Build Your Own Search Engine

An Introduction to Machine Learning Methods in Text Mining

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Executive Summary

Search pervades everyday life in the twenty-first century. Students, researchers, and ordinary people alike enjoy the privilege of record-speed information retrieval from the Internet. Despite its fundamental importance, search is complicated. It leans heavily on the subject of text mining—the extraction of pertinent information from natural language documents—a subject which faces many challenges, both theoretical and practical. How does one model the content of a textual document? The same words appear in many documents, but each document may use those words differently. What determines a document's semantics? What about semantic similarity? Most importantly, how can one measure the semantic features of a document in real time? It turns out that many of these questions can be addressed at least in part by machine learning. In particular, the issue of semantic categorization can be reduced to a multiple-classification problem, one that is accurately solved by support vector machines.

We address these questions in the present paper, namely the questions of textual document representation and categorization. We will discuss methods for representing textual data, isolating the semantics of a textual document, and predicting attributes of new documents with learned classifiers. Specifically, we will discuss: term-frequency/inverse-document-frequency representations of text and measures of similarity between different texts; semantic noise reduction of many documents via latent semantic analysis; and supervised categorization of textual documents using k-nearest neighbors and support vector machines. At the end, the reader will be tasked with building a simple search engine for the Reuters-21578 corpus, whose implementation will depend on the methods of text mining discussed in the background.

Contents

1 Background

1.1 Problem Description

Consider a law firm that wants to build a strong defense for its client: thousands of legal documents are available to the firm, but only a small fraction of the information contained in these documents is useful for the given case. How would lawyers go about gathering pertinent information or evidence? On one hand, the lawyers could read every word in every document available and then hope to remember it all by the end. On the other hand, such brute force is inhumanly difficult and exorbitantly expensive. More to the point, not all available information is useful. In fact, chances are that the vast majority of legal documents are irrelevant to the case at hand. For example, if the lawyers are handling a case on tax evasion, they probably do not want to read about domestic laws. The firm ultimately needs an automated and highly efficient method for organizing textual information and for retrieving the pertinent information quickly.

This problem of organizing textual information is not new. Humans began writing somewhere around 5000 B.C. and since then have created a monolithic amount of written information. For hundreds of years, the only way to research pertinent information was to visit a library and page through book after book. The process was slow and oftentimes unfruitful. Then, with the advent of the internet, the world rejoiced at the new availability of information and yet faced an organizational crisis. What should be done with this monolith of textual information? In what ways might all this information be useful? Most importantly, how could this information be made accessible to the general public? These questions emphasized the need for search engines, programs that could troll through large masses of web content and pick out only the relevant bits of information in real time. During the 90s, companies like Yahoo!, Ask Jeeves, and eventually Google sprang up, each offering unique methods for indexing webpages and providing speedy information retrieval. Search engines have since become a staple and keystone for day-to-day life in the twenty-first century.

Underpinning much of search is the concept of **text mining**. Text mining refers to the organization and extraction of information from natural language documents. Note that search and text mining are not one and the same. Search engines typically undertake a number of steps to produce results, including indexing and query enrichment. By contrast, text mining is a more general collection of methods for extracting data from textual documents. Some examples of text mining include part-of-speech tagging, concept extraction, and sentiment analysis. In this paper, we will focus on the representation of textual documents as vectors and the categorization of textual documents into predefined categories. We will see that, with the right precautions, document categorization works out to be a fairly straightforward classification problem.

Document categorization has many immediate applications. Lawyers may wish to automatically sort large quantities of text into semantic categories (domestic law, tax law, etc.); companies may wish to categorize incoming email and eliminate spam; search engines may want to categorize new queries to restrict their search space; and so forth. Despite its usefulness, document categorization is not trivial. It is not immediately obvious what determines whether a document belongs to a category. Language is by nature ambiguous. Some studies from cognitive science suggest that this ambiguity is evolutionarily advantageous because it eases the language learning process for young children, but that ease does not surface for computers. In particular, polysemy—the presence of multiple meanings for the same word—makes it difficult for computers to disambiguate between pairs of sentences like "I climbed the steep bank," and "I deposited a check in the bank." On the other hand, language is often redundant and dilutes information with unnecessary synonyms and clarifiers like pronouns and tense. Worse yet, the categorizer may have limited access to information about the documents. In the case of web search engines, the webpages will not come with category labels; it is up to the search engine to organize the data into semantic categories.

To address these problems, we need an effective way to encode the semantics of a document without getting lost in the ambiguity and noisy redundancy of natural language. We present methods for doing so in the proceeding section.

1.2 Representation of Textual Data

In order to extract information from documents, we must first define what we mean by document. For our purposes, a **document** is a sequence of natural language tokens. In this sense, the sentence "I ate an apple" is as much a document as the entire play *Hamlet*. It is worth noting that although our definition of document is limited to text, other definitions are typically more general so as to include any collection of information in written or electronic form. In fact, many of the methods for text document representation and categorization that we discuss here will apply to other types of documents as well.

