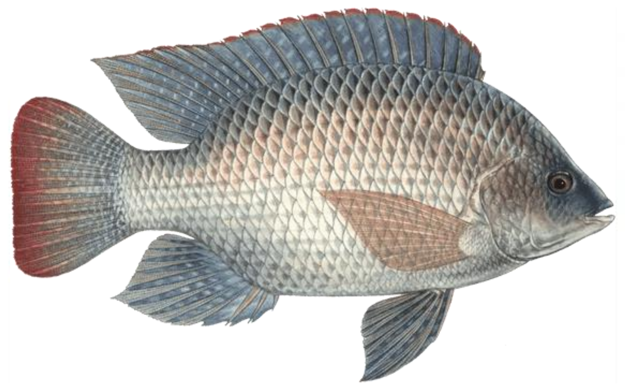

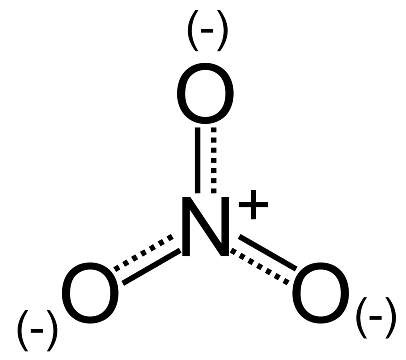

A Recursive MATLAB Model for Nitrate in a Decoupled Aquaponics System

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Introduction

- Recently, aquaponics has been at the forefront of sustainability efforts, combining both aquaculture and hydroponics to effectively use the waste of fish to grow plants. The self-design of such systems has prompted the development of specific fish diets to also be more sustainable, with some including the **reuse of food waste** from humans (*Bake et al., 2009, Nasser et al., 2018*).
- Models have been developed to help understand their nutrient and energy dynamics in the effort to improve these systems (*Debrota, K., 2017, Yogev, U. et. al., 2016*).
- Nitrogen cycling is crucial in aquaponics systems, as **nitrate** is a major nutrient desired by plants. **Nitrate availability** is impacted by many components of aquaponics setup and therefore, is often a nutrient of **concern** (*Wongkiew, S. et. al.*, 2017).
- Models have been developed to analyze nitrate concentrations (and closely related nitrogen-cycle compounds) in **coupled** aquaponics systems (*Shazila, E. et.al*., 2022).
- The nitrate added to an aquaponics system can be **proportionally related** to the protein percent of feed fed to the fish of the system (*Ebeling, J. M. et.al., 2006*). This relation suggests the possibility of improving the efficiency of a system, both plants and fish, by altering the protein content of the feed inputted to the system.

Objective

The objective of this project was to create a mathematical model that can predict nitrate concentration in both parts of a decoupled aquaponics system, the fish tank and plant containers. It aims to quantify how differences in protein levels of feed directly translate to the efficiency of decoupled systems.

Considerations of Model

Table 1: All variables included in the MATLAB model for predicting nitrate concentrations in both fish and plant components of a decoupled aquaponics system. The values of variables are specific to our system and can be readily changed in the model.

Variable	Value	Description	Units	Source
A	1.00 (100%)	Conversion efficiency for ammonia into nitrate (%).	-	*Ebeling et. al.*
C_F	-	Concentration of nitrate (ppm) in fish aquarium.	(days, ppm)	-
C_P	-	Concentration of nitrate (ppm) in kale tub.	(days, ppm)	-
D	1.00 (100%)	Percent nitrate left after denitrification in aquarium (%)	-	-
E	7	Transfer rate of water into kale tubs from aquarium (days)	days	-
F	0.05 (5%)	Percent bodyweight of feed fed to tilapia in given day (%)	-	FAO
I	62.3/72.6	Initial nitrate concentration of aquarium	ppm	-
K	28	Starting day of kale, where K is a positive multiple of E.	days	-
L	50	Length of experiment.	days	-
N_F	18	Number of tilapia.	-	-
N_P	6	Number of plants.	-	-
P	40%/25%	Protein percent of tilapia feed.	-	-
R	0.3525	Protein transfer rate from protein to ammonia.	-	*Ebeling et. al.*
S_F	-	Starting age of tilapia.	days	-
S_P	-	Starting age of kale.	days	-
V_A	9.00	Volume of additional water removed during water transfer.	liters	-
V_F	76.00	Volume of fish aquarium.	liters	-
V_P	9.00	Volume of kale tub.	liters	-

Table 2: All functions included in the MATLAB model for predicting nitrate concentrations in both fish and plant components of a decoupled aquaponics system. The values of the functions are specific to our system and can be readily changed in the model.

