The European Journal of Finance

ISSN: 1351-847X (Print) 1466-4364 (Online) Journal homepage: https://www.tandfonline.com/loi/rejf20

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The European Journal of Finance **8**, 187–205 (2002)

New evidence on the implied-realized volatility relation

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We consider the relation between the volatility implied in an option's price and the subsequently realized volatility. Earlier studies on stock index options have found biases and inefficiencies in implied volatility as a forecast of future volatility. More recently, Christensen and Prabhala find that implied volatility in at-the-money one-month OEX call options on the S&P 100 index in fact is an unbiased and efficient forecast of ex-post realized index volatility after the 1987 stock market crash. In this paper, the robustness of the unbiasedness and efficiency result is extended to a more recent period covering April 1993 to February 1997. As a new contribution, implied volatility is constructed as a trade weighted average of implied volatilities from both in-the-money and out-of-the-money options and both puts and calls. We run a horse race between implied call, implied put, and historical return volatility. Several robustness checks, including a new simultaneous equation approach, underscore our conclusion, that implied volatility is an efficient forecast of realized return volatility.

Keywords: index options, implied volatility, realized volatility, volatility forecasting, simultaneous equation estimation

1. INTRODUCTION

It is widely believed that the volatility implied in an option's price is the option market's forecast of future return volatility over the remaining life of this option. Under a rational expectations assumption, the market uses all the information available to form its expectations about future volatility, and hence the market option price reveals the market's true volatility estimate. Furthermore, if the market is efficient, the market's estimate, the implied volatility, is the best possible forecast given the currently available information. That is, all information necessary to explain future realized volatility generated by all other explanatory variables in the market information set should be subsumed in the implied volatility.

The hypothesis that implied volatility is an efficient forecast of the subsequently realized volatility has frequently been tested in the literature. In a cross-sectional analysis Latane and Rendleman (1976) and Chiras and Manaster (1978)

The European Journal of Finance
ISSN 1351-847X print/ISSN 1466-4364 online
http://www.tandf.co.uk/journals
DOI: 10.1080/13518470110071209

conclude that implied volatility is a more accurate forecast of future volatility than the historical volatility measures. They regress future volatility on the weighted implied volatility across a broad sample of Chicago Board Options Exchange (CBOE) stock options. Jorion (1995) concludes from a time series perspective that the implied volatility is an efficient estimator of future return volatility in the foreign exchange market. However, other studies cast doubt about the efficiency hypothesis. Lamoureux and Lastrapes (1993) who focus on individual stocks options, find that the historical volatility contains predictive information beyond that in the implied volatilities. Even in the context of the most active option market, namely the market for OEX options on the S&P 100 stock market index, the conclusions are mixed. Day and Lewis (1992) find that weekly implied volatility contains information, but it is not a better forecast of subsequent realized volatility than time-series models. Canina and Figlewski (1993) report that the implied volatility has little predictive power relative to the historical volatility.

However, Christensen and Prabhala (1998) (henceforth CP) find that in the period after the 1987 stock market crash, the implied volatility from one-month at-the-money OEX call options in fact is an unbiased and efficient forecast of ex-post realized index volatility. Recently, Gwilym and Buckle (1999) focus on forecasting future stock market volatility on a daily basis. They find that the implied volatility from options on the UK FTSE 100 index is a less accurate forecast of realized index return volatility than historical volatility. Furthermore, implied volatility is found to be a biased forecast of future realized volatility. Moreover, their results show that implied volatility overestimates realized volatility as time-to-maturity of the option decreases. Only historical volatility does not overestimate realized volatility as expiry of the option approaches. However, implied volatility in option prices contains more information about future realized volatility than historical volatility estimators judged by their regression results. They conclude that the best method used for forecasting can depend on the length of the forecasting period. The implied volatility becomes more volatile around the maturity of the option, and at this period it tends to overestimate the realized volatility. If we focus on a one-month forecasting horizon, then it appears from CP and Gwilym and Buckle (1999) that implied volatility is a good forecast of future stock return volatility in both the UK and US markets.

In this paper, the robustness of the unbiasedness and the efficiency of implied volatility is verified for the market for OEX options using a data set from Datastream. The data cover a more recent period from April 1993 to February 1997 than other studies on this market. Furthermore, we include in-the-money, at-the-money, and out-of-the-money options, and we use both puts and calls in a five day period to construct the implied volatility.

