system whose transfer function is, G (s). H (s) = 10/s (1 + s) (1 + 2s). And also find the steady state error for r (t) = 1 + t + $t_{2/2}$.

13. a) Sketch the Bode plot and hence find Gain cross over frequency, Phase cross over Frequency, Gain margin and Phase margin for the function

$$G(s) = \frac{10(s+3)}{s(s+2)(s^2+4s+100)}.$$

(OR)

b) Sketch the polar plot for the following transfer function and find Gain cross over frequency, Phase cross over frequency, Gain margin and Phase margin for G (s) = 400/s (s+ 2) (s + 10).

14. a) A unity feedback control system has an open loop transfer function G (s) = K (s + 9) / s (s^2 + 4s+ 11). Sketch the root locus.

(OR)

b) Determine the stability of closed loop system by Nyquist stability criterion, whose open loop transfer function is given by, G (s). H (s) = (s + 2) / (s+ 1) (s – 1).

15. a) Explain the concepts of controllability and observability.

(OR)

b) Obtain the complete solution of nonhomogeneous state equation using time domain method.

PART – C **(1×15=15 Marks)**

16. a) For the given system, G (s) = K / s (s + 1) (s + 2), design a suitable lag-lead compensator to give, velocity error constant = 10 sec-1, phase margin = 50°, gain margin ≥ 10 dB.

(OR)

b) Realize the basic compensators using electrical network and obtain the transfer function.

Reg. No. :

Question Paper Code : 41245

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Fifth Semester

Instrumentation and Control Engineering

IC 6501 – CONTROL SYSTEMS

(Common to Electrical and Electronics Engineering/Electronics and Instrumentation Engineering)

(Regulations 2013)

Time : Three Hours — Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. List the characteristics of negative feedback in control system.

2. Write the expression for mason's gain formula.

3. How is a system classified depending on the value of damping ?

4. What is steady state error ?

5. Give the advantages of frequency response analysis.

6. Define corner frequency.

7. Differentiate between gain margin and phase margin.

8. What is dominant pole ?

9. Write the advantages of state space analysis.

10. State the concept of observability.

PART – B (5×13=65 Marks)

11. a) Write the differential equations governing the mechanical system shown in Fig 11.a. Also draw the force voltage and force current analogous circuit and verify by writing mesh and node equations. (13)

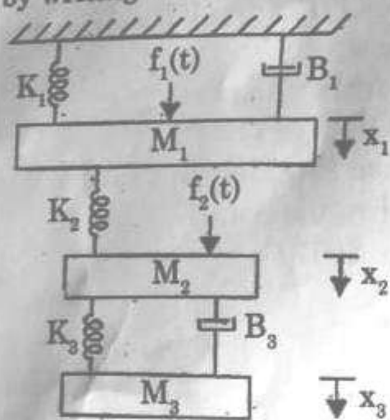

Fig 11. a)

(OR)

b) The block diagram of a closed loop system is shown in Fig 11. b). Using block diagram reduction technique, determine the closed loop transfer function. (13)

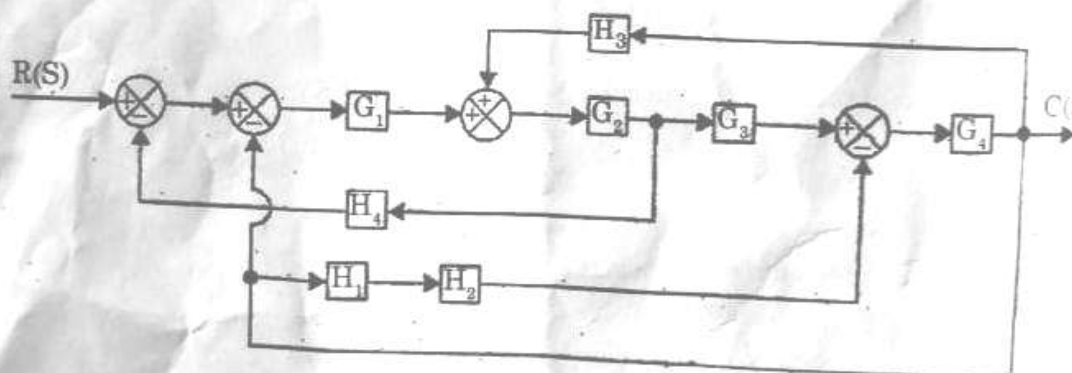

Fig. 11. (b)

