A Case for DLC and Content Based Price Discrimination: a Statistical Analysis of the Gaming Industry (2004 - 2008)

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Abstract

This report examines the gaming industry during the period 2004 to 2008, before microtransactions and addons became prevalent, focusing on finding the predictive factors for geographic game sales in North America, Japan and the European Union. To do this, unsupervised and supervised machine learning techniques were employed, including principle component analysis, classification and regression trees, random forests, gradient boosted trees, and lasso regression paths. The report identifies four distinct market segments with unique sales patterns, user engagement and market reach: Family Friendly Blockbusters, Industry Average & Low Performers, AAA Series Staples, and Niche Enthusiasts. The findings from the predictive models suggest that games offering both quick completion and an extended leisure play meet these diverse consumer demands. These observations, along with the regional preferences shown in the lasso regression models provide insight into consumer behavior that supports the shift to a more flexible pricing model in the games industry that maximizes both player enjoyment and revenue generation.

Introduction

In 1971, the first commercial video game "Computer Space" was released in what would now be called an arcade cabinet. Today, less than 55 years later, the gaming industry has grown to become the highest grossing entertainment industry in the world [1]. It now surpasses the gross revenues of the movie and music industries combined as well as that of most printed media [2]. Today, anyone can participate in the gaming industry, also known as the interactive entertainment industry, by being what is called an indie or an independent developer. Presently, there are many different ways of pricing a game, whether that's free to play, play to earn, subscription-based, or live service. Additionally, many games provide downloadable content (or DLC) as well as in-game microtransactions where a gamer can pay more money to have a longer or more rich game experience with additional content. Those who paid for the base game, but did not pay these additional fees, now get a slightly shorter and/or different play experience. These aforementioned microtransactions initially gained traction in April of 2006. The release of The Elder Scrolls 4: Oblivion's first set of expansion content provided for a new source of revenue for the game developers [3]. What this report seeks to find in the two year leading up to this release and the two years after, is evidence for why this change in pricing structure is observed today. by analyzing the time in the gaming industry when additional game content was not behind a pay wall as it is today and this transition period when that first began to change.

This report will be analyzing the sales of games that follow the prevalent model throughout the industry's history, called the premium model. In the premium model, players are charged a flat positive price for the game, and after paying that price, they have access to the full game for the product's lifetime. This is the same pricing most physical products bought at a store would have. This analysis seeks to find the strongest predictive model for how well a premium-model console game would sell in North America and other regions

of the world based on data available from 2004 to 2008. The game consoles released at that time and the basis for this report's analysis are the Nintendo Wii, Nintendo DS, Microsoft's Xbox 360, and Sony's PS3 and PSP. In finding these predictive models, the report focuses on whether the length of the game, either rushed, leisurely played, or the time to beat on average, according to https://howlongtobeat.com/ is predictive of game sales. The question at the center of this report is whether or not there is predictive evidence to suggest that making a game shorter and allowing consumers to pay for a longer experience could benefit the gaming industry. In other words, was there a case for interactive entertainment companies to restrict game content and charge additional prices for those who want a longer game. This question is analyzed using unsupervised and supervised machine learning including principle component analysis (PCA), variable importance plots, partial dependence plots, tree models, and regression paths.

Methods

The data set that will be analyzed is a merge of three data sets. The first dataset can be found on https://www.kaggle.com/datasets/sidtwr/videogames-sales-dataset/data [4]. It lists 16,720 game releases for 11,563 different games that were released on Consoles or on PCs. Each release/observation sold at least 10,000 copies between 1980 and December 22nd 2016 which is much of the industry's history. Due to the nature of the data set, the games listed do not include mobile games, and most if not all of the games listed follow the premium pricing model mentioned in the introduction. The data set has 7 categorical variables and 10 continuous variables. The columns used in this report are the platform, genre, publisher, sales, and review data.

The 2nd data set this report is using comes from Dr. Joe Cox at the University of Portsmouth [5]. The data set was compiled for a paper he wrote on what makes a blockbuster video game. The variables of interest used from this data set are the dummy variables listed below as well as the used sales prices of these titles.