We focus, like CP, on a lower (monthly) sampling frequency than earlier studies. Moreover, we use non-overlapping data on options with about 22 trading days to expiration, so that each option in our data set expires before the next option is sampled. Our results are similar to those in CP, thus establishing the robustness of the unbiasedness and the efficiency of implied volatility to changes in time to expiration and strike price.

As a natural extension of CP, we separate the implied volatility into a call and a put implied volatility to see if there is any difference in predictive power when forecasting future volatility. The information content of volatility implied in put options has not been examined previously. Among our robustness checks is a new simultaneous equation approach to volatility modeling.

The paper is organized as follows. Section 2 describes how we construct the implied and realized volatility series. In this section, we use a trade-weighted average of implied volatilities from in-the-money and out-of-the-money empirical puts and calls. In Section 3 we separate the implied volatilities from puts and calls and consider the resulting three-equation generalization of CP. Section 4 concludes.

2. THE INFORMATION CONTENT OF IMPLIED VOLATILITY

This study is based on call and put options on the S&P 100 index, with one month to expiration, the so-called OEX options. These options are American style options and they are traded at the Chicago Board Options Exchange. We sample our data from Datastream using a recent period from April 1993 to February 1997.

2.1 Implied volatility

Datastream reports implied call, σ_{it}^{c}, and the implied put volatility, σ_{it}^{p}. These implied volatilities are constructed as trading weighted averages of the individual Black (1976) implied volatilities of all near term call or put OEX options, traded over the last five days. Moreover, they are calculated in according to

$$\sigma_i = \left[\frac{\sum_d^5 \sum_{t=1}^s \sigma_{idt} N_{dt}}{\sum_d^5 \sum_{t=1}^s N_{dt}} \right] \tag{1}$$

where s is the number of series with one month to expiration. The first summation is taken because the last five days enter into the volatility calculation, N_{dt} is the number of trades d day(s) ago in series t, and σ_{idt} denotes the implied call or put volatility d day(s) ago on series t calculated using Black's (1976) option pricing model from current market prices.

The OEX options expire, by convention, on the Saturday immediately following the third Friday each month. To avoid overlapping observations, we move five days ahead and record the implied volatilities reported by Datastream. These volatilities are trade-weighted averages of the implied volatilities of the near term options, that is they have between 17 and 22 trading days to expiry.[1,2]

[1] The number of trading days is of course lower if there are mid-week holidays.

[2] Allowing longer terms to maturity would reduce the number of observations in the time series or introduce overlapping data. With shorter horizons, the proxy for the realized volatility (cf. Section 2.2) deteriorates. For studies using different horizons, see Canina and Figlewski (1993) and Gwilym and Buckle (1999).

We construct a third time series of implied volatilities by calculating the trade weighted average of the call and the put volatilities. The implied volatility measure therefore contains information from a broad sample of OEX options in a five day period.

The weighting scheme puts more weight on volatility information from the most actively traded options. Note that we use a weighting scheme for the volatilities as opposed to Day and Lewis (1988; 1992) who derive the implied volatilities from a trade weighted sum of call option prices.[3]

2.2 Realized volatility

We construct a time series of realized index return volatilities. First, a time series of daily S&P 100 closing index levels is sampled from Datastream in the period April 1993 to February 1997. Second, the realized volatility is calculated as the sample standard deviation of the index return for month t is then given by:

$$\sigma_{ht} = \sqrt{\frac{252}{\tau_t - 1} \sum_{k=1}^{\tau_t} (R_{t,k} - \overline{R}_t)^2} \tag{2}$$

where R_k is the daily return on the S&P 100 index and k runs from the Thursday following the third Saturday in month t to the Friday preceding the third Saturday in month $t + 1$. Following this procedure, we obtain a time series of non-overlapping data for a proxy for the realized index return volatility,[4] that covers the same period as the implied volatilities. Here τ_t denotes the number of trading days to maturity for options with expiry in month $t + 1$.[5]

2.3 Descriptive statistics

Table 1 presents a descriptive statistics for the two volatility series: the implied volatility for index options and the realized index return volatility. It is seen that the average realized return volatility is lower than the average implied volatility. This makes sense, since there are features in OEX options that may carry a premium beyond that accounted for in the Black (1976) option pricing model, namely, early exercise features and possibly price pressure on the puts stemming from the demand for portfolio insurance. The two volatility time series are plotted in Fig. 1.