12. a) i) Outline the time response of first order system when it is subjected to a unit step input. (8)

ii) Determine the response of the unity feedback system whose open loop transfer function is $G(s)=\frac{4}{s(s+5)}$ and when the input is unit step. (5)

(OR)

b) i) A unity feedback system has the forward transfer , function

$$G(s)=\frac{K_1(2s+1)}{s(5s+1)(1+s)^2}$$ when the input r(t) = 1 + 6t, determine the

minimum value of K_1 so that the steady error is less than 0.1. **(8)**

ii) Derive the transfer function of PID controller. **(5)**

13. a) Construct the polar plot and determine the gain margin and phase margin of a unity feedback control system whose open loop transfer function is,

$$G(s)=\frac{(1+0.2s)(1+0.025s)}{s^3(1+0.005s)(1+0.001s)}.$$ **(13)**

(OR)

b) Draw the bode diagram for the following transfer function,

$$G(s)=\frac{75(1+0.2s)}{s(s^2+16+100)}$$ **(13)**

14. a) i) Use the routh stability criterion, determine the range of K for stability of unity feedback system whose open loop transfer function is

$$G(s)=\frac{K}{s(s+1)(s+2)}.$$ **(10)**

ii) State Routh stability criterion. **(3)**

(OR)

b) Design a lead compensator for a unity feedback system with open loop transfer function, $G(s)=\frac{K}{s(s+1)(s+5)}$ to satisfy the following specifications

i) Velocity error constant, $K_v \geq 50$

ii) Phase margin $\geq$ 20 degrees. **(13)**

15. a) Determine the canonical state model of the system whose transfer function is

$$T(s)=\frac{2(s+5)}{(s+2)(s+3)(s+4)}.$$ **(13)**

(OR)

b) Consider a linear system described by the following transfer function, $\frac{Y(s)}{U(s)} = \frac{10}{s(s+1)(s+2)}$. Design a feedback controller with a state feedback so that the closed loop poles are placed at $-2, -1 \pm j1$. **(13)**

PART – C **(1×15=15 Marks)**

16. a) A unity feedback control system has an open loop transfer function,

$G(s) = \frac{k}{s(s^2+4s+13)}$. Sketch the Root Locus. (15)

(OR)

b) Construct the Nyquist plot for a system whose open loop transfer function is given by $G(s)H(s) = \frac{K(1+s)^2}{s^3}$, Find the range of K for stability. (15)

Question Paper Code : 91440

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

Fourth Semester

Electrical and Electronics Engineering

EE 2253/EE 44/EE 1253 A/080280033/10133 IC 401 — CONTROL SYSTEMS

(Common to Instrumentation and Control Engineering and Electronics and Instrumentation Engineering)

(Regulation 2008/2010)

(Also common to PTEE 2253 – Control Systems for B.E. (Part-Time) Third Semester – Electronics and Instrumentation Engineering – Regulation 2009 and 10133 IC 401 – Control System for B.E. (Part-Time) Third Semester – EEE – Regulation 2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What are the basic elements in control systems?

2. Define: transfer function.

3. What is the type and order of the system?

$$G(S) = \frac{K}{S(TS+1)}$$

4. Write the PID controller equation.

5. Write the expression for resonance frequency and peak in terms of time response specifications.

6. Define: Gain margin.

7. What are the location of roots in S – plane for stability?

8. What is meant by $+20db/dec$ slope change?

9. What is the need for compensators?

10. What are the desired performance criteria specified in compensator design?

PART B — (5 × 16 = 80 marks)

11. (a) With neat diagrams, explain the working of AC and DC servo motors. (16)

Or

(b) Using block diagram reduction rules, convert the block diagram of Fig. 1 to a simple loop.