The 3rd data set used in this analysis is from Dr. Austin Cory Bart at the University of Delaware when he was a PhD student at Virginia Tech [6]. His data set was compiled from using Joe Cox's data in addition to a scrape of HowLongToBeat.com. The variables of interest from this set were the length variables and the max players.

These three data sets were merged by the games name and by platform so that every release between 2004 and 2008 had the following variables:

Categorical Variables:

Name: The name of the game

Platform: what platform was the game released on (PSP, PS3, Xbox360, Wii, DS)

Max_Players: The number of people who can play the game on the same system at once

Genre: the genre of the game (Strategy, Action, etc.)

Publisher: the publishing studio of the game (Activision, Nintendo, etc.)

Age Rating: (E for Everyone, M for mature audiences, etc.) provided by the ESRB

Year: The year the game was released (2004 to 2008)

"Continious" Variables

Review Score: average critics score on https://www.metacritic.com/ (1 - 100)

Used Price: the average resale price of the game

NA Sales: sales in North America in millions

EU Sales: sales in the European Union in millions

JP_Sales: sales in Japan in millions

Other Sales: sales in the rest of the world in millions

Global scales: total sales in millions

User_Score: average user score on https://www.metacritic.com/ (1 - 10)

User_Count: number of users in the average score

"Continious" Variables from https://howlongtobeat.com/

which defines playstyles as Length_Median: median time to beat the game in hours for all playstyles

Length_Average: average time to beat the game in hours for all playstyles

Length_Leisure: time to beat the game in hours if played leisurely (all playstyles)

Length_Rushed: time to beat the game in hours if rushed through (all playstyles)

Length_Polled: number of people polled for all playstyles on HowLongToBeat.com

Dummy Variables

Sequel: 1 if the game is a sequel to another game, 0 otherwise

Online: 1 if the game connects to the internet, 0 otherwise

Licensed: 1 if it a licensed IP (e.g. NBA, etc), 0 otherwise

Handheld: 1 if the game is released on one of the 2 handheld concolses (DS, PSP)

Multiplatform: 1 if the game is on multiple consoles, 0 otherwise

Third_Person: 1 if the camera is not in the eyes of the playable character, 0 otherwise

The statistical analysis begins with a unsupervised machine learning approach using principal component analysis to see if the data can segment the gaming market into discernible groups naturally. The PCAs will be based on the length, sales and review variables. An interesting outcome of this is would be to see if any clusters emerge that can be identified and named based on popular discussions of the gaming industry. As a major part of this report is to see if the length of a game can help predict sales, attention will be put to the cluster sorting of the long games.

After the clusters are formed and classified, the report moves on to predictive modeling of game sales in the respective geographic regions: North America, Japan, and the European Union. The models used are classification and regression trees (CART), random forests, and gradient boosted trees. Each will be tested using the same 80/20 train-test-split and the out-of-sample root mean squared error (RMSE) will be calculated and compared to help discern what model is the better predictor for sales in the respective region. What's most interesting for this analysis is what variables are most predictive of sales, in particular how length predicts sales. For this, the output presented in the results section include the North American large CART model, the variance importance plots of the random forest models of all three regions, and partial dependence plots of leisure, rushed, and average length time-to-beat on game sales from the gradient boosted trees.

Lastly, the third section of the report uses a lasso regression to again estimate the most predictive variables for sales in the three geographic regions. By reducing the penalty to the model, the results find the first 10 variables that switch to having non-zero coefficients.