Realized volatility is almost twice as variable as implied volatility, as judged by the respective variances. This is in good accordance with the results found in the post-crash period by CP, even though our implied volatility is derived from a broad collection of index options. Both in- and out-of-the-money, and

[3] As noted by one referee, the volatility indices do not correspond to any given traded option. We are restricted to the somewhat arbitrary five day averaging and evidence on the usefulness of the Datastream measures in a forecast context.

[4] We thank an anonymous referee for pointing out that this is only a proxy for the true but unknown realized return volatility. Alternatively, we could have estimated the realized index return volatility along the lines of Andersen *et al.* (1999) using intraday data.

[5] Following Hull (2000) we multiply by 252 to obtain the volatilities in annual terms.

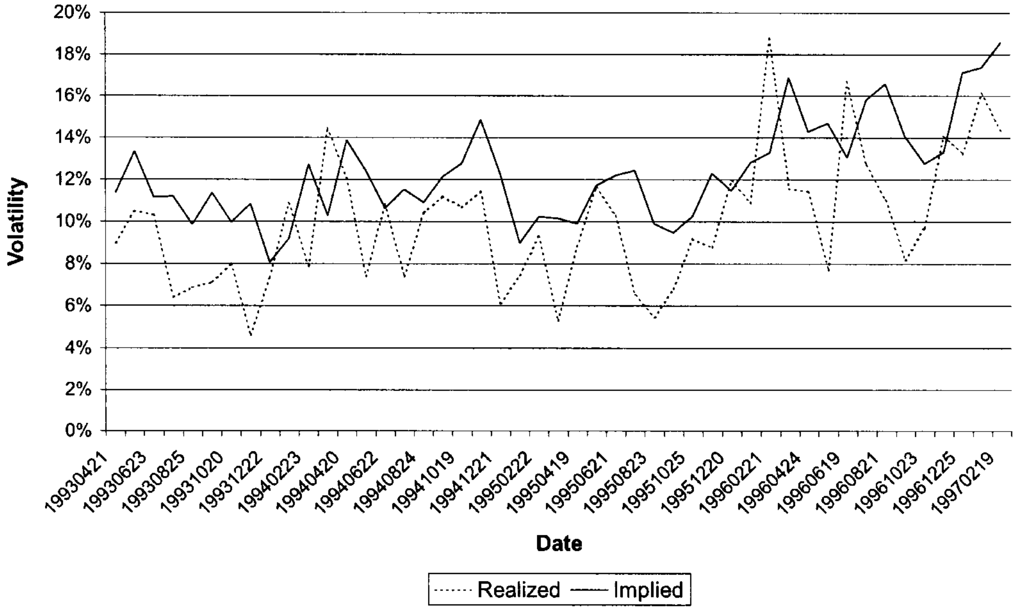

Fig. 1. Realized and implied volatility. Realized volatility is defined in Equation 2 as the sample standard deviation of the daily S&P 100 index returns within each month. The implied volatility is defined in Equation 1 as the trade weighted average of Black-Scholes implied volatilities from call and put OEX options of different moneyness. The date is shown on the *x*-axis according to the YYYYMMDD convention; April 1993 through March 1997

both put and call options. The most remarkable difference in our results compared to those presented by CP is that our variances are approximately three to four times smaller. This may be caused by the difference in sample periods. Our sample period covers only four years but on the other hand we have used the most recent data. The CP post-crash sample period covers the $7\frac{1}{2}$ years following the October 1987 crash. Our results indicate that the S&P 100 index after a period with relatively high volatility following the crash has become more stable. In fact, our realized index return volatility has almost the same variance as the one reported in CP from the pre-crash period.

In Table 1 we also see that the distributions of the implied volatility and the realized volatility are more skewed and leptokurtic for the raw volatility series than for the log-transformed series. The Jarque–Bera test for normality, *JB*, is significantly smaller for the log-transformed series than for the raw volatility series, indicating that the log-transformed series conform better to normality than the raw series.