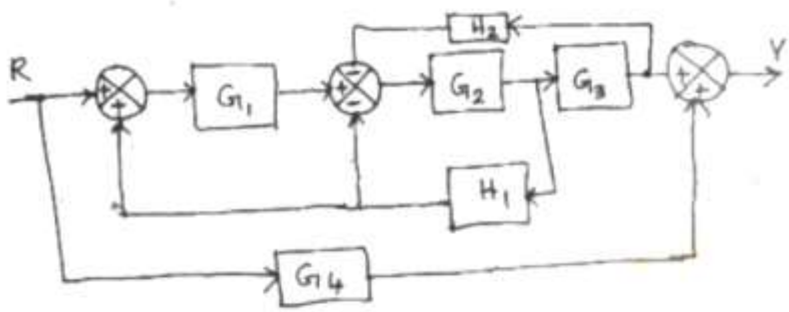

Fig. 1

12. (a) Derive the expression for unit step response of under damped second order system. (16)

Or

(b) Obtain the expression for dynamic error coefficients of the following system $G(S) = \frac{10}{S(1+S)}$ (16)

13. (a) Draw the Bode plot of the following system $GH(S) = \frac{10}{S(0.1\,S+1)(0.01S+1)}$ and hence obtain gain crossover frequency. (16)

Or

(b) Using polar plot, determine gain crossover frequency, phase crossover frequency, gain margin and phase margin of feedback system with open – loop transfer function (16)

$$G(S)H(S) = \frac{10}{S\,(1+0.2S)(1+0.002\,S)}$$

14. (a) Consider the closed – loop system shown in Fig. 2, determine the range of K for which the system is stable. (16)

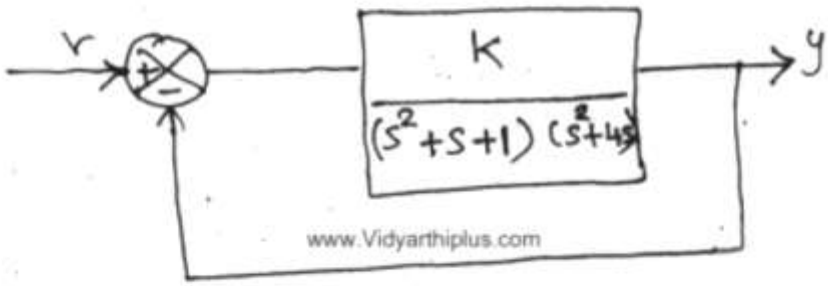

Fig. 2

Or

(b) Draw the root locus of the following system (16)

$$G(S).H(S) = \frac{K}{S(S+1)(S+2)}$$

15. (a) A unity feedback system has an open loop transfer function

$$G(S) = \frac{5}{S(S+1)(0.5S+1)}$$

Design a suitable compensator to maintain phase margin of at least 40°.

Or

(b) Consider the unity feedback system whose open – loop transfer function is $G(S) = \dfrac{k}{S\,(0.1S+1)(0.2\,S+1)}$

The system is to be compensated to meet the following specifications:

(i) Velocity error constant $k_v = 30$

(ii) Phase margin $\phi m \geq 50°$

(iii) Bandwidth $w_1 = 12\ rad/sec$.

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Question Paper Code : 51437

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2014.

Fourth Semester

Electrical and Electronics Engineering

EE 2253/EE 44/EE 1253 A/080280033/10133 IC 401 — CONTROL SYSTEMS

(Common to Instrumentation and Control Engineering and Electronics and Instrumentation Engineering)

(Regulation 2008/2010)

(Common to PTEE 2253 – Control Systems for B.E. (Part-Time) Third Semester – Electronics and Instrumentation Engineering Regulation 2009)

Time : Three hours Maximum : 100 marks

Note : Polar plot to be issued.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. List the major advantages and disadvantages of open-loop control systems.
2. What are the applications of synchros?
3. What are the standard test signals used in control systems?
4. What is the effect of PD controller on the performance of a system?
5. Define the terms: 'resonant peak', and 'resonant frequency'.
6. What is a constant M circle?
7. How are the locations of roots of characteristic equation related to stability?
8. State the Nyquist stability criterion.
9. What is the necessity of compensation in feedback control system?
10. Write the transfer function of lag-lead compensator.

