THE UNIVERSITY OF TENNESSEE KNOXVILLE

TheEmpireStrikesBack
SRM 678 D1 500 Pointer

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October 22, 2019

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The Problem

- Darth Vader wants to destroy all N planets in the galaxy.
- Darth Vader is aboard the Death Star, which has M missiles.
- Darth Vader needs to install a missile booster with strength S to use the missiles.
- Darth Vader wants to use the minimum strength booster $min(S)$ to destroy all planets.
- Given the planets' locations and M, find $min(S)$.

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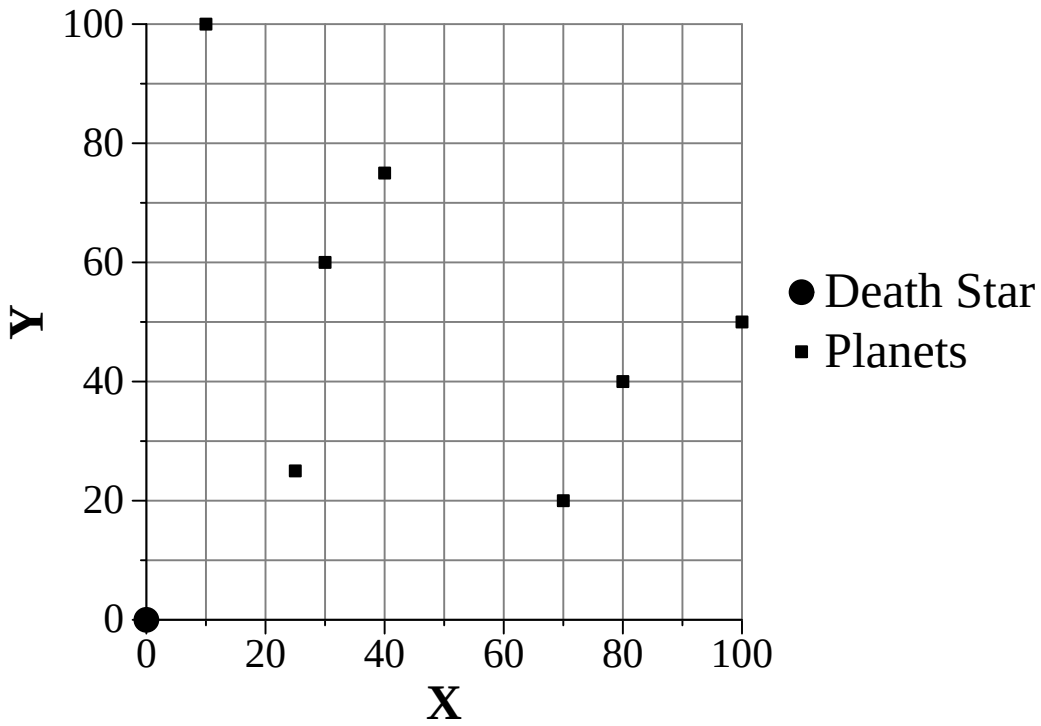
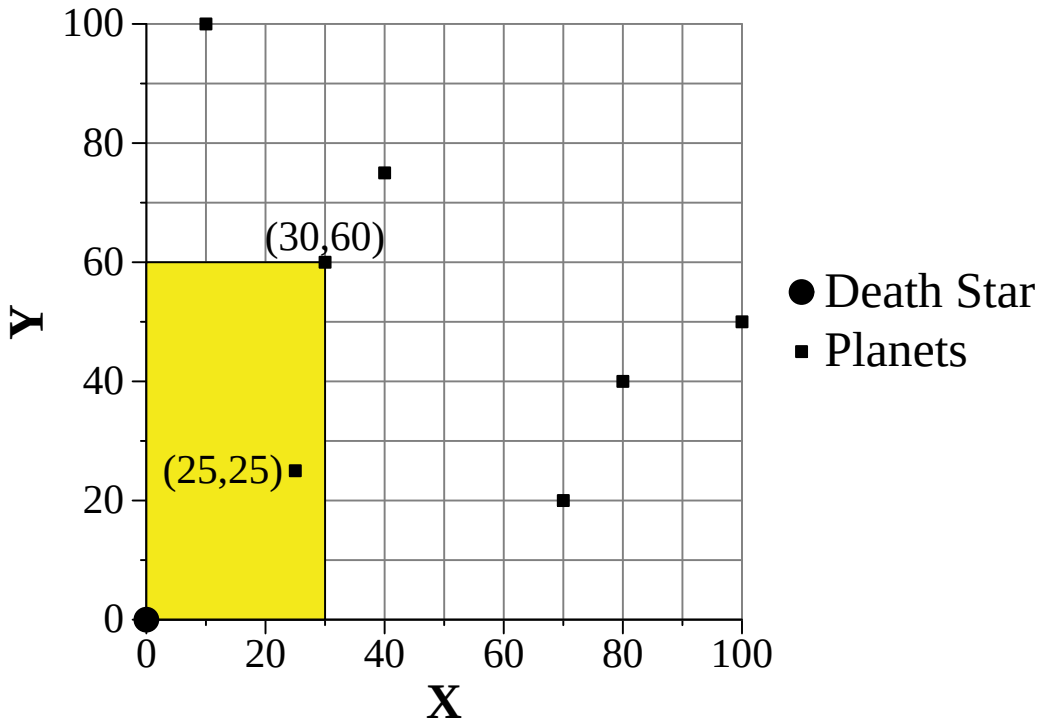
Galaxy Details

- The galaxy is in a 2D plane (first quadrant).
- Planets are located at some coordinate (x, y).
- The Death Star is always located at $(0, 0)$.



