

## August 3 Update

### Simulating a Modulated Signal

From Kian's thesis we have the following formula for signals with frequency modulation:

$$X_t = \sum_m \mu_m \cos(2\pi f_m t + 2\pi \int_0^t \phi_m(\tau) d\tau) + Z_t$$

Now let  $\theta(t) = 2\pi f_m t + 2\pi \int_0^t \phi_m(\tau) d\tau$ . Since  $\phi$  and  $\theta$  are both polynomials, we can simplify to:

$$\begin{aligned}\phi(\tau) &= \sum_{p=0}^P a_p \tau^p \\ \theta(t) &= 2\pi f_m t + 2\pi \int_0^t \sum_{p=0}^P a_p \tau^p d\tau \\ \theta(t) &= 2\pi f_m t + 2\pi \sum_{p=0}^P \frac{a_p}{p+1} t^{p+1}\end{aligned}$$

Using these formulas I wrote a function to simulate a modulated signal given a length  $N$ , number of frequencies  $m$ , and the maximum degree of polynomial to consider  $P$ .