TESTING RANGE ESTIMATORS OF HISTORICAL VOLATILITY

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This study investigates the relative performance of various historical volatility estimators that incorporate daily trading range: M. Parkinson (1980), M. Garman and M. Klass (1980), L. C. G. Rogers and S. E. Satchell (1991), and D. Yang and Q. Zhang (2000). It is found that the range estimators all perform very well when an asset price follows a continuous geometric Brownian motion. However, significant differences among various range estimators are detected if the asset return distribution involves an opening jump or a large drift. By adding microstructure noise to the Monte Carlo simulation, the finding of S. Alizadeh, M. W. Brandt, and F. X. Diebold (2002)—that range estimators are fairly robust toward microstructure effects—is confirmed. An empirical test with S&P 500 index return data shows that the variances estimated with range estimators are quite close to the daily integrated variance. The empirical results support

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INTRODUCTION

Volatility estimation has important practical implications in portfolio management, option pricing, and volatility trading. Because the volatility measures the risk of an investment in a stock, it is an important piece of information in constructing an optimal portfolio. Historical volatility is also used by option traders as a proxy for future volatility in evaluating options. Its value is directly related to the benchmark value of the option. Therefore, correct modeling of volatility is always desirable to both practitioners and researchers. Classically, historical volatility is computed as the standard deviation of daily returns within a certain period, say 3 months. One implicitly assumes that the volatility is a constant within the 3 months. However, it is unrealistic to assume that the volatility of asset returns remains constant during a long period;[1] therefore the volatility estimated with the classical estimator is essentially the average volatility over the specified period.

On the other hand, a separate line of literature focuses on estimating historical volatility of a security from its trading range.[2] Examples of such estimators include Parkinson (1980), Garman and Klass (1980), Rogers and Satchell (1991), and Yang and Zhang (2000). These volatility estimators are classified as range estimators because they use information on daily trading range. Recently, Alizadeh, Brandt, and Diebold (2002) extended the range-based method to estimate stochastic volatility models. Theoretically, range estimators are more efficient than the classical close-to-close estimator. It has been proven that the Parkinson estimator is five times more efficient than the classical estimator, and the Yang and Zhang estimator is 7.3 times more efficient than the classical volatility estimator. However, range estimators are built on the strict assumption that an asset price follows a geometric Brownian motion, which is certainly not the case in real markets. People often use the range estimators to study the volatility patterns of market data[3] without taking into account the assumptions made on developing the range

[1]For example, the so-called "leverage effect" (Nandi, 1998) indicates that the volatility of asset return is negatively correlated with the return process.
[2]The trading range is the difference between the recorded high and low price for a security over some time interval.
[3]For example, Chang, Jain, and Locke (1995) use the Parkinson estimator to study futures volatility and price changes of the S&P 500 index around the NYSE close. Fleming (1998) uses the Parkinson estimator to study the volatility of the S&P 100 index returns.

estimators. It is obvious that deviation from a geometric Brownian motion will affect the accuracy and efficiency of range estimators, but it is important to know the extent to which they remain useful in the analysis of real market behavior.

An attempt is made to fill this gap by conducting a sensitivity analysis to test how deviation from a geometric Brownian motion affects the performance of each of the four range estimators previously mentioned: Parkinson, Garman and Klass, Rogers and Satchell, and Yang and Zhang. These estimators all assume that the security price follows a geometric Brownian motion, but with slight differences. The Parkinson estimator and the Garman and Klass estimator are more restrictive, because they require continuous-time geometric Brownian motion with zero drift. The Rogers and Satchell estimator allows a nonzero drift in the continuous return path. The Yang and Zhang estimator is a multiperiod volatility estimator and it allows overnight price jumps.[4] One part of the comparative study is the sensitivity analysis of the range estimators on the different assumptions. The effects of nonzero drift, and opening jump on the accuracy and efficiency of each range estimator are examined. The impact of changing volatility on each range estimator is also tested. Because the range estimators need only a short period of data to estimate daily volatility, it is hypothesized that range estimators are more suitable than the classical volatility estimator to capture time-varying volatility. As the true variance of asset returns is unobservable in market data, but tractable in simulated data, the sensitivity analysis is based on Monte Carlo simulation.

