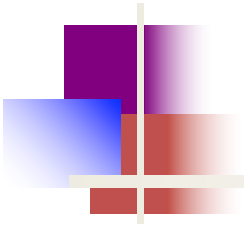


Relational Algebra

Chapter 4

Formal Query Languages

- Foundation for commercial query languages like SQL
- Two types
  - Declarative: Relational Calculus
    - Describe what a user wants, rather than how to compute it
  - Procedural: Relational Algebra
    - Operational, very useful for representing execution plans
- Query Languages != programming languages
  - QLs not expected to be “Turing complete”
  - QLs not intended to be used for complex calculations
  - QLs support easy, efficient access to large data sets

Understanding Algebra & Calculus is key to understanding SQL query processing!

Relational Algebra Preliminaries

- Query:
  - Input: Relational instances
  - Output: Relational instances

  - Specified using the schemas
    - May produce different results for different instances
    - But schema of the result is fixed

Numeric Algebra

- Input: Rational Numbers
- Output: Rational Numbers
- Operators: +, -, x, /
- Closed under these operators

Question??

The smallest set of numbers that is CLOSED with respect to the operators {+,-,x} is:

A. Natural Numbers
B. Integers
C. Rational Numbers
D. Real Numbers

Relational Operators

- Selection σ: Selects a subset of rows from relation
- Projection π: Deletes unwanted columns from relation

} 1 reln

- Cross-product $\times$: Allows us to combine two relations
- Set-difference $-$: Tuples in reln. 1, but not in reln. 2
- Union $\cup$: Tuples in reln. 1 and in reln. 2

} 2 reln

- Additional operations (constructed from basic ops – not essential, but very useful):
  - Intersection, **Join**, Division, Renaming
- Because algebra is closed, we can **compose** operators

Selection

- Retrieve **rows** that satisfy a logical condition

$$\sigma_{condition}(relation)$$

- Conditions are boolean combinations (∧, ∨) of terms
- Each term has one of the following forms:
  - Attr1 <oper> Attr2
  - Attr <oper> value

} where <oper> in {>, ≥, = ≤, <}

Athlete

aid	name	sport	country
1	Simone Biles	gymnastics	USA
2	Gabby Thomas	track	USA
3	Katie Ledecki	swimming	USA
4	Embla Njerve	pole vault	Norway
5	Zhang Boheng	gymnastics	China
6	Sunisa Lee	gymnastics	USA

$$\sigma_{sport='gymnastics' \wedge country='USA'}(Athlete)$$

aid	name	sport	country
1	Simone Biles	gymnastics	USA
6	Sunisa Lee	gymnastics	USA

Question??

A SET of items can have duplicate entries.

That is, the same item can occur more than once in a set. E.g. {4, 5, 4, 3, 2, 4, 2, 7}

A. True
B. False
C. Depends

Projection

- Selects columns
- Delete attributes that are not in projection list
- Remove **duplicates**!
  - Remember classical relational model is a **SET**

Athlete

aid	name	sport	country
1	Simone Biles	gymnastics	USA
2	Gabby Thomas	track	USA
3	Katie Ledecki	swimming	USA
4	Embla Njerve	pole vault	Norway
5	Zhang Boheng	gymnastics	China
6	Sunisa Lee	gymnastics	USA

$$\pi_{sport,country}(Athlete)$$

sport	country
gymnastics	USA
track	USA
swimming	USA
gymnastics	China
skating	Norway

Set Operations: Union (∪), Intersection(∩), Set-Difference (-)

- **Input:** Two union-compatible relations
  - Same number and type of attributes, in same order
- **Field names of result:** uses the name from the FIRST input

name
Simone Biles
Gabby Thomas
Katie Ledecki
Embla Njerve
Zhang Boheng
Sunisa Lee

−

AthleteName
Simone Biles
Zhang Boheng
Apollo Ono
Picabo Street
Chen Yibing
Katie Ledecki

=

name
Gabby Thomas
Embla Njerve
Sunisa Lee

Cross-Product (Cartesian Product)

- **Result Schema:**
  - All fields from both relations, names inherited if possible

Athlete

aid	name	sport	country
1	Simone Biles	gymnastics	USA
2	Gabby Thomas	track	USA

×

Venue

vid	venue
1	Paris
2	London

=

aid	name	sport	country	vid	venue
1	Simone Biles	gymnastics	USA	1	Paris
2	Gabby Thomas	track	USA	1	Paris
1	Simone Biles	gymnastics	USA	2	London
2	Gabby Thomas	track	USA	2	London

Question??

