Premise 1

Experimentally, we are targeting the model learning of the NV-centre Hamiltonian

$$\mathcal{H}_n = \mathcal{H}_{\mathrm{hf}} + \mathcal{H}_{\mathrm{nZ}} + \mathcal{H}_{\mathrm{Q}}.$$

Where:

$$\mathcal{H}_{\mathrm{hf}} = \hat{S} \cdot \bar{A} \cdot \hat{I} \qquad \text{with} \quad \bar{A} = \begin{pmatrix} A_\perp & 0 & 0 \\ 0 & A_\perp & 0 \\ 0 & 0 & A_\| \end{pmatrix}$$

$$\mathcal{H}_{\mathrm{nZ}} = \hbar \tilde{\gamma}_n \mathbf{B} \cdot \hat{I},$$

And the quadrupole splitting HQ contribution has been neglected.

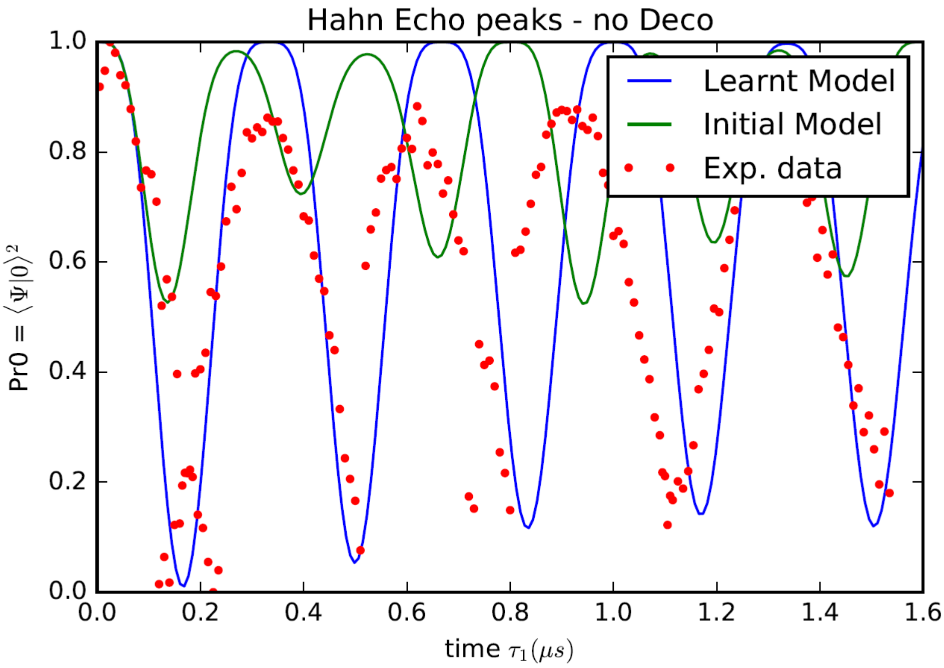
Experimental data for this model learning task are “Hahn peaks” (see previous email), where we vary tau compared to tau’, thus decreasing the decoupling effect from the environment, obtained via the standard Hahn echo sequence.

Phenomenologically this leads to an exponential decrease $e^{-(\frac{t}{\tau_c})^4}$