Another goal of this study is to examine the applicability of range estimators to market data. Up to now, few studies have empirically tested the accuracy of range estimators with the use of market data. The major problem is that the true variance of market data is unknown, so the comparison lacks a benchmark. Rogers, Satchell, and Yoon (1994) compare the performance of the classical variance estimator with range estimators with the use of the prices of five British stocks. They find that there is no significant difference between range estimators, but that the variances estimated are significantly smaller than those estimated with the classical variance estimator. However, the daily return square is a very noisy estimator of the daily variance. Recent literature (see, e.g., Andersen, 2000; Andersen & Bollerslev, 1997), advocates the use of high-frequency data to extract daily return volatility. Andersen (2000) shows that the sum of intraday return squares, named "integrated variance," is a good proxy of

[4]Trading on real markets typically stops at night and on various trading holidays. If information flows during these periods, the subsequent opening price may deviate significantly from the previous closing price. This phenomenon is often called an *opening jump*.

daily return variance. However, the daily integrated variance will be biased if returns are serially correlated. More recent work by Alizadeh et al. (2002) reveals that in the presence of a bid/ask spread, intraday returns tend to be negatively correlated, and the integrated variance will overestimate the true variance because of the cumulation of noise effects. The present simulation results confirm that the integrated variance indeed overestimates the true variance when micro-structure noise is incorporated into the return data generating process. Although which one is a more appropriate proxy for the daily variance cannot be determined, a comparison of the performance between range estimators and daily integrated variance is still meaningful. If the degree of bias is almost the same, one would advocate the use of range estimators, because range estimators are more efficient and less data demanding.

In the rest of the article the various variance estimators are described, the simulation results and the empirical results are both discussed, and a conclusion is provided.

DATA AND METHODOLOGY

Various Variance Estimators

Let a security price, S_t, at time t, follows the geometric Brownian motion

$$dS_t = \mu S_t\,dt + \sigma S_t\,dB_t \tag{1}$$

where μ is a drift term, σ is a constant volatility parameter, and B_t is the standard Brownian motion. By Ito's lemma, the natural logarithm of the security price is:

$$d\ln S_t = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma\,dB_t \tag{2}$$

Various variance estimators have been developed under this assumption about the distribution of a security price. Here, the same notation used in Yang and Zhang (2000) is used, and the following definitions are given:

σ = volatility to be estimated
C_t = closing price on date t
O_t = opening price on date t
H_t = high price on date t
L_t = low price on date t
$c_t = \ln C_t - \ln O_t$, the normalized closing price
$o_t = \ln O_t - \ln C_{t-1}$, the normalized opening price
$h_t = \ln H_t - \ln O_t$, the normalized high price
$l_t = \ln L_t - \ln O_t$, the normalized low price

The classical sample variance estimator of variance σ^2 is then given by

$$\hat{\sigma}_c^2 = \frac{1}{n-1}\sum_{i=1}^{n}[(o_i + c_i) - (\overline{o+c})]^2 \tag{3}$$

where

$$(\overline{o+c}) = \frac{1}{n}\sum_{i=1}^{n}(o_i + c_i)$$

n is the total number of dates within the period, and the subscript i denotes the ith date. The classical estimator gives the average volatility over a period of n dates.

Parkinson (1980) proposes the use of the high and low values to estimate the variance. His estimator is

$$\hat{\sigma}_P^2 = \frac{1}{4\ln 2}(h - l)^2 \tag{4}$$

Garman and Klass (1980) extend the Parkinson's approach by adding the opening and closing prices into the equation. Their practical estimator is

$$\hat{\sigma}_{GK}^2 = 0.511(h - l)^2 - 0.019[c(h + l) - 2hl] - 0.383c^2 \tag{5}$$

Both the Parkinson estimator and the Garman and Klass estimator are built under the assumption that the expected return is equal to zero. Rogers and Satchell (1991) release this restriction and develop an estimator that allows a nonzero drift term. Their estimator is given by

$$\hat{\sigma}_{RS}^2 = h(h - c) + l(l - c) \tag{6}$$

Equations (4)–(6) are based on the daily data. An n-period variance estimator is the average of n daily variances:

$$\hat{\sigma}^2 = \sum_{i=1}^{n}\sigma_i^2/n$$

Yang and Zhang (2000) prove that an unbiased variance estimator independent of both the drift and the opening jump must be multiperiod based. They construct an estimator that has the minimum variance among all estimators that have the same property. Their estimator is given by

$$\hat{\sigma}_{YZ}^2 = \hat{\sigma}_o^2 + k\hat{\sigma}_c^2 + (1 - k)\hat{\sigma}_{RS}^2 \tag{7}$$

where $\hat{\sigma}_o^2$ and $\hat{\sigma}_c^2$ are the variances estimated by the classical estimator with the use of daily opening and closing prices, respectively. The constant k is set to be

$$k = \frac{0.34}{1.34 + (m + 1)/(m - 1)} \tag{8}$$

where m is the number of days. Yang and Zhang (2000) prove that this range estimator reaches its highest efficiency when $m = 2$. Therefore $m = 2$ is used in this article.