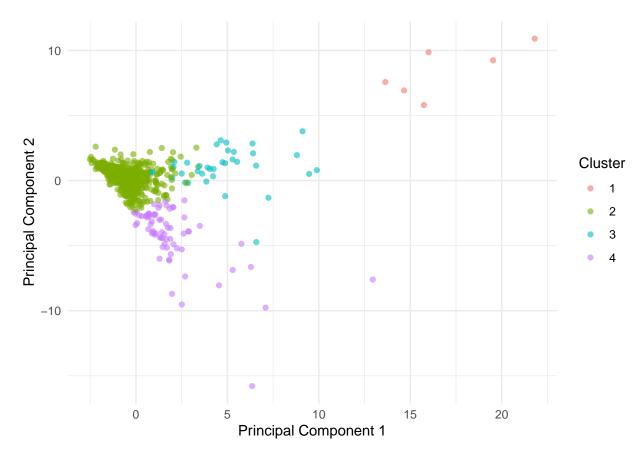
Results

Part 1: Principal Component Analysis

In clustering, the elbow method results were used to help determine the best numer of cluster (k), as shown in appendix A. The elbow seems to be not very pronounced, but a subtle change can be observed in the slope after k=4. For this reason, the modeling will make for 4 clusters in the PCA.

After the data is de-meaned and scaled, the cluster labels are attached to the original data. The following values represent the variable representations of the centroids of each of the four clusters in terms of standard deviations. These clusters are then plotted and graphically shown below using the first two principle components along with the size of each cluster above the plot.

```
##
     Review_Score Used_Price Length_Average Length_Leisure Length_Median
## 1
        0.7821945
                  1.5813627
                                -0.154807941
                                                 -0.1257322
                                                              -0.24921609
## 2
       -0.1328851 -0.1398689
                                -0.250403605
                                                 -0.2458172
                                                              -0.25234586
##
  3
        1.3513584
                   1.2781319
                                0.006707539
                                                  0.6770543
                                                                0.04627033
                                2.372644951
                                                  1.9870138
##
        0.5026896 0.5381867
                                                                2.37889146
     Length Polled Length Rushed
                                                            JP Sales Other Sales
##
                                    NA Sales
                                                 EU Sales
## 1
        0.01410217
                      -0.3179870
                                  8.47110056
                                               9.47989605
                                                           8.1239710
                                                                        8.5565310
       -0.16827244
## 2
                      -0.2458480 -0.16840785 -0.15432582 -0.1428825
                                                                       -0.1677067
## 3
        2.92462082
                                  1.83825752
                                              1.44535576
                                                           0.4665720
                      -0.1042757
                                                                        1.8174774
## 4
        0.10382492
                       2.3997186 -0.05874102 -0.07766034
                                                           0.4252230
                                                                       -0.0620407
     Global_Sales
                                User_Count Third_Person
##
                   User_Score
                   0.57010194
## 1
      9.317755565
                               0.462445904 0.185096349
## 2 -0.170139578 -0.08303287 -0.181345019 -0.006413727
      1.596428153
                   0.65056580
                               3.279482588 -0.136971298
     0.008670002
                   0.40550768 0.009367803 0.114332190
```



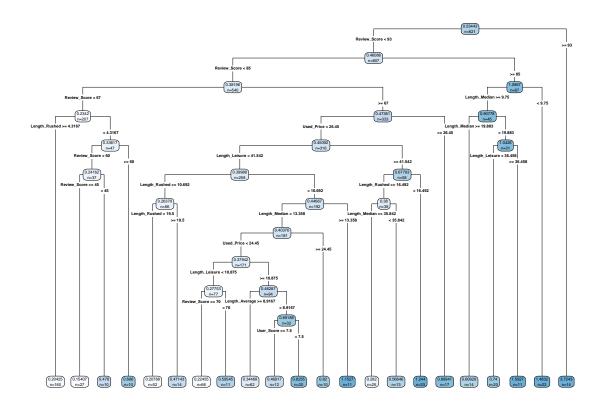
The values associated with each of the centroids of each of the clusters, along with other evaluations of other points in the clusters, and the analysis of what makes up principle components 1 and 2 are used to segment the market into four market clusters described in the conclusion. The 6 games in cluster 1 were found to be outliers in terms of sales for geographic areas and will be excluded from the data moving forward for the predictive modeling and lasso regressions. After removing these six titles, User_Count becomes the most predictive of sales in most countries, which may be particularly obvious, as the more people have the game, the more people there are to review it, so User_Count and Length_Polled will be removed from the data set for the remaining parts as well.