Function	Initial Value	Description	Units	Source
$U(t)$	-	Uptake rate of nitrate (ppm) for one kale plant on given day	(days, ppm/day)	-
$W(t,P)$	0.69	Weight of one tilapia (in grams) on given day, t while being fed food of protein percent, P.	(days, grams)	-

Important Assumptions to Note:

1. Conversion of ammonia to nitrate is assumed to be 1:1 (follows from *Ebeling, J. M. et.al., 2006*).
2. All protein-entered ammonia is converted during a single day, not by a determined rate. It then follows that the given nitrate concentration determined by the model is suited for only integer days, as rate is unknown, but the maximum amount of nitrate increase is capped.
3. Kale nitrate uptake rates follow our data from Summer 2023, although growing conditions differed in the fall (lower light levels).

Definition of Model

By our considerations, in a **coupled** aquaponics system, the concentration, C, of nitrate (ppm) can be roughly represented with the differential equation :

$$\frac{dC}{dt} = \left[(\mathrm{ADFN}_F\mathrm{PR})\left(\frac{1000}{V} * W(t,P)\right) - (N_P U(t))\right]$$

However, we can adjust this for our assumed, **decoupled** system with this **recursive** model for nitrate in the fish aquarium, $\mathbf{C_F}$, and nitrate in the kale tub, $\mathbf{C_P}$: Suppose $L \in \mathbb{Z}_{\geq 0}$. $K \in \{k \in \mathbb{Z}_{\geq 0} | k \leq L\}$. $E \in \{e \in \mathbb{Z}_{\geq 0} | K mod(e) = 0\}$.
Then for any $t \in \{z \in \mathbb{Z}_{\geq 0} | 0 \leq z \leq L\}$, we have:

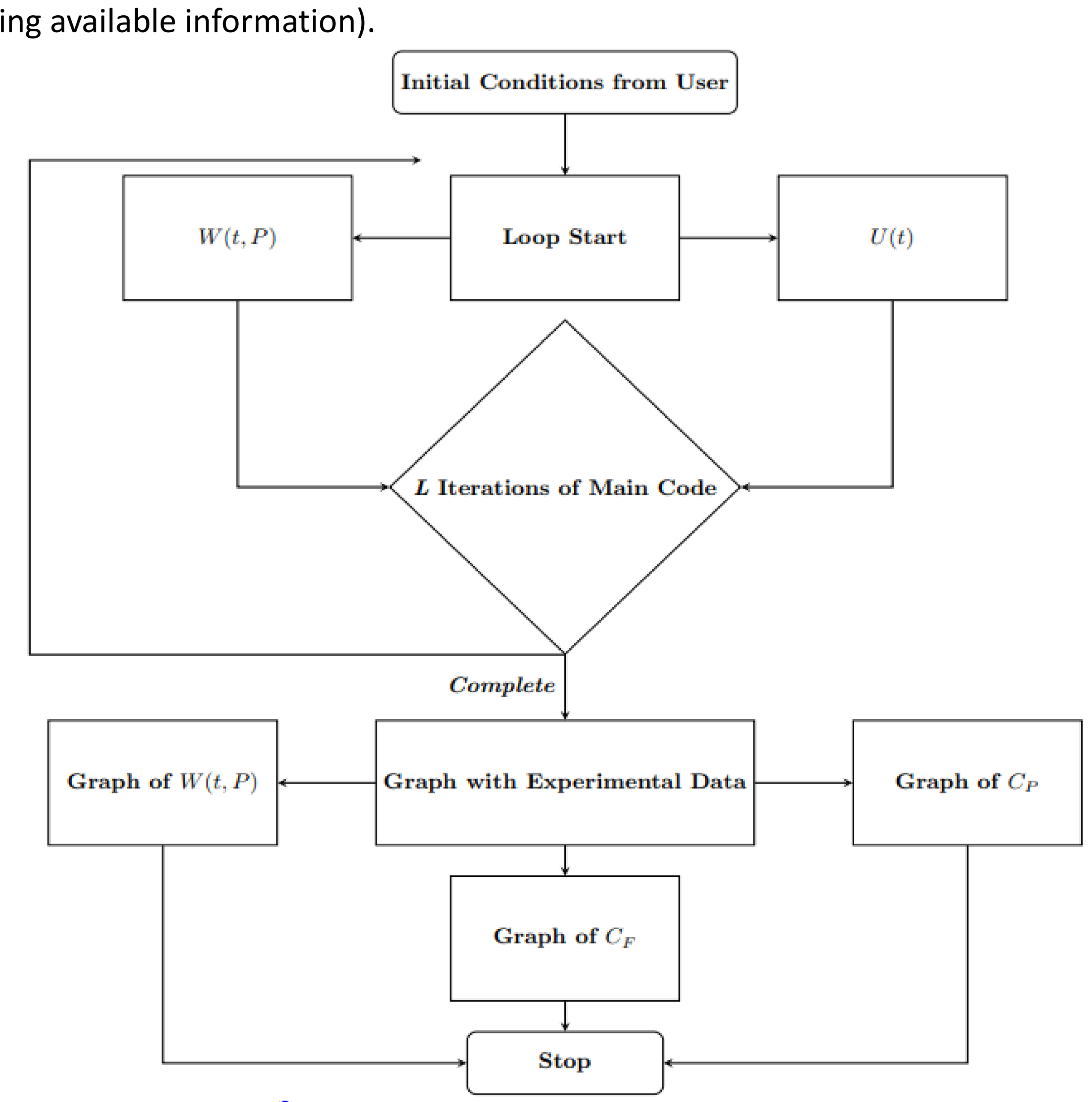
$$C_{F_t} = \begin{cases} I, & t = 0 \\ C_{F_{t-1}} + \left[\mathrm{ADFN}_F\mathrm{PR}\left(\frac{1000}{V_F}\right) * W(S_F + t - 1, P)\right], & 1 \leq t \leq L \\ \left[C_{F_{t-1}} + \left[\mathrm{ADFN}_F\mathrm{PR}\left(\frac{1000}{V_F}\right) * W(S_F + t - 1, P)\right]\left(\frac{V_F - (V_P + V_A)}{V_F}\right)\right], & 1 \leq t \leq L : t mod(E) = 0 \end{cases}$$

$$C_{P_t} = \begin{cases} 0, & t \leq K \\ max(C_{P_{t-1}} - N_P * U((t - K - 1) + S_P), 0), & t > K \\ C_{F_t}, & t \geq K : t mod(E) = 0 \end{cases}$$

Where these equations are processed **top-down** computationally.
This definition served as the basis for which we developed a computational model to predict nitrate concentration. The main concern of this model is understanding relative nitrate concentration on **a given day**, not necessarily continuously.

Creation and Implementation of Model

To construct the mathematical model computationally, we developed an approximately **250-line MATLAB** code that can be readily molded for various scenarios and conditions (assuming available information).