Monte Carlo Simulation

Suppose that the logarithm of the asset price follows the Brownian motion in Equation (2). A discrete approximation of $d \ln S$ is

$$\ln S_t - \ln S_{t-1} = (\mu - 0.5 \times \sigma_t^2)/N + \sigma_t \times \Delta B_t, \quad t = 1, 2, \ldots, N \quad (9)$$

where $\Delta B_t \sim n(0, 1/N)$ is a normal random number with a mean of zero and a variance of $1/N$, and N is the total number of price movements per day.

The simulations are conducted under various assumptions concerning the asset return distribution. An initial test investigates how a changing drift affects the performance of each range estimator. On each day, the asset price is assumed to move 400 times and 400 random numbers with distribution $N[(\mu - 0.5 * \sigma^2)/400, \sigma^2/400]$ are generated. For this case, the drift is allowed to vary. For most stocks and indices, the drift will be small, particularly when the subject is daily returns.[5] However, the drift can be very large in a short period, particularly when the market is moving in one direction.

The next test examines how the changing volatility affects the performance of each range estimator. Because the range estimators only require a short time period to give estimations, it is hypothesized that they are able to capture the short-run dynamics of volatility changes. Therefore, the volatility σ_t is simulated in Equation (9) with three different models—the constant volatility model, the deterministic volatility model, and the jump volatility model.

1. Constant volatility model: This model takes daily volatility be a constant 0.01, which is close to the average daily volatility of the S&P 500 within the period of January 1, 1995 to December 31, 1999.
2. Deterministic volatility model: It is well known that volatility changes over time.[6] This model assumes volatility changes each day, but it remains a constant within the day. In particular, it is assumed that the volatility is determined by the following equation:

$$\sigma_t = 0.01 - 0.0001 \times \ln S_{t-1} \quad (10)$$

[5]For example, the average daily return of the S&P 500 index over the entire estimation period is about 0.00092 per day; therefore the drift is only $\mu = 0.000972$ ($\mu - 0.5 \times \sigma^2 = 0.000922$).
[6]See French, Schwert, and Stambaugh (1987), Nandi (1998), and so on.

The starting stock price S_0 is set to 459.2, which is the same as the S&P 500 index level on January 3, 1995.

3. Jump volatility model: Sudden crashes and booms in a market due to unexpected information are not unusual. The classical estimator assumes that volatility remains constant during the whole estimation period, so it may not be able to reflect these short-run changes in volatility. The jump volatility model tests the performance of each range estimator by setting the true daily volatility to be 0.01 with 95% probability and 0.02 with 5% probability.

Finally, the impact of opening jump on the performance of each range estimator is tested. For this case, it is assumed that the market is moving continuously, but that data can only be observed when the market is open. It is assumed that on each day there are 500 price movements. The steps of the price movements while the market is closed are set to 200, 150, 100, 50, and 0 steps, respectively. One thousand days of prices are generated for each simulation, and each simulation is repeated 100 times.

Comparison Criteria

When comparisons are made, there are two major concerns. The first one is the accuracy of the estimation. The accuracy error is defined as

$$\text{error} = \text{estimated variance} - \text{known true variance}$$

Three kinds of errors were computed. The first kind is the mean error (ME), which is the average difference between the estimated variance and the true variance. The second kind is the mean-square error (MSE), which is the average of the squared error. The third kind is the relative error, which is defined as the percentage of difference between the mean estimated variance and the known true value of the variance.

The second concern is the efficiency of the estimators. The variance of an estimator measures the uncertainty of the estimation. The estimator with the minimum variance is considered to be the most efficient estimator. In order to make an easy comparison, the variance of the Parkinson estimator is used as the benchmark. The relative variance efficiency ratio of an estimator is defined by the ratio of the variance of the Parkinson estimator to the variance of the current estimator, denoted as V,

$$\text{Eff} = \frac{Var(\text{Parkinson})}{Var(V)}$$

The larger the ratio is, the more efficient the estimator is.

Data and Empirical Study

The range estimators are also tested with S&P 500 index to examine the extent to which they are useful with real market data. The tick-by-tick data, which are supplied by Tick Data, Inc., cover the period from January 1, 1995 to December 31, 1999. There are a total of 1263 trading days. A preliminary study shows that the average daily return for the entire period is 0.000922 and the standard deviation of the daily return is 0.01. Unlike the simulation, the true variance of market data is unobservable, so the comparison lacks a benchmark. Therefore, the performance of each range estimator is compared with the performance of the daily integrated variance, defined as:

$$\hat{\sigma}^2_{in\_vol} = \frac{1}{\Delta}\left[\frac{1}{N}\sum_{i=1}^{N}(R_{t+i\Delta})^2\right] \tag{11}$$

where Δ is the length of the period, and N is the total number of periods within a day.

