OLIN COLLEGE OF ENGINEERING
Controls: Theory and Practice
ENGR 3370

Laboratory 2 — Issued 12 Mar 2015
Motor Lab — Due 26 Mar 2015

Wet or Dry

This lab may be completed in two different ways:

1. If your design project will include a DC motor as part of your control system, you may use this lab as an opportunity to characterize that motor. Follow the basic instructions that follow to develop a complete electrical and mechanical model for your motor and inertial load.

2. Otherwise, you may use the data included at the end of this handout to complete the data analysis for a hypothetical motor-lab station.

Disclaimer

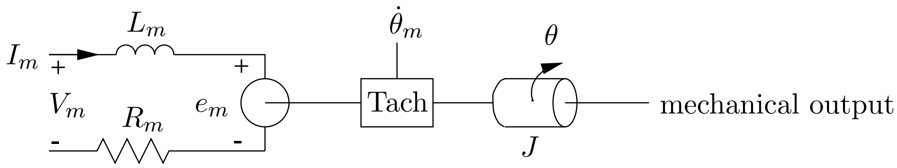
Completing the wet version of this lab requires some initial preparation and planning. Ideally, to extract a complete motor model, your motor mechanism should include:

- A brushed, permanent-magnet DC motor (not a stepper motor)
- A motor-driver amplifier with voltage drive and current drive
- A position sensor
- A velocity sensor
- A removable flywheel

These instructions assume that you have all of these features. Not all of these features are necessary in every application, but a large subset of them is required for good modeling. Start work on it now.

Background

The purpose of this lab is to specify a mathematical model for a physical system. The physical system explored in this lab is a simple servo-mechanical apparatus. The electrical model employed to describe the motor is the following:



where the back voltage (or back emf) of the spinning motor is

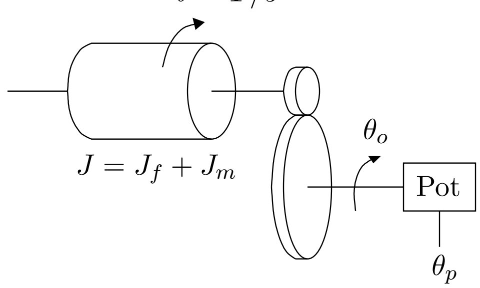
$$e_m = K_e \dot{\theta}$$

and the output voltage of the tachometer is

$$\dot{\theta}_m = K_{tach} \dot{\theta}.$$

The mechanical output of the servo-mechanism is modeled by

$$\ddot{\theta} = T/J$$



where the output torque of the motor is

$$T = K_t I_m$$

and the position of the output shaft is

$$\theta_o = \theta/n$$

where n is the gear-train ratio. The output voltage of the position sensor is

$$\theta_p = K_p \theta_o.$$

Note that the motor shaft angle is θ, and the output shaft angle is θ_o. The flywheel, with inertia J_f, is mounted directly on the motor shaft. The output shaft is geared to the motor shaft with a gear ratio n. The servo-potentiometer is connected to the output shaft, and the voltage across the potentiometer is θ_p.

This lab assignment will consist of determining values for the model parameters R_m, L_m, K_e, K_{tach}, J_m, J_f, n, K_t, and K_p.

Prelab Exercises

The following questions concern the modeling of the motor which will be used as part of this servo-mechanism lab.

1. Draw a block diagram for the motor, with V_m as the input and $\dot{\theta}_m$ and θ as the outputs of the system. Clearly label the blocks as well as the following intermediate variables: I_m, T, e_m. (Hint: K_e should appear in the feedback path.)

2. Derive the following transfer functions: $\frac{\dot{\Theta}}{I_m}(s)$, $\frac{\dot{\Theta}}{V_m}(s)$, and $\frac{\Theta}{V_m}(s)$.

3. Consider the addition of a damping term B, which represents the viscous damping component of the load (proportional to velocity). Draw a new block diagram for the motor which appropriately includes the damping term, and give the corresponding expressions for $\dot{\Theta}/I_m$ and $\dot{\Theta}/V_m$.

4. What change does damping cause in the $\dot{\Theta}/I_m$ transfer function? What is the significance of this change?

5. What is the electrical time constant for a circuit with an inductor L_m and a resistor R_m?

6. Assuming that the electrical time constant is much faster (smaller) than the mechanical time constant, simplify the transfer function $\dot{\Theta}/V_m$ (without damping) to that of a first order system. What is the time constant of this transfer function?