Part 2: Predictive Tree Modeling for Sales

To examine predictive evidence of game length on sales, the model predicts geographic game sales in absolute terms (not log terms). We split the data into a training set that accounts for 80% of the releases and a testing set that is the remaining 20% of the observations. The models calculate their predictions on the testing set for the CART, random forest, and gradient-boosted trees and compare the out-of-sample RMSE and find that the gradient boosted tree has the smallest out of sample RMSE in predicting game sales in Europe and Japan and the random forest is the best predictor for North America.

1: CART

We begin with the CART model that predicts sales in North America. For brevity in the results section, the CART models for Japan and Europe are provided in Appendix B. The full tree below includes all numerical variables in the data set. We have the tree set so the smallest split has at least 10 observations.



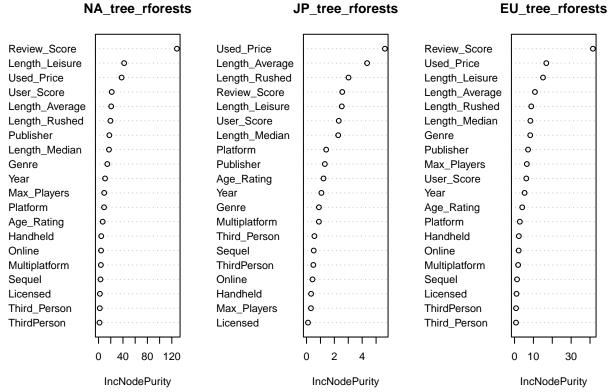
When the tree is pruned at the 1 standard error complexity level, the tree becomes too simple to be of any use of explanation. For Japan, the tree is just the average game sales of all games, suggesting that Japan has a low variability in game sales in the first place. In Europe the tree is only the first split where the review score is above or below 93. For North America, the smaller tree is its first 2 splits, if the review score if above or below 93, then if below 93, if it's above or below 85. This is the application of the 1SE rule to pick a tree that is simple but whose performance is not discernibly different from the best performing model. As all pruned trees are simply described here, they are not pictured here.

Although the pruned tree is a much simpler tree than the larger ones, the tree_cart_big is the best cart model compared to pruned one both for learnin the predictive variables and preformance. Thus, the big trees will be used in our comparisons as the RMSE for the large tree cart pictured is less than that of the CART pruned by 1 standard error.

2: Random Forest

Next, we provide some information on our random forest prediction results. Random forests are effective, fast and require little or no tuning via CV, and we found that the default settings do well for predictions. The variable importance plot showing when we leave out a variable, how much does that increase mean squared error is represented below for each of the three geographic regions.

Variance Importance Plots

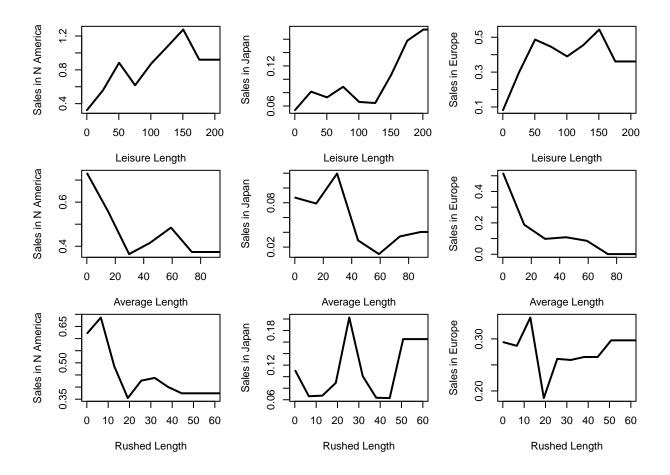


Here the largest increase in mean squared error would be achieved if the review score for North America and Europe and the used price for Japan were left out of the respective models. Additionally it can be seen that the predictive strength of the review score is potentially stronger relative to the other variables for North America and Europe than the used price is for Japan. The variable importance plot is made using the training data but the RMSE was calculated using the testing data is is shown below the next section about the gradient boosted tree.