PART B — (5 × 16 = 80 marks)

11. (a) Write the differential equations for the mechanical system shown in Fig. 1. Obtain an analogous electric circuit based on force-current analogy.

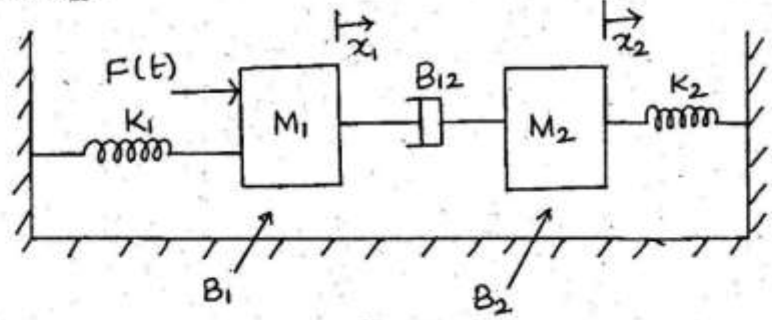

Fig. 1

Or

(b) Consider the signal flow graph shown in Fig. 2. Obtain the closed loop transfer function $\frac{C(s)}{R(s)}$ by the use of Mason's gain formula.

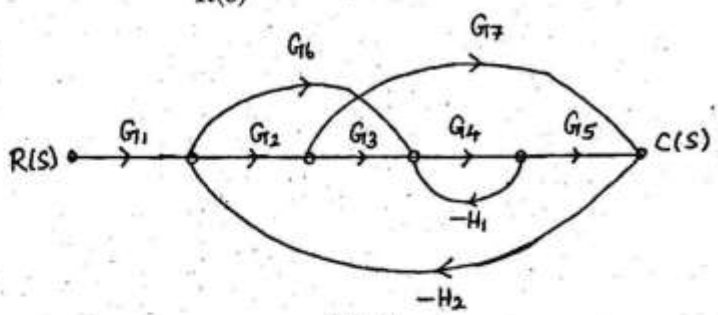

Fig. 2

12. (a) A unity feedback system is characterized by an open loop transfer function $G(s) = \frac{K}{s(s+10)}$.

Determine the gain K so that the system will have a damping ratio of 0.5. For this value of K, determine settling time, peak overshoot and time to peak overshoot for a unit step input.

Or

(b) The open loop transfer function of a servo system with unity feedback is

$$G(s) = \frac{10}{s(0.1s+1)}.$$

Evaluate the static error constants (K_p, K_v, K_a) for the system. Obtain the steady state error of the system when subjected to an input given by the polynomial $r(t) = a_0 + a_1 t + \frac{a_2}{2} t^2$.

13. (a) Sketch the Bode plot showing the magnitude in decibels and phase angle in degrees as a function of log frequency for the transfer function

$$G(s) = \frac{75(1 + 0.2s)}{s(s^2 + 16s + 100)}.$$

From the Bode plot, determine the gain cross-over frequency.

Or

(b) (i) Discuss the correlation between time and frequency response of second order system. (8)

(ii) How the closed loop frequency response is determined from the open loop frequency response using Nichols chart? Explain how the gain adjustment is carried out on the Nichols chart. (8)

14. (a) The open loop transfer function of a unity feedback control system is given by $G(s) = \frac{K}{(s+2)(s+4)(s^2 + 6s + 25)}$.

By applying Routh criterion, discuss the stability of the closed loop system as a function of K. Determine the values of K which will cause sustained oscillations in the closed loop system. What are the corresponding oscillation frequencies?

Or

(b) Sketch the root locus plot of a unity feedback system with an open loop transfer function $G(s) = \frac{K}{s(s+2)(s+4)}$. Find the value of K so that the damping ratio of the closed loop system is 0.5.

15. (a) Explain the electric network realization of a lead compensator and also its frequency response characteristics.

Or

(b) Describe the procedure for the design of lag compensator using Bode plot.

