14.1 - Multivariable Functions

Recall from previous lectures we discussed possible types of inputs/outputs for functions

$f: \mathbb{R} \to \mathbb{R}$ Calc 1, 2

☆☆ $f: \mathbb{R} \to \mathbb{R}^n$ Chapter 13

(Multivariable functions) $\boxed{f: \mathbb{R}^n \to \mathbb{R}}$ Now

$f(x,y) = z$

$f(x,y,z) = t$

$f: \mathbb{R}^5 \to \mathbb{R}$ $\underline{f(x_1, x_2, x_3, x_4, x_5) = y}$

A multivariable function $f: \mathbb{R}^n \to \mathbb{R}$ is a mapping $\underline{\text{(rule)}}$ that pairs each point/vector in $\mathbb{R}^n$ to a $\underline{\text{unique}}$ number in $\mathbb{R}$

Will $f: \mathbb{R}^2$

Will usually look at $\underline{f : \mathbb{R}^2 \to \mathbb{R}}$

$$\star \quad f(x,y) = z$$

Recall interpretation. z is a quantity that depends on x, y.

Ex: Bracelet. Cost depends on the market price of gold, diamonds

indep. var

Depend. var

$$f(x,y) = z$$

The inputs of function are the <u>independent variables</u>. The output is <u>dependent variable</u>.

$\mathbb{R}$

$$\star \quad f(x) = x^2 + 4x + \ln(x)$$

Ex:

$$f(x,y) = x^2 + 3^x \ln(y) = \quad \mathbb{R}$$

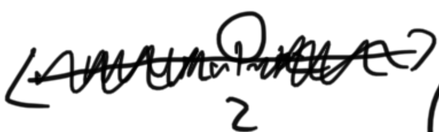
$$z = x^2 + 3^x \ln(y)$$

Compare to $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

Note: Some of equations we saw in 12.6 were multivariable functions.

Some were not. Will discuss later.

$$f: \mathbb{R} \to \mathbb{R}$$

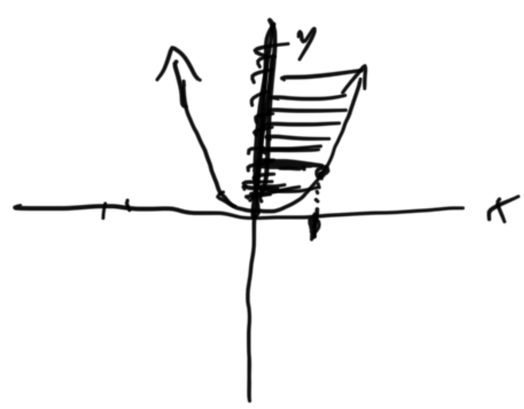


$$f(x) = \frac{1}{x-2} \qquad (-\infty, 2) \cup (2, \infty)$$

$$f(x) = x^2 \qquad \text{Domain} = (-\infty, \infty)$$

<u>Range</u>

outputs



$$\text{Range} = [0, \infty)$$

Just like all other types of functions, these multivariable functions have <u>domains</u> and <u>ranges</u>.*

✗ Division by zero

✗ logs of zero or negative numbers

✗ Square roots of negatives

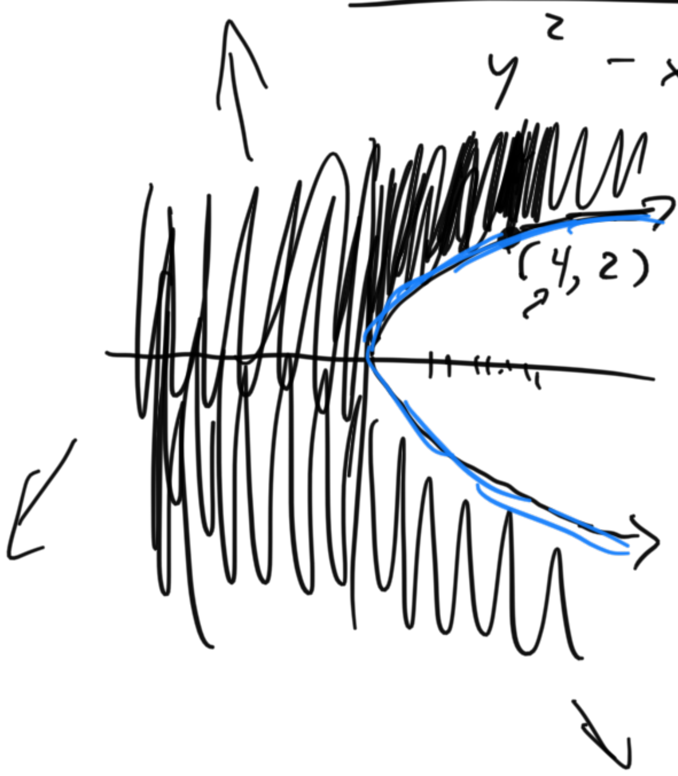
Ex: Find domains

(a) $z = \dfrac{\sqrt{x+y+1}}{x-1}$

(b) $f(x,y) = x\ln(y^2 - x)$

$$\boxed{y^2 - x > 0}$$

$y^2 - x > 0$ $\quad y^2 > x$



(x, y)