You compute the cartesian product of two sets, with cardinalities 3 and 4 respectively.

The cardinality of the result is:

A. 7
B. 12
C. Not possible to determine with the information given.

Cross-Product (Cartesian Product)

- **Result Schema:**
  - All fields from both relations, names inherited if possible
  - If both input relations have a field with the same name, use the rename operator ρ

Athlete

aid	name	sport	place
1	Simone Biles	gymnastics	USA
2	Gabby Thomas	track	USA

$\times$

Venue

vid	place
1	Paris
2	London

$=$

aid	name	sport	country	vid	place
1	Simone Biles	gymnastics	USA	1	Paris
2	Gabby Thomas	track	USA	1	Paris
1	Simone Biles	gymnastics	USA	2	London
2	Gabby Thomas	track	USA	2	London

Rename Operator

$$\rho_{AFTER}(BEFORE) \quad or \quad \rho(AFTER,\ BEFORE)$$

- Use cases:
  - Shorthand
  - Self-joins
  - If two relations have the same field

aid	name	sport	place
1	Simone Biles	gymnastics	USA
2	Gabby Thomas	track	USA

aid	name	sport	country
1	Simone Biles	gymnastics	USA
2	Gabby Thomas	track	USA

Derived Operators: Joins

- Most common way of combining information from two tables
- **Conditional join or Θ-join**
  - $R \bowtie_c S = \sigma_c(R \times S)$, where *c* is a condition
- **Equijoin**
  - Join condition consists only of equalities
- **Natural Join**
  - Like equijoin but drops the duplicate columns
  - $R \bowtie S$
- Despite equivalence, joins faster than cross-product!

Example

Examples: Writing Queries in RA

```
Sailors (sid, sname, rating, age)
Reserves (sid, bid, day)
Boats (bid, bname, color)
```

Sailors

sid	sname	rating	age
22	Dustin	7	45.0
29	Brutus	1	33.0
31	Lubber	8	55.5
32	Andy	8	25.5
58	Rusty	10	35.0
64	Horatio	7	35.0
71	Zorba	10	16.0
74	Horatio	9	35.0
85	Art	3	25.5
95	Bob	3	63.5

Reserves

sid	bid	day
22	101	10/10/98
22	102	10/10/98
22	103	10/8/98
22	104	10/7/98
31	102	11/10/98
31	103	11/6/98
31	104	11/12/98
64	101	9/5/98
64	102	9/8/98
74	103	9/8/98

Boats

bid	bname	color
101	Interlake	blue
102	Interlake	red
103	Clipper	green
104	Marine	red

Find names of sailors who have reserved boat #103

Example

Find names of sailors who have reserved boat #103

Solution 1: $\pi_{sname}((\sigma_{bid=103} \text{Reserves}) \bowtie \text{Sailors})$

Solution 2:

$$\rho\ (Temp1,\ \sigma_{bid=103}\ \text{Reserves})$$

$$\rho\ (Temp2,\ Temp1 \bowtie \text{Sailors})$$

$$\pi_{sname}\ (Temp2)$$

Solution 3: $\pi_{sname}(\sigma_{bid=103}(\text{Reserves} \bowtie \text{Sailors}))$

Example

Find names of sailors who have reserved a red boat

$$\pi_{sname}((\sigma_{color='red'} Boats) \bowtie Reserves \bowtie Sailors)$$

Sailors

sid	sname	rating	age
22	Dustin	7	45.0
29	Brutus	1	33.0
31	Lubber	8	55.5
32	Andy	8	25.5
58	Rusty	10	35.0
64	Horatio	7	35.0
71	Zorba	10	16.0
74	Horatio	9	35.0
85	Art	3	25.5
95	Bob	3	63.5

Reserves

sid	bid	day
22	101	10/10/98
22	102	10/10/98
22	103	10/8/98
22	104	10/7/98
31	102	11/10/98
31	103	11/6/98
31	104	11/12/98
64	101	9/5/98
64	102	9/8/98
74	103	9/8/98

Boats

bid	bname	color
101	Interlake	blue
102	Interlake	red
103	Clipper	green
104	Marine	red

Example

Find names of sailors who have reserved a red boat

$$\pi_{sname}((\sigma_{color='red'} Boats) \bowtie Reserves \bowtie Sailors)$$

An equivalent solution:

π_{sname} (π_{sid} ((π_{bid} $\sigma_{color='red'}$Boats) ▷◁ Res) ▷◁ Sailors)

The query optimizer chooses from the (equivalent) expressions and chooses one for efficiency of evaluation.

