Introduction to Quantum Mechanics

Quantum Mechanics Foundation Course:
Problem Sheet 1

1 De Broglie Wavelength

(i) An electron microscope uses an operating voltage of 200 kV (kilo-volts) to accelerate electrons. Calculate the (non-relativistic) De Broglie wavelength of the accelerated electrons.

(ii) A fly having mass $m = 12$ mg flies through a 1m wide door at a speed $v = 5$ km/h. Why don't we observe any quantum interference as the fly goes through the door?

2 The Schrödinger equation

(i) Write down the time-dependent Schrödinger equation and explain what each of the terms means.

(ii) Verify that the plane wave $\psi(x,t) = e^{i(kx-\omega t)}$ is indeed a solution of the time-dependent Schrödinger equation, and check that the dispersion relation $E = p^2/2m$ is correctly reproduced.

3 Wavefunctions

A particular solution to the Schrödinger equation for a particle in a 1-D infinite potential well ("box") of size $a = 10$ is the wavefunction,

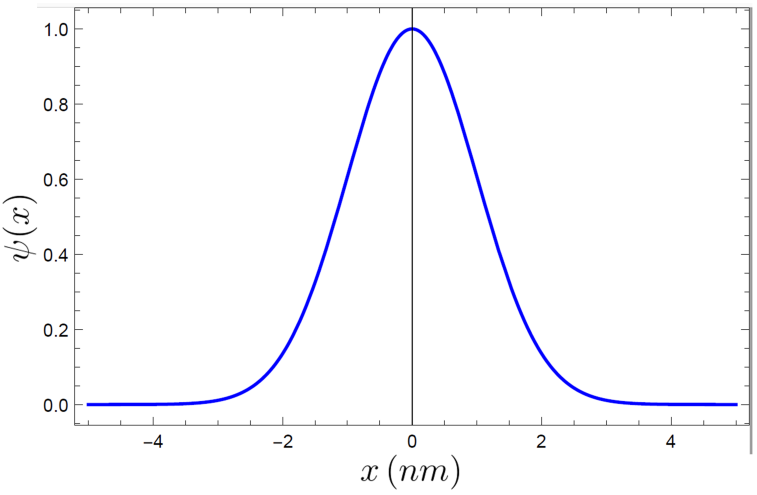
$$\psi(x) = \begin{cases} e^{i\omega t}\sqrt{\frac{2}{10}}\sin\frac{3\pi x}{10} & \text{for,} \quad 0 \le x \le 10, \\ 0 \quad \text{, otherwise.} \end{cases}$$

(i) Show that this wavefunction is correctly normalized (i.e. show that the probability of finding the particle described by this wavefunction anywhere on the real axis is 1).

(ii) What is the probability of finding the particle between $x = 1$ and $x = 3$?

4 Uncertainty Principle

Estimate the momentum spread of the following wavefunction:



5 Extra: Size of the hydrogen atom

(Adapted from the Feynman lectures on Physics, vol III, p. 2-6)

The uncertainty principle can be used to estimate the radius of the hydrogen atom. Using the fact that the potential energy of an electron in the field of a proton is,

$$U \approx -\frac{e^2}{a}$$

where a is the average distance between the electron ad the proton, minimise the total energy (potential and kinetic) to estimate a. (*hint:* you may use the uncertainty principle to estimate the typical momentum of the electron).

6 Extra: The Photoelectric Effect

A titanium metal surface has the work function $W = 4.3\,eV$ and is illuminated by light with frequency $\nu = 1.6 \times 10^{15}$ Hz. Are photoelectrons emitted? If so, calculate the maximum speed of the emitted elections.