Domain and range

(c) $f(x,y) = \sqrt{9-x^2-y^2}$

$\sqrt{9-0^2-3^2}$

$9 - x^2 - y^2 \geq 0$

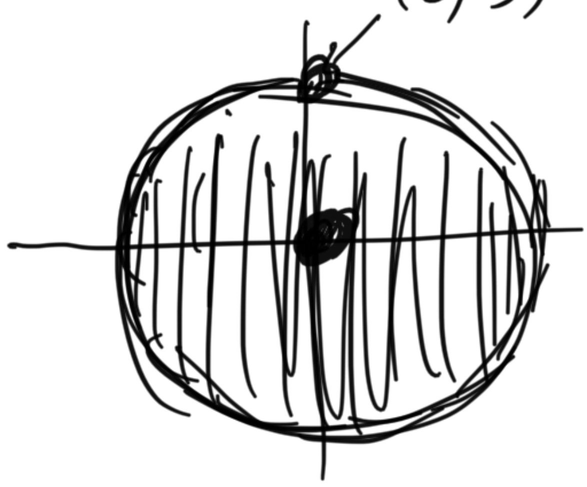
$= \sqrt{0}$

$= 0$

$9 \geq x^2 + y^2$

$\boxed{x^2 + y^2 \leq 9}$

(0, 3) $x^2 + y^2 \leq 9$



$0 = x$

$0 = y$

$\sqrt{9-0^2-0^2}$

$= \sqrt{9}$

$= 3$

(0, 3)

$f(x,y) = x^2 + y^2$ ✓

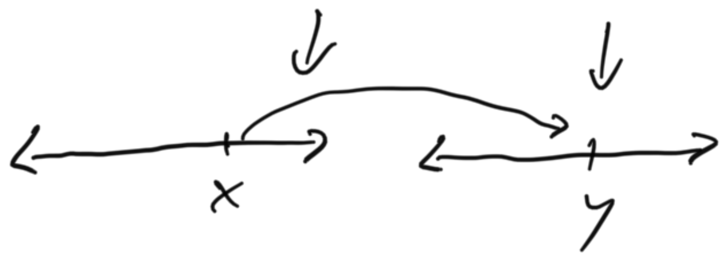
Real numbers

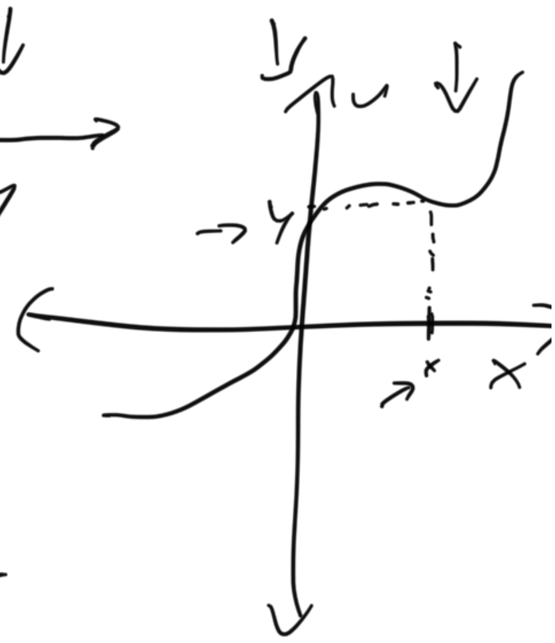
Range = $[0, \infty)$ ✓

Visualizing / Graphing

Input	Output	Graph

$f : \mathbb{R} \to \mathbb{R}$



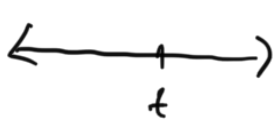


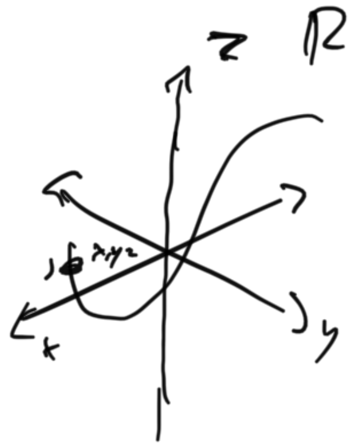
Ind var

(x, y)