In what follows, we focus on the log transformed volatility series. To ease the notation, we let i_t denote the natural logarithm of the implied volatility measure, from both call and put options, and we let h_t denote the natural logarithm of the realized volatility.

Table 1. Descriptive statistics

Statistic	impl. vol.	realized vol.	log impl. vol.	log realized vol.
Mean	0.1238	0.0997	–2.107	–2.353
Variance*100	0.0589	0.0968	3.632	9.846
Skewness	0.7071	0.6568	0.2705	–0.1250
Kurtosis	3.0869	3.3882	2.6909	2.7003
Jarque-Bera	3.931	3.674	0.760	0.298

The table reports descriptive statistic for different time series of implied volatility on the S&P 100 stock index. It is based on 47 monthly observations on each volatility series collected from Datastream and covers the period from April 1993 to February 1997.

2.4 Empirical results

We now turn to the results of our empirical analysis. We start out with a conventional analysis to asses the characteristics of the relationship between implied and realized volatility.

In Table 2, we examine the information content of implied volatility. We estimate the encompassing regression:

$$h_t = \alpha_0 + \alpha_i i_t + \alpha_h h_{t-1} + \varepsilon_t \tag{3}$$

Panel A reports the OLS estimates of specification (3). There are three testable hypotheses of main interest. First, if implied volatility contains *some* information about future realized volatility, then the coefficient of the implied volatility, α_i, should be non-zero. Second, if implied volatility is an *unbiased* forecast of future realized volatility, then α_i should be equal to one. Finally, if implied volatility is an *efficient* predictor of future realized volatility, then the coefficient of the historical volatility, α_h, should be zero, and the residuals ε_t should be white noise and thereby uncorrelated with any variable in the market's information set.

We see that implied volatility does contain information about future realized volatility, whereas past realized volatility does not enter significantly in the regression. We cannot reject the hypothesis of $\hat{\alpha}_i = 1$ and $\hat{\alpha}_h = 0$ (the F-test takes the value 0.79, on 2 and 45 degrees of freedom, corresponding to a p-value of 45%). Thus, implied volatility appears both unbiased and efficient. This confirms the results by CP and contradicts earlier studies that found inefficiencies in implied volatility as a forecast of future volatility (cf. Day and Lewis, 1992 and Canina and Figlewski, 1993).

In this and the following tables, we report Durbin's alternative, d (cf. Maddala, 1992, p.249) as a supplement to the Durbin–Watson statistic in the regressions, where the lagged left-hand side variables are included as regressors. Durbin's alternative is implemented by regressing the residuals on the explanatory variables and the lagged residual, and d is the coefficient on the latter (t-statistic in parenthesis). The null of no serial correlation is $d = 0$.

Table 2. Information content of average implied volatility: OLS and 2SLS estimates

		Panel A: OLS Estimates Dependent variable: h_t			
Intercept	i_t	h_{t-1}	Adj. R^2	DW	d
–0.3209	1.2966[a]	–0.2970	26.30%	2.032	–0.464
(–0.67)	(2.97)	(–1.11)			(–0.16)
Intercept	i_t	H_{t-1}	Adj. R^2	DW	d
–0.6107	1.1136[a]	–0.2583	25.66%	2.224	–0.132
(–1.27)	(3.20)	(–0.86)			(–0.85)
Intercept	i_t		Adj. R^2	DW	d
–0.5112	0.8734[a]		25.90%	2.207	–0.046
(–1.13)	(4.09)				(–0.16)
		Panel B: 2SLS Estimates Dependent variable: i_t			
Intercept	i_{t-1}	h_{t-1}	Adj. R^2	DW	d
–0.3956[a]	0.3312[a]	0.4222[a]	82.66%	1.750	–0.104
(–2.76)	(4.07)	(10.09)			(–0.69)
		Dependent variable: h_t Instruments for i_t: i_{t-1}, h_{t-1}			
Intercept	i_t		Adj. R^2	DW	
–0.7011	0:7806[a]		25.40%	2.111	
(–1.41)	(3.29)				
		Dependent variable: h_t Instrument for i_t: i_{t-1}			
Intercept	i_t		Adj. R^2	DW	
–0.4342	0.9100[a]		25.66%	2.233	
(–0.62)	(2.76)				