Andersen and Bollerslev (1997) show that the integrated variance is a good proxy of daily return variance. However, the accuracy of this volatility estimator is affected by the bid/ask spread, particularly when data are sampled at very high frequency. Therefore, intraday returns are calculated at 1-, 10-, 15-, and 30-minute intervals to test how sensitive the daily realized volatility is to the sampling frequency. It is shown that the daily realized volatility becomes stabilized when the sampling period is longer than 15 minutes.

DISCUSSION OF RESULTS

Simulation Results

Table I presents the simulation results under various drift sizes. When the drift term is small (0.005 or less), range estimators are fairly close to the true variance, and the maximum bias is less than 10%. There are no significant differences among the range estimators; each lies in the other's 95% confidence interval. When the drift term is large, the Parkinson estimator and the Garman and Klass estimator severely overestimate the true variance. The degree of pricing error increases as the drift term increases. In an extreme case, when the drift increases to 0.02, the Parkinson estimator is almost 133% higher than the true variance, and the Garman and Klass estimator is 33% higher than the true variance. The Rogers and Satchell estimator and the Yang and Zhang

TABLE I

Testing the Effect of Drift on Alternative Range Estimators

	Parkinson	*GK*	*RS*	*YZ*
Panel A: $\mu = 0$				
Mean	9.37E-05	9.08E-05	9.05E-05	9.11E-05
Mean error	−6.34E-06	−9.21E-06	−9.53E-06	−8.90E-06
Median	9.15E-05	9.01E-05	8.94E-05	9.00E-05
Sample variance	4.29E-10	2.53E-10	3.23E-10	2.84E-10
MSE	4.69E-10	3.38E-10	4.13E-10	3.63E-10
Relative error	−6.34%	−9.21%	−9.53%	−8.90%
MSE ratio	1.00	1.39	1.14	1.29
Panel B: $\mu = 0.001022$				
Mean	9.38E-05	9.14E-05	9.15E-05	9.23E-05
Mean error	−6.22E-06	−8.57E-06	−8.48E-06	−7.71E-06
Median	9.14E-05	8.94E-05	8.96E-05	9.08E-05
Sample variance	3.96E-10	2.58E-10	3.24E-10	2.82E-10
MSE	4.34E-10	3.32E-10	3.95E-10	3.41E-10
Relative error	−6.22%	−8.57%	−8.48%	−7.71%
MSE ratio	1.00	1.31	1.10	1.27
Panel C: $\mu = 0.005$				
Mean	1.02E-04	9.35E-05	9.06E-05	9.15E-05
Mean error	1.51E-06	−6.46E-06	−9.41E-06	−8.47E-06
Median	9.82E-05	9.22E-05	9.02E-05	9.06E-05
Sample variance	5.74E-10	2.77E-10	3.19E-10	2.85E-10
MSE	5.76E-10	3.18E-10	4.07E-10	3.56E-10
Relative error	1.51%	−6.46%	−9.41%	−8.47%
MSE ratio	1.00	1.81	1.41	1.62
Panel D: $\mu = 0.01$				
Mean	1.31E-04	1.03E-04	8.94E-05	9.03E-05
Mean error	3.07E-05	2.90E-06	−1.06E-05	−9.70E-06
Median	1.28E-04	1.02E-04	8.84E-05	8.89E-05
Sample variance	9.58E-10	3.44E-10	3.24E-10	2.81E-10
MSE	1.89E-09	3.52E-10	4.36E-10	3.75E-10
Relative error	30.65%	2.90%	−10.59%	−9.70%
MSE ratio	1.00	5.39	4.35	5.06
Panel E: $\mu = 0.015$				
Mean	1.74E-04	1.17E-04	8.85E-05	8.96E-05
Mean error	7.40E-05	1.66E-05	−1.15E-05	−1.04E-05
Median	1.71E-04	1.15E-04	8.71E-05	8.83E-05
Sample variance	1.64E-09	4.31E-10	3.44E-10	3.04E-10
MSE	7.11E-09	7.06E-10	4.75E-10	4.11E-10
Relative error	74.00%	16.62%	−11.48%	−10.40%
MSE ratio	1.00	10.07	14.98	17.28
Panel F: $\mu = 0.02$				
Mean	2.34E-04	1.33E-04	8.49E-05	8.61E-05
Mean error	1.34E-04	3.34E-05	−1.51E-05	−1.39E-05
Median	2.31E-04	1.33E-04	8.37E-05	8.48E-05
Sample variance	2.3E-09	5.17E-10	3.92E-10	3.4E-10
MSE	2.02E-08	1.63E-09	6.18E-10	5.33E-10
Relative error	133.71%	33.45%	−15.06%	−13.89%
MSE ratio	1.00	12.34	32.62	37.86

Note. Panels D–F are cases with larger drift. There are 1000 days with 400 price movements per day. The simulation is repeated 100 times. The true daily variance is 0.0001. GK is the Garman-Klass estimator; RS is the Rogers-Satchell estimator; YZ is the Yang-Zhang estimator. Relative error is the percentage difference between the mean estimation and the true value. Mean error is the mean of the difference between the estimated variance and the true variance. The MSE is the mean squared difference between the estimated variance and the true variance. The MSE ratio is the ratio of the MSE of the Parkinson estimator to the MSE of an alternative volatility estimator.