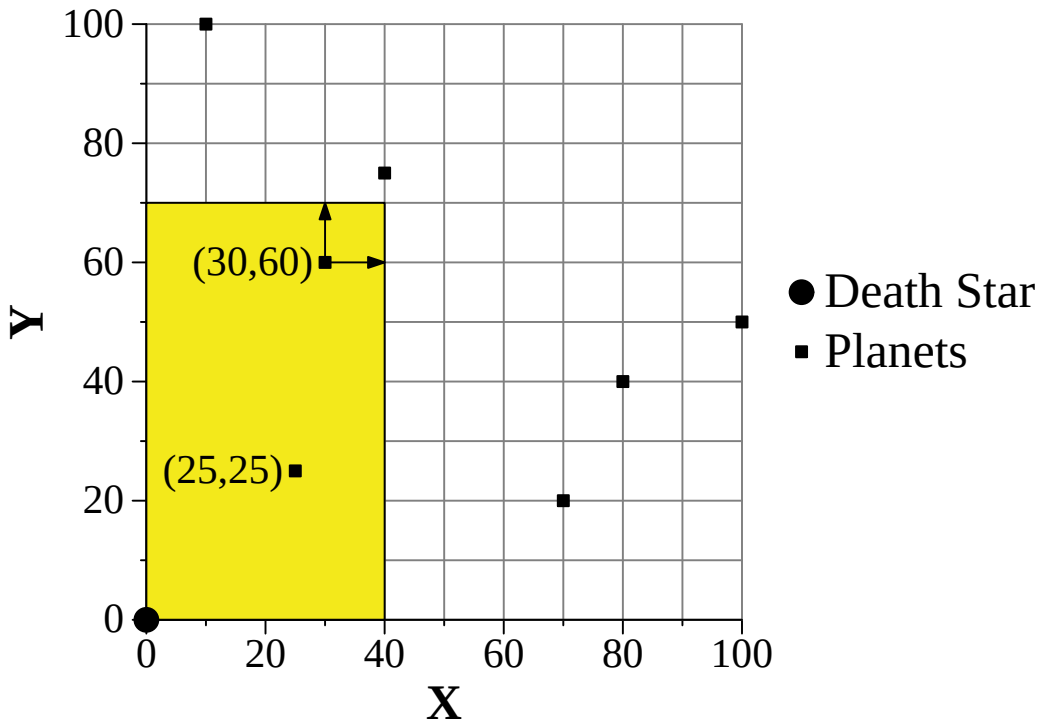
Missile Details

- Missiles can only target a planet's location.
- When a missile strikes (x, y), it destroys all planets within the bounding box specified by $(0, 0)$ and (x, y), inclusively.
- Example: striking (30,60)



Booster Details

- The missile booster strength S may change the bounding box.
- The bounding box is now specified by $(0, 0)$ and $(x + S, y + S)$.
- Example: striking (30,60) with booster strength $S = 10$



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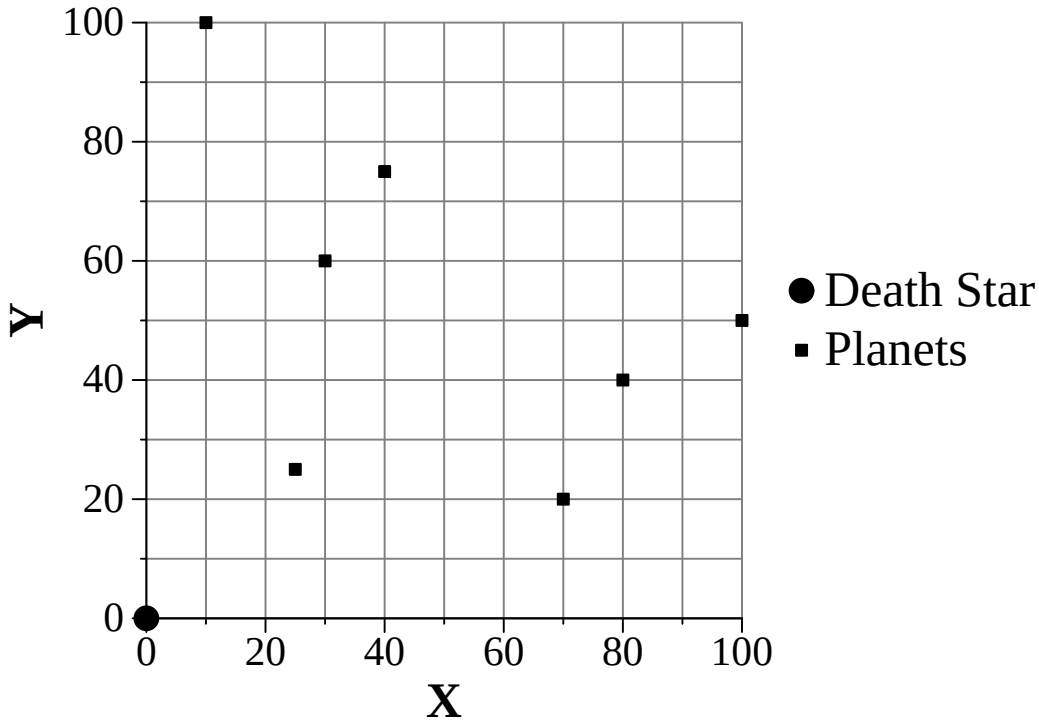
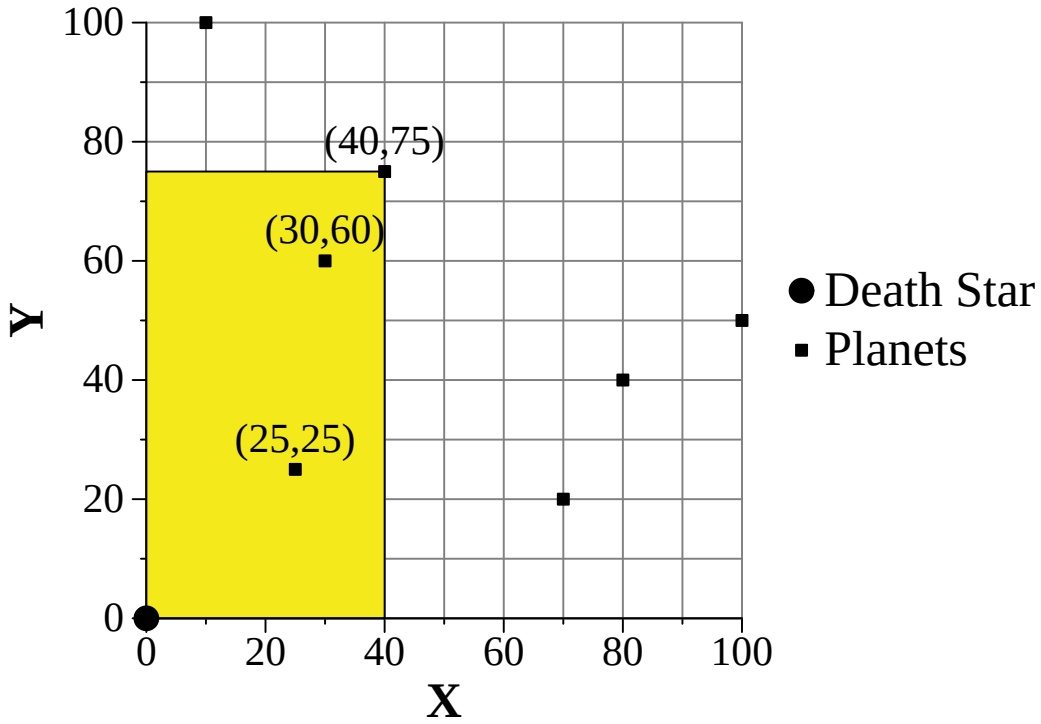
Example

Planets are at locations $(10, 100)$, $(25, 25)$, $(30, 60)$, $(40, 75)$, $(70, 20)$, $(80, 40)$, $(100, 50)$.