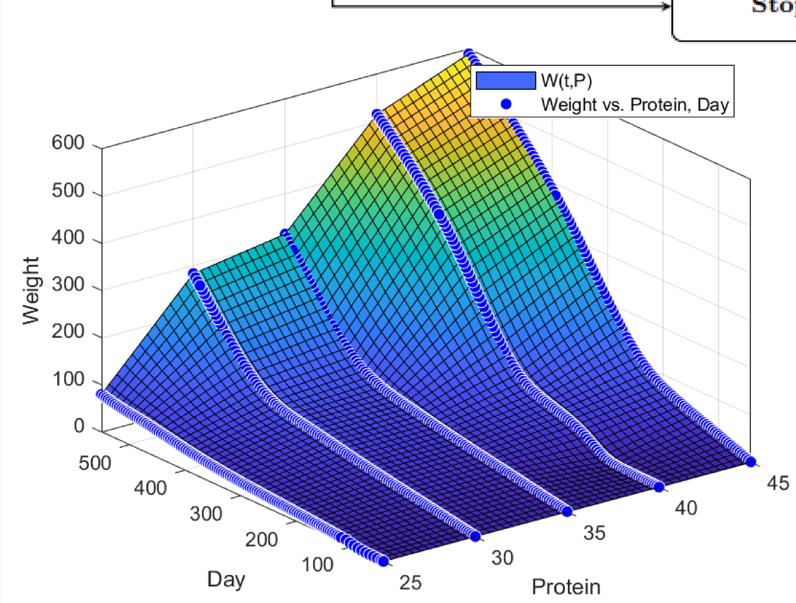


Figure 1: Linearly interpolated MATLAB fit used for predicting fish weight (grams) based on protein percentage fed. Data for the model fit was collected from our Spring 2023 experiment, Fall 2024 experiment, and Hafedh, Y. S. A,1999.

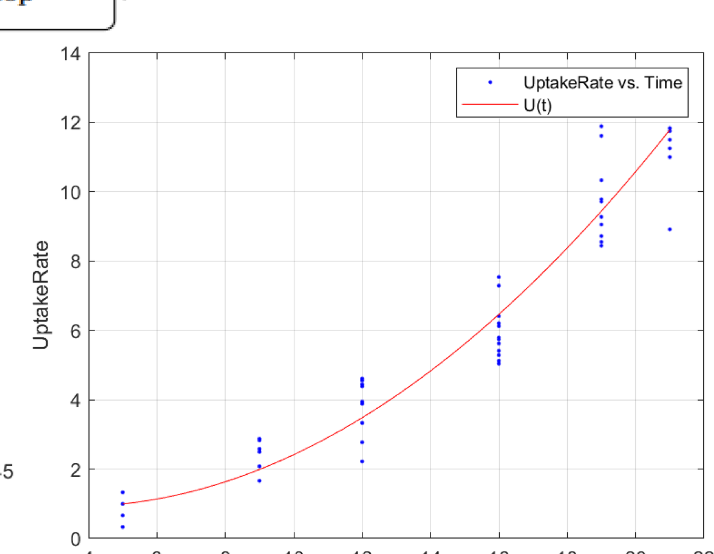


Figure 2: Polynomial regression MATLAB fit used for predicting kale uptake rate (ppm/day) based on determined age. Data for the model fit was collected from our Summer 2023 experiment.

Results of Adjusted Model

The **BLUE** line is the model prediction. The "*" are scatter plot data of experimental data. The **PURPLE** line is the mean plotting of the experimental data.

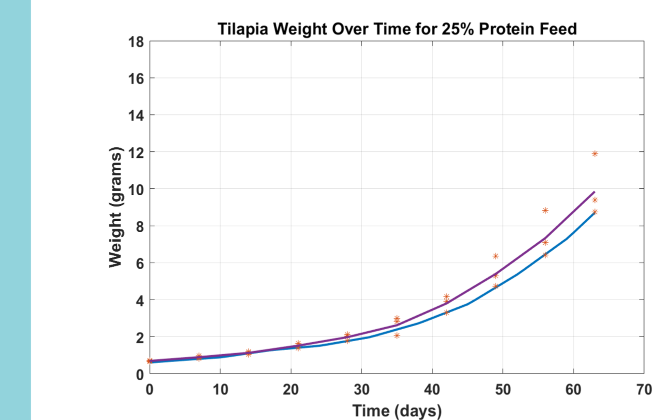


Figure 3: Model-predicted, individual tilapia weights (grams) versus experimentally collected weights for 25% diet (n=3).