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Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 80554

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Fifth Semester

Electrical and Electronics Engineering

IC 6501 — CONTROL SYSTEMS

(Common to Electronics and Instrumentation Engineering/Instrumentation and Control Engineering)

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Why negative feedback is preferred in control systems?

2. What are the differences between a Synchro transmitter and a Synchro control transformer?

3. Give the relation between static and dynamic error coefficients.

4. State the basic properties of root locus.

5. What does a gain margin close to unity or phase margin close to zero indicate?

6. What are the effects and limitations of phase-lag control?

7. What are two notions of system stability to be satisfied for a linear time-invariant system to be stable?

8. Why frequency domain compensation is normally carried out using the Bode plots?

9. For a first order differential equation described by $\dot{x}(t) = a\,x(t) + b\,u(t)$, draw the block diagram form of state diagram.

10. State the limitations of state variable feedback.

PART B — (5 × 16 = 80 marks)

11. (a) Write the differential equations governing the mechanical system shown in Fig. Q 11(a). Draw the force-voltage and force-current electrical analogous circuits. (16)

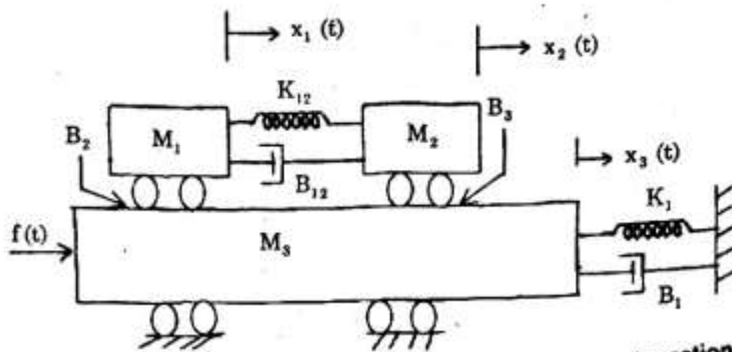

Fig. Q 11 (a)

Or

(b) (i) Derive the transfer function of AC servomotor. (8)

(ii) Construct a block diagram for the simple electrical network shown in Fig. Q 11 (b) (ii) and hence, obtain the signal flow graph and the transfer function $\frac{E_0(s)}{E_i(s)}$. (8)

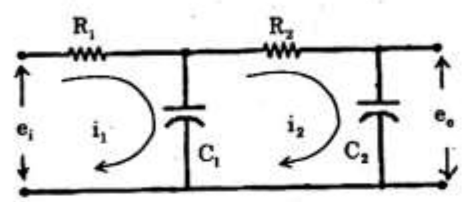

Fig. Q 11 (b) (ii)

12. (a) Draw the root locus for a system is given by $G(s) = \frac{K(s+1)}{s(s^2+5s+20)}$. (16)

Or

(b) (i) The overall transfer function of a control system is given by $\frac{C(s)}{R(s)} = \frac{16}{(s^2+1.6s+16)}$. It is desired that the damping ratio be 0.8. Determine the derivative rate feedback constant K_t and compare rise time, peak time, maximum overshoot and steady state error for unit ramp input function without and with derivative feedback control. (10)

(ii) Compare P, I and D controller.

13. (a) The open loop transfer function of a unity feedback system is given by $G(s) = \frac{1}{s(s+1)(2s+1)}$. Sketch the polar plot and determine the gain margin and phase margin. (16)

Or

(b) Draw the Bode plot for the transfer function $G(s) = \frac{1}{s(s^2+3s+5)}$. Determine the gain margin and phase margin. (16)

14. (a) (i) Determine the range of values of K for which the system described by the following characteristic equation is stable. (10)

$$s^3 + 3Ks^2 + (K+2)s + 4 = 0.$$

(ii) State and explain Nyquist stability criterion. (6)

Or

(b) Design a lead compensator for a unity feedback system with an open loop transfer function $G(s) = \frac{K}{s(s+1)}$ for the specifications of $K_v = 10\,\text{sec}^{-1}$ and phase margin $\phi_m = 35°$. (16)

