12.6 - Quadratic Surfaces

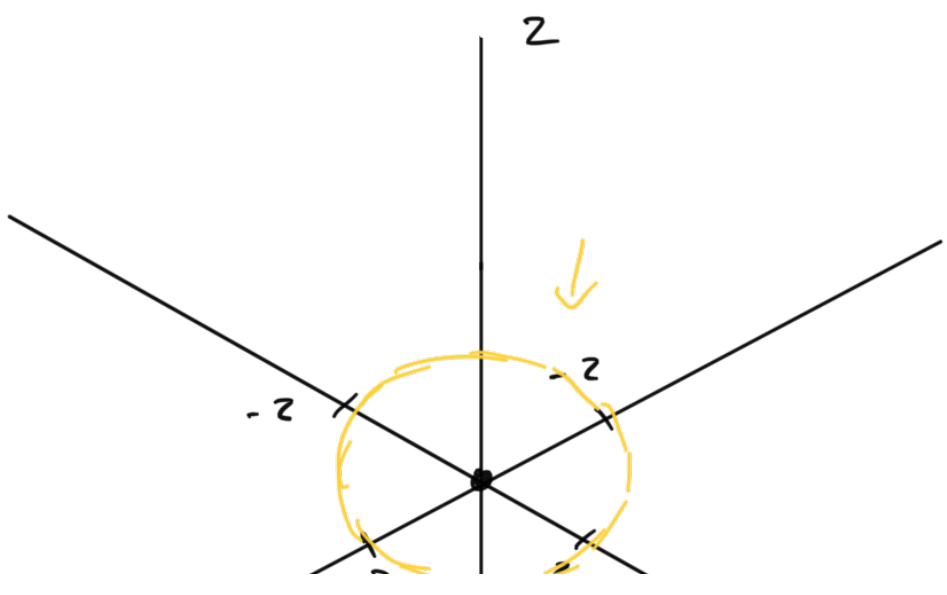
Graphing more complicated curves/surfaces will formalize a technique we have already briefly seen.

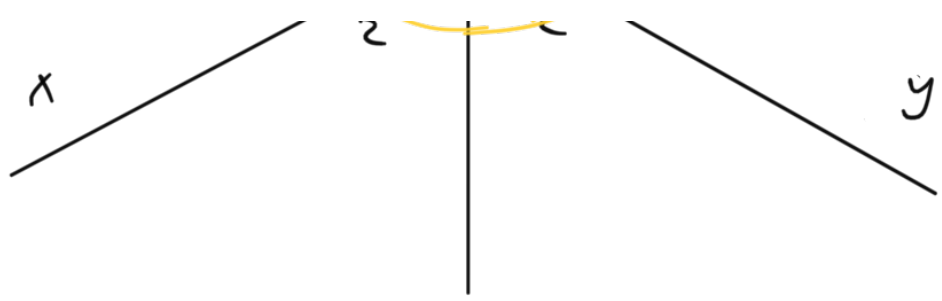
A **trace** is the intersection of a surface and a plane parallel to one of coordinate planes.

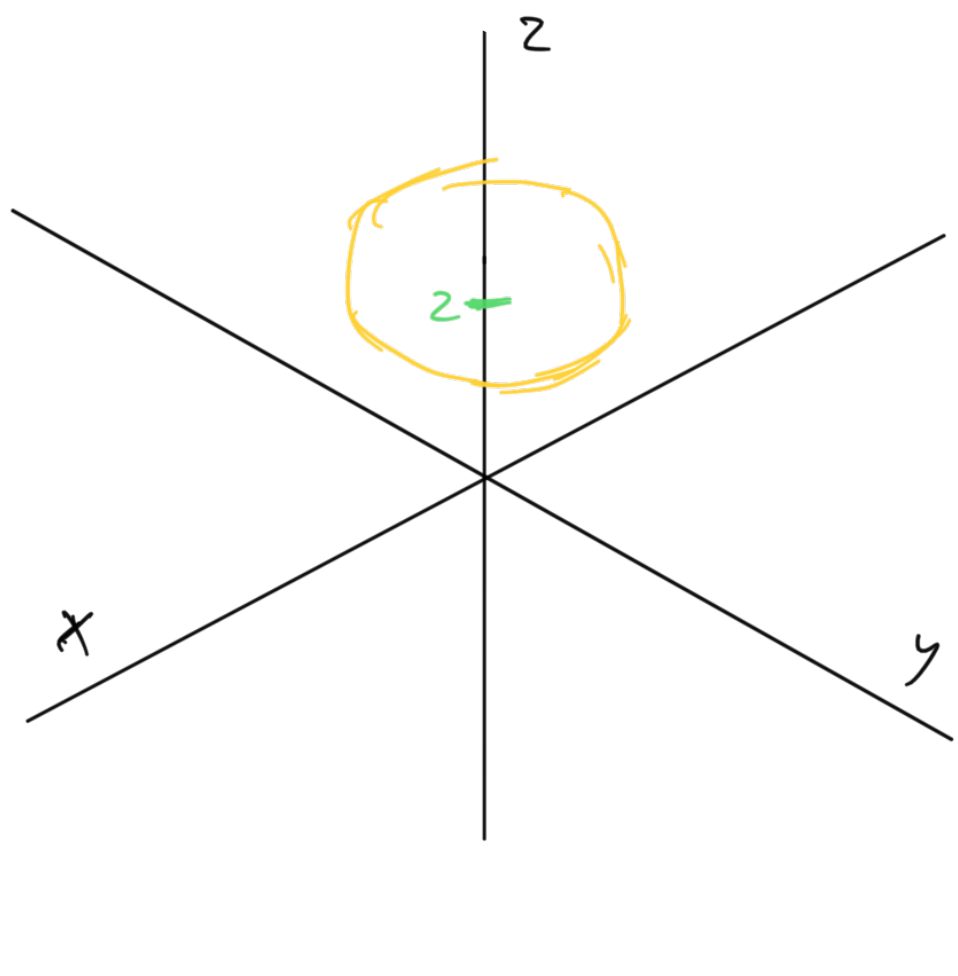
Think of them like slices of the surface. Give us hints of what surface looks like.