As one might expect, raw strings are not conducive to mathematical manipulation and therefore are not conducive to large-scale search. Performing string comparisons is both slow and inaccurate; the user is looking for a document semantically similar to his query, not for an exact textual match to his query. The alternative is to represent documents as vectors by defining a global list of features, and then encoding each document in terms of these features. Thus, for a corpus with m features, each document will be encoded as an m-dimensional vector. The list of features is typically determined by the characteristics of the entire corpus.

Here, we will employ the term-document model. In the term-document model, each feature corresponds to a term from the collection of all terms taken over all documents. Its value is given by the number of times that term appears in the document. For example, let $d = [d_1 \ d_2 \ \ldots \ d_m]^T$ be the feature vector given by some new document, and $\mathcal{V}$ the collection of all terms (features) in the corpus. If term $v \in \mathcal{V}$ appears in the document k times, we have $d_v = k$, noting that we let each term correspond to some index of the

vector. Unfortunately, this encoding is still meaningless: a document might contain the word "the" in 1256 different places, giving it a large feature value, but that does not mean the word "the" is semantically salient. As a first step to remedying this, we need to weight each term inversely to how often it is used throughout the corpus, under the assumption that rarer words better identify the semantics of a document. This encoding is called the **term frequency-inverse document frequency (tf-idf)** because each term frequency feature is multiplied by the inverse of its corpus frequency.

This encoding gives rise to a natural similarity measure for documents, namely, the normalized dot product or cosine similarity measure. For documents d_i and d_j in the same corpus, the similarity of d_i and d_j is given by $\frac{d_i^\mathsf{T} d_j}{||d_i|| \cdot ||d_j||}$. Any features that are not shared by both documents will be zeroed out, so the result is entirely determined by tf-idf values for terms that appear in both documents. Moreover, since common terms will have low weights, they will contribute little to the dot product, while shared rare terms like "interpolate" or "Nietzsche" will contribute a great deal to the dot product. This is in line with our assumption that rare words probably have semantic salience. Note that we can also obtain a measure of term similarity in this way: if we arrange each document vector as a row in a matrix, we see that the columns form feature vectors for terms, where the ith feature is the tf-idf of that term in document i. This is an intuitive representation because terms that are semantically similar will likely appear in the same document. As one might expect, the normalized dot product of two term vectors gives a rough similarity measure of those terms.

The term-document model is good at highlighting the semantic hotspots of a document— that is, the rare and potentially salient words—but it does not completely remove the redundancy of natural language. We must take other steps to reduce the noise from each document. One cheap solution is to remove short words from the feature space. This works because most short words are either articles or acronyms that are spelled out elsewhere in the text; however, there is still the risk of eliminating important information. Supposing we chose to remove all words of length three or less, say, in an effort to remove "the" from consideration, we might in the process eliminate the word "bee" from a text on endangered species, which is clearly undesirable. A more effective approach is to remove all **stop words** from the corpus, where a stop word is defined to be any word that contributes no information to the text, like "the." The collection of stop words varies from case to case.

Finally, before featurizing a corpus, we should make sure to remove any unnecessary prefixes or suffixes from each word, so that the words "walk" and "walked" are not considered separate features. This is called **stemming**. We omit the details.

1.3 Latent Semantic Analysis

In the previous section, we hinted that an entire corpus can be represented as a single matrix, where each row is a document vector. In fact, this representation exposes information about the corpus as a whole, and provides a means for extracting and simplifying that information.

The astute reader will have noticed that the term-document model results in sparse vectors with many dimensions. Some of these dimensions are virtually useless, while

others match the semantic category of a document but are zeroed out because the term represented by that dimension does not appear in the document. Despite our efforts at normalization with the term-document model, we still have not accounted for the problem of synonomy. We want to uncover the latent semantic structures that underly a corpus. In other words, we would like to discover the semantic relationships between terms and to combine synonyms such that our term vectors (the columns of the corpus matrix) represent something closer to a semantic category than a specific word. For example, "chihuahua" and "dog" are different words but both belong to the same semantic category. From a search perspective, we would like to consider a document that mentions chihuahuas as similar to a query for dogs.