Sailors

sid	sname	rating	age
22	Dustin	7	45.0
29	Brutus	1	33.0
31	Lubber	8	55.5
32	Andy	8	25.5
58	Rusty	10	35.0
64	Horatio	7	35.0
71	Zorba	10	16.0
74	Horatio	9	35.0
85	Art	3	25.5
95	Bob	3	63.5

Reserves

sid	bid	day
22	101	10/10/98
22	102	10/10/98
22	103	10/8/98
22	104	10/7/98
31	102	11/10/98
31	103	11/6/98
31	104	11/12/98
64	101	9/5/98
64	102	9/8/98
74	103	9/8/98

Boats

bid	bname	color
101	Interlake	blue
102	Interlake	red
103	Clipper	green
104	Marine	red

Example

$$\pi_{sname}(Reserves \bowtie Sailors)$$

Sailors

sid	sname	rating	age
22	Dustin	7	45.0
29	Brutus	1	33.0
31	Lubber	8	55.5
32	Andy	8	25.5
58	Rusty	10	35.0
64	Horatio	7	35.0
71	Zorba	10	16.0
74	Horatio	9	35.0
85	Art	3	25.5
95	Bob	3	63.5

Reserves

sid	bid	day
22	101	10/10/98
22	102	10/10/98
22	103	10/8/98
22	104	10/7/98
31	102	11/10/98
31	103	11/6/98
31	104	11/12/98
64	101	9/5/98
64	102	9/8/98
74	103	9/8/98

Boats

bid	bname	color
101	Interlake	blue
102	Interlake	red
103	Clipper	green
104	Marine	red

Example

Find names of sailors who have reserved at least one boat

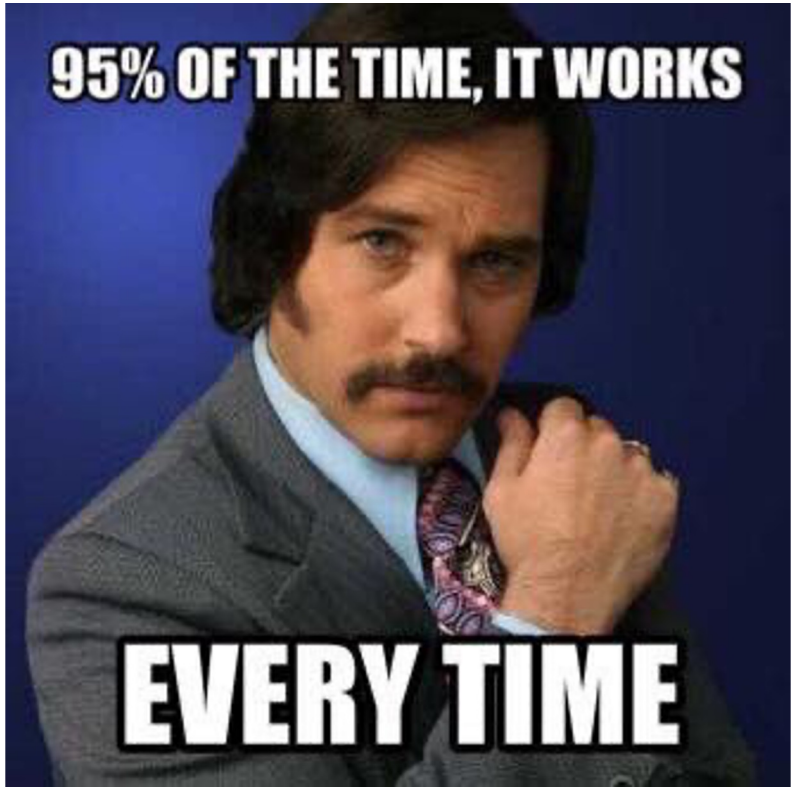
$$\pi_{sname}(Reserves \bowtie Sailors)$$

Sailor appears in this intermediate relation only if there is at least one Reserves tuple with same sid.