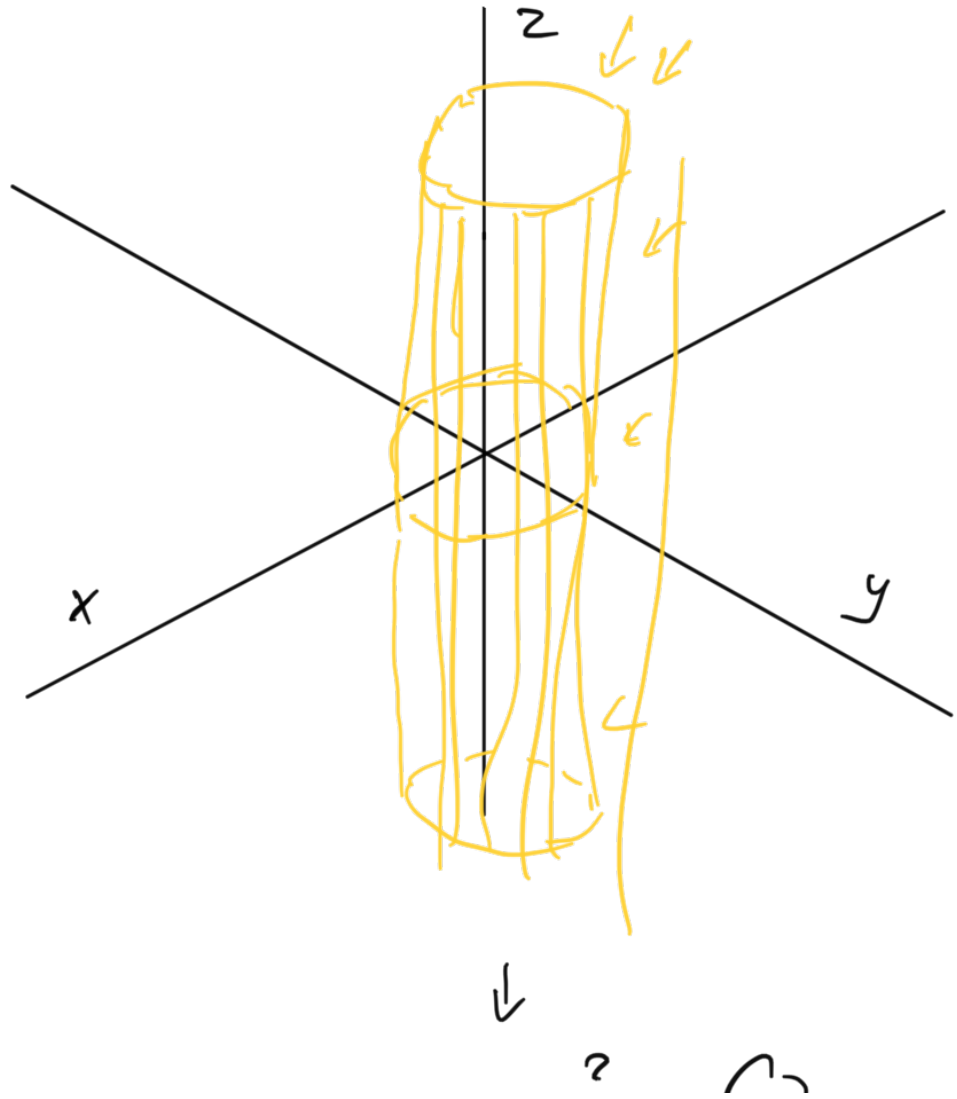
Ex: $x^2 + y^2 = 4$

First, restrict ourselves to $z = 0$

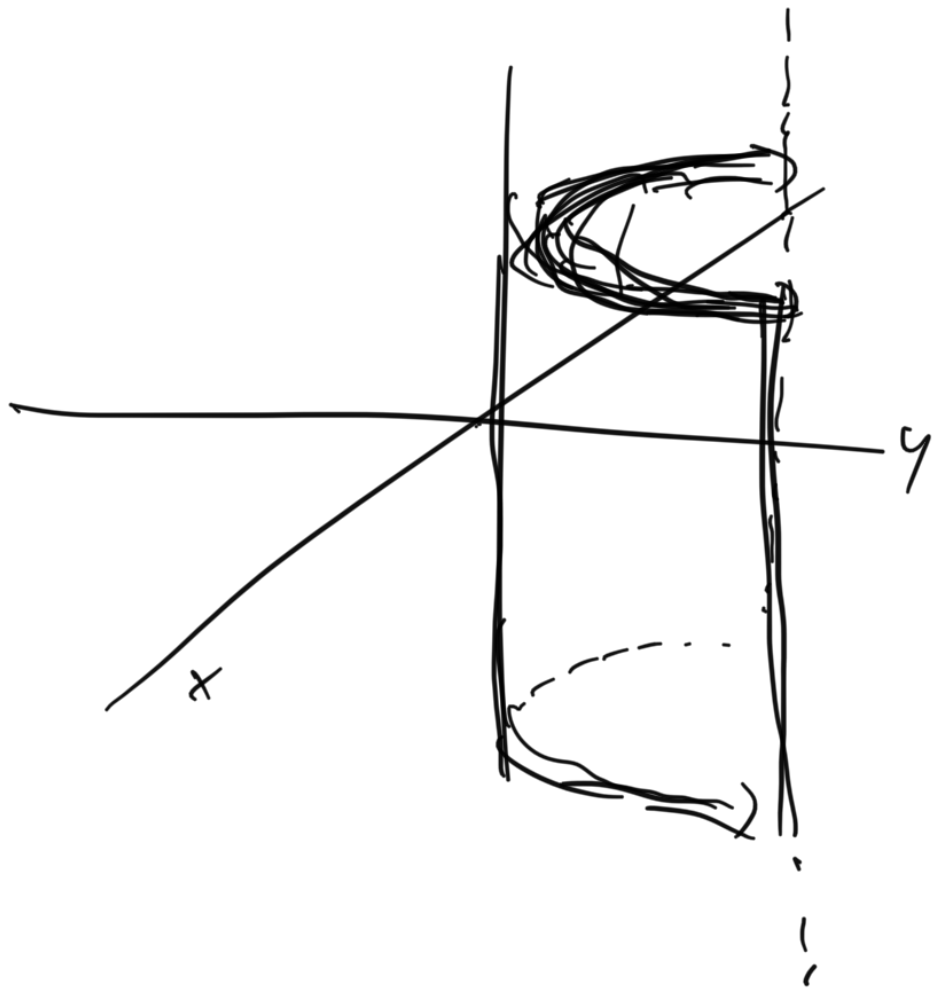



x
y


z
2
x
y


z
x
y

$y - x = 0$