We can achieve this dimensionality reduction by performing **latent semantic analysis (LSA)** on the corpus matrix. Latent semantic analysis involves finding a low-rank approximation of the corpus matrix. To do this, we apply the Eckart-Young theorem, which states that the best k-rank approximation for an $n \times m$ matrix, $k < m$, over some unitarily invariant norm is given by the outer product of the the first k singular values and vectors of the original matrix. Formally, if the singular value decomposition of a matrix A is given by $\sum_{i=1}^{r} \sigma_i u_i v_i^\mathsf{T}$, then the best k-rank approximation to A is:

$$A_k = \sum_{i=1}^{k} \sigma_i u_i v_i^\mathsf{T}$$

We can think of this approximation in a number of ways. On one hand, some dimensions (terms) are being zeroed out, namely those dimensions with the least semantic significance across all documents, so we are reducing the sway of redundant or unhelpful term features. On the other hand, terms that were previously zeroed out in certain documents may now be nonzero. We can think of this as stretching terms into k of the most salient semantic categories across the entire corpus, where each term has varying membership to each category. Notice that this analysis depends heavily on the assumption that similar words in semantic space will generally co-occur in documents.

Typically, we choose $k \approx 100$, though studies have shown that there are good results with k as small as 50 or as large as 1000.

1.4 Categorization of Textual Documents

We now have the tools to discuss methods of document categorization. As we have suggested, document categorization boils down to a supervised classification problem with a few modifications. We will discuss and compare three different classifiers as they relate to document categorization: k-nearest neighbors (kNN), least squares (LS), and support vector machines (SVMs).

1.4.1 Formalization

We begin by formalizing document categorization as a classification problem. Let $\mathcal{D}$ denote the set of all documents in the corpus and $\mathcal{C}$ the set of all semantic categories, which we take to be predefined. Formally, our input is a set of tuples (d_i, C_i), where $d_i \in \mathcal{D}$ denotes the tf-idf vector for document i and $C_i \subset \mathcal{C}$ denotes the set of categories to which document i belongs. Notice that the labels are not individual objects but sets. It is easy to imagine cases where a document belongs to multiple semantic categories. Thus, for a new, unlabeled document d_* we would like to predict the set of semantic categories C_* to which it belongs.

Already it should be clear that this problem is more complicated than other classification problems. The classifiers we will discuss are prototypically binary classifiers—that is, new examples belong to exactly one of two classes—so we will have to find an adequate modification that (a) can classify examples into more than just two categories and (b) allows examples to belong to more than one category. This is called the problem of **multiple classification**, and we address it for each classification method in turn.

1.4.2 k-Nearest Neighbors

The first classification scheme we will discuss is **k-nearest neighbors**. We can imagine our document vectors as points in m-dimensional semantic space, where spatial proximity correlates with semantic similarity. The kNN classification scheme grounds itself in this idea and assumes that new, unlabeled examples will be similar to the labeled examples that are nearby in feature space. Specifically, for $k > 0$, the kNN classifier chooses the k points from the training data that are closest in Euclidean distance to a new data point and takes the majority class of the nearest neighbors to be the class of the new example. To visualize this, consider figure X: if $k = 3$, then the new point would be labeled as red since two of its three closest neighbors are red; if $k = 5$, the new point would be labeled blue.

kNN has the advantage of naturally addressing the mutliple-classification problem. For each of the k nearest neighbors, we increment the vote count for each class to which that neighbor belongs. We then ignore classes whose vote counts are lower than some constant threshold and choose the r most-voted of the remaining classes to be our classification.

This kNN classification scheme has been shown to work well for document categorization. In fact, it is the most common method of document categorization because it is easy to implement and because it does not require us to train a classifier. The latter advantage is also a significant disadvantage, however, because new training documents must be compared to every training document to obtain a classification, which results in $O(nm)$ classification time complexity for corpora with n documents and m term features. This complexity is especially bad when we consider that both n and m are typically quite large. Another disadvantage of kNN is that the classification results may vary as you change k. We could compute the optimal k by trying many values of k and choosing the one that minimizes classification error—this results in a pareto curve where k is graphed on the x-axis and classification error is graphed on the y-axis. Unfortunately, this computation is slow and expensive for large datasets.

1.4.3 Least Squares

Next we will discuss the **least squares** classifier. Least squares classification grounds itself in the assumption that classes are determined as a linear combination of term features. We want to learn the optimal coefficients for said linear combination. To do this, we minimize the squared loss between predicted classes and actual classes in a set of training data. Formally, given our document matrix A, a vector w of weights, and a vector b of numerical labels, the predicted labels are given by $\text{sign}(Aw)$, where the sign is taken element-wise, and we wish to find $\text{argmin}_w ||Aw - b||_2^2 = \hat{w}$. We can then classify a new document a by evaluating $\text{sign}(a^\intercal \hat{w})$, where again the sign is element-wise. Practically, we can solve this problem by computing the pseudoinverse of A and multiplying it by b.