7. Draw a block diagram for the mechanical output to the motor. Label clearly θ, θ_o, and θ_p. (Ignore the viscous damping that was modelled in Question 3.)

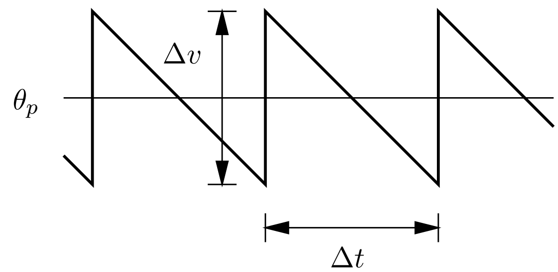
8. The outputs $\dot{\theta}_m$ and θ_p are electrical quantities (in volts) which you can measure on an oscilloscope. Given this, please indicate the units (using standard SI units) for all of the following system parameters. (Hint: K_t, K_e, and K_{tach} are not unitless!)

   $R_m, L_m, I_m, V_m, \theta_p, \dot{\theta}_m, K_{tach}, K_e, K_t, K_p, T, J_m, J_F, B, n, \theta, \theta_o$.

Measurements: Part I

Part I of this lab is completed on a motor lab station that includes the flywheel J_F.

1. Drive the motor with a constant (DC) voltage. The output θ_p should look like this:

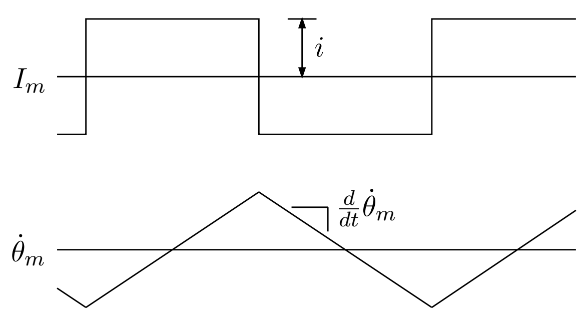
   Note that the θ_p waveform is discontinuous — the monitor voltage "wraps around" when the output shaft passes through an angle of $\pm\pi$ radians (the measured voltage θ_p is continuous for output shaft angles in the range $-\pi < \theta_o < +\pi$). Measure the size of the discontinuity Δv.

2. Drive the motor with a constant (DC) voltage. Measure and record the monitor values V_m, I_m, and $\dot{\theta}_m$. Also measure the output shaft speed by recording Δt, the time between wraps, in the θ_p waveform. Repeat this measurement for five different voltages in the range 0 V $\leq V_m \leq$ 10 V. To what does the time between the wraps correspond?

   Note that all of the monitor outputs are voltages. A reading of 0.5 V on the I_m output corresponds to a motor current of 0.5 A, since the scale factor for the I_m monitor is 1 V/A.

   Recall that the monitor voltage $\dot{\theta}_m$ is related to the motor shaft speed $\dot{\theta}$ by the scale factor K_{tach}. As part of this lab you will determine the value of K_{tach}; note that it has units of V-sec/rad.

3. Drive the motor with a square wave of current (with no offset). The motor current and angular velocity should look something like the following picture:

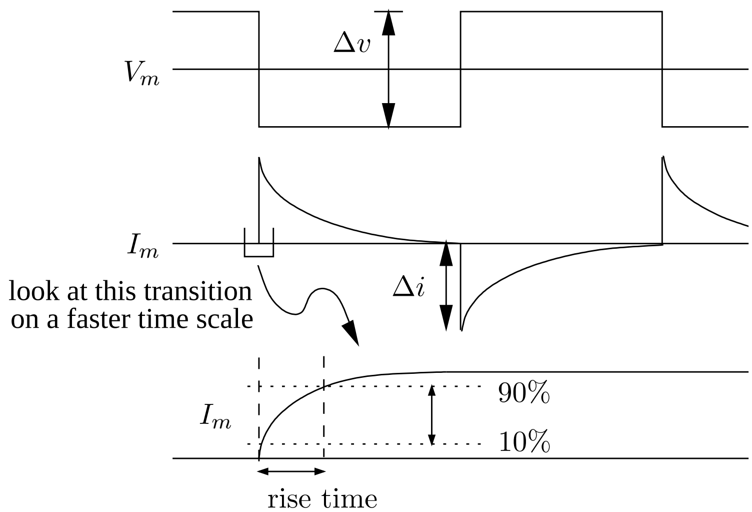
   

   Measure and record the values of I_m and $\frac{d}{dt}\dot{\theta}_m$. Be sure that you are not saturating the amplifier when making these measurements (I_m should be a clean square wave). Drive the system fast enough that the $\dot{\theta}_m$ waveform is made up of relatively straight line segments. Repeat this for five different values in the range 0 A $\leq I_M \leq$ 1.5 A.

Measurements: Part II

Part II of this lab is completed without the flywheel J_F.

1. Drive the motor with a relatively slow square wave of voltage (with no offset). The motor voltage and current waveforms should look something like the following artist's conception:



   Measure and record the values of Δi and Δv. Also measure the 10%–90% risetime of the motor current waveform. Make sure that you are measuring the rise time, as shown in the figure, and not the fall time.

   Repeat this measurement for five different Δv values in the range $0 \text{ V} \le \Delta v \le 10 \text{ V}$.

2. Drive the motor with a square wave of voltage and measure the 10%–90% risetime of the motor velocity monitor, $\dot{\theta}_m$. Repeat this for five different voltages.

3. Drive the motor with a square wave of current (with no offset). Repeat the measurement from Part I–3.

Data Reduction

We will use a linear-least-squared-error approximation technique to reduce the measurement data and obtain the desired model parameter values. In a perfect world (like maybe on an exam) this technique wouldn't be necessary; a single set of measurements in the lab would sufficiently and uniquely determine the parameter values. However, measurement errors and modeling deficiencies limit the accuracy of parameter calculation in the real world. To improve our accuracy, several measurements are "averaged" to yield the desired model parameters.

K_{tach} **calculation:** Ideally, $\dot{\theta}K_{tach} = \dot{\theta}_m$. We can obtain several measurements of θ_o from the time between wraparounds in the $\dot{\theta}_p$ waveform (corresponding to an output shaft rotation θ_o equal to 2π). Be sure to convert $\dot{\theta}_o$ to $\dot{\theta}$ by taking the gear ratio into account. We also have the corresponding tachometer monitor voltage measurements of $\dot{\theta}_m$. Thus, in vector form, we can write:

$$\begin{aligned} \begin{pmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_5 \end{pmatrix} K_{tach} &= \begin{pmatrix} \dot{\theta}_{m1} \\ \vdots \\ \dot{\theta}_{m5} \end{pmatrix} \\ \vec{\dot{\theta}} K_{tach} &= \vec{\dot{\theta}}_m \end{aligned}$$

Determine the best-fit value of K_{tach} by minimization of the squared error, a la

$$K_{tach} = \frac{\vec{\dot{\theta}}^T \vec{\dot{\theta}}_m}{\vec{\dot{\theta}}^T \vec{\dot{\theta}}}$$

R_m **calculation:** Once again in an ideal world, $R_m \Delta i = \Delta v$. From Part II–1 we have several measurements of Δi, along with the corresponding Δv values; calculate the best-fit value for R_m using the least squares estimation technique as was used for K_{tach}.

L_m **calculation:** The time constant of an exponential response is equal to the 10%–90% risetime divided by a factor of 2.2. Average the five risetime measurements and divide by 2.2 to obtain the average time constant, which corresponds to the L_m/R_m time constant in the motor model. The value of L_m is calculated by multiplying the average time constant by the best-fit R_m value.

K_e **calculation:** $K_e\dot{\theta}$ should equal $(V_m - I_m R_m)$. We have obtained several measurements of $\dot{\theta}$ for the K_{tach} calculation above. We also have the corresponding V_m and I_m measurements, and the best-fit value of R_m. Once again, use the LLSE to determine the best-fit value for K_e.

n **(gear ratio) calculation:** Counting gear teeth isn't terribly interesting or educational, so we have done this for you. The gear on the motor shaft has 32 teeth, and the output shaft gear has 216 teeth. Thus the value of n is 6.75.