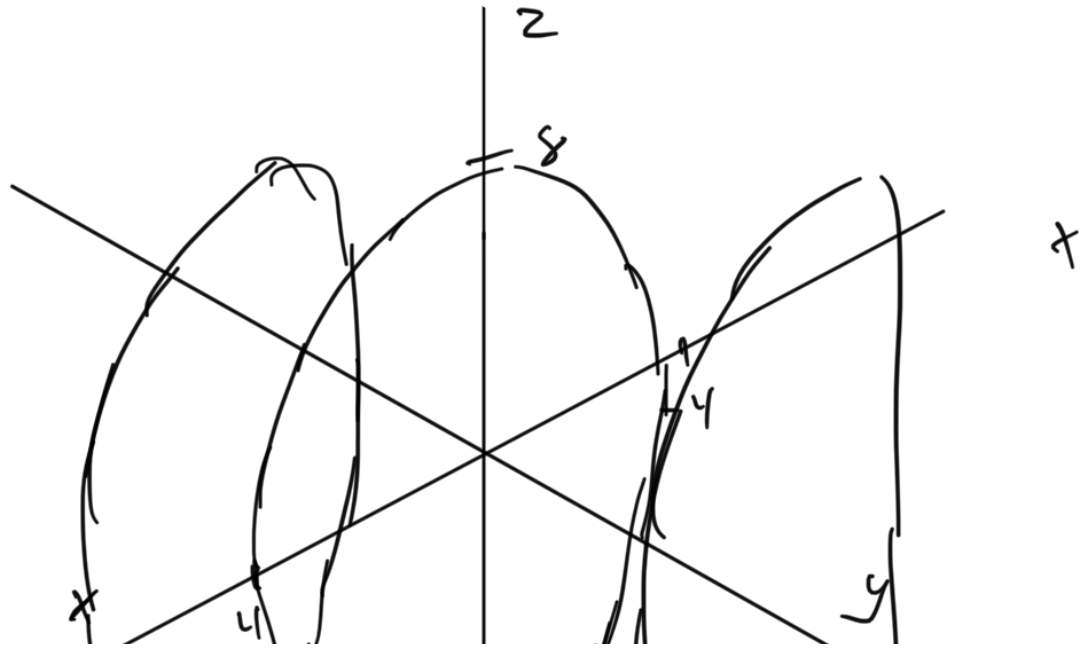


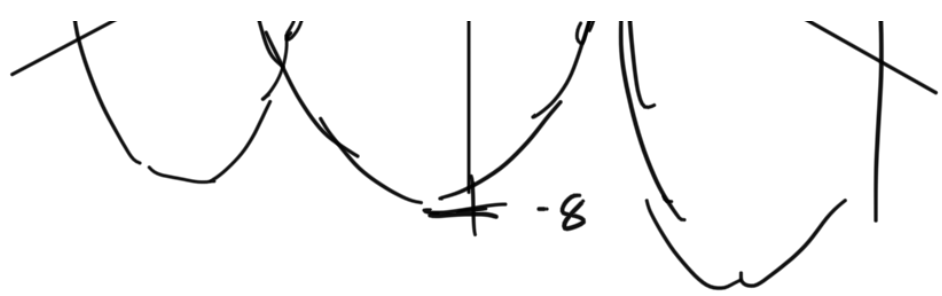
This gives us a cylinder. But cylinder is actually general term for all parallel lines that pass through a plane (2-D) curve