As we've already seen, least squares on its own is not sufficient for document categorization because it cannot accurately distinguish more than two categories. In practice, we have two choices for multiple-classification schemes with binary classifiers. The first scheme is called **one-vs-all classification**, where one classifier is trained per category that learns only to distinguish its category from all others, and each classifier returns a continuous measure of class membership as opposed to a strict yes/no answer. Thus, if there are k classes, then the one-vs-all schema trains k different classifiers. The second schema is called **one-vs-one classification**, where $k(k-1)/2$ binary classifiers are trained, one for each unique pair of categories. Each classifier learns to distinguish between its two given categories. To classify a new example, we apply each classifier and choose the class with the majority vote.

Both schemas have advantages and disadvantages. However, one-vs-one classification tends to be the method of choice. Although it requires us to train more classifiers, we train each classifier on fewer examples. Moreover, the voting system helps to eliminate noise in the data and gives us a means of determining multiple-class membership for new examples—we need only iterate through the class votes.

If we again think of document vectors as points in semantic space, then least squares finds the optimal hyperplanes for separating each pair of categories. Of course, the data might not be linearly separable, but we can make it separable lifting each point into a higher-dimensional space with kernel methods, and then we can apply least squares in the new feature space. We can do the same with the SVM. We omit the details.

Least squares tends to perform well on document categorization tasks, particularly when the semantic structure of a corpus has been compacted with latent semantic analysis. Although the kernelized SVM is preferred to least squares in most applications, the two classification schemes have comparable performance. In both cases, the loss function is generally regularized with respect to the L_2 norm so as to ensure that no one term carries too much weight. This makes sense because the semantics of a document are almost never determined by a single term—rather, the semantics depend on the culmination of all terms and concepts presented in the document.

1.4.4 Support Vector Machines

The last classifier we will mention is the **support vector machine**. Formally, the support vector machine learns the optimal weight vector w such that the hinge loss given by $\sum_{i=0}^{n}(1 - y_i a_i^\mathsf{T} w)_+$ is minimized, where y_i is the label of the ith document, a_i the feature vector for the ith document, and the last subscript represents a soft threshold.

Like least squares, the SVM finds a separating hyperplane between two classes. However, least squares struggles with outliers in the data. If a point is classified "too correctly," that is, it is classified correctly but is far from the decision boundary, then its squared distance from the hyperplane will be large and thus the least squares hyperplane will be pulled toward that data point. By contrast, the SVM does not penalize data that are classified very well. This is because the SVM finds the hyperplane with the greatest margin between the two classes. The resulting hyperplane depends only on the points closest to it.

As we have suggested, the SVM implemented with a one-vs-one multiple classification schema is the preferred classifier for document categorization. One reason for this is that we expect documents to be linearly separable in semantic space, and SVMs are known to learn robust linear separators. In fact, studies and experiments with the Reuters-21578 corpus (for which we will be constructing a search engine) show that almost every semantic category in the corpus is linearly separable by using an SVM. Additionally, SVMs lend themselves to high-dimensional feature spaces like textual corpora because they are built to combat overfitting. This is a result of the large margins enforced by SVMs—we can think of SVMs as leaving a lot of room for new examples.

SVMs perform even more favorably in conjunction with Tikhonov regularization. After we perform LSA, we expect few features to be irrelevant. Most or all of the remaining term features should contribute to the meaning of the document. As such, we want to avoid regularizers like the L_1 norm that promote sparse solutions. We also should not regularize with a quantizing regularizer like the L_∞ norm because we do expect some terms to be more important than others in determining a document's semantics.

2 Lab

In this lab, you will construct a small and simple search engine for the Reuters-21578 corpus. The Reuters corpus was released in 2000 and since then has become the most widely used dataset for research on text categorization. It is called Reuters-21578 because it contains 21578 articles taken from Reuters News. Each article is labeled with a number of semantic categories ranging from acquisitions to coconuts. Note that there is a lot of overlap between categories, and some categories have significantly more articles than others. The corpus has already been partitioned into a training set and testing set.

Real search engines are complicated and involve many intermediate steps like intention mapping, query enrichment, etc. We will focus on the material covered in the background and implement a simplified search engine. The search engine will organize the Reuters Corpus by category, with each category having its own training document matrix and testing document matrix. During initialization, the search engine will perform latent semantic analysis on all document matrices under each category, reducing the rank of each matrix by at most half. The search engine will then train a one-vs-one collection of SVM classifiers to categorize new documents. It will use this classification schema to categorize vectorized queries and limit its search space to promising categories. Finally, the search engine will compare the vectorized query to every document in the majority-vote categories.

To make your life easier, we have provided a code skeleton on top of which you will implement your search engine. Below is a brief description of the files we have provided.

File	Useful Functions	Read it?	Description
`pre.py`	None	No	Physically loads and vectorizes the Reuters corpus. Caches the results.
`corpus.py`	`train_matrix`, `test_matrix`, `complete_matrix`	Yes	Gets the Reuters corpus, vectorizes each document, and organizes the documents into category matrices, where each document in a category c's matrix belongs to category c. Provides utility for getting the training/testing partitions of a category. **You will use this code to help you test.**
`search_engine.py`	`search`	No	Uses the functions from the lab skeleton to implement the search steps described above.
`main.py`	`main`	No	Runs a simple interface for the search engine.