TABLE II

Testing Four Range Estimators with the use of Simulated Data Without an Opening Jump

	Parkinson	*GK*	*RS*	*YZ*
Panel A: Constant volatility model. True daily variance 0.0001. Daily variance estimated by the classical estimator with daily closing price 9.78E-05				
Mean	9.38E-05	9.14E-05	9.15E-05	9.23E-05
Mean error	−6.22E-06	−8.57E-06	−8.48E-06	−7.71E-06
Sample variance	3.96E-10	2.58E-10	3.24E-10	2.82E-10
MSE	4.34E-10	3.32E-10	3.95E-10	3.41E-10
Relative error	−6.22%	−8.57%	−8.48%	−7.71%
MSE ratio	1.00	1.31	1.10	1.27
Variance efficiency ratio	1.00	1.53	1.22	1.40
Panel B: Deterministic volatility model, $\sigma_t = 0.01 - 0.0001 \times \ln S_{t-1}$. True average daily variance 8.72E-05. Daily variance estimated by the classical estimator with daily closing price 8.43E-05				
Mean	8.05E-05	7.85E-05	7.84E-05	7.89E-05
Mean error	−6.66E-06	−8.70E-06	−8.80E-06	−8.20E-06
Sample variance	3.01E-10	2.00E-10	2.60E-10	2.27E-10
MSE	3.45E-10	2.75E-10	3.37E-10	2.94E-10
Relative error	−7.64%	−9.99%	−10.07%	−9.43%
MSE ratio	1.00	1.25	1.02	1.15
Variance efficiency ratio	1.00	1.51	1.16	1.33
Panel C: Jump volatility model. Daily volatility is 0.01 with 95% probability and 0.02 with 5% probability. True average daily variance 1.11E-04. Daily variance estimated by the classical estimator with daily closing price 1.10E-04				
Mean	1.06E-04	1.03E-04	1.04E-04	1.04E-04
Mean error	−5.24E-06	−7.33E-06	−7.24E-06	−6.34E-06
Sample variance	1.01E-09	7.62E-10	8.21E-10	7.48E-10
MSE	7.07E-10	5.05E-10	5.83E-10	5.09E-10
Relative error	−4.72%	−6.60%	−6.52%	−5.71%
MSE ratio	1.00	1.40	1.21	1.39
Variance efficiency ratio	1.00	1.33	1.23	1.35

Note. There are 1000 days with 400 price movements per day. The simulation is repeated 100 times. Relative error is the percentage difference between the mean estimation and the true value. Mean error is the mean of the difference between the estimated variance and the true variance. The MSE is the mean squared difference between the estimated variance and the true variance. The MSE ratio is the ratio of the MSE of the Parkinson's estimator to the MSE of an alternative estimator. The variance efficiency ratio is the ratio of the variance of Parkinson's estimator to the alternative estimator variance.

estimator are designed to be drift independent, so their pricing accuracy is not significantly affected by the size of the drift terms.[7]

Table II reports the performance of each range estimator when the data-generating process involves time-varying volatility. Panel A corresponds to the constant volatility case, Panel B corresponds to the case

[7]To give the reader some idea of the magnitude of the drift size, please notice that the average drift of S&P 500 index daily return during this sample period is 0.000972, but drift can be large during short periods, particularly when asset price moves along one direction.

when the daily variance is generated by Equation (10), and Panel C corresponds to the case when the daily variance may experience sudden jumps. A notable finding is that the degree of bias does not worsen as volatility increases. Actually, the mean distance between the estimated variance and the true variance is even smaller for the jump volatility model than for the constant volatility model. Among the four range estimators, the Parkinson estimator has the highest accuracy, followed by the Yang and Zhang estimator. The Garman and Klass and the Rogers and Satchell estimators have relatively higher pricing errors, but the degree of bias is not severe. The maximum pricing errors are less than 10%. The Garman and Klass estimator had a maximum pricing error of 8.57% under the constant volatility model, but this decreased to 6.6% under the jump volatility model. This result confirms the hypothesis that range estimators are able to capture short-run dynamics of volatility changes. Thus they are suitable to be used to estimate volatility when the return-generating process involves time-varying volatility. In terms of efficiency, the Garman and Klass estimator generally has the highest efficiency.[8] The improvement in efficiency can be as large as 50% over the Parkinson's estimator. The Yang and Zhang estimator has the second-highest efficiency, and the Parkinson's estimator is the least efficient. The present results show that changing volatility has no apparent effect on the performance of range estimators. Range estimators can provide good estimation of the true variance as long as the underlying asset price follows a geometric Brownian motion with no opening jump, either with constant volatility or time-varying volatility.