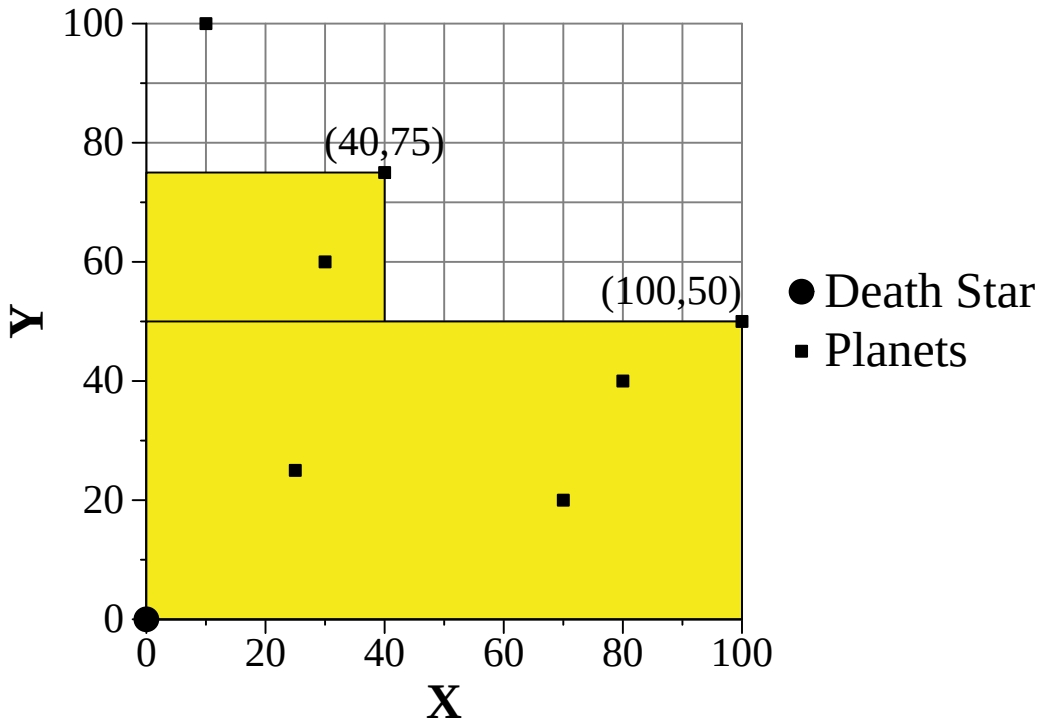
Example

What happens if we strike planet at $(40, 75)$ with a missile? Booster strength $S = 0$.



Example

If we want to eliminate most of the planets, we can strike the outer planets of the galaxy.



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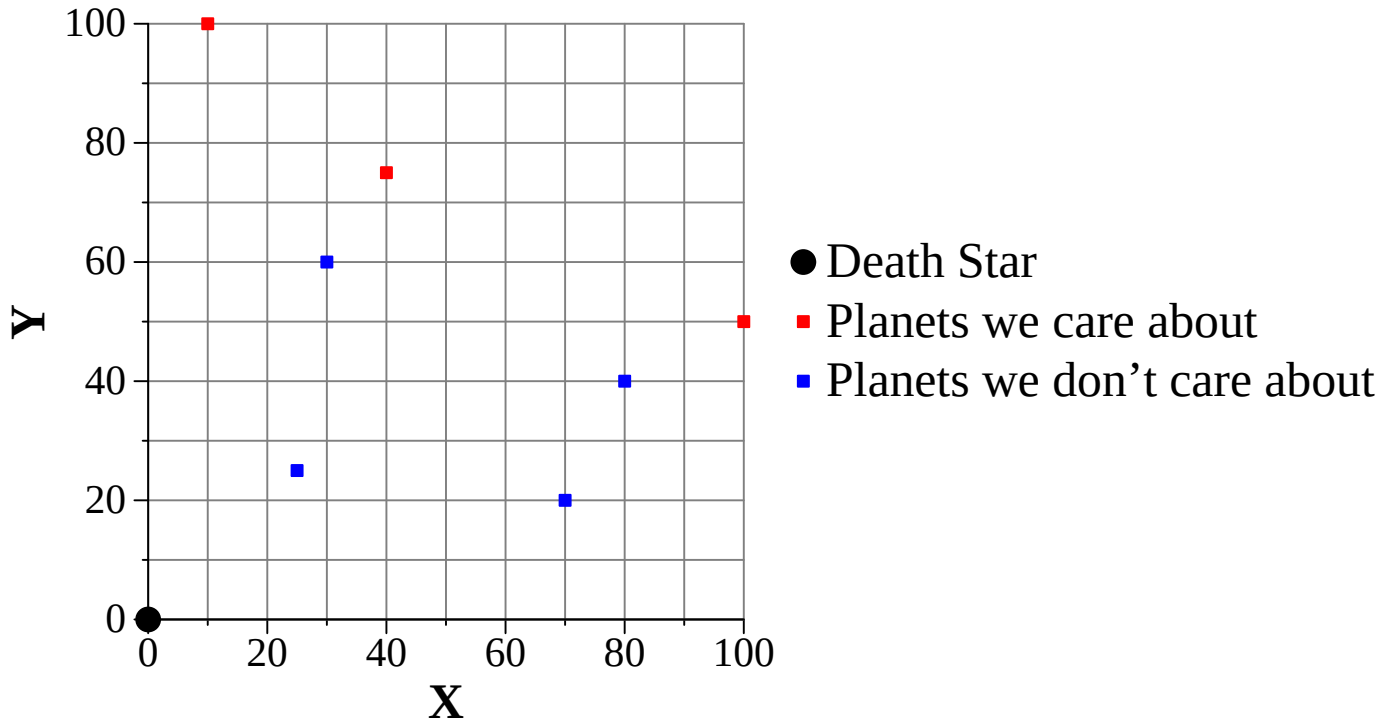
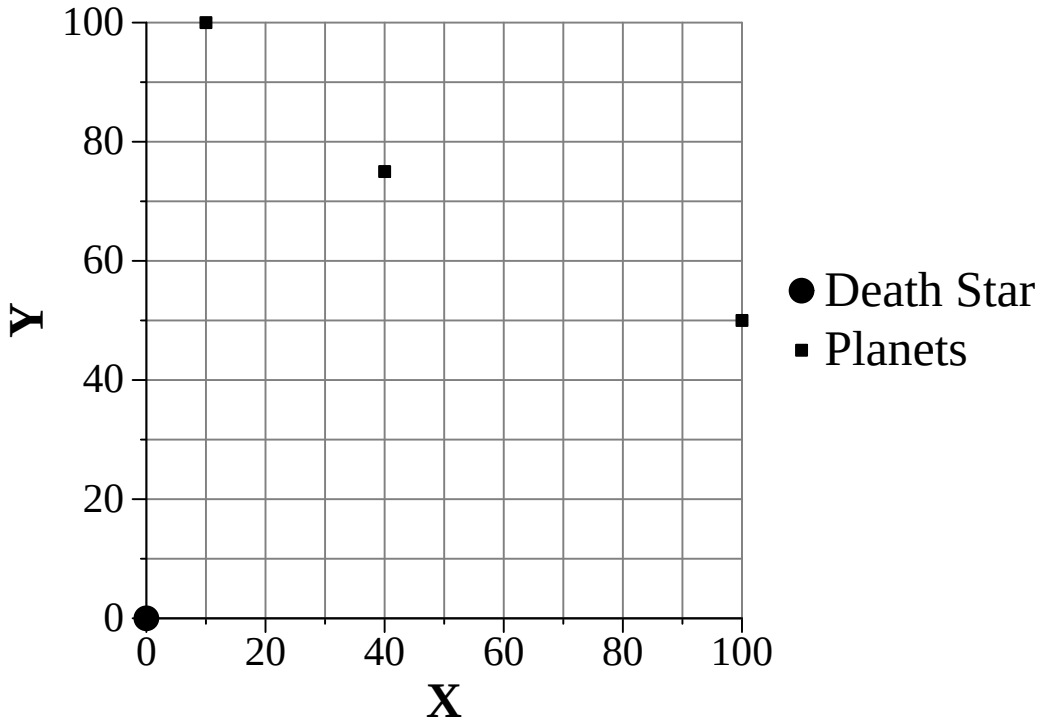
Example