Dep

$$\boxed{f : \mathbb{R} \to \mathbb{R}^3}$$

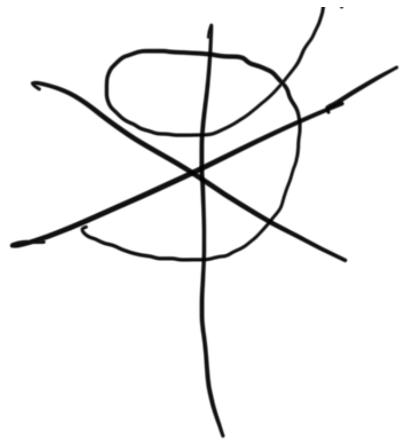




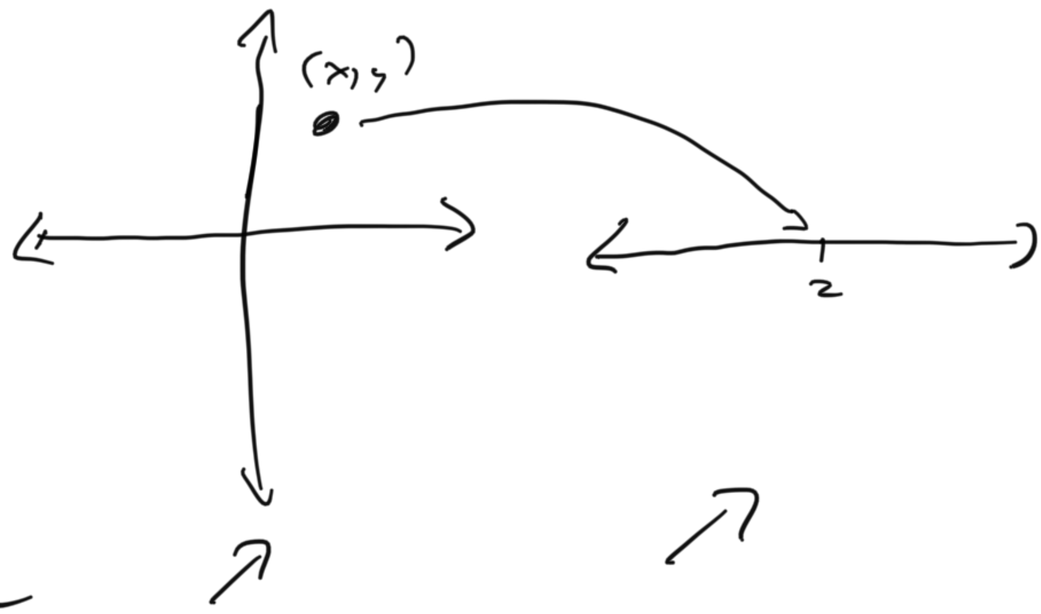
$\mathbb{R}^3$ outputs only

No graph

$\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$



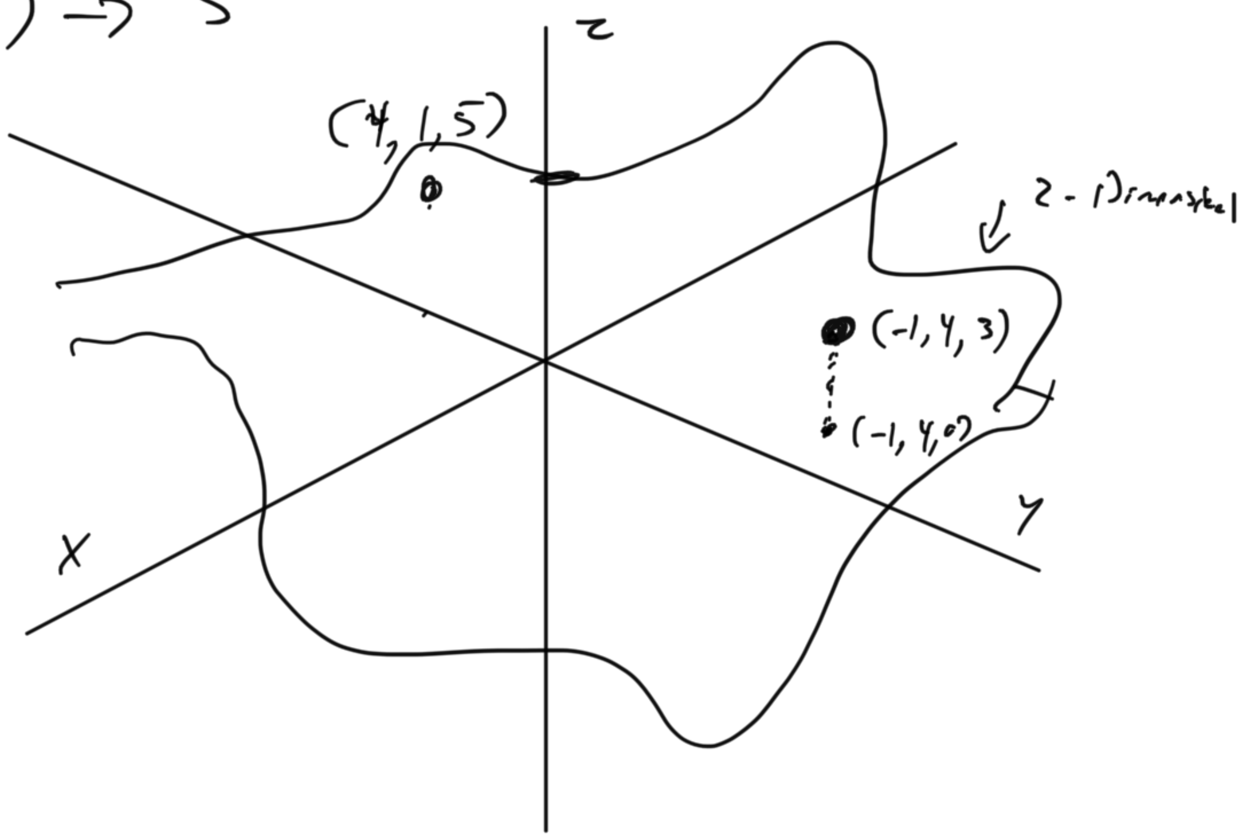
$f : \mathbb{R}^2 \to \mathbb{R}$



$f(4,1) \to 5$

$\downarrow \quad \downarrow$

$(4,1,5)$



Have already seen how to graph $f : \mathbb{R}^2 \to \mathbb{R}$