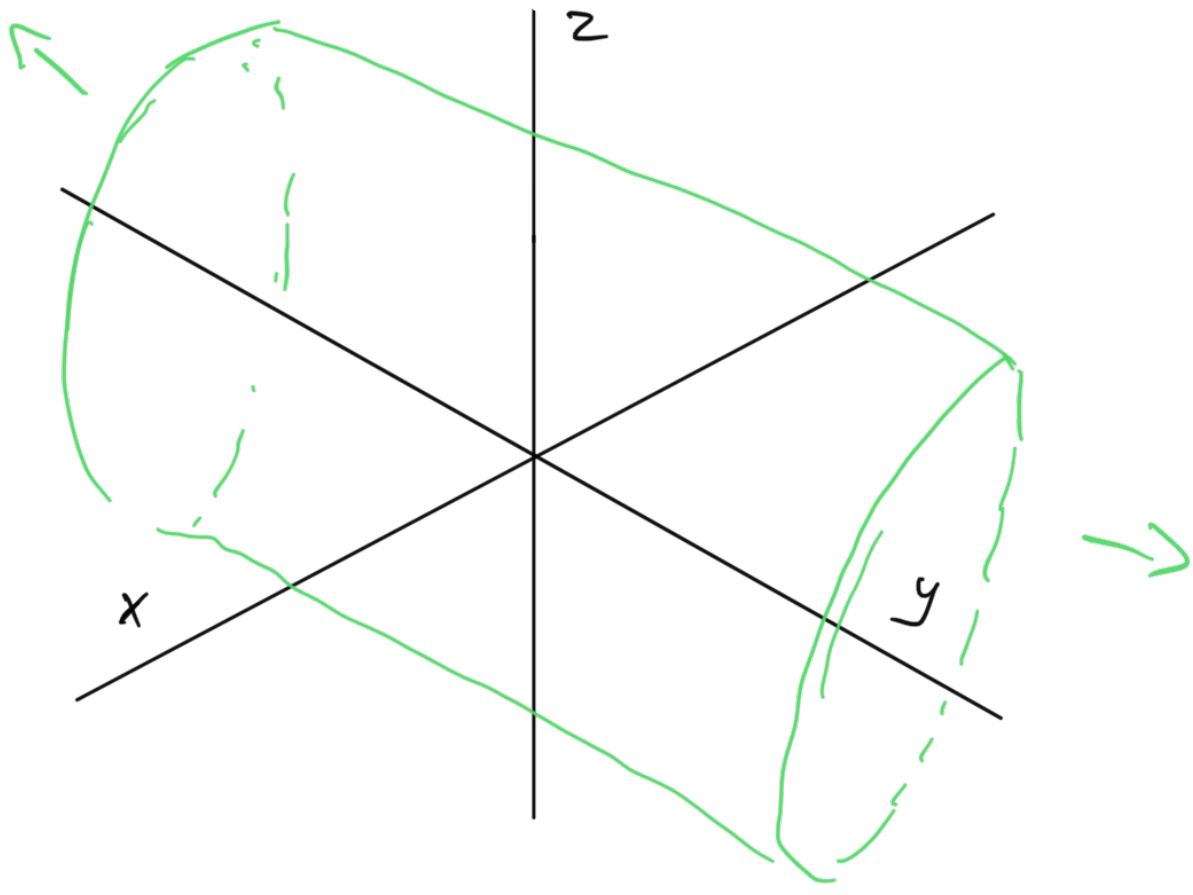
Ex: $\frac{x^2}{2} + \frac{z^2}{8} = 8$

$\frac{z^2}{8} = 8$ $\quad z^2 = 64$

$z = \pm 8$







Quadratic Surfaces

Fancy name, don't be intimidated.

Have equation in x, y, z and at least one of those is squared.

Ex:

$$x^2 + y^2 + z^2 = 1$$

$$x^2 + y^2 + 3z = 0$$

$$-x + y + 2z^2 = 4$$

etc.

call the graphs of this type of equation a quadratic surface.

Don't need to know properties, just how to graph them.

Use traces!

Graphing Steps

Given an equation: ☆

① Pick a direction to take traces (slices)

Ex: along z-axis

② Pick a value along that axis, plug it into equation to get trace

Ex: choose $z = 3$. Plug 3 in for z in equation. Graph that 2-D curve in plane $z = 3$.

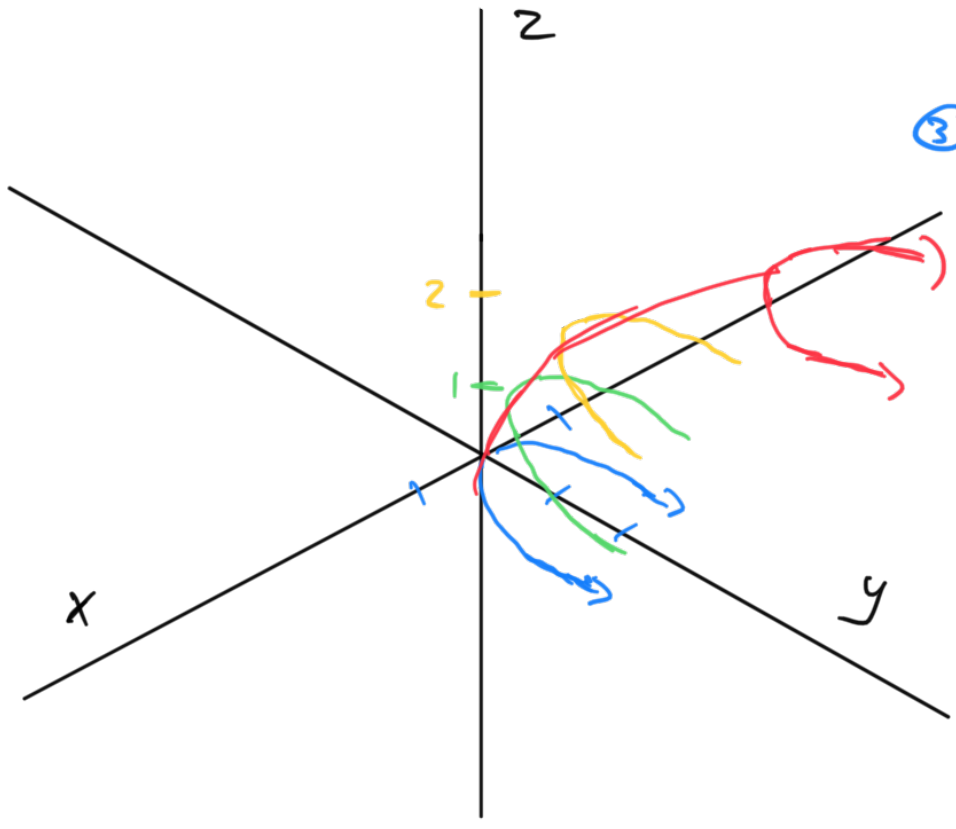
③ Repeat for more along same axis.