In particular, we ask that you spend some time looking at the functions in `corpus.py`. After instantiating a `Corpus` object, you can easily access the training matrix for the 'acq' category, for example, by calling `my_corpus.train_matrix('acq')`. Likewise, you can access the testing matrix for the 'acq' category by calling `my_corpus.test_matrix('acq')`, or the complete matrix by calling `my_corpus.complete_matrix('acq')`. If you want, you can create a dictionary whose keys are category names and whose values are training, testing, or complete matrices. The `Corpus` class also provides static methods for getting a document's categories or raw text.

Please note that your code will take a long time to run the first time. This is because our skeleton is vectorizing the corpus. After the first time, we cache the data in `pkl` files, so subsequent instances will run much faster.

2.1 Setup

First, you will need to download our skeleton code and make sure you have the required software and libraries to run it. We used Python 2.7.11 and pip 9.0.1 on OSX 10.0.1, but these versions for Python and pip are not strictly required. If you do run into problems, we recommend that you just upgrade to the latest versions of pip and Python 2.7. The installation steps are as follows:

1. Install Python and pip.

2. Install Python libraries dill, nltk, sklearn, and numpy through pip.

3. Download the data by running

   ```
   python -m nltk.downloader punkt stopwords reuters
   ```

   or by using `nltk.downloader()` from within Python and installing the punkt, stopwords, and reuters datasets.

4. Finally, you'll need the code we've provided, which you can find in the appendix or at https://github.com/EvanFredHernandez/byose.

2.2 Warmup

We begin with some routine exercises. These should expose you to our skeleton. Feel free to create a separate python file for these questions—they will not be used in the actual lab.

1. Create a `Corpus` object. Get the document vector with the document id 'training/309' calling the `document_vector` function of your Corpus instance. The 'training/309' document is about gold mines in South Africa. The document vector is long, but take a look and explain what the vector represents (what do the columns

correspond to?) and why there are so many zeros. If you're curious about the content of the document, you can call the static method `Corpus.document_text(doc_id)` to have a look.

2. Now get the document vectors for 'training/309' and 'training/448' and compute the normalized dot product between them. The 'training/448' document is about Brazilian gold mines. Now compute the normalized dot product between 'training/309' and 'training/3358' and compare this result to the previous. The 'training/3358' document is about agriculture and cereal ingredients. What do the dot products mean? Do the results make sense, relative to each other?

3. Now suppose that you have a matrix X that?s composed of stacked document vectors, like the ones you retrieved in the preceding parts. What would be the meaning of a matrix $XX^\intercal$ and each of its elements? What about $X^\intercal X$?

2.3 Build Your Own Search Engine

You will now implement the functions used by the search engine. Note that each question has its own skeleton file. For example, the skeleton for question one can be found in `part1.py`. We will often refer to functions defined in the skeleton files. Feel free to add helper functions.

1. Implement the `k_rank_approximate` function. It takes two inputs: a matrix whose rows are document vectors, and an integer k, which specifies the desired rank for the resulting matrix. After you have finished this method, implement `test_k_rank_approximate` by finding a 2-rank approximation of the 4×4 identity and then finding a 300-rank approximation of the complete matrix for the 'acq' category. Verify that the resulting ranks are correct.

2. We must now teach our search engine to categorize documents. Let's first try categorizing documents the old-fashioned way using k-nearest neighbors.

   a) Implement the `knn` function. Do not count the given document as a neighbor of itself.

   b) Find document with ID 'training/1684' and read the text. Use the `knn` function to find its 3 nearest neighbors (not including itself) and read the text of those documents. Are they similar? Does these results make sense?

   c) Now test the accuracy of your `knn` function by implementing `test_knn`. For one or more categories in the Reuter's corpus, try to classify every document in the category, and determine the accuracy of your kNN classifier.

3. As you have probably observed, k-nearest neighbors classification is slow. Such latency is unacceptable when users expect real-time answers to queries. Even though we have do not have the technical means for implementing real-time search, we can

at least try to do better. Let's try training least squares and SVM classifiers so that our classification time is $O(m)$, where m is the number of term features.

a) Implement `train_ls_classifier` using your favorite iterative method. Your implementation should use Tikhonov regularization and should run for 10000 iterations, with regularization parameter $\lambda = 0.01$ and learning rate $\gamma = 0.001$. Also implement `compute_categorization_error`. Then, in `test_ls_classifier`, train a least squares classifier to distinguish between the 'money-fx' and 'acq' categories. How does your classifier perform? Train another least squares classifier to distinguish the 'money-fx' and 'money-supply' categories. How does your classifier perform now? Is there a performance difference? Why?

b) Implement `train_svm_classifier` using gradient descent and Tikhonov regularization. Repeat the experiments above. How do your results compare to the least squares classifier?

c) Use these functions to implement the functions `train_one_vs_one_classifier` and `one_vs_one_classify`. We will represent one-vs-one classifiers as the tuple (<weights>, <+1 category>, <-1 category>).

