

# STAT 3010: Assignment 3

Spring 2025: Due March 7

## Problem 1

Part (a)

$$c \sum_{x=0}^4 (5-x) = 1 \quad c(15) = 1 \quad c = \frac{1}{15}$$

Part (b)

$$P(X \geq 1) = 1 - P(X = 0) \quad P(X = 0) = c(5) = \frac{1}{15}(5) = \frac{1}{3} \quad 1 - \frac{1}{3} = \frac{2}{3}$$

Part (c)

$$\mathbb{E}[X] = \sum_{x=0}^4 x \left( \frac{1}{15}(5-x) \right) = \frac{1}{15} \sum_{x=0}^4 x(5-x) \quad \mathbb{E}[X] = \frac{1}{15}(20) = \frac{4}{3}$$

## Problem 2

Part (a)

$$Z = \frac{48 - 55}{6} = \frac{-7}{6} \approx -1.1667 \quad P(Z < -1.17) = \mathbf{0.1210}$$

Part (b)

$$Z_1 = \frac{60 - 55}{6} = \frac{5}{6} \approx 0.83, \quad Z_2 = \frac{65 - 55}{6} = \frac{10}{6} \approx 1.67$$

$$P(60 < H < 65) = P(0.83 < Z < 1.67) = P(Z < 1.67) - P(Z \leq 0.83) = 0.9525 - 0.7977 = \mathbf{0.1548}$$

Part (c)

$$h = \mu + Z_{0.90}(\sigma) = 55 + 1.28(6) = 55 + 7.68 = \mathbf{62.68}$$

Part (d)

$$P(H < 54) = P\left(Z < \frac{54 - 55}{6}\right) = P(Z < -0.17) = \mathbf{0.4325}$$

Part (e)

$$Q_1 = 55 + (-0.6745) \times 6 = 55 - 4.047 \approx 50.95 \quad | \quad Q_3 = 55 + 0.6745 \times 6 = 55 + 4.047 \approx 59.05$$

$$\text{IQR} = Q_3 - Q_1 \approx 59.05 - 50.95 = \mathbf{8.10}$$

## Problem 3

$$X \sim N(\mu = 1500, \sigma = 300) \text{ and } P(X \geq 2100 | X \geq 1900) = \frac{P(X \geq 2100)}{P(X \geq 1900)}, \text{ so}$$

$$P(X \geq 2100) = P\left(Z \geq \frac{2100 - 1500}{300}\right) = P(Z \geq 2) = 0.0228$$

$$P(X \geq 1900) = P\left(Z \geq \frac{1900 - 1500}{300}\right) = P(Z \geq 1.33) = 0.0918$$

$$\frac{0.0228}{0.0918} \approx \mathbf{0.2489}$$