15. (a) Obtain the time response of the system described by

$$[\dot{x}(t)] = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t)$$

with the initial conditions $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $y(t) = [0 \quad 1] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$. (16)

Or

(b) Determine whether the system described by the following state model is completely controllable and observable (16)

$$[\dot{x}(t)] = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} u(t); \; y(t) = [1 \quad 0 \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}.$$

Reg. No. :

Question Paper Code : 72004

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Fifth Semester

Electrical and Electronics Engineering

IC 6501 — CONTROL SYSTEMS

(Common to Electronics and Instrumentation Engineering/Instrumentation and Control Engineering)

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Why negative feedback is preferred in closed loop control system?

2. What is block diagram? State its components.

3. Define maximum peak overshoot.

4. Determine type and order of the following system $G(s)H(s) = 10/[S^3(S^2 + 2s + 1)]$

5. What is meant by frequency response?

6. State about Lead-Lag compensation.

7. What is characteristic equation?

8. State Nyquist stability criterion.

9. Draw the block diagram representation of a state model.

10. What is controllability?

PART B — ($5 \times 16 = 80$ marks)

11. (a) Write the differential equations governing the mechanical translational system shown in figure1. Draw the electrical equivalent analogy circuits. (16)

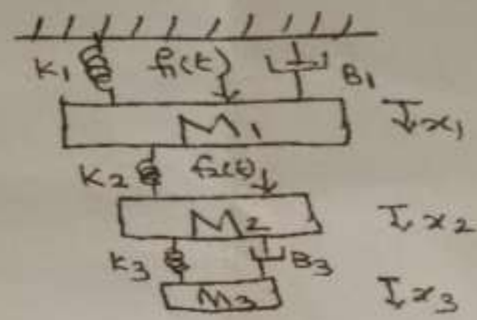

Figure.1.

Or

(b) (i) With its operating principle derive the transfer function of AC servo motor in control system. (12)

(ii) Compare open loop and closed loop control systems. (4)

12. (a) Derive the time response of Undamped and Critically damped second order system for unit step input. (16)

Or

(b) (i) A unity feedback control system has an open loop transfer Function $G(s) = 10/[s(s+2)]$ Find the rise time, peak time, percentage overshoot and settling time for step input of 12 units. (8)

(ii) For servomechanisms, with open loop transfer function given below explain what type of input signal give rise to a steady state error and calculate their values.

(1) $G(s) = [20(s+2)]/s(s+1)(s+3)]$

(2) $G(s) = 10/[(s+2)(s+3)]$. (8)

13. (a) Plot the Bode plot for the following transfer function and determine the phase and gain cross over frequencies. $G(s) = 10/[s(1+0.4s)(1+0.1s)]$. (16)

Or

(b) The open loop function of a unity feedback system is given by $G(s) = 1/[s(1+s)(1+2s)]$. Sketch the polar plot and determine the gain and phase margin. (16)

14. (a) (i) Using Routh criterion, determine the stability of a system representing the characteristic equation $S^4+8S^3+18S^2+16S+5=0$ Comment on location of the roots of the characteristics equation. (6)

(ii) Write down the procedure for designing Lag compensator using Bode plot. (10)

Or

(b) Explain in detail the realization of Lag, Lead and Lag-Lead electrical networks. (16)

15. (a) Check the controllability and observability of the system whose state space representation is given as (16)

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-1 & 0 & 0\\ 1 & -2 & 0\\ 2 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}10\\ 1\\ 0\end{bmatrix}u \quad y=\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Or

(b) (i) What are state variables? Explain the state space formulation with its equation. (8)

(ii) Given that

$A_1=\begin{bmatrix}\sigma & 0\\ 0 & \sigma\end{bmatrix}$; $A_2\begin{bmatrix}0 & w\\ -w & 0\end{bmatrix}$; $A=\begin{bmatrix}\sigma & w\\ -w & \sigma\end{bmatrix}$ Compute state transistion matrix. (8)

Reg. No. A 2 1 6 1 3 1 0 5 0 2 0

Question Paper Code : 27300

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fifth Semester

Electrical and Electronics Engineering

IC 6501 – CONTROL SYSTEMS

(Common to Instrumentation and Control Engineering and Electronics and Instrumentation Engineering)

(Regulations 2013)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Draw the electrical analog of a thermometer.

