Section 4.2

In last section we saw that if $\vec{v}_1 ... \vec{v}_k \in V$ then spen $\{\vec{v}_1,... \vec{v}_m\}$ subspace of V. Will see some special examples of this type.

Consider the homogeneous equation $A\vec{x} = \vec{G}$.

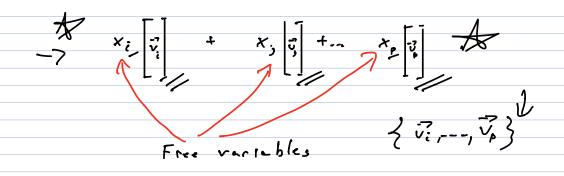
 $A \quad \overline{x}' = \overline{0}$

Recall we discussed set of all solutions to this eq.

3eR^ Nul A= 2 x e R^: A= 3}

• 3e Nul A √

There is always the trivial solution but these may be nontrivial solutions as well. In that case we can write solutions like so:



Argue that Nul A is a subspace of 127

From previous know that if we take $\vec{v}_i ... \vec{v}_p$ from the above then Spen $\vec{v}_i ... \vec{v}_p$ is

a subspace of 127

But does Nul A= Spon & vi ... vij 3?

Assume $\vec{y} \in Spen \{\vec{v}_i ... \vec{v}_p\}$. Then \vec{y}_i is a linear combination of the rectors. $\vec{y}^2 = x_i \vec{v}_i^2 + ... \times \vec{v}_p \qquad 4\vec{y}^2 = \vec{O}$

By linearity of metrix equation we have

 $A \vec{y} = x_i A \vec{v_i} + \dots x_r A \vec{v_r}$ $= x_i \vec{O} + \dots \times_r \vec{O}$ $= \vec{O}$

So is also a solution to Azio

Thus every element in Spentive -- vy 3

Theorem: The null space of an man metrix

A is a subspace of IR. Equivalently, the

set of all solutions to the system AF-B

of m hemogeneous linear equations in a

naknowns is a subspace of IR.

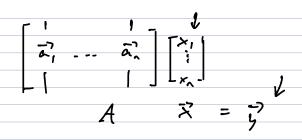
Describing Nul A:

Have seen that if we salve $A\bar{x}^2 = 0$ get something like:

And Span 2 2. ... 2 3 13 some as Nul A.

2 v2 .-- vp } is the key information here, call
it the spanning set. This is after most

convenient way to describe North.
Note: {ai,,ai} is a spenning set of subspece H if H = span {ai, ain}
We can associate another important subspace
with the man metrix A. Assume A is
made up of columns a, a
$A = \begin{bmatrix} 1 & 1 & 1 \\ a_1 & \dots & a_n \end{bmatrix}$
Can consider Span & a, an }. This will also
be a subspace, called Column Space of A
(c) A = Span { a, a, }
What does element of Col A look like?
A linear combination of a) an
y 6 Cal 1 = Span 3 a, an 3
Vecher $y^2 = x_1 \overline{a_1} + x_2 \overline{a_2} + \times_n \overline{a_n}$
Rewrite as matrix transfermedian



So elements in Column Space are the vectors you get when you malliply of by some inpart vector \tilde{x}^2

In alver words Col A is all the possible vectors \vec{b} such that $A\vec{x} = \vec{b}$

Cal A may be some as all of IRM.

of 12 if and only if 4x = b has solution for all be 12 (i.e. A B Onto)

Like Column Space, there is also

Row Space, but is not very often used.

Will omit it here.

Whet is	ds Merence	between Nul A, Col A?
		, page 217 A
Renembe	c there	is a close relationskip
between (metrices	linear fre. Ax? are line	is formations and matrices ar transformations and finear transformations by matrices)
S1: 5 h f ly	different	terminology.
<u>M</u> .		rous Linear Transformations
	$A\bar{\chi}'$	T(式)
	Nul A	Kemel T Range T