Note: Make sure you are computing $k(k-1)/2$ classifiers and **not** k^2 classifiers. You do not have to write tests for these functions, but you can if you wish.

4. Finally, implement the `find_closest_documents` function. You have now completed your search engine! To test your code, execute the `main.py` file, initialize the search engine with the `i` command, and then enter a few of your own queries. You can also try searching 'bahia cocoa zone.' How does it work?

3 Final Thoughts

Internet usage has exploded over the past 20 years. More than half the world's population depends on the Internet for quick access to reliable information. We can attribute much of this growth and dependence to the availability of search engines and more generally to the advancement of text mining. Indeed, the Internet sits atop a mountain of information, thousands of terabytes in magnitude. With all this data floating around, scientists and ordinary people alike need the ability to extract relevant information from natural language text in real time. Document categorization offers just one technique for doing so.

In general, text mining and natural language processing are growing fields. Language can be difficult even for human adults, and in many ways language is a complete mystery to computers. Filled with ambiguity and loose structure at every corner, language is akin to an ever-morphing labyrinth. As such, text mining is left with many open questions, some of which already have patchwork solutions while others appear to be more or less out of the question for the foreseeable future. Regardless, text mining, natural language processing, and information science are exciting fields of research. We hope you agree after completing this lesson.

Bibliography

A Appendix

We have included sample solutions to each exercise. The skeleton code and provided APIs are omitted for concision, but can be found at .

A.1 Warmup Solutions

1. The document vector is a vector of the tf-idf statistics for one document. There are many zeros because there is one feature for every meaningful word in the corpus, but only a small subset of all words appears in each document. Thus, there will be a zero tf-idf value for most words.

2. The normalized dot product measures how "well aligned" the two document vectors are in the feature space. This is a measure of how similar the two documents are. A larger normalized dot product means that the documents shared many terms with large tf-idf weights, so they are semantically similar. The results do make sense because both documents are about gold mines—they likely share many significant terms related to mining gold, giving them a large normalized dot product. On the other hand, the document about agriculture will have a small normalized dot product with the document on gold mining because they are unlikely to share many significant terms. We would hope this is the case, as they're documents on completely different subjects!

3. $XX^{\intercal}$ is a matrix where element ij is the dot product of d_i and d_j. Thus it is a matrix of non-normalized document similarities. $X^{\intercal}X$ would be a matrix where element ij is the non-normalized dot product of the term i and term j vectors. In this case we have a matrix of non-normalized term similarities.

A.2 Lab Solutions

We provide code solutions for each part of the lab. Some of the exercises can be solved in many ways!

A.2.1 Part 1

You should see that the resulting matrices have rank 2 and rank 300, respectively.

Listing A.1: Part 1 Code Solutions

```python
import numpy as np
from corpus import Corpus

def k_rank_approximate(doc_matrix, k):
    """Finds a k-rank approximation of the document vectors.

    Args:
        doc_matrix: An (n, m) matrix where each row is a document vector.
        k: The rank to approximate.

    Returns:
        An (n, m) matrix with rank k formed by the first k singular
        values/vectors of doc_matrix.
    """
    if np.linalg.matrix_rank(doc_matrix) <= k:
        raise Exception('Given document matrix is too low rank.')
    u, s, v = np.linalg.svd(doc_matrix)
    s = np.diag(s)
    return  np.dot(np.dot(u[:, :k], s[:k, :k]), v[:k, :])

def test_k_rank_approximate(corpus):
    """Test your k-rank approximation function.

    First, sanity check your function by finding a 2-rank approximation
    of the 4-dimensional identity. Then, find a 300 rank approximation
    of the acq category matrix and verify that it's rank is actually 300.

    Args:
        corpus: A corpus object, used for convenient access to the Reuters
    corpus.
    """
    I4 = np.identity(4)
    I4_approx = k_rank_approximate(I4, 2)
    print 'First approximation rank:', np.linalg.matrix_rank(I4_approx)

    doc_matrix = corpus.complete_matrix('acq')
    doc_matrix_approx = k_rank_approximate(doc_matrix, 300)
    print 'Second approximation rank:', np.linalg.matrix_rank(
    doc_matrix_approx)
```