④ Do same for other two axes.

⑤ Connect the traces.

Ex:

$$y = 2x^2 + z^2$$



① z-axis

② $z = 0$

$y = 2x^2$

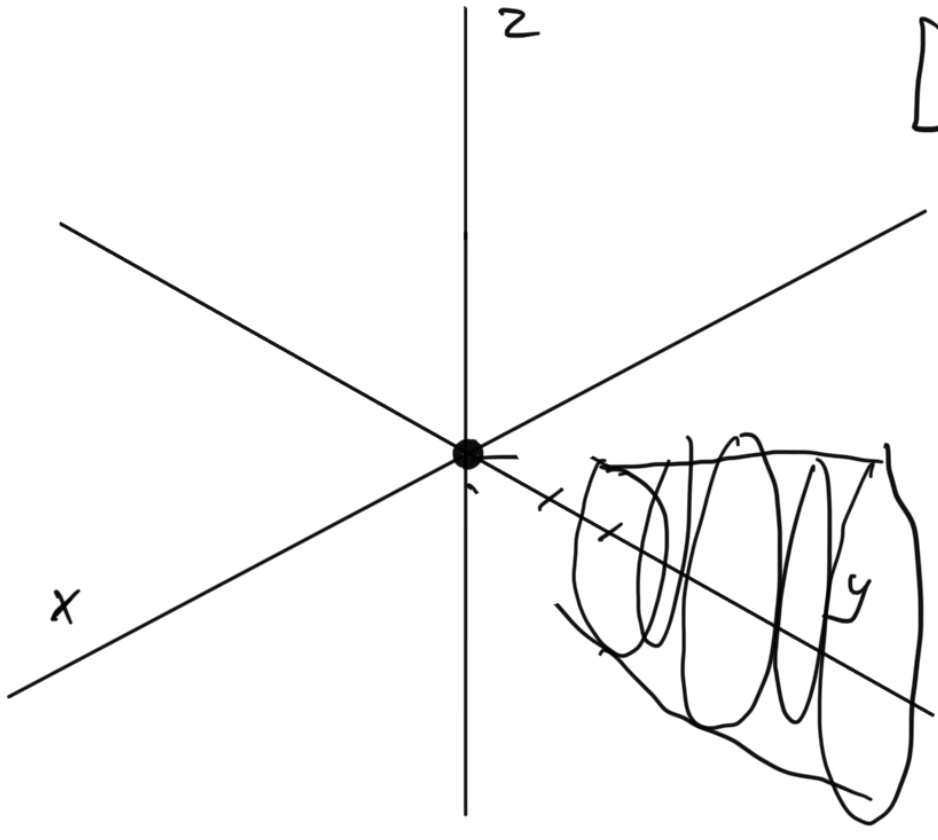
③ $z = 1$ plane

$y = 2x^2 + 1$

$z = 2$ plan

$y = 2x^2 + 4$