Table III presents the simulation results under various opening jump sizes. The simulation generates 500 price movements, and it is assumed that the exact price movements for the trading and nontrading periods are known. The nontrading period is set to be 0, 50, 100, 150, and 200, respectively. It is found that when there is big gap between the closing price and the subsequent opening price, only the Yang and Zhang estimator is able to give an accurate estimation. The other three range estimators are severely downward biased. The degree of the bias increases monotonically with the size of the opening jump. Actually, if the 6–8% estimation error is assumed to be caused by a discretization problem regardless of whether there is an opening jump or not, then the bias caused by the opening jump is almost proportional to the size of opening jump.

[8]The variance efficiency ratio for the Yang and Zhang estimator is 1.35 under the jump volatility model, slightly better than that of the Garman and Klass estimator, which is 1.33. But the difference is not significant.

TABLE III

Testing the Effect of Opening Jump on Alternative Range Estimators

	Parkinson	*GK*	*RS*	*YZ*
Panel A: No opening jump, 500 price movements per day				
Mean	9.14E-05	9.03E-05	9.06E-05	9.09E-05
Mean error	−8.62E-06	−9.72E-06	−9.37E-06	−9.10E-06
Median	8.91E-05	8.94E-05	8.97E-05	9.00E-05
Sample variance	3.57E-10	2.26E-10	2.84E-10	2.49E-10
MSE	4.31E-10	3.20E-10	3.72E-10	3.32E-10
Relative error	−8.62%	−9.72%	−9.37%	−9.10%
MSE ratio	1.00	1.34	1.16	1.30
Panel B: 450 price movements when market is open and 50 price movements when market is closed				
Mean	8.29E-05	8.13E-05	8.13E-05	9.18E-05
Mean error	−1.71E-05	−1.87E-05	−1.87E-05	−8.16E-06
Median	8.08E-05	7.96E-05	7.99E-05	9.05E-05
Sample variance	2.77E-10	1.81E-10	2.37E-10	2.29E-10
MSE	5.68E-10	5.32E-10	5.86E-10	2.96E-10
Relative error	−17.06%	−18.73%	−18.70%	−8.16%
MSE ratio	1.00	1.07	0.97	1.92
Panel C: 400 price movements when market is open and 100 price movements when market is closed				
Mean	7.46E-05	7.29E-05	7.28E-05	9.35E-05
Mean error	−2.54E-05	−2.71E-05	−2.72E-05	−6.49E-06
Median	7.23E-05	7.18E-05	7.20E-05	9.24E-05
Sample variance	2.59E-10	1.63E-10	1.98E-10	2.48E-10
MSE	9.01E-10	8.95E-10	9.34E-10	2.90E-10
Relative error	−25.37%	−27.08%	−27.16%	−6.49%
MSE ratio	1.00	1.01	0.96	3.11
Panel D: 350 price movements when market is open, 150 price movements when market is closed				
Mean	6.41E-05	6.29E-05	6.32E-05	9.32E-05
Mean error	−3.59E-05	−3.71E-05	−3.68E-05	−6.81E-06
Median	6.25E-05	6.23E-05	6.26E-05	9.24E-05
Sample variance	1.84E-10	1.25E-10	1.53E-10	3.05E-10
MSE	1.47E-09	1.50E-09	1.51E-09	3.51E-10
Relative error	−35.91%	−37.10%	−36.80%	−6.81%
MSE ratio	1.00	0.98	0.98	4.20
Panel E: 300 price movements when market is open, 200 price movements when market is closed				
Mean	5.48E-05	5.37E-05	5.39E-05	9.30E-05
Mean error	−4.52E-05	−4.63E-05	−4.61E-05	−6.98E-06
Median	5.36E-05	5.32E-05	5.28E-05	9.07E-05
Sample variance	1.37E-10	8.8E-11	1.18E-10	3.87E-10
MSE	2.18E-09	2.23E-09	2.25E-09	4.35E-10
Relative error	−45.16%	−46.28%	−46.13%	−6.98%
MSE ratio	1.00	0.98	0.97	5.00

Note. There are 1000 days with 500 price movements per day. The simulation is repeated 100 times. The true daily variance is 0.0001. Panels D and E reflect a large opening jump size. Relative error is the percentage difference between the mean estimation and the true value. Mean error is the mean of the difference between the estimated variance and the true variance. The MSE is the mean squared difference between the estimated variance and the true variance. The MSE ratio is the ratio of the MSE of the Parkinson estimator to the MSE of an alternative estimator.