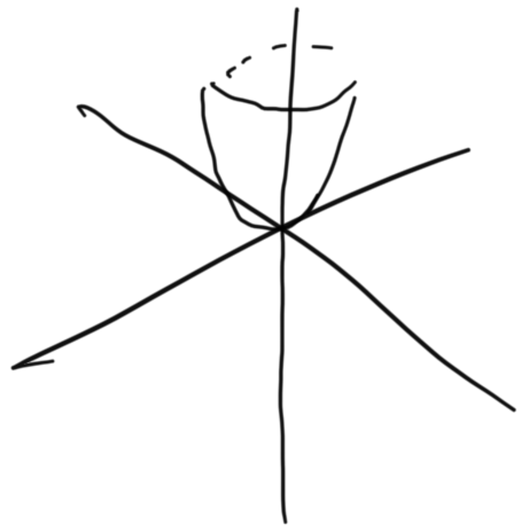
We learned in 12.6. But <u>not</u> all of these equations were functions $f : \mathbb{R}^2 \to \mathbb{R}$

Ex: Domain, range, graph of:

$h(x,y) = x^2 + y^2$

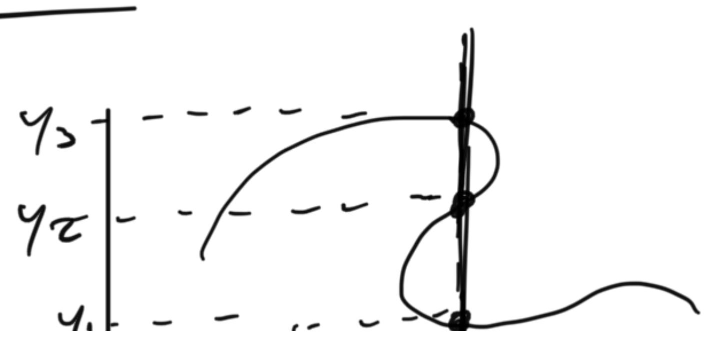
$z = x^2 + y^2$

12.6
Elliptic paraboloid

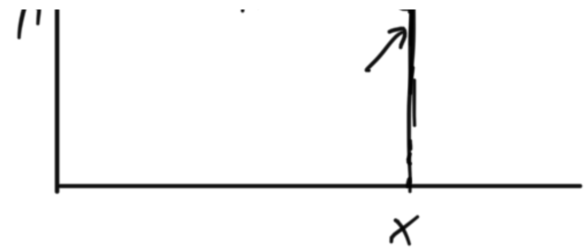


Can now talk about why some of equations in 12.6 were functions, some were not.