We don't really care about planets within the strike of other planets.



Example

This simplifies the problem. Now we only have planets at locations $(10, 100)$, $(40, 75)$, $(100, 50)$.



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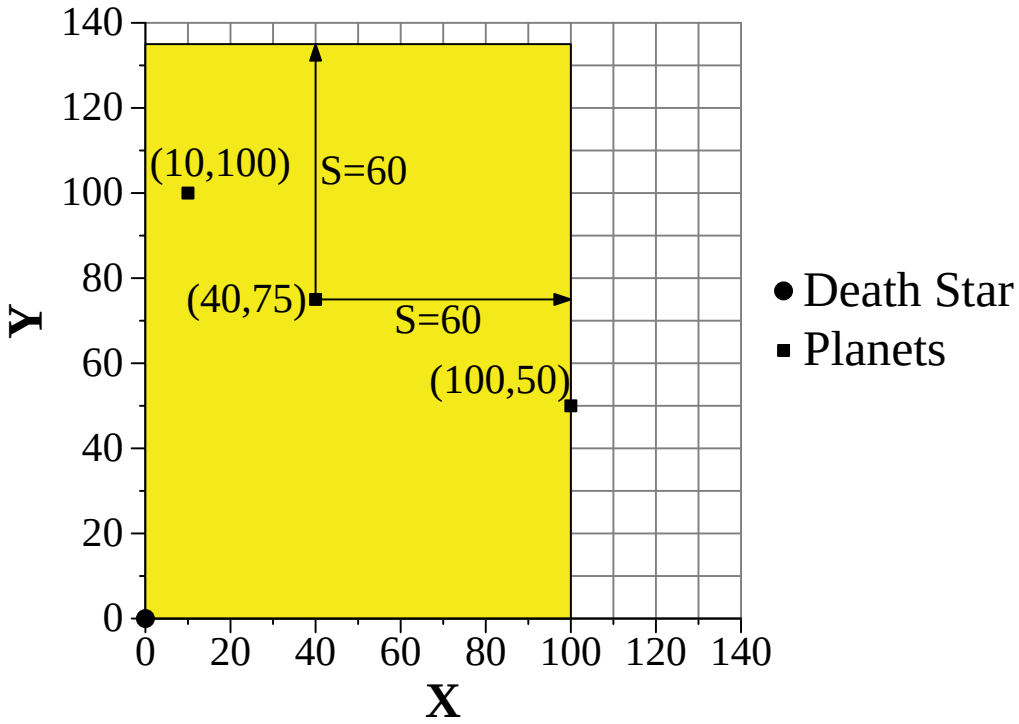
Example