3: Gradient-Boosted Tree

Next we find the predictions of a gradient boosted tree. The error curves that were used to find the amount of boosting steps are provided in Appendix C. We initially set the depth of each tree to 4, using 500 trees, the shrinkage factor at it's typical 0.05. The gradient boosted trees were used to compile the partial dependence plots.

Below are the partial dependence plots of the time it takes to beat a game on the gradient boosted machine across the different geographic regions. The first row provides the partial dependence on length to beat leisurely, the 2nd row is the partial dependence to beat the game on average, and the third row is the partial dependence plot of the rushed time to beat the game. The 1st column is these effects on sales in North America, the second column is the effect on sales in Japan, and the third column is the effect on sales in Europe.



4: Compare the Models

We compile the root mean squared errors of the three methods which are provided below.

```
## NA_RMSE JP_RMSE EU_RMSE
## cart_big 0.7486385 0.4480365 0.5203784
## random forest 0.6520718 0.3922706 0.4493635
## boosted 0.6437356 0.4298612 0.4502848
```

Part 3: Lasso Regression

For the last portion of the analysis, a lasso regressions were run to predict the geographic sales, without using other regional sales for predictions, or user count or length polled. The graphs of he paths are provided in appendix D. What is presented for the results are the 1st 10 non-zero coefficients that arise as the penalty on the lasso regression is reduced.

##		North_America	Japan	Europe
##	1st	Review_Score	PublisherNintendo	Review_Score
##	2nd	Used_Price	Length_Average	Used_Price
##	3rd	Length_Leisure	PublisherSquare Enix	Age_RatingM
##	4th	Handheld1	Used_Price	Length_Leisure
##	5th	Age_RatingM	Length_Leisure	Handheld1
##	6th	PublisherMicrosoft Game Studios	PublisherCapcom	PublisherNintendo

##	7th	GenreMisc	${\tt GenreMisc}$	GenreRole-Playing
##	8th	PublisherActivision	Handheld1	Max_Players8
##	9th	Max_Players4	GenreStrategy	${\tt GenreMisc}$
##	10th	PublisherLucasArts	Third_Person	Age_RatingT

Conclusion

Part 1: Market segmentation

Based on the PCA clustering shown in the results we can segment the gaming market into 4 segments. These groups are very natural groups that arise in popular discussions regarding the gaming industry. These clusters can help game developers understand how their game is positioned to maximally appeal to each market segment.

Cluster 1: Family Friendly Blockbusters (High Sales Achivers): Cluster 1 is high on PC1 and high on PC2. This cluster has extremely high values for NA_Sales, EU_Sales, JP_Sales, Other_Sales, and Global_Sales, significantly higher than any other cluster. This cluster also has above average reviewer and user scores as well as a higher used price. Only 6 games in the data set are in this category and all 6 are published by Nintendo including 2 Mario Kart games, Nintendogs, a mainline Mario game, Wii Fit, and Wii Play. As these games are all produced by Nintendo, they are all exclusive to Nintendo Consoles are are not multi-platform. As the data set has its observations by release, rather than game, this may give the entry higher values for sales as it's not divided across multiple consoles. Anyone who wants to play these games also needs to buy a Nintendo console providing a "tie-in" effect that results in an increased likelihood of purchasing other Nintendo games. For example, those who buy Mario Kart for the Wii would likely also play a standard (mainline) Mario game. All of the games in this segment are also rated E for everyone and can all involve multiple players on the same game at the same time.

Cluster 2: Industry Average & Low Performers:

Cluster 2 is very low on PC1, and moderate on PC2. This cluster makes up the vast majority of games released from 2004 to 2008 (670 releases). The games in this cluster have below average scores across all sales variables and also unperformed in both types of review scores and resale price. This cluster includes game series involving many intellectual properties (Lego, Harry Potter, Spongebob, Sonic, NFL, and many more) and likely represents how the industry has a long right tail distribution in popularity and is very skewed. Most games did not sell many millions of copies at this time like those in Cluster 1 and are not as well received as the large studios like Nintendo and those in Cluster 3.