$y = 2x^2 + z^2$



$y = 0$

$0 = 2x^2 + z^2$

$y = -1$

$-1 = 2x^2 + z^2$

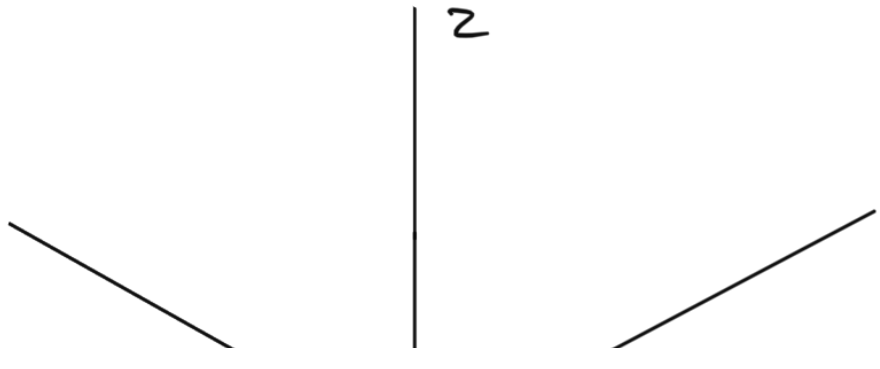
$y = 2$

$2 = 2x^2 + z^2$

$y = 4$

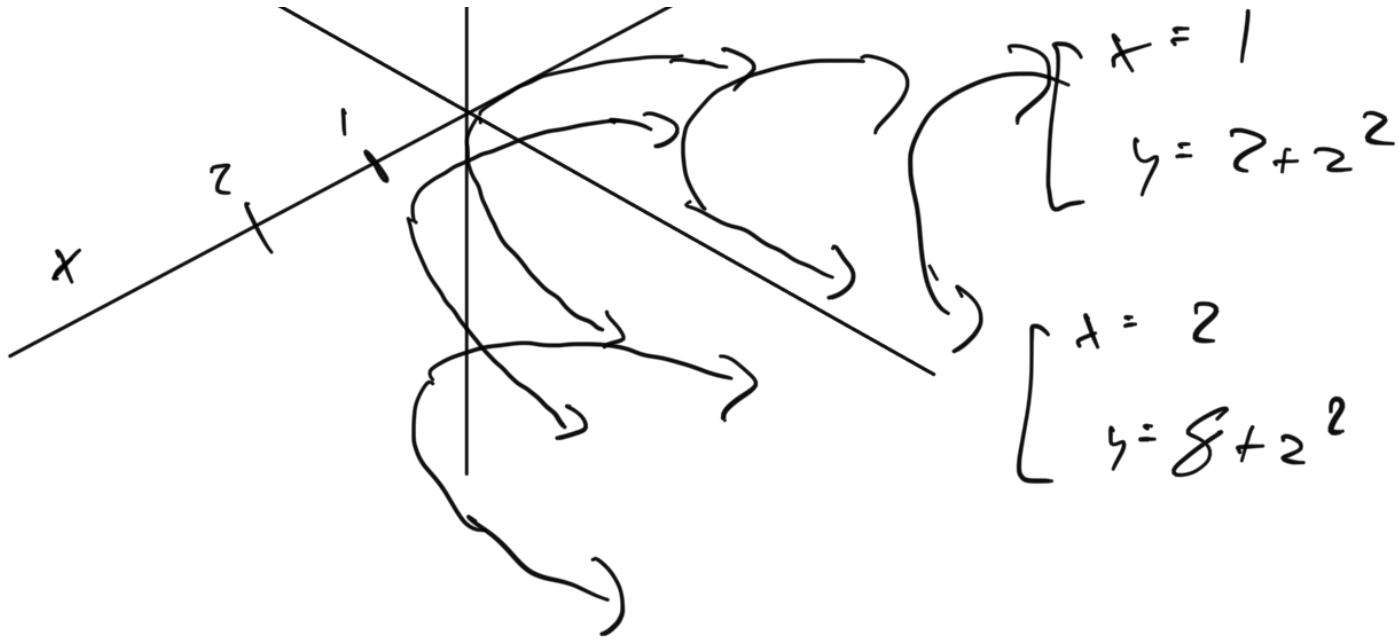
$4 = 2x^2 + z^2$

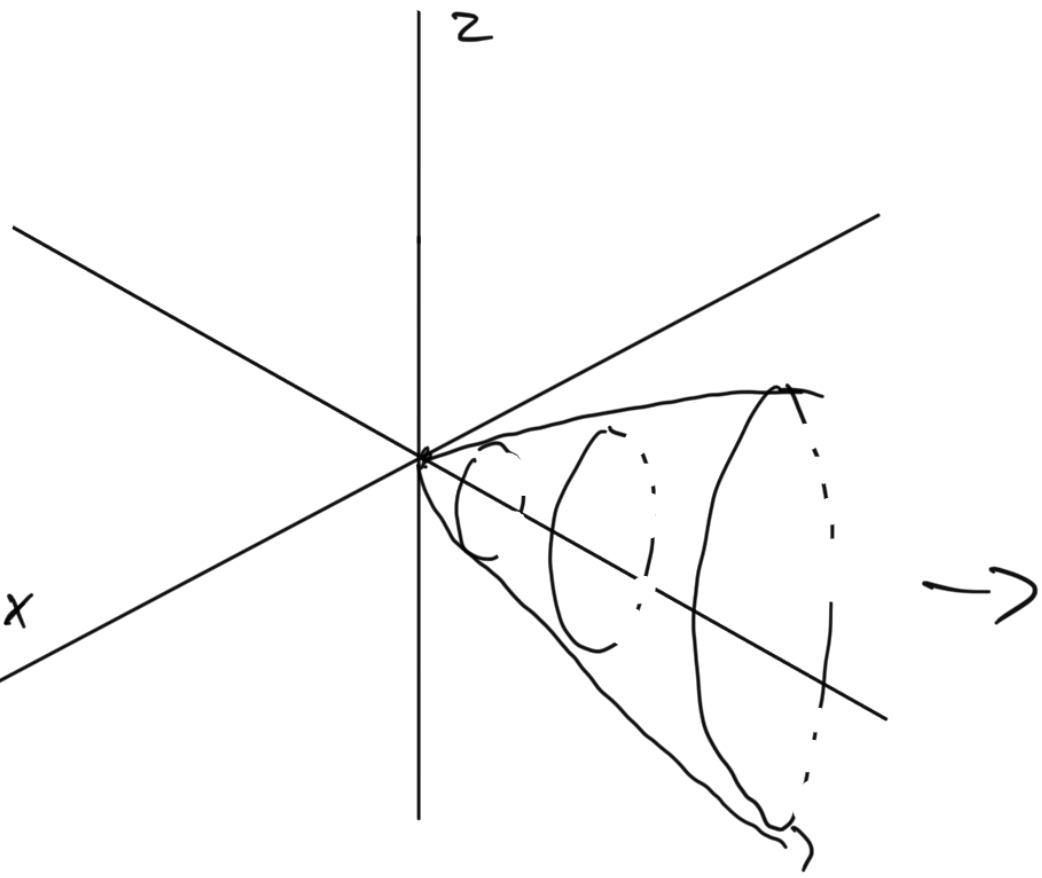