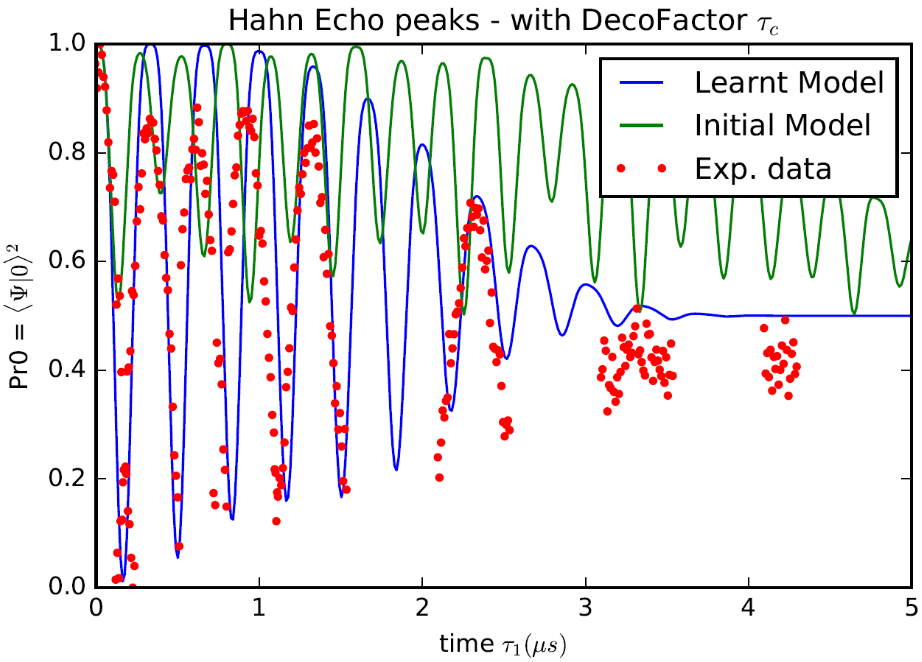
In the photoluminescence from the sample (and therefore the overlap of the final state with $|0\rangle$).

Experimental data and preliminary (QML + CLE) learning

Learning the 6 parameters model (QML) from the data collected does a decent job in reproducing the dynamics (data are truncated to avoid the decoherence to affect too much the experimental data used for training here):



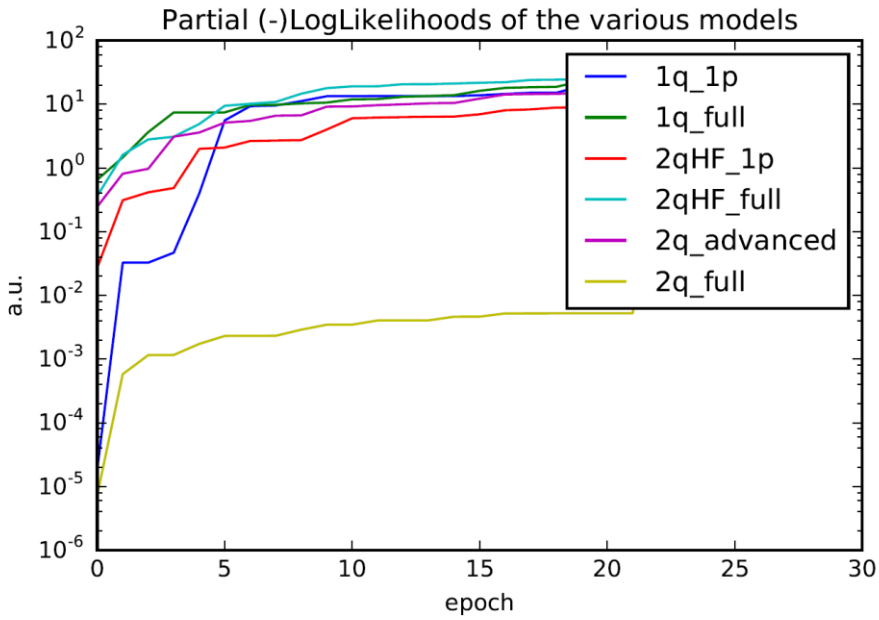
Also, trying to learn tau_c after the parameters for the model have "stabilised" appears to succeed:



On a more “model-learning side”, the Bayes factor after a single run appear to favour extremely clearly the full H_n Hamiltonian.