Recall in previous courses had <u>vertical line test</u>



Not a



Function

What is the "math" behind vertical line test?

$$x \to y_1, \quad x \to y_2$$

$$f : \mathbb{R} \to \mathbb{R}$$

Each point in domain $\mathbb{R}$ paired with unique number in $\mathbb{R}$

Have similar definition for $f : \mathbb{R}^2 \to \mathbb{R}$ and so get a similar vertical line test

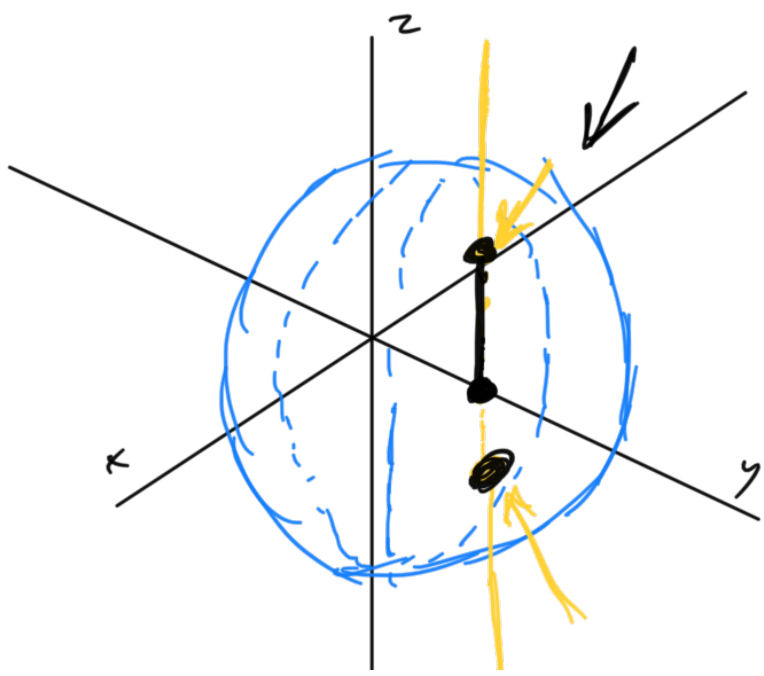
$$f : \mathbb{R}^2 \to \mathbb{R}$$

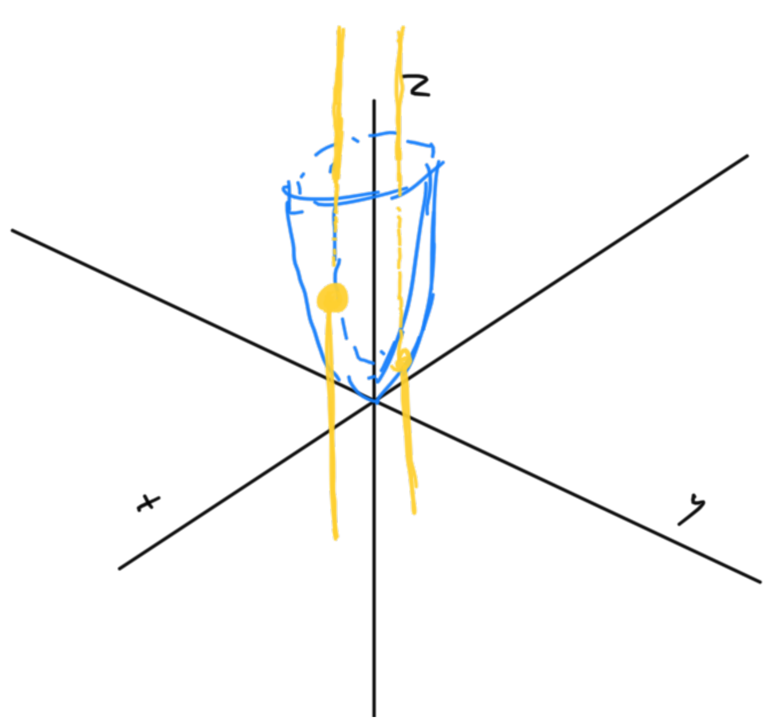
Pair each (x, y) to a unique z

$x^2 + y^2 + z^2 = 9$

☆ $z = x^2 + y^2$

Not a function





Level Curves

Level curves are useful tool to help visualize our surfaces.

$$f(x,y) = c$$

Have $f(x,y)$. Set $\underline{f(x,y) = c}$, a constant.
$f(x,y) = c$ will be an equation in $\mathbb{R}^2$, can plot in $\mathbb{R}^2$

Result will be all points $\underline{(x,y)}$ in $\mathbb{R}^2$ such that $\underline{f(x,y) = c}$.