2. What is electrical zero position of a synchro transmitter?

3. For the system described by $\frac{C(s)}{R(s)} = \frac{16}{s^2+8s+16}$; find the nature of the time response.

4. Why is the derivative control not used in control systems?

5. Draw the approximate polar plot for a Type 0 second order system.

6. What is the basis for the selection of a particular compensator for a system?

7. How are the roots of the characteristic equation of a system related to stability?

8. Draw the electric lag network and its pole-zero plot.

9. What is meant by 'State' of a dynamic system?

10. When do you say that a system is completely state controllable?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Explain open loop and closed loop control systems with examples. (8)

(ii) Derive the transfer function of an armature controlled DC servomotor. (8)

Or

(b) (i) For the mechanical system shown in Fig. Q 11(b)(i).

(1) Draw the mechanical network diagram and hence write the differential equations describing the behaviour of the system.

(2) Draw the force-voltage and force-current analogous electrical circuits. (6+4)

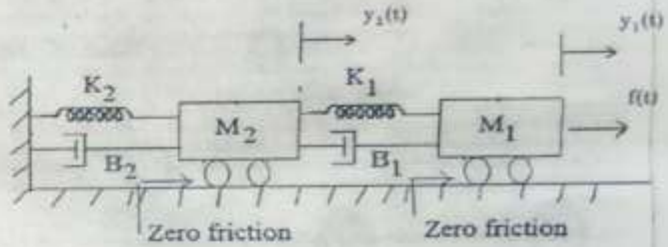

Fig. Q 11(b)(i)

(ii) For a nonunity negative feedback control system whose open loop transfer function is $G(s)$ and feedback path transfer function is $H(s)$, obtain the control ratio using Mason's gain formula. (6)

12. (a) (i) Derive the expressions for the unit step response of a second order

(1) underdamped, and

(2) undamped systems. (8+4)

(ii) Explain briefly the PID controller action with block diagram and obtain its transfer function model. (4)

Or

(b) (i) The open loop transfer function of a unity feedback system is given by $G(s) = \frac{1}{s(1+s)}$. The input to the system is described by $r(t) = 4 + 6t$. Find the generalized error coefficients and steady state error. (6)

(ii) Explain the rules to construct root locus of a system. (10)

13. (a) Construct Bode plot for the system whose open loop transfer function is given below and determine (i) the gain margin, (ii) the phase margin, and (iii) closed-loop system stability.

$$G(s) = \frac{4}{s(1+0.5s)(1+0.08s)}$$ (16)

Or

(b) (i) Explain the use of Nichol's chart to obtain closed loop frequency response from open loop frequency response of a unity feedback system. (8)

(ii) Describe the correlation between time and frequency domain specifications. (8)

14. (a) (i) By use of the Nyquist stability criterion, determine whether the closed-loop system having the following open-loop transfer function is stable or not. If not, how many closed- loop poles lie in the right-half s-plane.

$$G(s)H(s) = \frac{s+2}{(s+1)(s-1)}$$ (6)

(ii) Explain the procedure for the design of a lead compensator using Bode plot. (10)

Or

(b) (i) The open loop transfer function of a unity feedback system is given by $G(s)H(s) = \frac{K}{(s+2)(s+4)(s^2+6s+25)}$. By applying the Routh criterion, find the range of values of K for which the closed loop system is stable. Determine the values of K which will cause sustained oscillations in the closed loop system. What are the corresponding oscillation frequencies? (10)

(ii) Derive the transfer function of the lag-lead compensator. (6)

15. (a) (i) Obtain the state model of the mechanical system shown in Fig.Q11(b)(i) in which $f(t)$ is the input and $y_2(t)$ is the output. (10)

(ii) State and prove the properties of State Transition Matrix. (6)

Or

(b) Check for controllability and observability of a system having following coefficient matrices. (8+8)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix};\ B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix};\ C^T = \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix}$$