Cluster 3: AAA Series Staples (Critically Acclaimed):

Cluster 3 is moderate on both PC1 and PC2. This cluster has well above average sales across all geographic regions as well as the highest user and critic scores. Although they have less sales than those in cluster 1, they have many more people reviewing the game and sharing how long the game took to beat; these games have high user engagement and positive reception. This cluster includes games from series like The Legend of Zelda, Assassins Creed, Call of Duty, Grand Theft Auto, and Bioshock. These series are what the gaming community refers to as AAA (Triple A titles) referring to the massive studios and publishers with large budgets for their games. Most of the games in this cluster are action and adventure games, often rated M for mature audiences.

Cluster 4: Long form games & RPGs (Niche Enthusiasts)

Cluster 4 is low to moderate for PC 1 and low for PC2. These games are by far and away longer games than those on the other clusters, from rushed play to leisurely enjoying. These games are more popular in Japan than the other regions comparatively and mostly include role playing games (or RPGs) or strategy games. Some examples of the types of game series in this cluster are The Sims, Animal Crossing, The Elder Scrolls, and Final Fantasy. These games offer longer engagement and usage times allowing the consumers of the product to have more use out of their game than other clusters.

Summary

The takeaway of this exercise in unsupervised learning is that the gaming industry is very right-skewed when it comes to market performance in terms of sales. No games in this list sold more than 20 million copies in a single release other than the 6 Nintendo games in cluster one, so moving forward, those 6 games will be disregarded from future predictive models to avoid massive errors in predicting sales. Additionally, although the algorithm had no knowledge of publisher or genre, the sorting into clusters by only only the sales, review, and length related variables essentially also grouped the games by genre and publisher. The massive publishers were sorted into Cluster 1, 3, and 4 and the smaller studios were sorted into Cluster 2. Party games were sorted into Cluster 1, action and adventure games were put into Cluster 3, and strategy games and RPGs were sorted into Cluster 4. It's enlightening to see that these natural market segments present themselves in the data naturally and seem to accurately reflect the public discussion about the gaming industry.

Part 2: Predictive Tree Models

The out-of-sample root mean square error of the boosted trees is a bit better at predicting sales than the other two models for Japan and Europe. The Random Forest is a little stronger for North America. In the gradient boosted tree, it's constructed as an ensemble of trees in such a way that the later the model gets in the tree, the more aware it is in the error of the earlier trees that it can do better. As the model predicts the count outcomes, making these trees assumes that the count outcomes are Gaussian may not always be accurate.

First, using only the numerical variables, the CART models are able to predict game sales for all three regions and there appears to be a strong predicting factor of the length variables in sales for the highest selling games. It seems longer games predict those higher sales. For every instance of length leisure used in the North American plot, the game with the higher length leisure variable predicts higher sales. This is not necessarily the case for the other length variables. In Japan's CART tree, all but one split involving length variables has the longer game sell better. This agrees with Cluster 4's discussion earlier. Interestingly, the reviewer score for the game appears to be a far better predictor of game sales than the user score. User scores are only used in the North America CART tree 10 splits down from the root and the review scores are at the very top. This may imply that either consumers tend to respond in the same direction as review scores or that reviews may accurately predict the sales of a game.

Next, the random forests provide helpful variable importance plots that capture how much the mean squared error changes when a variable is removed. Here, the mentioned importance of the review score in the CART trees are emphasized for Europe and North America as the review score points lie much further to the right than the other variables, meaning they are very predictive. This is good news for the game reviewers. What's good news for the purposes of this report is that game length variables show very highly on the variable importance plots. Again, it is noted that Japan seems to particularly like the longer games by the length average being further to the right. However, as the variance generally for sales in Japan and Europe are much less than that of North America, the scales are much smaller than that of the US.

Lastly, for gradient-boosted trees, the partial plots are attempting to show the relationship between the length to beat the game either leisurely, on average, or rushed and the sales the game receives in each of the

geographic regions. It's attempting to take into account the joint effect of all of the other features in the data set.