For a selection of partial 1qubit H_Rabi, or H_f + H_nZ 2qubit terms against the full H_n (“2q_full” below) we obtained after a single run for a selection of the entertained models:

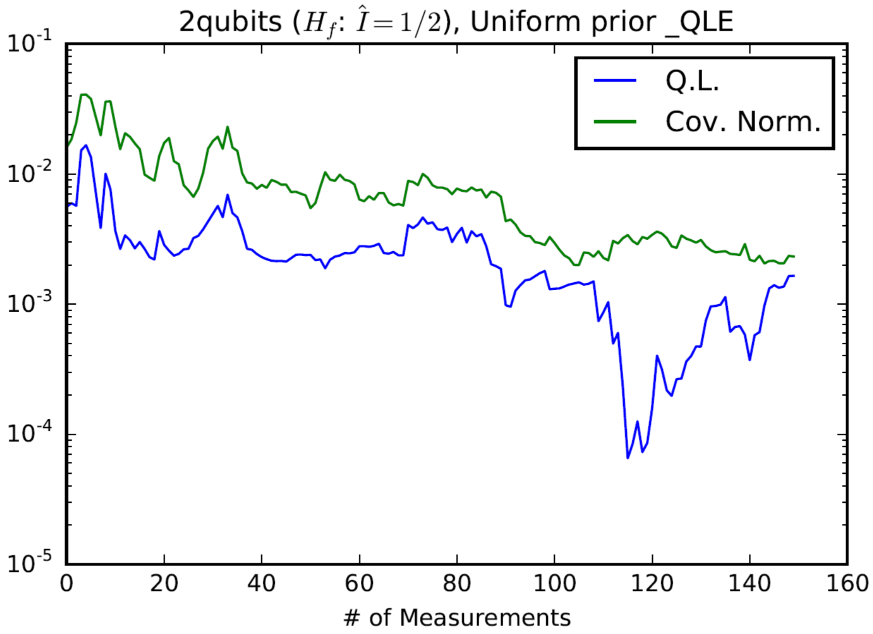
```
'2q_full VS 1q_1p': 3110229842481.2217,
 '2q_full VS 1q_full': 1.279945694188824e+24,
 '2q_full VS 2qHF_1p': 79376992444673.891,
 '2q_full VS 2qHF_full': 1.9106781702023922e+36,
 '2q_full VS 2q_advanced': 1.5406929129358649e+22
```



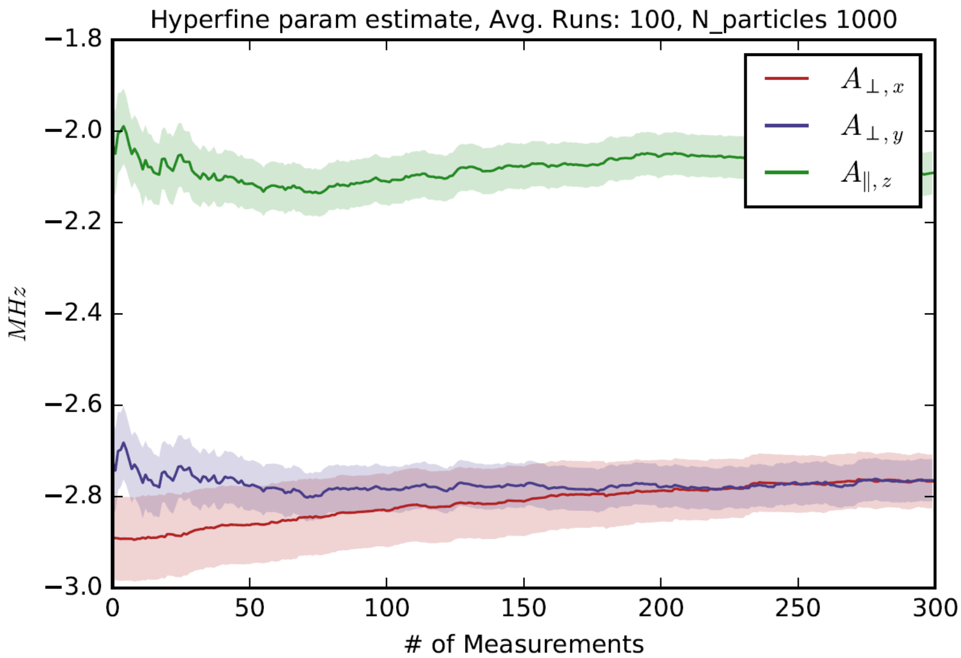
Open Problems

The estimate for the parameters after the QML is not particularly accurate.