Derived Operators: Division

- Useful for queries like:

  "Find the sailors who have reserved **all** boats."

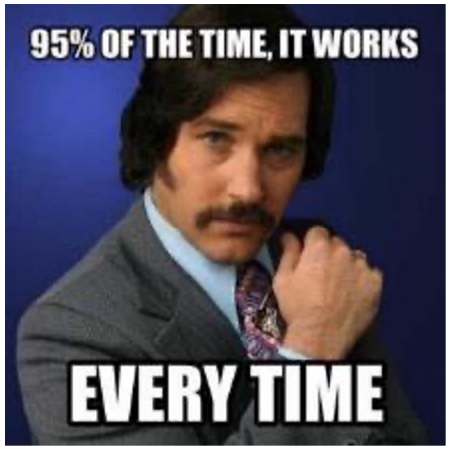


Derived Operators: Division

- Useful for queries like:

  "Find the sailors who have reserved **all** boats."

- Let *A* have 2 fields, *x* and *y* ; *B* have only field *y* :
  - A/B = {<x> | ∀<y>∈ B, <x, y>∈A}
  - all x tuples (sailors) s.t. for **every** y tuple (boat) in B, there is an <x,y> tuple in A



Example

Examples of Division A/B

$A/B = \{\langle x \rangle \mid \forall \langle y \rangle \in B, \langle x, y \rangle \in A\}$

cid	bname
c1	b1
c1	b2
c1	b3
c1	b4
c2	b1
c2	b2
c3	b2
c4	b2
c4	b4

A

bname
b2

B1

bname
b2
b4

B2

bname
b1
b2
b4

B3

cid
c1
c2
c3
c4

A/B

1

cid
c1
c4

A/B

2

cid
c1

A/B

3

Expressing A/B Using Basic Operators

- Can be equivalently expressed using basic operators
- **Idea:** For A/B, compute all x values that are not disqualified by some y value in B
  - x value is disqualified if by attaching y value from B, we obtain an <x,y> tuple that is not in A

Can you express this operator using basic operators?

A/B: $\pi_x(A) - \pi_x((\pi_x(A) \times B) - A)$

Disqualified *x* values

Example

Examples of Division A/B

$$\pi_x(A) - \pi_x((\pi_x(A) \times B) - A)$$

cid	bname
c1	b1
c1	b2
c1	b3
c1	b4
c2	b1
c2	b2
c3	b2
c4	b2
c4	b4

A

bname
b2
b4

B

cid	bname
~~c1~~	~~b2~~
~~c1~~	~~b4~~
~~c2~~	~~b2~~
c2	b4
~~c3~~	~~b2~~
c3	b4
~~c4~~	~~b2~~
~~c4~~	~~b4~~

-A

cid
c1
c4

A/B

$$\pi_x(A) \times B$$

Example

Find names of sailors who have reserved a red OR green boat

- Identify all red or green boats, then
- find sailors who have reserved one of these boats

Sailors

sid	sname	rating	age
22	Dustin	7	45.0
29	Brutus	1	33.0
31	Lubber	8	55.5
32	Andy	8	25.5
58	Rusty	10	35.0
64	Horatio	7	35.0
71	Zorba	10	16.0
74	Horatio	9	35.0
85	Art	3	25.5
95	Bob	3	63.5

Reserves

sid	bid	day
22	101	10/10/98
22	102	10/10/98
22	103	10/8/98
22	104	10/7/98
31	102	11/10/98
31	103	11/6/98
31	104	11/12/98
64	101	9/5/98
64	102	9/8/98
74	103	9/8/98

Boats

bid	bname	color
101	Interlake	blue
102	Interlake	red
103	Clipper	green
104	Marine	red