This will feel similar to 12.6 cross sections, but is different.

Ex: Draw level curves for

$f(x,y) = x^2 + y^2$ $\boxed{z = x^2 + y^2}$

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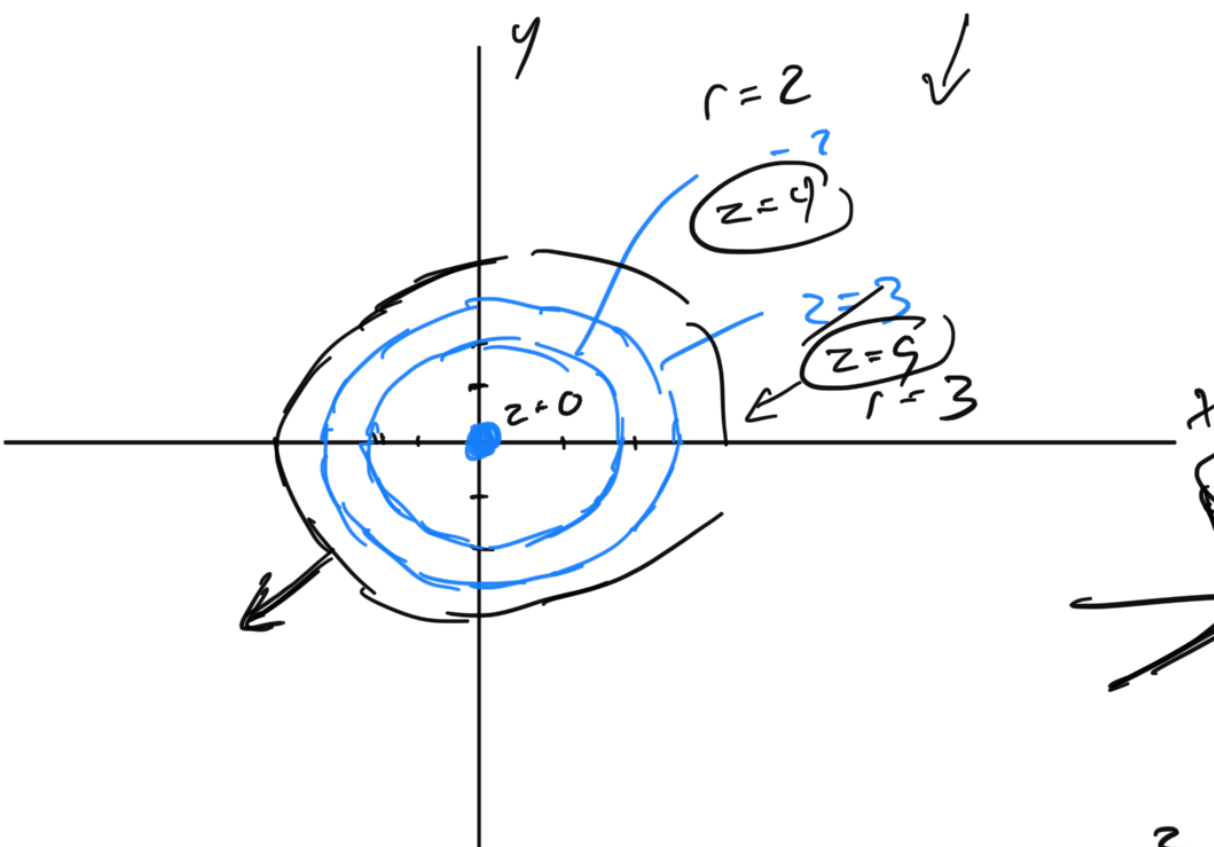
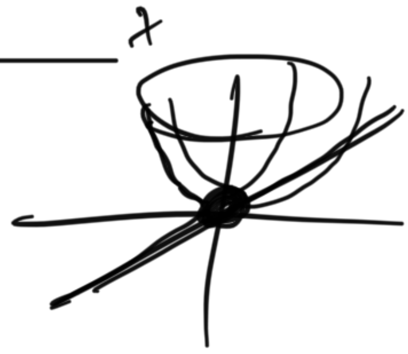
$\underline{z = 0}$: $0 = x^2 + y^2$

