STAT 3010: Assignment 2

Spring 2025: Due Feb 20

1. The problem is reminiscent of Bayes' Rule examples from the Ch5_3 powerpoint, so I'll use that.
   P(C): The probability a woman over 40 has breast cancer = 0.0035
   P(positive | C): The probability of a positive test given the woman has cancer = 1-0.11 = 0.89
   P(positive | C'): The probability of a positive test given the woman doesn't have cancer = 0.07
   P(C | positive): The probability of the woman having cancer given a positive test.

   By Bayes' theorem, $P(C|positive) = \frac{P(positive|C)\times P(C)}{P(positive|C)\times P(C)+P(positive|C')\times P(C')}$

   Plugging in, $P(C|positive) = \frac{0.003115}{0.003115+0.069755} \approx \frac{0.003115}{0.07287} \approx \mathbf{0.0427\ or\ 4.3\%}$

2. (a) $\frac{\binom{4}{2}}{\binom{12}{2}} = \frac{6}{66} = \mathbf{\frac{1}{11}}$

   (b) $\frac{\binom{7}{2}}{\binom{12}{2}} = \frac{21}{66} = \mathbf{\frac{7}{22}}$

   (c) $1 - \frac{\binom{9}{2}}{66} = 1 - \frac{6}{11} = \mathbf{\frac{5}{11}}$

   (d) $\mathbf{0}$

   (e) $\frac{\binom{4}{2}+\binom{5}{2}+\binom{3}{2}}{66} = \mathbf{\frac{19}{66}}$

3. Obviously it's an exaggeration, but I'll use Bayes' Rule again to prove that the probability isn't 0%.
   P(L): The probability of a patient having Lupus = 0.02
   P(positive | L): The probability of a positive test given the patient has Lupus = 0.98
   P(positive | L'): The probability of a positive test given the patient doesn't have Lupus = 0.26
   P(L | positive): The probability of the patient having Lupus given a positive test.

   By Bayes' theorem, $P(L|positive) = \frac{P(positive|L)\times P(L)}{P(positive|L)\times P(L)+P(positive|L')\times P(L')}$

   Plugging in, $P(L|positive) = \frac{0.0196}{0.0196+0.2548} = \frac{0.0196}{0.2744} = \mathbf{0.0714\ or\ 7.14\%}$