Section 3.2

First theorem just about how row operations affect determinants. Why do we care?
Start with a matrix with difficult determinant. Row reduce to upper triangular. Account for row reductions, determinant easier to find.

Theorem: A a square matrix $(n \times n)$

① If a multiple of one row added to another to get matrix B, $\det B = \det A$. (so det. unchanged)

② If two rows of A exchanged to produce B then $-\det A = \det B$.

③ If row of A multiplied by k to get B then $\det B = k \det A$

Add multiples of row 1 to other rows

Ex $A = \begin{vmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{vmatrix} \sim \begin{pmatrix} 1 & -4 & 2 \\ 0 & 0 & -5 \\ 0 & 3 & 2 \end{pmatrix}$

row switch

$\sim \begin{pmatrix} 1 & -4 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & -5 \end{pmatrix} = B$

$$\det B = -15$$

$$-\det A = \det B$$

$$\det A = 15$$

☆ **Theorem:** A square matrix A is invertible if and only if $\det A \neq 0$.

☆ **Theorem:** If A is an $n \times n$ matrix then $\det A = \det A^T$

Theorem: If A, B are $n \times n$ matrices then $\det(AB) = \det(A)\det(B)$.