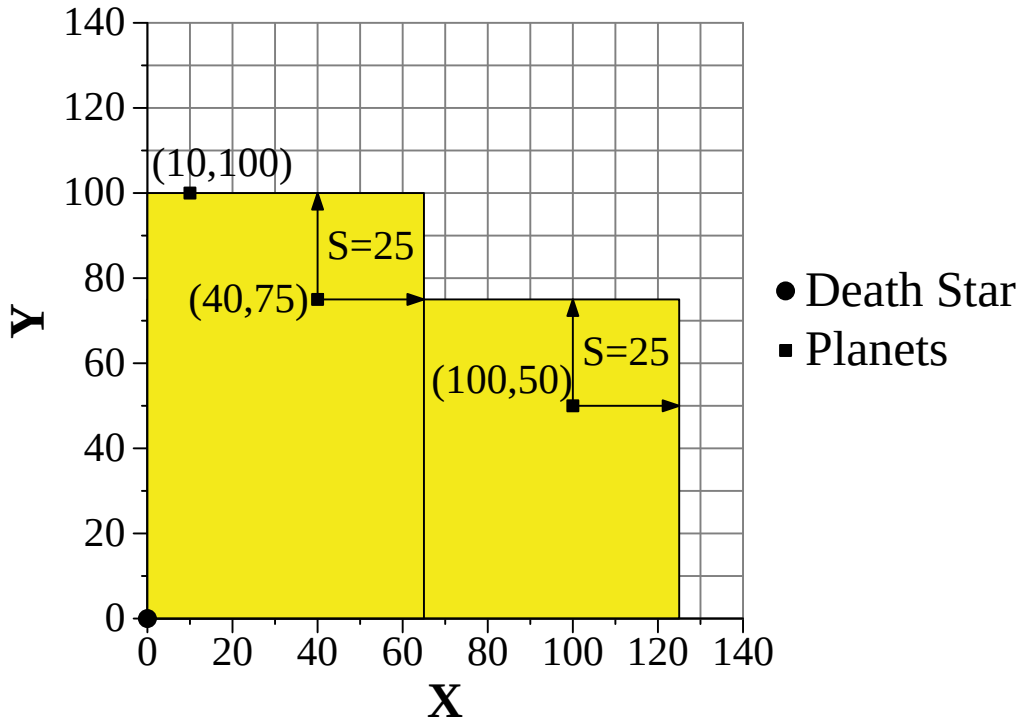
What if we are given $M = 1$ missile?



$min(S) = 60$

Example

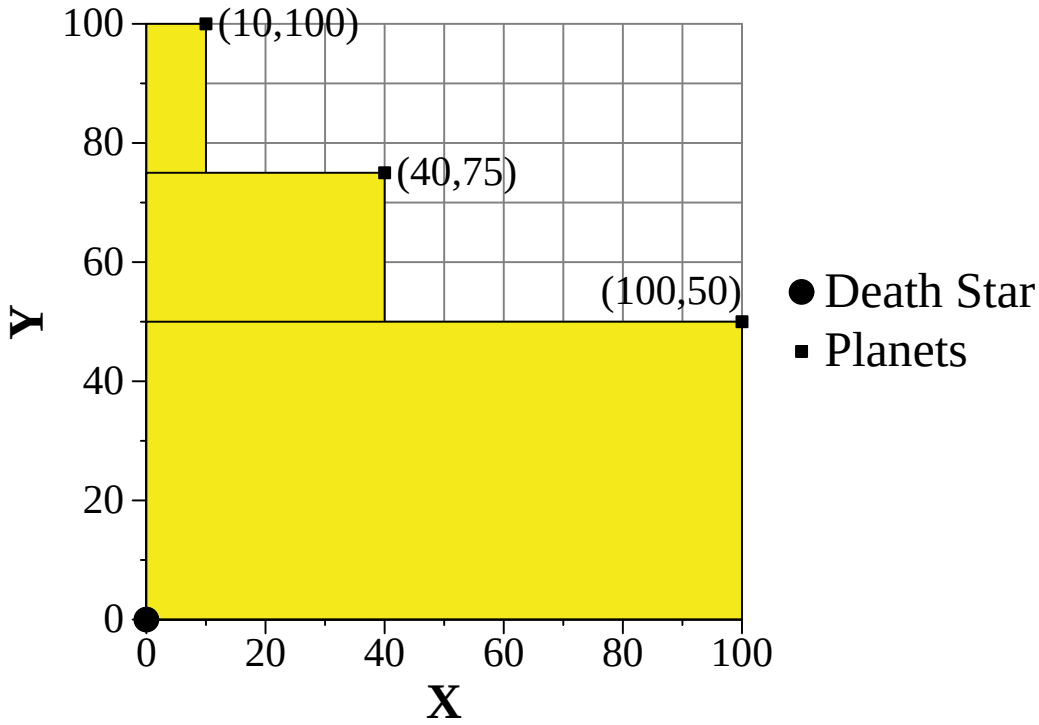
What if we are given $M = 2$ missiles?



$min(S) = 25$

Example

What if we are given $M = 3$ missiles?



$min(S) = 0$

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Prototype

- **Class name:** TheEmpireStrikesBack
- **Method:** find()
- **Parameters:**

AX	int	determines planets' locations
BX	int	determines planets' locations
CX	int	determines planets' locations
AY	int	determines planets' locations
BY	int	determines planets' locations
CY	int	determines planets' locations
N	int	number of planets
M	int	number of missiles

- **Return Value:** int

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Constraints

- AX, BX, CX, AY, BY, CY is between 0 and 10^9.
- N is between 1 and 10^5.
- M is between 1 and N.

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Something to notice

- We know N is between 1 and 100,000
- Because we can only do roughly 10,000,000 operations, N^2 is too slow

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Algorithm

- Find all the planets locations given N, AX, BX, CX, AY, BY, and CY. $O(N)$
- Insert the planets into a multimap keyed on x. $O(Nlog(N))$
- Eliminate planets that we do not care about. $O(N)$
- Put the remaining planets into a vector. $O(N)$
- Perform binary search. Given M and remaining planets, determine if S is valid. $O(Nlog(maxsize(S)))$

