Berkeley ONLINE MPH

Effect Measure Modification on Different Scales

Moon Choi

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EMM scales

- Additive scale
  - Measures of association on the absolute scale:
    - E.g., Risk difference
  - RERI
- Multiplicative scale
  - Measures of association on the relative scale:
    - E.g., Cumulative incident ratio, Incidence density ratio, Odds ratio, Prevalence ratio

Reminder:

Statistical interaction and EMM correspond to each other when no bias is present.

EMM on one or the other scale

- Mathematically speaking, if EMM is not present one scale, it *will* be present on the other scale!

Example

	Disease	No Disease	Total
Exposed	80	20	**100**
Unexposed	416	384	**800**
Total	**496**	**504**	**900**

Absolute scale

CI in exposed: 80/100 = 0.8

CI in unexposed: 416/800 = 0.52

RD: 0.8-0.52 = 0.28

Relative scale

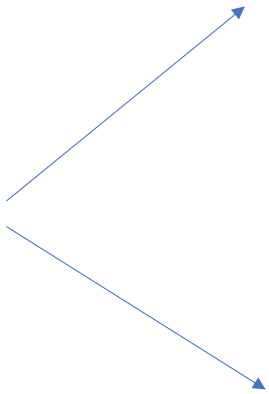
CI in exposed: 80/100 = 0.8

CI in unexposed: 416/800 = 0.52

CIR: 0.8/0.52 = 1.54

Example

	Disease	No Disease	Total
Exposed	80	20	**100**
Unexposed	416	384	**800**
Total	**496**	**504**	**900**



W=1	Disease	No Disease	Total
Exposed	60	10	**70**
Unexposed	340	260	**600**
Total	**400**	**370**	**670**

W=0	Disease	No Disease	Total
Exposed	20	10	**30**
Unexposed	76	124	**200**
Total	**96**	**134**	**230**

W=1	Disease	No Disease	Total
Exposed	60	10	**70**
Unexposed	340	260	**600**
Total	**400**	**370**	**670**

Additive scale for W=1

CI in exposed: 60/70 = 0.86

CI in unexposed: 340/600 = 0.57

$RD_{W=1}$: 0.86-0.57 = 0.29

W=0	Disease	No Disease	Total
Exposed	20	10	**30**
Unexposed	76	124	**200**
Total	**96**	**134**	**230**

Additive scale for W=0

CI in exposed: 20/30 = 0.67

CI in unexposed: 76/200 = 0.38

$RD_{W=0}$: 0.67-0.38 = 0.29

W=1	Disease	No Disease	Total
Exposed	60	10	**70**
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Exposed	20	10	**30**
Unexposed	76	124	**200**
Total	**96**	**134**	**230**

Additive scale for W=0

CI in exposed: 20/30 = 0.67

CI in unexposed: 76/200 = 0.38

$RD_{W=0}$: 0.67-0.38 = 0.29

For the sake of simplicity, the RDs are the same.

A chi-square test of homogeneity is needed to know whether the stratum-specific MoA are considered equal.

W=1	Disease	No Disease	Total
Exposed	60	10	**70**
Unexposed	340	260	**600**
Total	**400**	**370**	**670**

Multiplicative scale for W=1

CI in exposed: 60/70 = 0.86

CI in unexposed: 340/600 = 0.57

$CIR_{W=1}$: 0.86/0.57 = 1.51

W=0	Disease	No Disease	Total
Exposed	20	10	**30**
Unexposed	76	124	**200**
Total	**96**	**134**	**230**

Multiplicative scale for W=0

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$CIR_{W=0}$: 0.67/0.38 = 1.76

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Total	**96**	**134**	**230**

Multiplicative scale for W=0

CI in exposed: 20/30 = 0.67

CI in unexposed: 76/200 = 0.38

$CIR_{W=0}$: 0.67/0.38 = 1.76

For simplicity, assume that we conducted a chi-square test of homogeneity and failed to reject the null hypothesis, i.e., the two CIRs are not equal.

CIs	W=0	W=1
E=0	0.38	0.57
E=1	0.67	0.86

W=1

CI in exposed: 60/70 = 0.86

CI in unexposed: 340/600 = 0.57

$CIR_{W=1}$: 0.86/0.57 = 1.51

W=0

CI in exposed: 20/30 = 0.67

CI in unexposed: 76/200 = 0.38

$CIR_{W=0}$: 0.67/0.38 = 1.76

CIs	W=0	W=1
E=0	0.38	0.57
E=1	0.67	0.86

The cells do not contain counts! They are cumulative incidences that we calculated in the slides before.

CI_{11}: 0.86
CI_{10}: 0.67
CI_{01}: 0.57
CI_{00}: 0.38

Note that this is not the 2x2 table we are used to seeing!

This table is a way to organize the CIs for each level of the exposure and covariate of interest.

CIs	W=0	W=1
E=0	0.38	0.57
E=1	0.67	0.86

CI_{11}: 0.86
CI_{10}: 0.67
CI_{01}: 0.57
CI_{00}: 0.38

CI_{11}/CI_{00} **?** $CI_{10}/CI_{00} * CI_{01}/CI_{00}$

CI_{11} **?** $CI_{10}/CI_{00} * CI_{01}$

CI_{11} **?** $(CI_{10} * CI_{01})/CI_{00}$

0.86 **?** (0.67 * 0.57)/0.38
0.86 **?** 1.01

0.86 **<** 1.01

Note that the expected CI_{11}=1.01. This is not the same thing as the crude CIR (1.54) that was calculated earlier.

CIs	W=0	W=1
E=0	0.38	0.57
E=1	0.67	0.86

CI_{11}: 0.86
CI_{10}: 0.67
CI_{01}: 0.57
CI_{00}: 0.38

CI_{11}/CI_{00} **?** $CI_{10}/CI_{00} * CI_{01}/CI_{00}$

CI_{11} **?** $CI_{10}/CI_{00} * CI_{01}$

CI_{11} **?** $(CI_{10} * CI_{01})/CI_{00}$

0.86 **?** (0.67 * 0.57)/0.38
0.86 **?** 1.01

0.86 **<** 1.01

In the absence of multiplicative interaction, we would expect to see CI_{11} = 1.01.
Instead, we see 0.86, which is less than 1.01.
This indicates that there is antagonistic interaction on the multiplicative scale.