The key note here is that for all of the leisure length plots, the longer the game takes leisurely, the higher the effect on sales as an overall trend for all three regions. Fascinatingly, the reverse trend is true for the average time to beat the game. The longer the game takes to beat on average, it predicts the game to sell less or have a smaller effect on sales. This observation is likely the strongest evidence to suggest that intentionally making a game shorter appeals to the observations seen the the second row of the plots while at the same time providing on option to have a longer experience, through DLC and extra content appeals to the observations in the first row. Gaming companies are able to provide additional content in exchange for money in a way to capture both of these partial effects.

Part 3: Lasso Regression Predictors

The Lasso regression predictors indicate more unique characteristics about the respective geographic regions. For Instance, in Japan, the top 10 predictors include whether the game was published by Nintendo, Capcom, and Square Enix, all three are large Japanese gaming companies. Similarly, in North America, some of the best predictors of sales are whether it's published by Microsoft, Activision, or Lucas Arts, all three are large American gaming companies. Some other interesting predictors is that a game being rated M is a good predictor of sales in the US and Europe but not in Japan, and that strategy games and average length are good predictors for Japan sales and role playing games are good predictors for European sales. This agrees with the cluster 4 in the market segmentation work where the long strategy games seemed to preform well in Japan. Also leisure length is in the top five of each of the geographic regions and the only other length type is average length in Japan. This may also provide evidence that knowing a game has more content, which would take more time to play leisurely, may be more important in the decision process to purchase the game than generally how long it takes to beat normally. It's again also interesting to see that the review score plays such a big role in game sales (top 1 for both America and Europe but not in the top 10 for Japan). As critic reviews are released before a game is released to public, it seems Americans and Europeans seems to be quite receptive to positive reviews, in a non-causal but predictive sense.

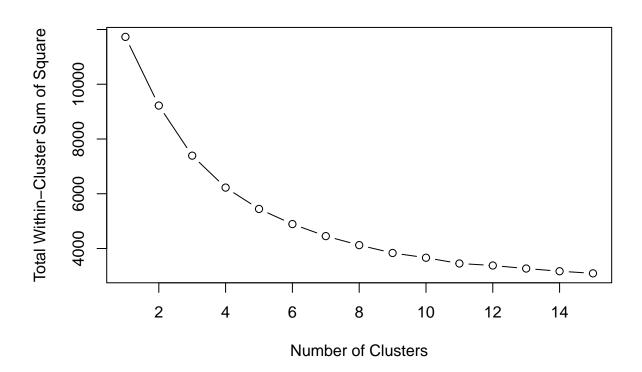
Final Takeaway

Put all together, it seems there is sufficient evidence to suggest that consumers are indeed looking for games with more content to play on their free time, and that they can sink many hours into playing. Additionally, there's another group, or maybe the same group, that wants the game to be beatable quick enough to be rewarding. If we potentially think of these consumers as the casual players that want a more quick experience, they'll buy the family friendly party games and the AAA classics in Cluster 1 and 3 and play for a white to beat it. The gaming enthusiasts want a longer experience. They are buying the longer form games in cluster 4 and are wanting more "bang for their buck" and may be willing to pay for additional content to increase the length of their favorite games. Therefore, it appears that the gaming industry did have a good reason to suppose that a more dynamic pricing model of game sales after this time period would work. It seems this would allow them to take advantage of this discrepancy shown in the partial plots that gamers want games that can be beat quickly, but also games that have enough to keep them playing for 150-200+. This is done today (post-2008 to present) by providing a smaller game at sticker price then providing additional add-ons and DLC at an additional fee, attempting to satisfy all market clusters. Even for those folks that are in the early peak in average time and rushed time as shown in the partial dependence plots, game companies have micro-transactions in games that allow for content that makes the game progress faster and allows a game to be beaten quicker for those who are willing and want to pay for a shorter experience.