$y = 2x^2 + z^2$



$A = 0$

$y = z^2$

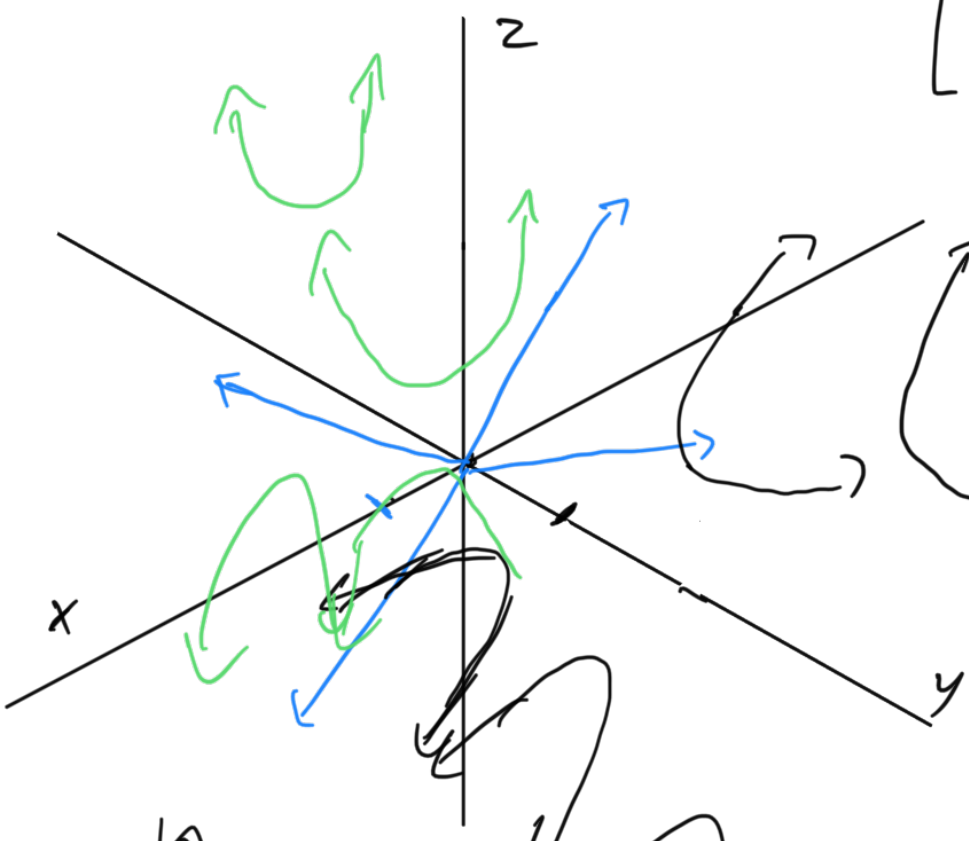




Need to be able to sketch

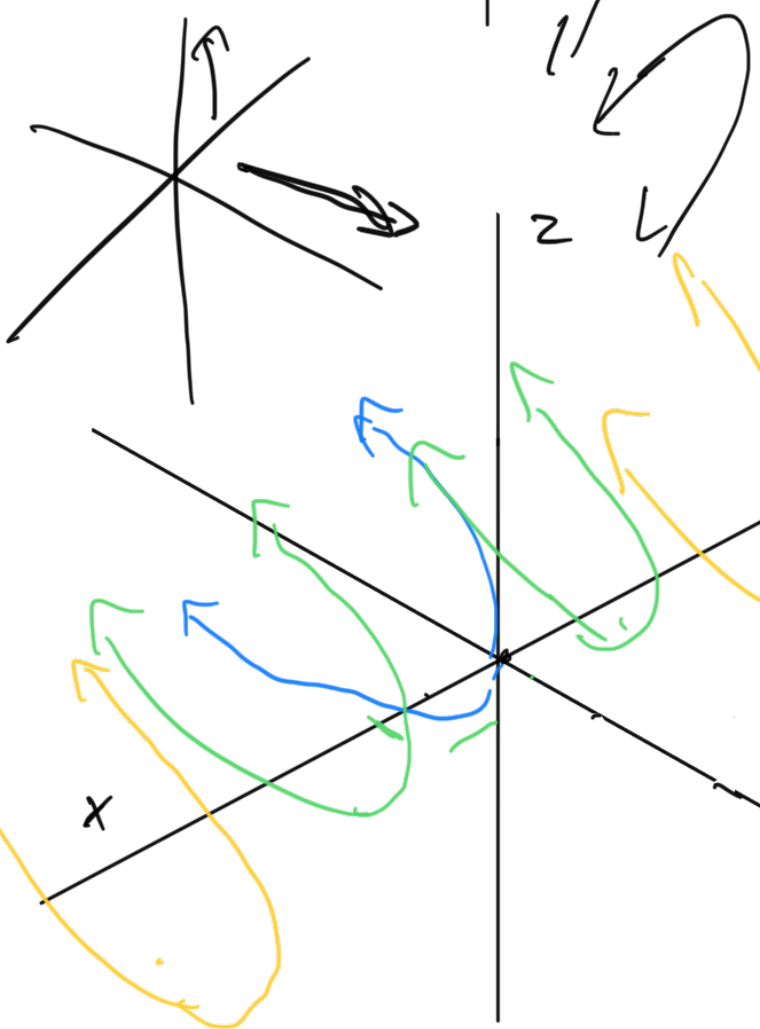
- Lines
- Circles
- Ellipses
- Parabolas
- Hyperbolas