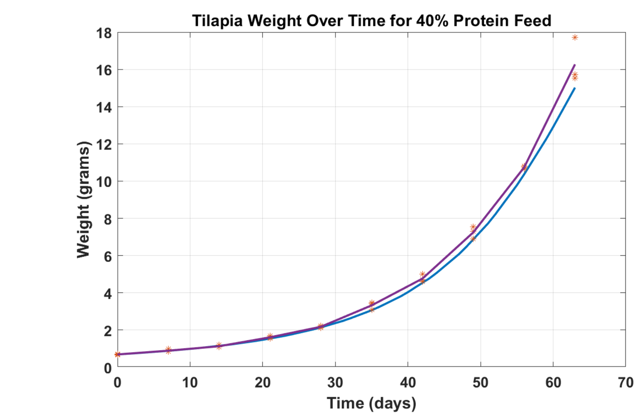


Figure 4: Model-predicted, individual tilapia weights (grams) versus experimentally collected weights for 40% diet (n=3).

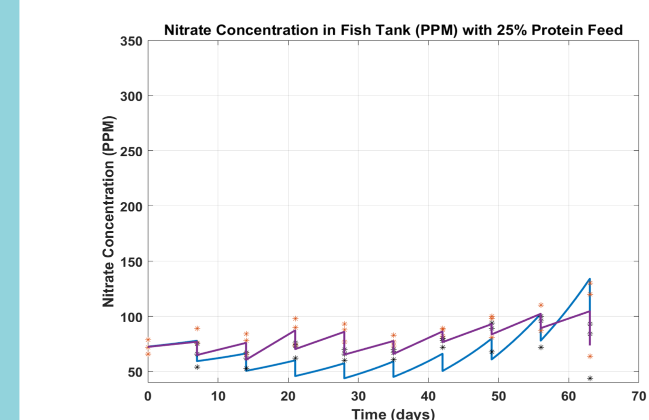


Figure 5: Model-predicted, individual fish aquarium nitrate concentrations (ppm) versus experimentally collected concentrations for 25% diet where "" is before dilution and "*" is after (n=3).*

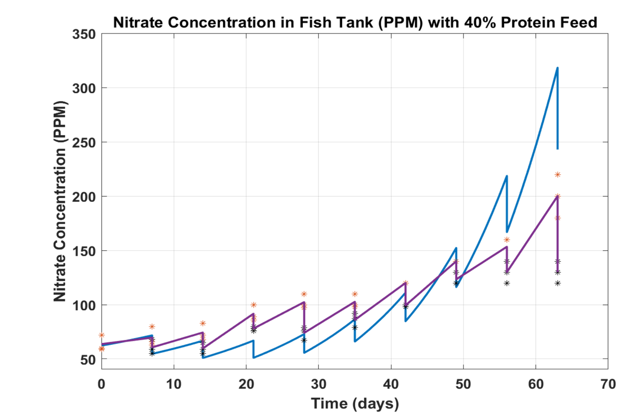


Figure 6: Model-predicted, individual fish aquarium nitrate concentrations (ppm) versus experimentally collected concentrations for 40% diet where "" is before dilution and "*" is after (n=3).*

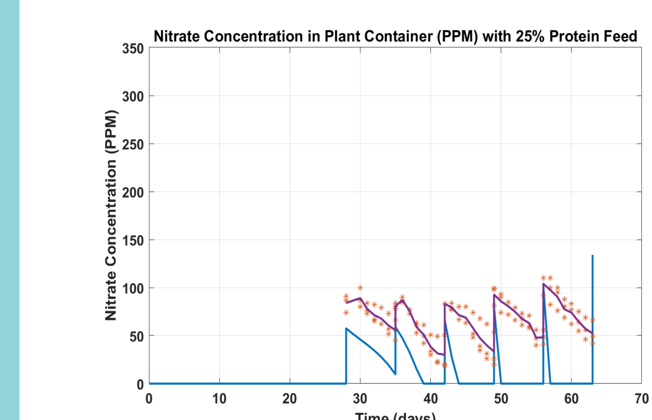


Figure 7: Model-predicted, individual kale tub nitrate concentrations (ppm) versus experimentally collected concentrations for 25% diet (n=3).