Example

Find names of sailors who have reserved a red OR green boat

- Identify all red or green boats, then
- find sailors who have reserved one of these boats:

$$\rho\ (Tempboats, (\sigma_{color='red' \vee color='green'} Boats))$$

$$\pi_{sname}(Tempboats \bowtie Reserves \bowtie Sailors)$$

Equivalent:

$$\rho(Tempboats, (\sigma_{color='red'}(Boats) \cup \sigma_{color='green'}(Boats)))$$

$$\pi_{sname}(Tempboats \bowtie Reserves \bowtie Sailors)$$

Example

Find names of sailors who have reserved a red AND green boat

?

$$\rho\ (Tempboats, (\sigma_{color='red' \wedge color='green'} Boats))$$

$$\pi_{sname}(Tempboats \bowtie Reserves \bowtie Sailors)$$

Example

Find names of sailors who have reserved a red AND green boat

- Identify
  - sailors who have reserved red boats
  - sailors who have reserved green boats
- Then find the intersection (sid is a key for Sailors):

Sailors

sid	sname	rating	age
22	Dustin	7	45.0
29	Brutus	1	33.0
31	Lubber	8	55.5
32	Andy	8	25.5
58	Rusty	10	35.0
64	Horatio	7	35.0
71	Zorba	10	16.0
74	Horatio	9	35.0
85	Art	3	25.5
95	Bob	3	63.5

Reserves

sid	bid	day
22	101	10/10/98
22	102	10/10/98
22	103	10/8/98
22	104	10/7/98
31	102	11/10/98
31	103	11/6/98
31	104	11/12/98
64	101	9/5/98
64	102	9/8/98
74	103	9/8/98

Boats

bid	bname	color
101	Interlake	blue
102	Interlake	red
103	Clipper	green
104	Marine	red

Example

Find names of sailors who have reserved a red AND green boat

- Identify
  - sailors who have reserved red boats
  - sailors who have reserved green boats
- Then find the intersection (sid is a key for Sailors):

$$\rho\,(Tempred,\ \pi_{sid}((\sigma_{color='red'}Boats)\bowtie Reserves))$$

$$\rho\,(Tempgreen,\ \pi_{sid}((\sigma_{color='green'}Boats)\bowtie Reserves))$$

$$\pi_{sname}((Tempred \cap Tempgreen)\bowtie Sailors)$$

Example

Find the SIDs of sailors over age 20 who have not reserved a red boat

$$\pi_{sid}(\sigma_{age>20}Sailors) -$$

$$\pi_{sid}((\sigma_{color='red'}Boats) \bowtie Reserves)$$

Sailors

sid	sname	rating	age
22	Dustin	7	45.0
29	Brutus	1	33.0
31	Lubber	8	55.5
32	Andy	8	25.5
58	Rusty	10	35.0
64	Horatio	7	35.0
71	Zorba	10	16.0
74	Horatio	9	35.0
85	Art	3	25.5
95	Bob	3	63.5

Reserves

sid	bid	day
22	101	10/10/98
22	102	10/10/98
22	103	10/8/98
22	104	10/7/98
31	102	11/10/98
31	103	11/6/98
31	104	11/12/98
64	101	9/5/98
64	102	9/8/98
74	103	9/8/98

Boats

bid	bname	color
101	Interlake	blue
102	Interlake	red
103	Clipper	green
104	Marine	red

Example

Find the names of sailors who have reserved **all** boats

- Uses division; schemas of the input relations must be carefully chosen:

$$\rho\ (Tempsids, (\pi_{sid,bid} \text{Reserves}) / (\pi_{bid} Boats))$$

$$\pi_{sname}(Tempsids \bowtie Sailors)$$

Sailors

sid	sname	rating	age
22	Dustin	7	45.0
29	Brutus	1	33.0
31	Lubber	8	55.5
32	Andy	8	25.5
58	Rusty	10	35.0
64	Horatio	7	35.0
71	Zorba	10	16.0
74	Horatio	9	35.0
85	Art	3	25.5
95	Bob	3	63.5

Reserves

sid	bid	day
22	101	10/10/98
22	102	10/10/98
22	103	10/8/98
22	104	10/7/98
31	102	11/10/98
31	103	11/6/98
31	104	11/12/98
64	101	9/5/98
64	102	9/8/98
74	103	9/8/98

Boats

bid	bname	color
101	Interlake	blue
102	Interlake	red
103	Clipper	green
104	Marine	red

Review

- Selection
- Projection
- Cross-product
- Set-difference
- Union
- Join