Listing A.2: Part 2 Code Solutions

```python
import heapq
import numpy as np
from corpus import Corpus

N_LARGEST = 4
VERBOSE = False

def knn(doc_matrix, ids, doc_vector, k):
    """Finds the k most similar docs in the doc matrix to the given doc
    vector.

    Args:
        doc_matrix: An (n, m) matrix where each row is a vectorized
    document.
        ids: An (n, 1) array where the ith element corresponds to the ith
    row in doc_matrix.
        doc_vector: An (m, 1) vector that represnts a vectorized document.
        k: The number of clusters to create.

    Returns:
        List of unique categories of the k nearest neighbors of the
    doc_vector
    """
    best = []
    for i in range(k):
        heapq.heappush(best, (-float("inf"), (i, None)))

    for i in range(len(doc_matrix)):
        distance = -np.linalg.norm(doc_matrix[i] - doc_vector) ** 2
        end_priority, _ = best[0]
        if distance > end_priority and distance < -.01:
            heapq.heapreplace(best, (distance, (i, ids[i])))

    n_largest = heapq.nlargest(N_LARGEST, best)

    if VERBOSE:
        for i in range(N_LARGEST):
            print n_largest[i]
            print ids[n_largest[i][1][0]]
            print Corpus.document_text(n_largest[i][1][1])

    categories = []

    for i in range(k):
        categories.extend(Corpus.categories_of(best[i][1][1]))

    categories = set(categories)

    return categories

def test_knn(corp):
    """ Tests the knn classification function. Implement this however you
    like.

    Args:
        corpus: A corpus object, used for convenient access to the Reuters
    corpus.
    """
    ids, complete_matrix = corp.complete_matrix(include_ids=True)
```

```python
        category_map = corp.complete_matrix_dict()
        categories = category_map.keys()
        errors = 0
        document_count = 0

        for _, category in enumerate(categories):
            documents_in_category = category_map[category]
            for i in range(len(documents_in_category)):
                print document_count
                document_count += 1
                if not category in knn(complete_matrix, ids,
        documents_in_category[i], 10):
                    errors += 1
            print (errors, float(errors) / document_count, category)
```