Algorithm - Determine if S is valid. `isValid(S, M, planets)`

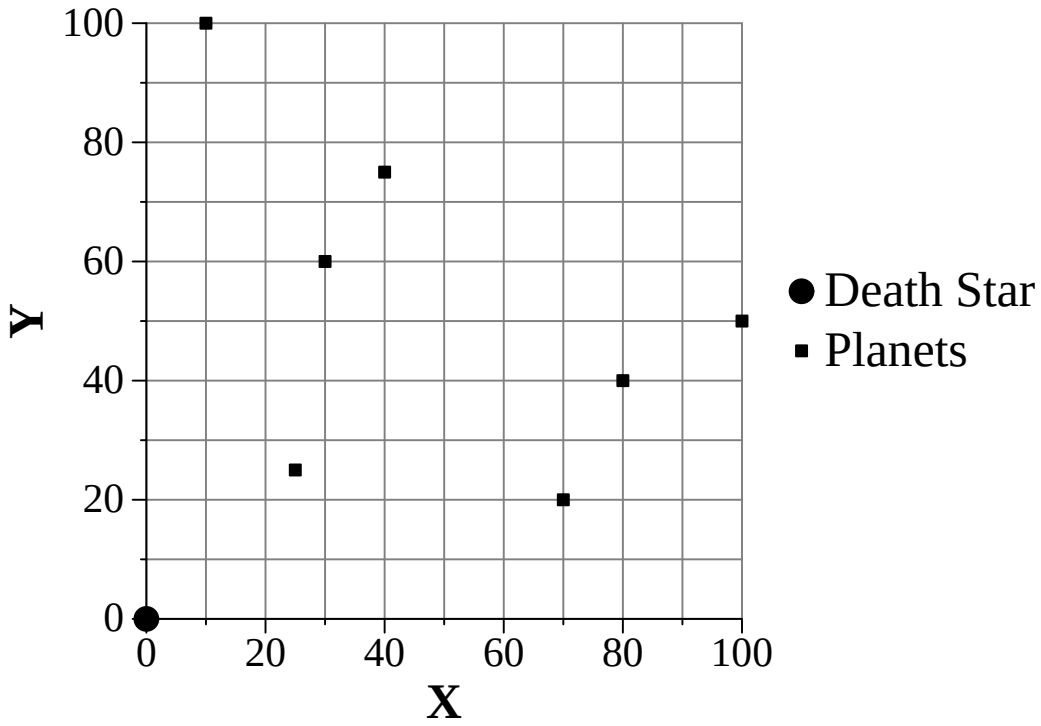
Loop:

- If M is equal or larger than the number of planets left, we **can** destroy them all.
- If $M = 0$, we **cannot** destroy them all.
- Strike first planet of vector, and eliminate planets that are also hit by missile.
- Decrement M.

This is linear in the size of planets.

Algorithm Example

- Planets are at locations $(10, 100)$, $(25, 25)$, $(30, 60)$, $(40, 75)$, $(70, 20)$, $(80, 40)$, $(100, 50)$.
- $M = 2$



Algorithm Example - Insert the planets into a multimap

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Key	Value
10	100
25	25
30	60
40	75
70	20
80	40
100	50

Algorithm Example - Eliminate planets

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Key	Value
10	100
25	25
30	60
40	75
70	20
80	40
100	50

Algorithm Example - Eliminate planets

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Key	Value
10	100
25	25
30	60
40	75
70	20
80	40
100	50

Algorithm Example - Eliminate planets

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Key	Value
10	100
25	25
30	60
40	75
70	20
80	40
100	50

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Algorithm Example - Binary search

- The binary search will call `isValid` 63 times if S can be a `long long`.
- Let's do a few examples of `isValid`.

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Algorithm Example - `isValid`

$S = 10$

x	y
10	100
40	75
100	50

M=2

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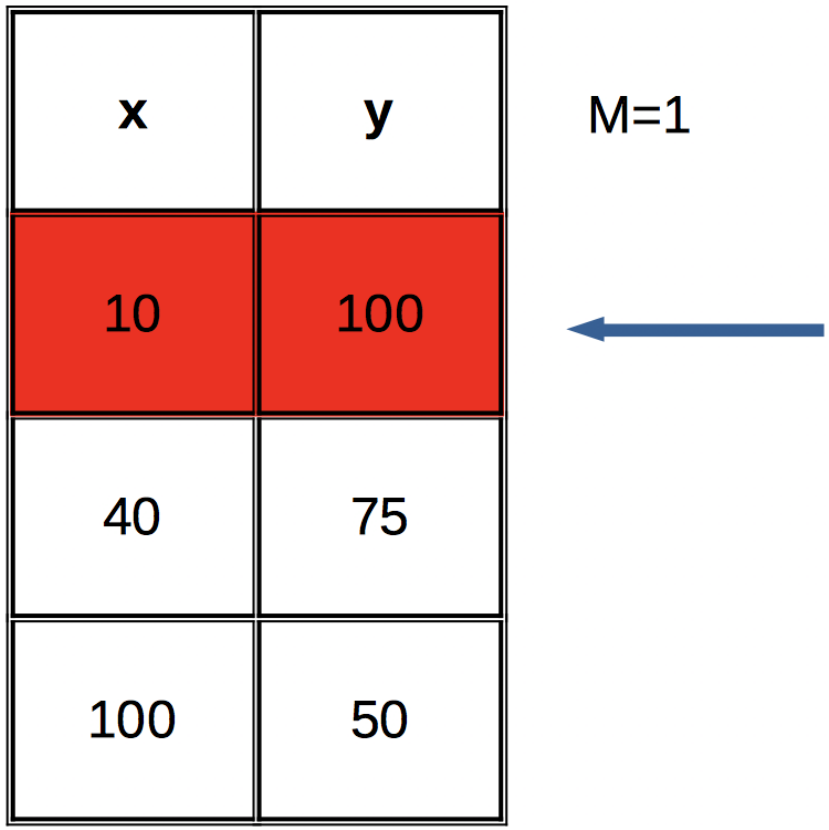
Algorithm Example - `isValid`

$S = 10$



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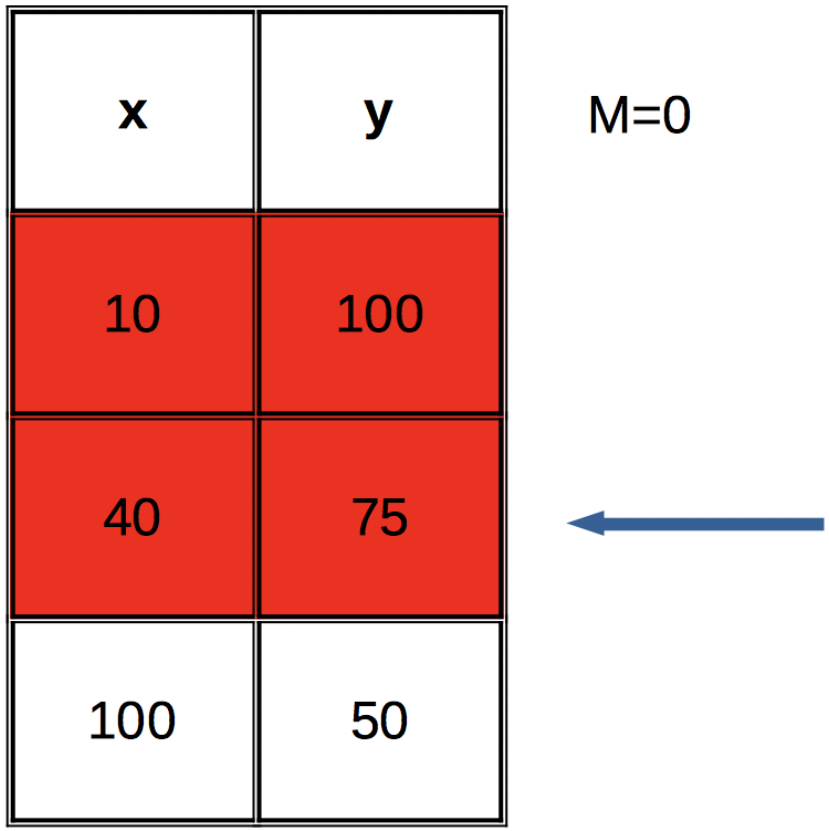
Algorithm Example - `isValid`