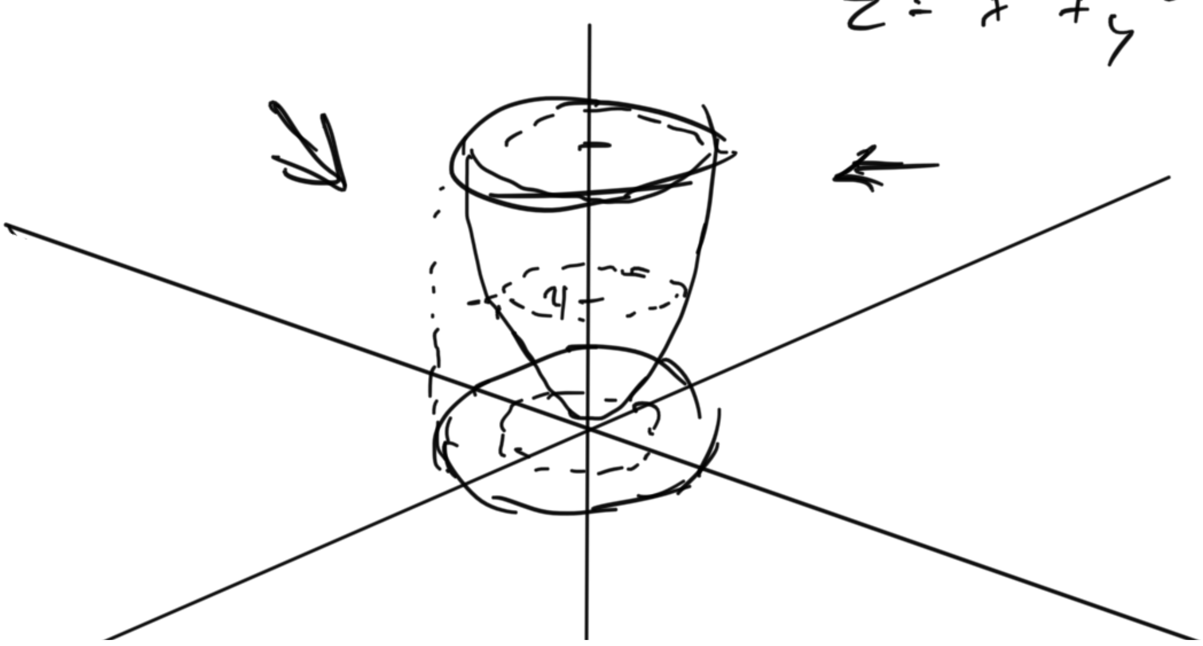
$\underline{z = 4}$: $4 = x^2 + y^2$ $\mathbb{R}^2$

$\underline{z = 9}$: $9 = x^2 + y^2$





$z = x^2 + y^2$

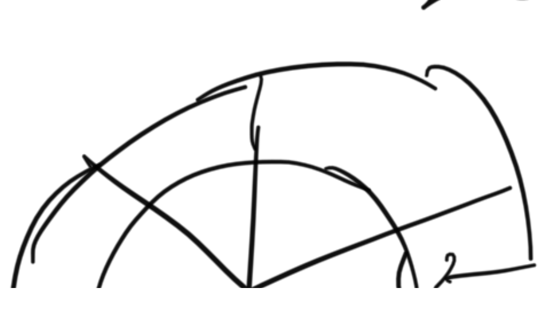


Level Surfaces: Same concept but get plots in $\mathbb{R}^3$.