Empirical Results

The same range estimators are also applied to the S&P 500 index return data, with the use of the daily high, low, opening, and closing prices. Unlike in the numerical test, the true volatility of asset returns in the real markets is not known here, so the comparison lacks a benchmark. Recent literature (see, e.g., Andersen, 2000; Andersen & Bollerslev, 1997), advocates the use of high-frequency data to extract daily return volatility. Andersen (2000) shows that the sum of intraday return squares, named "integrated variance," is a good proxy of daily return variance. However, this is only true when there is no market microstructure noise. In the presence of a bid/ask spread, the observed price will deviate from the true price by half of the bid/ask spread. The bid/ask bounce in observed price increases the measured volatility of the intraday returns and hence the sum of intraday return squares. Alizadeh et al. (2002) show that the daily integrated variance is very sensitive to market microstructure noise. In the presence of a bid/ask spread, the accuracy of the daily integrated variance will be greatly reduced. On the other hand, range estimators are less sensitive to microstructure noise, because there are no cumulation effects. The performance of the range estimators is compared with the daily integrated variance with and without a bid/ask spread with the use of a Monte Carlo simulation. The true price is simulated by Equation (9), and the observed price is defined as

$$(\text{True} - 0.0625) \times q + (\text{True} + 0.0625) \times (1 - q)$$

where q is a random number that follows a Bernoulli distribution.

Panel A in Table IV presents the results of the different estimators with the use of the true prices, that is, the prices that are generated without a bid/ask spread. The daily integrated variance is very close to the true variance, with a relative error less than 0.01%. The relative pricing error for range estimators varies from 6% to about 9%. In terms of efficiency, daily integrated variance is about 80 times more efficient than the Parkinson estimator. The story is quite different when there are microstructure effects. The results in Panel B show that in the presence of bid/ask spread, the daily integrated variance is about 6% larger than the true variance due to the accumulation of noise. The variance of the daily integrated variance also increases greatly because of noise from the bid/ask spread; therefore, the efficiency drops dramatically from about 80 times higher to only 3 times higher than the Parkinson estimator. The current results are consistent with those of Alizadeh et al. (2002), and further confirm that although the daily integrated variance provides good

TABLE IV
The Effect of Microstructure Noise on the Four Range Volatility Estimators and on the Daily Integrated Variance

	Integrated variance	*Parkinson*	*GK*	*RS*	*YZ*
Panel A: Daily variance estimated from the true prices without bid/ask spread					
Mean	1.00E-04	9.40E-05	9.12E-05	9.11E-05	9.22E-05
Mean error	−6.96E-09	−6.00E-06	−8.80E-06	−8.89E-06	−7.77E-06
Median	1.00E-04	9.38E-05	9.12E-05	9.10E-05	9.22E-05
Sample variance	5.14E-13	4.15E-11	2.33E-11	2.97E-11	2.64E-11
MSE	5.14E-13	7.75E-11	1.01E-10	1.09E-10	8.68E-11
Relative error	−0.01%	−6.00%	−8.80%	−8.89%	−7.77%
Variance efficiency ratio	80.73	1.00	1.78	1.40	1.57
Panel B: Daily variance estimated from the observed prices with bid/ask spread					
Mean	1.07E-04	9.44E-05	9.17E-05	9.16E-05	9.27E-05
Mean error	6.80E-06	−5.63E-06	−8.30E-06	−8.38E-06	−7.27E-06
Median	1.06E-04	9.42E-05	9.17E-05	9.14E-05	9.27E-05
Sample variance	1.29E-11	4.17E-11	2.34E-11	2.99E-11	2.66E-11
MSE	5.92E-11	7.34E-11	9.23E-11	1.00E-10	7.95E-11
Relative error	6.80%	−5.63%	−8.30%	−8.38%	−7.27%
Variance efficiency ratio	3.22	1.00	1.77	1.39	1.57

Note. There are 1000 days with 400 price movements per day. The simulation is repeated 100 times. The true variance is 0.0001. The integrated variance is the sum of 400 intraday return squares. Relative error is the percentage difference between the mean estimate and the true value, where the true value is the average daily integrated variance. The variance efficiency ratio is the ratio of the variance of the Parkinson estimator to the alternative variance estimators.

proxy of daily variance, its accuracy and efficiency will decrease sharply in the presence of a bid/ask spread that cannot be ignored in the real market. On the other hand, range estimators are not sensitive to a bid/ask spread. The pricing accuracy and efficiency of range estimators do not change very much between Panel A and Panel B. The simulation results show that in the presence of bid/ask spread, the daily integrated variance tends to overestimate the true variance, and the range estimators tend to underestimate the true variance. It is hard to detect which one is superior, but it is much easier to compute with range estimators than to compute with the daily integrated variance, because range estimators require much less information.