Listing A.3: Part 3 Code Solutions

```python
import operator
import itertools
import numpy as np

from corpus import Corpus

def train_ls_classifier(A, y):
    """Trains the document category classifier by minimizing L2-regularized
     least squares loss.

    Args:
        A: An (n, m) matrix of training data where the rows are document
    vectors.
        y: An (n, 1) vector of labels (+1 or -1).

    Returns:
        An (m, 1) vector of the optimal classification weights.
    """
    w = np.zeros((A.shape[1], 1))
    num_iterations = 10000
    learning_rate = 0.001
    regularization_weight = 0.01
    for i in range(num_iterations):
        z = w - learning_rate * A.transpose() * (A * w - y)
        w = (1/(1 + learning_rate * regularization_weight)) * z
    return w

def train_svm_classifier(A, y):
    """Trains the document category classifer by minimizing L2-regularized
    hinge loss.

    Args:
        A: An (n, m) matrix of training data where the rows are document
    vectors.
        y:  An (n, 1) vector of labels (+1 or -1).

    Returns:
        An (m, 1) vector of the optimal classification weights.
    """
    w = np.zeros((A.shape[1], 1))
    num_iterations = 10000
    learning_rate = 0.001
    regularization_weight = 0.01
    for i in range(num_iterations):
        # Compute the gradient.
        grad = np.zeros((A.shape[1], 1))
        for i in range(A.shape[0]):
            if y[i] * A[i] * w < 1:
                grad -= y[i, 0] * A[i, :].transpose()
        grad -= regularization_weight * 2 * w
        # Update the weight vector.
        w = w - learning_rate * grad
    return w

def classification_error(A, w, y):
    """Calculates the classification error for the given classifier weights
    .

    Args:
```

```python
         A: An (n, m) matrix of test n m-dimensional examples
56       w: An (n, 1) vector of learned weights for classification.
         y: Actual labels (+1 or -1) for each of the n examples.
58
     Returns:
60       Number of misclassified examples.
     """
62   return np.sum(0.5 * (np.sign(A * w) - y))

64 def train_one_vs_one_classifier(category_train_documents):
     """Computes a one-vs-one classifier schema for each pair of categories.
66
     Args:
68       category_train_documents: A dict from categories to document
     matrices.
70   Returns:
         A dict of dicts, where dict['category_1']['category_2'] is the
     weight vector
72       that represents the one-vs-one classifier for category_1 and
     category_2.
     """
74   category_classifiers = []
     for cat_1, cat_2 in itertools.combinations(category_train_documents.
     keys(), 2):
76       print 'Now training classifier for', cat_1, 'and', cat_2
         train_1 = category_train_documents[cat_1]
78       train_2 = category_train_documents[cat_2]
         A = np.concatenate((train_1, train_2))
80       y = np.concatenate((np.ones((train_1.shape[0], 1)),
                             -np.ones((train_2.shape[0], 1))))
82       category_classifiers.append((train_svm_classifier(A, y), cat_1,
     cat_2))
     return category_classifiers
84
 def classify(one_vs_one_classifiers, doc):
86   """Given a one-vs-one classifier schema, classifies the example.
88   Args:
         one_vs_one_classifiers: The one-vs-one classifier schema.
90       doc: The vectorized document to classify.
92   Returns:
         A 2-tuple of the two majority-vote categories.
94   """
     votes = {}
96   for tup in one_vs_one_classifiers:
         classifier = tup[0]
98       cat_1 = tup[1]
         cat_1 = tup[2]
100      prediction = np.sign(np.inner(doc, classifier)[0])
         if prediction == 1:
102          votes[cat_1] = 0 if cat_1 not in votes else votes[cat_1] + 1
         else:
104          votes[cat_2] = 0 if cat_2 not in votes else votes[cat_2] + 1
     best = sorted(votes.items(), key=operator.itemgetter(1), reverse=True)
106  return (best[0][0], best[1][0])

108 def test_ls_classifier(corp):
     """Test your LS classifier!
110
```

```
        First, train an LS classifier to distinguish between the coconut and
        coconut-oil
112     categories. How does it perform on the testing data? Then train another
         LS classifier
        to distinguish between the coconut and copper categories. Again, how
        does it perform?
114     Why do you see these results?

116     Args:
            corp: A corpus object, used for convenient access to the Reuters
        corpus.
118     """
        print 'Testing LS classifier on coconut vs. copper...'
120     test_classifier_on_categories('money-fx', 'acq', train_ls_classifier,
        corp)

122     print 'Testing LS classifier on coconut vs. coconut-oil...'
        test_classifier_on_categories('money-fx', 'money-supply',
        train_ls_classifier, corp)

124
    def test_svm_classifier(corp):
126     """Test your SVM classifier!

128     Train an SVM classifier for the two pairs of categories given above.
        How does it compare
        to the LS classifier?

130
        Args:
132         corp: A corpus object, used for convenient access to the Reuters
        corpus.
        """
134     print 'Testing LS classifier on coconut vs. copper...'
        test_classifier_on_categories('money-fx', 'acq', train_svm_classifier,
        corp)

136
        print 'Testing LS classifier on coconut vs. coconut-oil...'
138     test_classifier_on_categories('money-fx', 'money-supply',
        train_svm_classifier, corp)

140 def test_classifier_on_categories(category_1, category_2, train_classifier,
         corp):
        """A utility function for testing the LS and SVM classifiers.
142
        Trains a classifier using the given function to distinguish between the
144     two given categories. Prints the classification error calculated on the
        corresponding test datasets.
146
        Args:
148         category_1: The first category to test against.
            category_2: The second category to test against.
150         train_classifier: Callable for training the classifier.
            corp: A corpus object, used for convenient access to the Reuters
        corpus.
152     """
        train_matrix_1 = corpus.train_matrix(category_1)
154     train_matrix_2 = corpus.train_matrix(category_2)
        train_doc_matrix = np.concatenate((train_matrix_1, train_matrix_2))
156     train_labels = np.concatenate((
            np.ones((train_matrix_1.shape[0], 1)),
158         -np.ones((train_matrix_2.shape[0], 1))
        ))
```

```
160    classifier = train_classifier(train_doc_matrix, train_labels)
       test_matrix_1 = corpus.test_matrix(category_1)
162    test_matrix_2 = corpus.test_matrix(category_2)
       test_doc_matrix = np.concatenate((test_matrix_1, test_matrix_2))
164    test_labels = np.concatenate((
           np.ones((test_matrix_1.shape[0], 1)),
166        -np.ones((test_matrix_2.shape[0], 1))
       ))
168    error = classification_error(test_doc_matrix, classifier, test_labels)
       print category_1, 'vs.', category_2, 'error:', error
```

Listing A.4: Part 4 Code Solutions

```python
import numpy as np
import part3_sols as p3

def find_closest_documents(query, category_classifiers, category_docs,
    category_ids):
    """Finds the four documents that best match the query vector.

    For simplicity, this function categorizes the query and then finds the
    two
    best-matching documents in each category.

    Args:
        query: The vectorized query.
        one_vs_one_classifier: The one-vs-one classifier schema.
        category_docs: A dict that maps category names to document matrices
    .
        category_ids: An dict that maps category names to an array of ids,
            where the ith element in the array corresponds to the ith row
    of
            the category matrix.

    Returns:
        List of IDs of the four best-matching documents.
    """
    best_docs = []
    query_categories = p3.classify(category_classifiers, query)
    for category in query_categories:
        similarities = category_docs[category] * query
        best_docs.append(category_ids[category][np.argmax(similarities)])
    return best_docs
```