K_p **calculation:** The discontinuity at an output shaft angle of $\pm\pi$ radians corresponds to a jump of 2π radians. Thus the value of K_p is $\Delta v/2\pi$.

J_f **calculation:** The flywheel inertia J_f is calculated from basic physics. The flywheel is a cylinder made of aluminum, which has a density of 2.7 g/cm^3. The thickness of the flywheel is 6.35 mm, and its diameter is 63.5 mm.

K_t **and** J_m **calculation:** Consider first the data set taken in Part II–3 without the extra flywheel inertia. Note that the inertias of the motor armature, the gears, the potentiometer, etc., are all lumped together into a single effective inertia which we call the 'motor inertia', J_m. Once again considering the ideal scenario, $K_t i = J_m \frac{d}{dt}\dot{\theta}$. We have several measurements of $\frac{d}{dt}\dot{\theta}_m$ (which can be converted to $\frac{d}{dt}\dot{\theta}$ values through division by the model parameter K_{tach}), along with the corresponding i values. Calculate the best-fit value for the ratio K_t/J_m using the LLSE technique.

Repeat the above procedure for the data set taken in Part I–3 with the flywheel inertia J_f on the motor shaft. Note that now you are solving for the ratio $K_t/(J_m + J_f)$. We now have an expression for K_t/J_m and and expression for $K_t/(J_m + J_f)$. Since the value for J_f is known, we have two equations in two unknowns. Solve these equations for K_t and J_m.

τ_m **calculation:** Average the five risetime measurements for $\dot{\theta}_m$ from Part II–2 and divide by 2.2 to determine the motor time constant τ_m. Explain why this calculation gives you the motor time constant.

Questions:

1. In the prelab, you found an expression for the motor time constant. How well does the measured value agree with a calculated value based upon your model parameters? (Remember that the flywheel was *removed* for these measurements.)

2. Two of your motor parameters should be the same. Which two? Why? Are they?

3. How does the inertia of the flywheel compare to that of the big gear? Why?

Just to give you an idea...

Here are some order of magnitude approximations for the parameters you will be concerned with:

$$\begin{aligned}
R_m &\sim 10^0 \\
L_m &\sim 10^{-3} \\
K_e &\sim 10^{-2} \\
K_{tach} &\sim 10^{-2} \\
J_f &\sim 10^{-5} \\
J_m &\sim 10^{-6} \\
K_t &\sim 10^{-2} \\
K_p &\sim 10^0
\end{aligned}$$

These are listed in SI units (which you determined in the prelab).

Dry-Lab Data

Measurements: Part I

- The discontinuity is 20 volts.
- Constant voltage drive: V_m, I_m, $\dot{\theta}_m$, and Δt

Measurement	1	2	3	4	5	Units
V_m	1.9002	3.9850	5.9546	7.9687	10.2702	V
I_m	0.0112	0.0235	0.0342	0.0436	0.0553	V
$\dot{\theta}_m$	3.8886	7.5883	11.0163	14.9751	19.2254	V/s
Δt	0.4754	0.2511	0.1593	0.1278	0.0958	s

- Square wave of current: I_m and $\frac{d}{dt}\dot{\theta}_m$

Measurement	1	2	3	4	5	Units
I_m	0.2053	0.3875	0.5872	0.7673	1.0076	V
$\frac{d}{dt}\dot{\theta}_m$	6.7782	13.3743	19.8200	27.0090	33.7718	V/s

Measurements: Part II

- Square wave of voltage: Δv, Δi, and t_r

Measurement	1	2	3	4	5	Units
Δv	2.0358	4.0543	6.2671	7.7671	10.2093	V
Δi	0.2782	0.5497	0.8663	1.1372	1.4227	V
t_r	0.9581	0.9683	0.9287	0.9581	0.9350	ms

- Square wave of voltage: t_r

Measurement	1	2	3	4	5	Units
t_r	0.0915	0.0915	0.0864	0.0895	0.0893	s

- Square wave of current: I_m and $\frac{d}{dt}\dot{\theta}_m$

Measurement	1	2	3	4	5	Units
I_m	0.2008	0.4148	0.5859	0.7854	0.9619	V
$\frac{d}{dt}\dot{\theta}_m$	69.4887	135.0599	199.2733	269.9538	334.2939	V/s