$S = 10$



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Algorithm Example - `isValid`

$S = 25$

x	y
10	100
40	75
100	50

M=2

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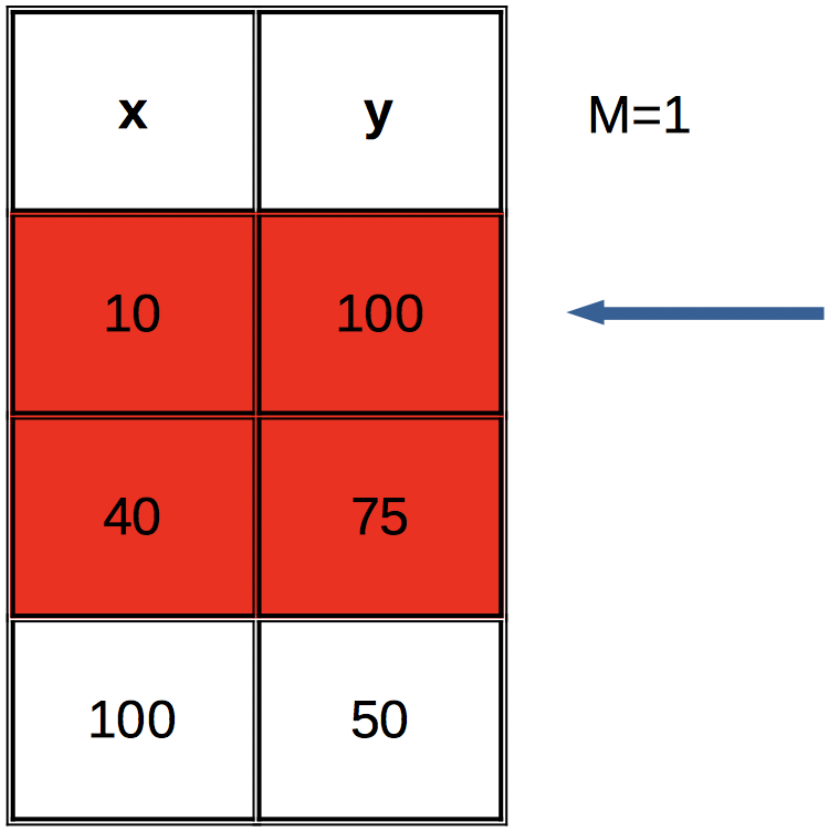
Algorithm Example - `isValid`

$S = 25$



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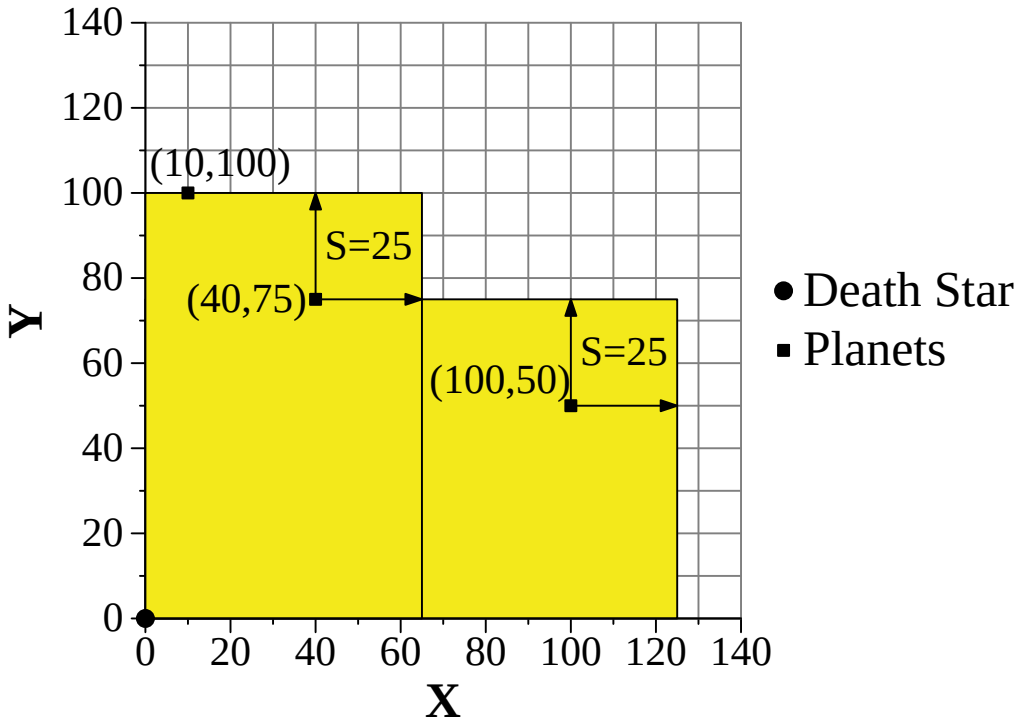
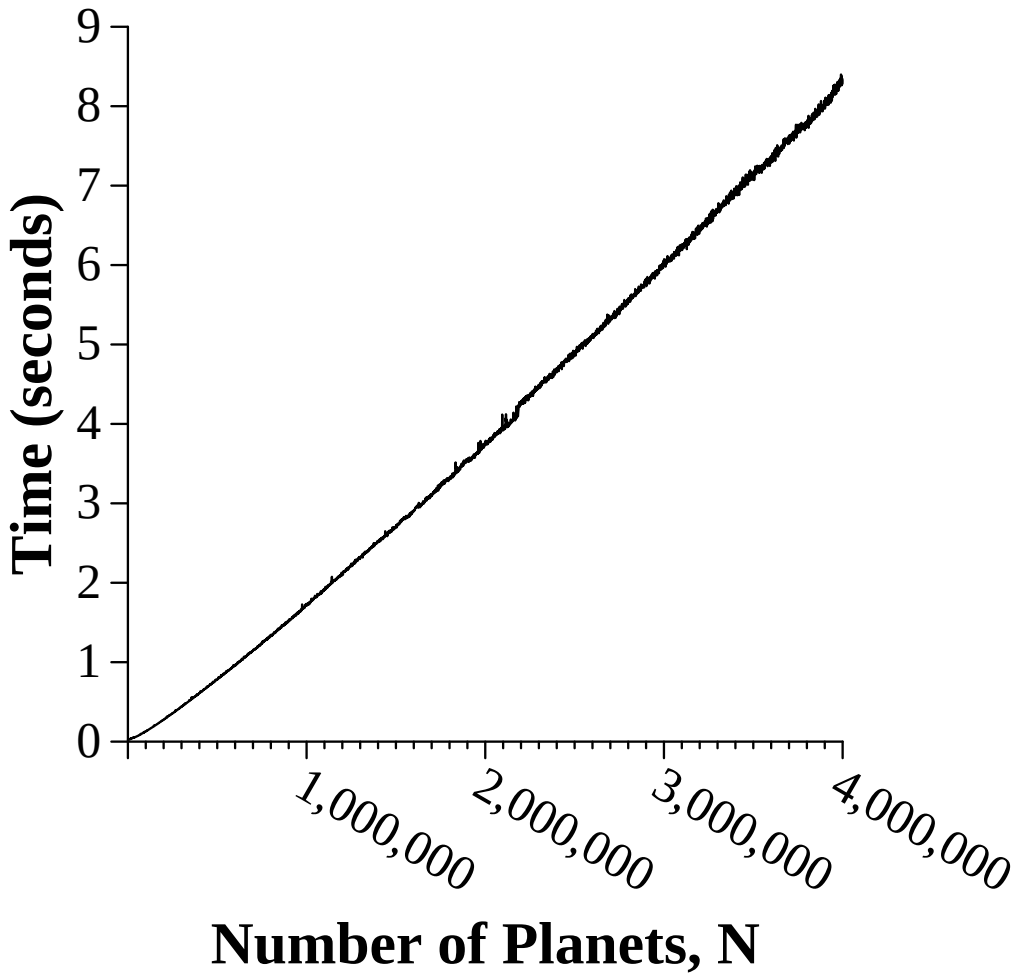
Algorithm Example