Ex. $y = x^2 - z^2$



$$\begin{cases} y = 0 \\ 0 = x^2 - z^2 \\ x^2 = z^2 \\ |x| = |z| \end{cases}$$

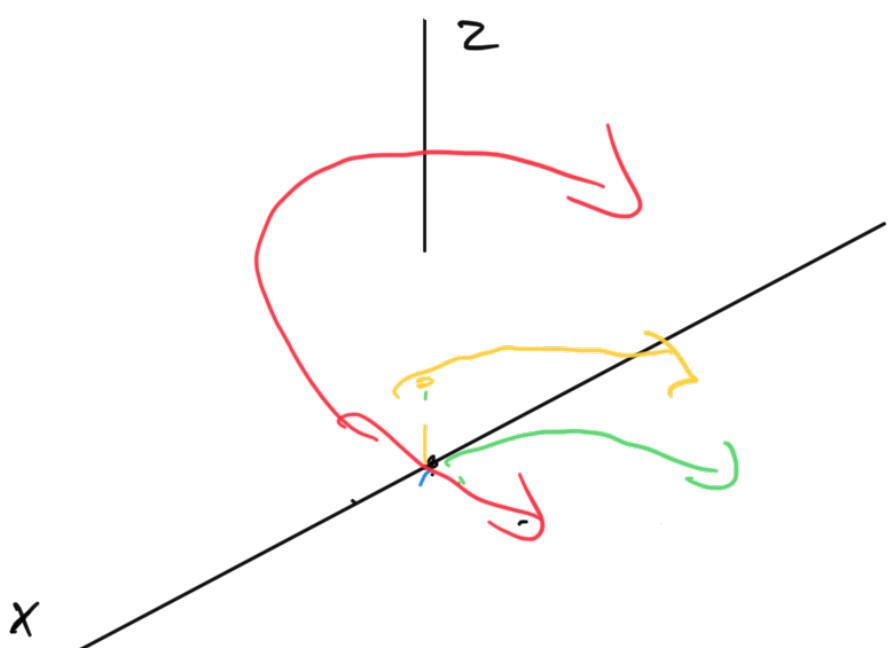
$$\begin{cases} y = -1 \\ -1 = x^2 - z^2 \\ 1 = z^2 - x^2 \end{cases}$$



$$y = x^2 - z^2$$

$$\begin{cases} x = 0 \\ y = -z^2 \end{cases}$$

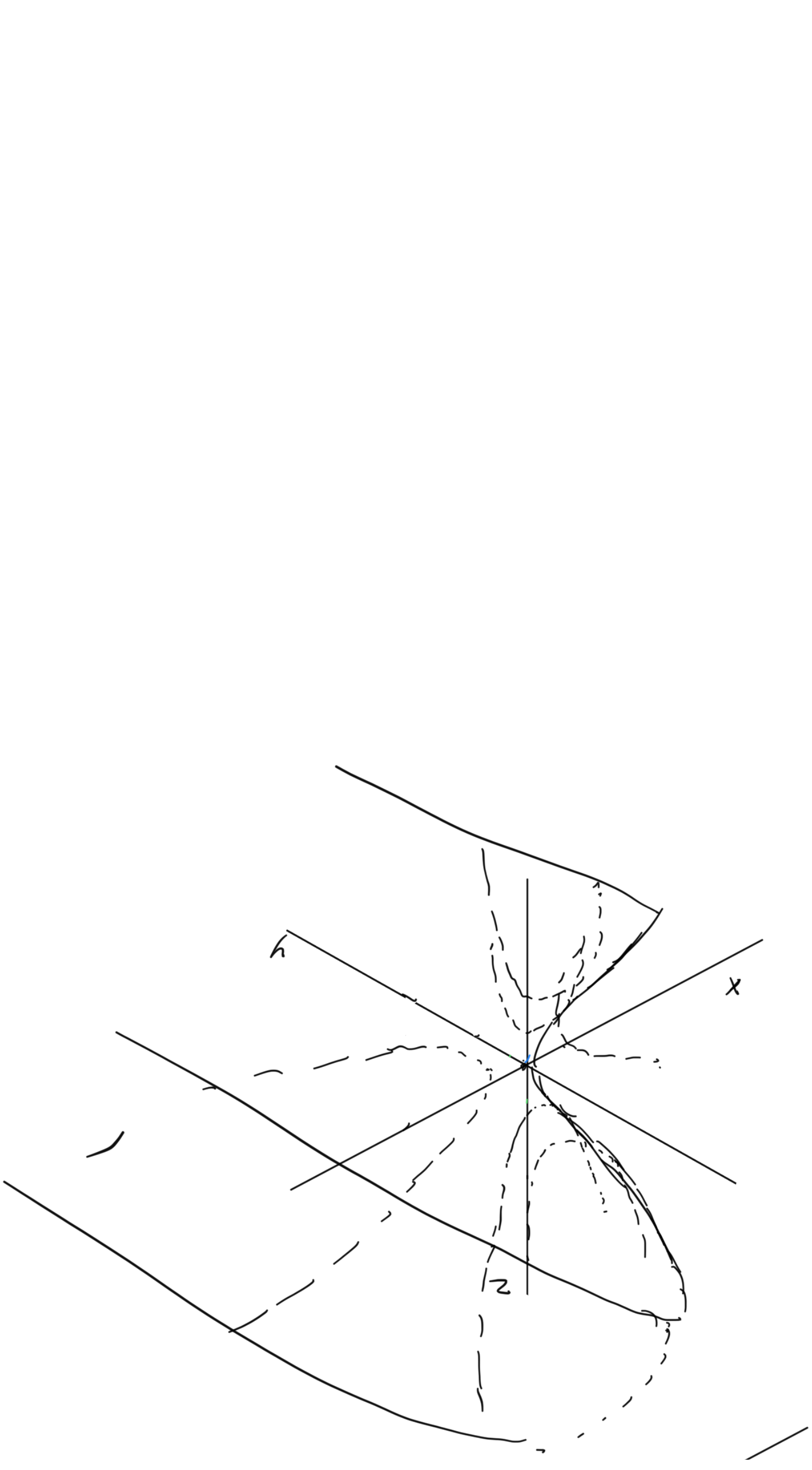
$$\begin{cases} x = 1 \\ y = 1 - z^2 \\ y = -z^2 + 1 \end{cases}$$



$$y = x^2 - z^2$$

$$\begin{cases} z = 0 \\ y = x^2 \end{cases}$$

$$\begin{cases} z = 1 \\ y = x^2 - 1 \end{cases}$$


h
x
z