References

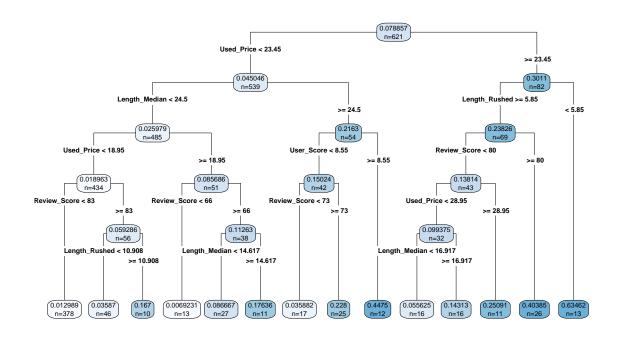
- 1. Reader TMP. Before Pong, There Was Computer Space. In: The MIT Press Reader [Internet]. 15 Oct 2021 [cited 25 Apr 2024]. Available: https://thereader.mitpress.mit.edu/before-pong-there-was-computer-space/
- 2. Helplama. Game Industry Usage and Revenue Statistics 2023. In: Helplama.com [Internet]. 14 Feb 2023 [cited 29 Apr 2024]. Available: https://helplama.com/game-industry-usage-revenue-statistics/
- 3. Williams M. The Harsh History Of Gaming Microtransactions: From Horse Armor to Loot Boxes. In: VG247 [Internet]. 11 Oct 2017 [cited 29 Apr 2024]. Available: https://www.vg247.com/the-harsh-history-of-gaming-microtransactions-from-horse-armor-to-loot-boxes
- 4. Video Games Sales Dataset. [cited 29 Apr 2024]. Available: https://www.kaggle.com/datasets/sidtwr/videogames-sales-dataset
- 5. Cox J. What makes a blockbuster video game?: an empirical analysis of US sales data. Managerial & Decision Economics. 2014;35: 189–198. doi:10.1002/mde.2608. Available: https://researchportal.port.ac.uk/en/datasets/video-games-dataset
- 6. Bart A. C. CORGIS Datasets Project. [cited 29 Apr 2024]. Available: https://corgis-edu.github.io/corgis/csv/video_games/

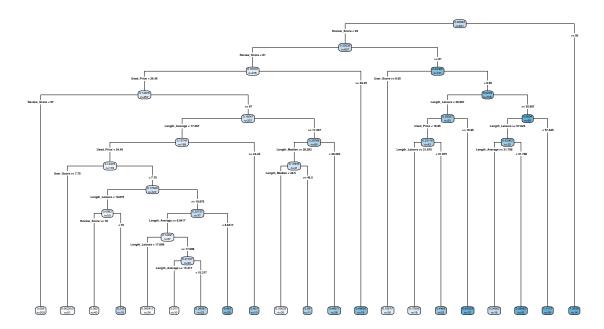
Appendix A: Elbow Plot



Appendix B: Japan & Europe CART Models

Below are the CART models of the non-pruned trees for Japan (top) and Europe (bottom).



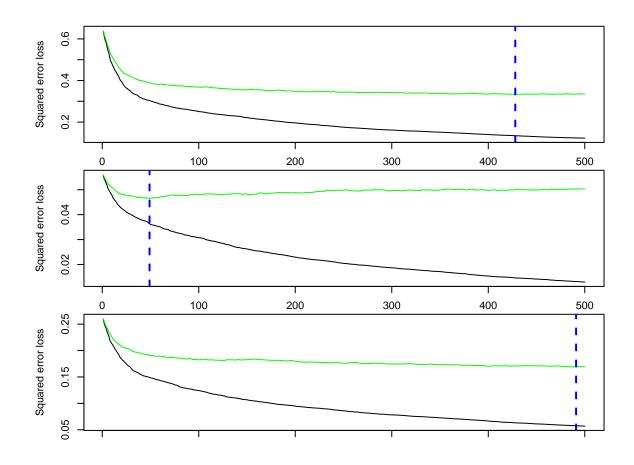


Appendix C: Gradient Boosted Tree Loss Curves

Below are the loss curves for North America (top), Japan (middle), and Europe (bottom) along with the number of trees used by each gradient boosted method.

[1] 428

[1] 49



[1] 491

Appendix D: Lasso Regression Paths

Below are the lasso regression paths for North America (top), Japan (middle), and Europe (bottom) which represent how the coefficients on the variables become non-zero as the penalty to the lasso model decreases.