The binary search using `isValid` finds the minimum S. $S = 25$



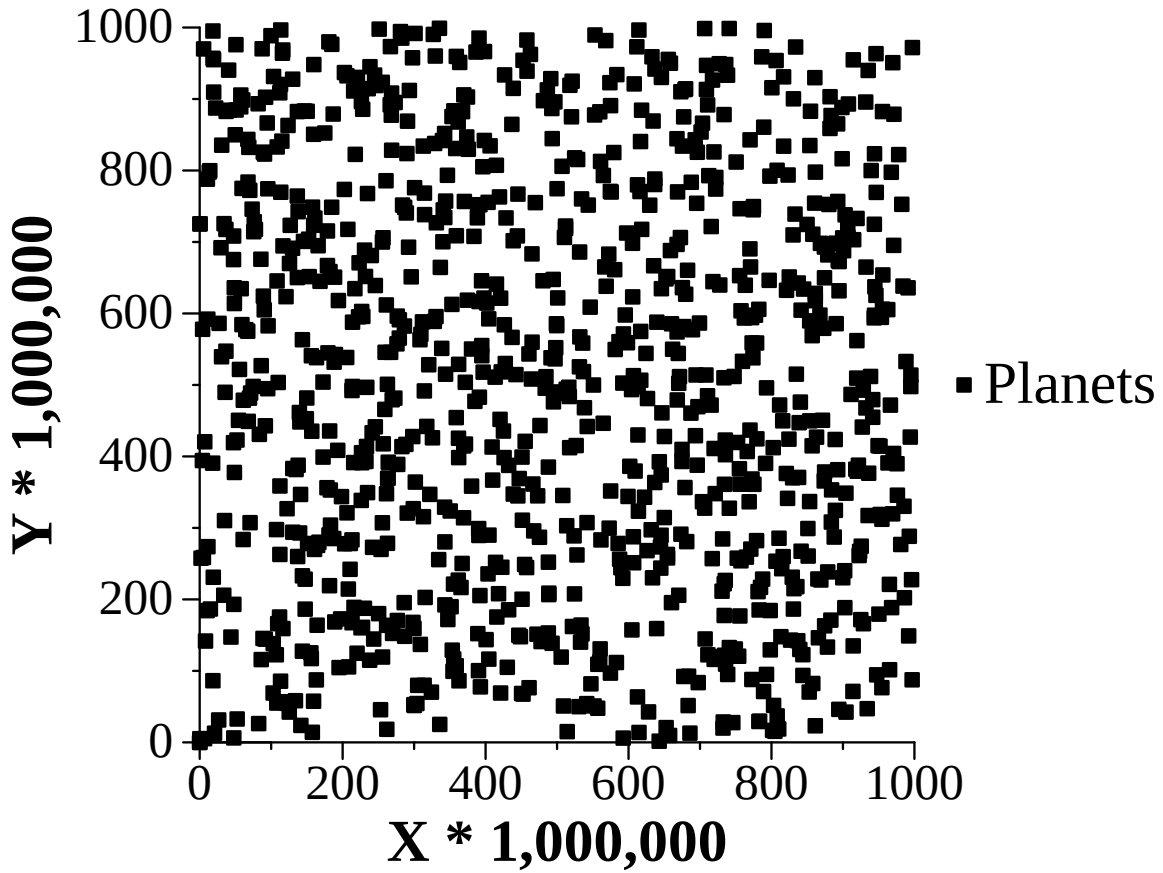
Performance

- Frobnitzem, my Linux machine
- Intel i5 at 2.6GHz



Test Galaxy

- $AX = 10,\ BX = 20,\ CX = 30$
- $AY = 40,\ BY = 50,\ CY = 60$
- $N = 1000$



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How did the Topcoders do?

- 255 Topcoders opened the problem.
- 119 (46%) submitted a solution.
- 99 (83%) of the submissions were correct.
- Success rate was 38%
- Best time was 8:11
- Average correct time was 29:35

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