Before range estimators are compared with the daily integrated variance, notice that the accuracy of the daily integrated variance is sensitive to return sampling frequency. Therefore, the stability of the daily integrated variance constructed from various return periods is tested first. The results in Table V indicate when the intraday return is sampled at a very high frequency level, for example, minute by minute, the estimated

TABLE V

Descriptive Statistics for Daily Integrated Variance Estimated from Various Intraday Return Periods of S&P 500 Index Data

	1 minute	*5 minute*	*10 minute*	*15 minute*	*30 minute*
Mean	3.96E-05	6.54E-05	7.32E-05	7.23E-05	7.20E-05
Sample variance	1.56E-07	3.24E-08	3.79E-08	1.48E-08	1.34E-08
Meidan	1.90E-05	3.81E-05	4.17E-05	4.27E-05	4.22E-05
Minimum	1.63E-06	2.72E-06	2.42E-06	2.89E-06	2.11E-06
Maximum	1.39E-02	5.51E-03	5.79E-03	1.93E-03	1.91E-03

Note. The sample period is from January 1, 1995 to December 31, 1999. The daily integrated variance is estimated as the sum of daily intraday return squares.

TABLE VI

Testing Four Range Volatility Estimators with the use of S&P 500 Index Data

	Integrated variance	*Parkinson*	*GK*	*RS*	*YZ*
Mean	7.23E-05	7.7E-05	6.9E-05	7E-05	7.3E-05
Sample variance	1.48E-08	1.9E-08	1.3E-08	1.7E-08	1.8E-08
Minimum	2.89E-06	2.6E-06	8.8E-07	0	2.5E-09
Maximum	1.93E-03	0.0021	0.00195	0.00195	0.00237
Relative error	0%	6.5%	−4.56%	−3.18%	1.00%
Variance efficiency ratio	1.28	1.00	1.46	1.11	1.05

Note. The sample period is from January 1, 1995 to December 31, 1999. The average daily integrated variance is 7.23E-05. The integrated variance is the sum of daily 15-minute return squares. Relative error is the percentage difference between the mean estimate and the true value, where the true value is the average daily integrated variance. The variance efficiency ratio is the ratio of the variance of the Parkinson estimator to the alternative variance estimators.

daily integrated variance is very small. This is probably because of the strong negative autocorrelation between minute-by-minute intraday returns. However, the integrated variance becomes quite stable when the intraday return period is 15 minutes or longer. Therefore the integrated variance estimated from 15-minute returns is used as a proxy of the true daily variance. This variance serves as a benchmark to compare the performance of range estimators with the use of market data.

Table VI provides the comparison results. The range estimators are found to be quite close to the daily integrated variance. The maximum relative pricing error is 6.5%, which corresponds to the Parkinson estimator. The Yang and Zhang estimator clearly leads the other three, with a relative error of only 1%. In terms of efficiency, the Garman and Klass estimator has the minimum variance, and the variances of the other three estimators are larger than that of the integrated variance. But because range estimators only need a short period of data to achieve this performance, the computational effort is significantly reduced. This finding is quite different from previous studies. Rogers, Satchell, and Yoon (1994) compare the range estimators with the historical variance computed by using the squared daily return. They find that the variance estimated by the range estimators is much smaller than the historical variance. This result changes completely if the daily variance is defined as the integrated variance estimated from the intraday returns. By setting up a more accurate measurement for realized volatility, this study shows that range estimators can accurately estimate historical volatility over short time frames.

CONCLUSIONS

In this study, the relative performances of four range-based variance/volatility estimators were compared. Numerical tests with Monte Carlo simulation show that the accuracy of range estimators depends on the assumption of the asset return distribution. If a stock price follows a geometric Brownian motion with a small drift and with no opening jump, the four range estimators all provide good estimation of the true variance. If the drift term is large, the Parkinson estimator and the Garman and Klass estimator will significantly overestimate the true variance, whereas the Rogers and Satchell estimator and the Yang and Zhang estimator are drift independent. If there is a large opening jump, only the Yang and Zhang estimator is able to give an accurate estimation. The other three estimators are downward biased. The degree of bias is proportional to the size of opening jump. When volatility is time varying, the average estimation error is even smaller than the constant volatility case. This result shows that the range estimators are able to capture the short-run dynamics of volatility variation. By adding microstructure noises to the Monte Carlo simulation, the Alizadeh et al. (2002) finding that the range estimators are fairly robust toward microstructure effects is confirmed. An empirical test with S&P 500 index return data shows that the variances estimated with range estimators are quite close to the daily integrated variances computed with the use of the sum of 15-minute squared returns. Their maximum relative difference is about 6.5%. But range estimators require less information to achieve this performance. The empirical result is supportive of the use of range estimators in estimating historical volatility.

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