$$f(x_1, x_2) = y$$

$$f(x_1, x_2, x_3) = y$$

$$c = \underbrace{f(x_1, x_2, x_3)}$$

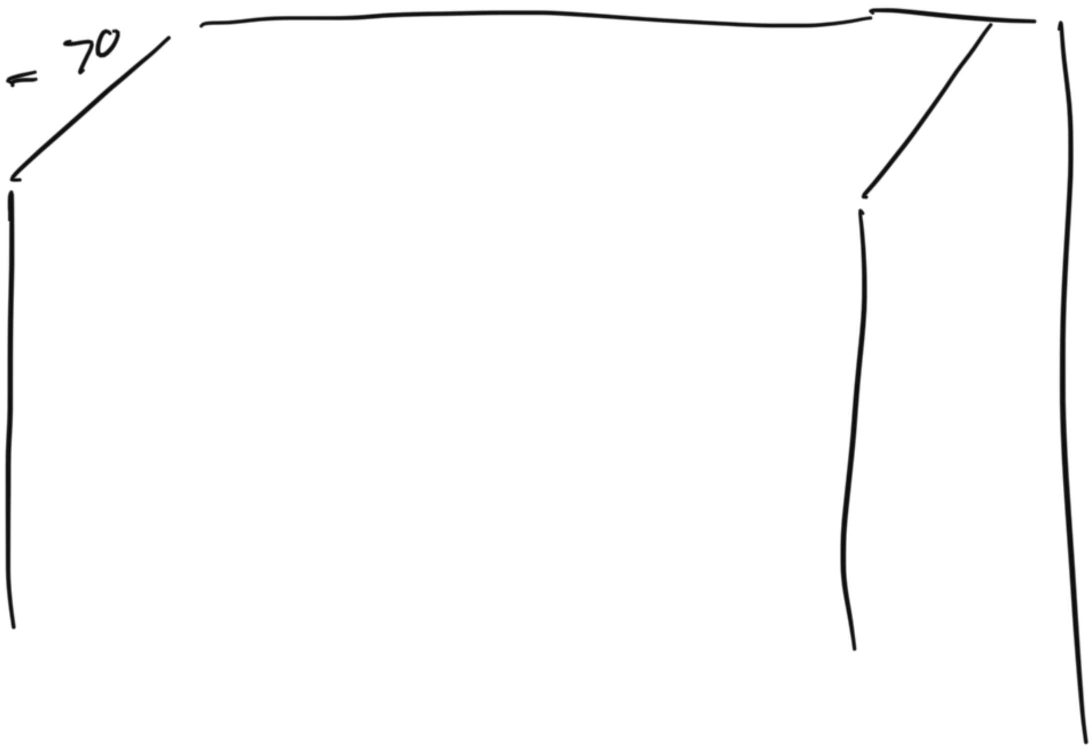


$f(x_1, x_2, x_3) = 4$

$f(x_1, x_2, x_3) = 3$

$f(x_1, x_2, x_3) = 70$

$= 60$



$$f(\overbrace{x_1, x_2, \dots x_n}) = y$$

① n different inputs

1 output (number)

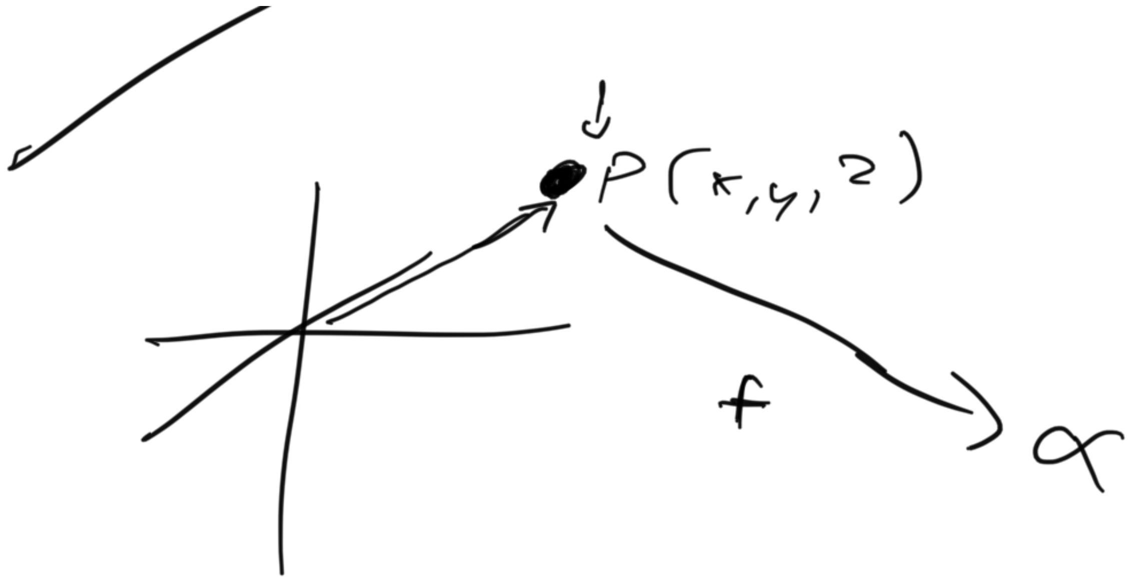
②

$$f : \mathbb{R}^3 \to \mathbb{R}$$

$$f(x, y, z) = \alpha$$

③ vector input

D



☆ Linear function — Example 5

☆ Half space — Example 14