From preliminary **simulations** with the same Hamiltonian we already expect QLE to be much less reliable and quick in the learning compared to IQLE, e.g. for single runs again:

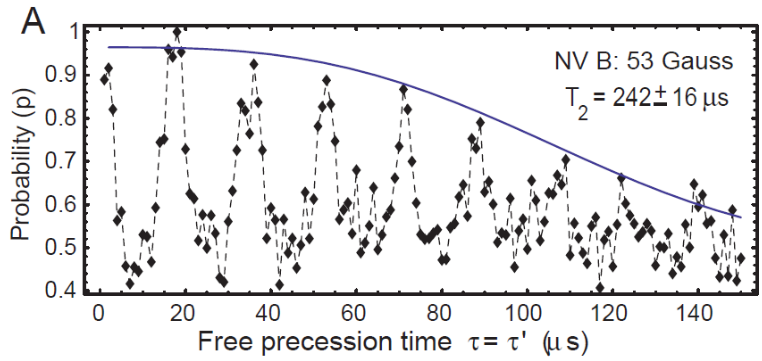


Very interestingly, we expect theoretically two of the hyperfine tensor parameters to be equal (see above), however the QML tends to learn – for a reasonable number of measurements – values quite different. This appears to be fixed by longer CLE runs on top (see below), but they may be too long to run experimentally on the photonic chip as well.

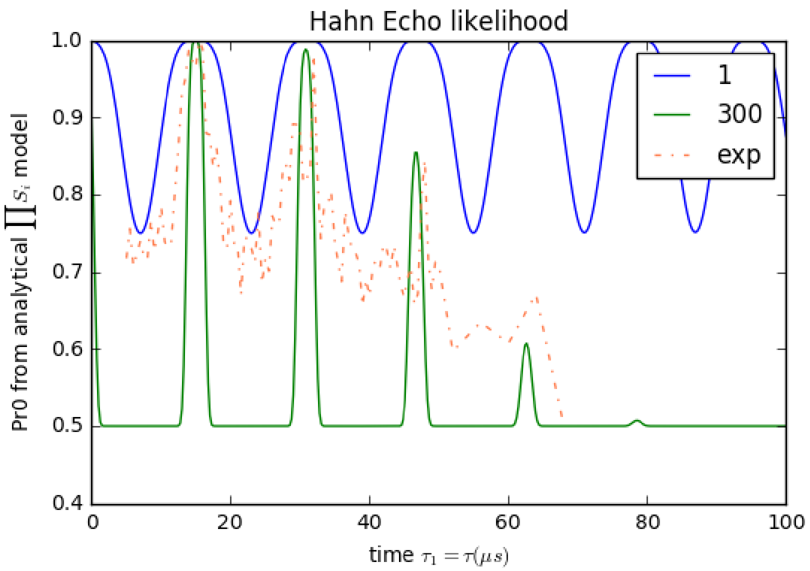


Premise 2

The second idea was to learn the same model for “Hahn-echo signal” experiments (i.e. where tau=tau’).



However 1 single environmental qubit seems really not enough alone for learning a reasonable model.



We considered the analytical model for these experiments:

$$S_j(\tau) = 1 - \frac{2|\mathbf{B}_0^{(j)} \times \mathbf{B}_1^{(j)}|^2}{|B_0^{(j)}|^2 |B_1^{(j)}|^2} \sin^2(\omega_{j,0}\tau/2) \sin^2(\omega_{j,1}\tau/2).$$

That gives the final “Pr0”:

$$p = (S+1)/2 \text{ with } S = \prod_j S_j.$$

Tweaking it, in order to have less parameters to learn.

This would only work as a CLE task though... (discussion to follow).