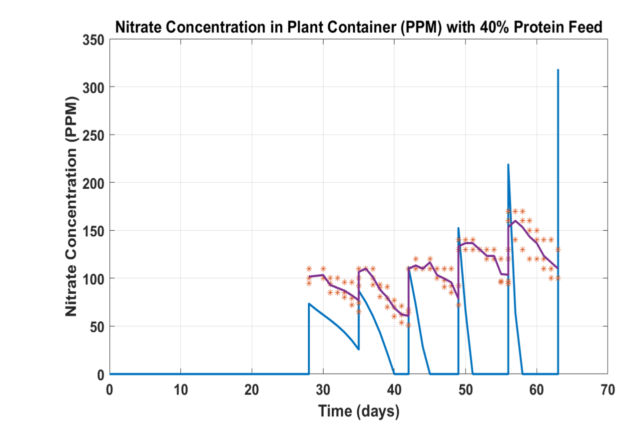


Figure 8: Model-predicted, individual kale tub nitrate concentrations (ppm) versus experimentally collected concentrations for 40% diet (n=3).

Conclusion and Future Research

- With our R (protein transfer rate from protein to ammonia) adjustment, the projections follow well to the model predictions for nitrate in the fish aquarium.
- Kale uptake rates are overestimated, which is also consistent with the conditions of our current kale in comparison to the sample data.
- Further research determining the exact R will provide an even more accurate model for our specific system.
- Future research will also focus on the refining of our model and possibly, extending its output capabilities to also predict plant biomass. This aspect could prove very useful in an economic sense, where high plant yield is desirable.

Sources Cited

Bake, G.G., Endo, M., Akimoto, A. *et al*. Evaluation of recycled food waste as a partial replacement of fishmeal in diets for the initial feeding of Nile tilapia *Oreochromis niloticus*. *Fish Sci* **75**, 1275–1283 (2009). https://doi.org/10.1007/s12562-009-0133-x.
Debrota, K. (2017). *Mechanistic Modeling of an Aquaponic Controlled-Environment Agriculture System: Nutrient and Water Dynamics, Harvest Productivity, and Waste Treatment A Dissertation Presented to The Academic Faculty*. https://repository.gatech.edu/server/api/core/bitstreams/25032e72-38e1-4c8f-9428-098cf650455e/content
Ebeling, J. M., Timmons, M. B., & Bisogni, J. J. (2006). Engineering analysis of the stoichiometry of photoautotrophic, autotrophic, and heterotrophic removal of ammonia–nitrogen in aquaculture systems. *Aquaculture*, *257*(1-4), 346–358. https://doi.org/10.1016/j.aquaculture.2006.03.019
Hafedh, Y. S. A. (1999). Effects of dietary protein on growth and body composition of Nile tilapia, Oreochromis niloticus L. *Aquaculture Research*, *30*(5), 385–393. https://doi.org/10.1046/j.1365-2109.1999.00343.
Nasser, N, Abiad, MG, Babikian, J, Monzer, S, Saoud, IP. Using restaurant food waste as feed for Nile tilapia production. *Aquac Res*. 2018; 49: 3142–3150. https://doi.org/10.1111/are.13777.
Shazila, E., Roni, M., & Ali, A. (2022). Mathematical Modelling of an Aquaponics System in the Presence of Nitrite. *Proceedings of Math and Science*, *9*(9), 49–63.
Wongkiew, S., Hu, Z., Chandran, K., Lee, J. W., & Khanal, S. K. (2017). Nitrogen transformations in aquaponic systems: A review. *Aquacultural Engineering*, *76*, 9–19. https://doi.org/10.1016/j.aquaeng.2017.01.004
Yogev, U., Barnes, A., & Gross, A. (2016). Nutrients and Energy Balance Analysis for a Conceptual Model of a Three Loops off Grid, Aquaponics. *Water*, *8*(12), 589. https://doi.org/10.3390/w8120589

Images:
Blue Tilapia, 2023. Bingcom. [accessed 2023 Apr 27]. https://th.bing.com/th/id/OIP.qRP_H7SE8caSFVX-y9V6JQHaE7?pid=ImgDet&rs=1.
Nitrate, Techno-Science.net. [Accessed 2024 Apr]. www.techno-science.net/illustration/Definition/inconnu/n/Nitrate-ion.png.

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