[a] p-value $<$ 0.01; [b] p-value $\in$ [0.01; 0.05]; [c] autocorrelation

Panel A reports the OLS estimates of the specification: $h_t = \alpha_0 + \alpha_i i_t + \alpha_h h_{t-1} + e_t$. Panel B reports the two-stage least squares estimates, where we first have used i_{t-1} and h_{t-1} as instruments and second we have only used i_{t-1} as instrument. Here i_t denotes the natural logarithm of the average of the volatility implied in the call and put options on the S&P 100 index calculated by Datastream; and h_t denotes the natural logarithm of the ex-post realized daily return volatility of the S&P 100 index calculated over the remaining lives of the relevant options; and H_t is a long run level of h_t. Durbin's alternative, d, is reported as a supplement to the Durbin–Watson statistic *DW* in the regressions, where the lagged left-hand side variables are included as regressors. Data consist of 47 monthly observations on each volatility series constructed in a non-overlapping fashion, covering the period from April 1993 to February 1997. The numbers in parentheses denote asymptotic t-statistics.

One might wonder whether a historical return volatility estimated from more data may have a greater chance of entering significantly in the regression. In the light of this hypothesis we construct a long-run level of the three previous periods' realized volatilities H_{t-1}. We interchange the past realized volatility with the long-run level in the regression. It is seen that long-run lagged return volatility has even weaker explanatory power than the original variable, h_{t-1}, and we henceforth use the latter for our historical volatility measure. We also estimate the final model, restricting the coefficient on past realized volatility to zero (last line of Panel A). The slope coefficient ($\hat{\alpha}_i = 0.87$) is estimated to be less than unity. Although the slope is insignificantly different from unity (t-value $= -0.59$), this may be an indication of an errors-in-variable (EIV) problem in the implied volatility time-series.

There are several sources of measurement errors involved when deriving the implied volatility. (See Harvey and Whaley, 1991, 1992; Jorion, 1995; and CP for a detailed list of measurement errors.) The problem is also addressed by CP, who cannot reject the hypothesis that the coefficient on implied volatility was downwards biased due to measurement error. The EIV problem induces a bias in both α_i and α_h, stemming from potential correlation between measured implied volatility and the error term. Moreover, α_i is biased downwards towards zero. Under the efficiency null (i.e. $\alpha_h = 0$), the bias in α_i is even greater when lagged realized volatility is included in the regression than when it is left out. Assuming that the slope coefficient on implied volatility and the correlation between implied and past realized volatility are positive, which is supported by our empirical investigations in Table 2, the slope associated with the past realized volatility α_h is biased upwards. Even under the efficiency null, the probability limit of the OLS estimate of $\hat{\alpha}_h$ is still positive.

Consistent estimation in the presence of the EIV problem may be achieved using an instrumental variables (IV) method.[6] Thus, instruments for implied volatility are called for. The lagged values of implied and realized volatility are natural candidates. Therefore, we consider the following additional equation:

$$i_t = \beta_0 + \beta_i i_{t-1} + \beta_h h_{t-1} + e_t \tag{4}$$

This specification allows for the possibility that historical volatility provides information for option prices and therefore for implied volatility. Estimates for Equation 4 are presented in the first line of Table 2, Panel B. Both explanatory variables are highly significant. This confirms the result of CP that implied volatility may be precisely forecasted (note the high adjusted R^2, 83%) using a parsimonious set of variables in the market's information set.

Two-stage least squares (2SLS) estimates of the realized volatility equation are obtained by using the fitted values from the implied volatility Equation 4 as an instrument. The results appear in the second line of Panel B. It turns out that the slope for implied volatility is no stronger than in the corresponding OLS regression. Here the Hausman (1975) test indicates that the EIV problem does not matter in our case. However, further analysis actually reveals that the Hausman (1975) test indeed indicates the presence of an EIV problem if the IV

[6] See Greene (2000, Chapter 9) for an outline of the IV method.

method is implemented slightly differently. In particular, one might argue that using lagged realized volatility in the instrumentation for implied volatility gives an inadequate advantage to implied volatility in the estimation. On the other hand, if only lagged implied volatility is allowed as an instrument, then all explanatory power ascribed to implied volatility in the estimation is based on information backed out of option prices. To analyse this issue, we repeat the IV procedure, but using fitted values from the implied volatility equation with lagged realized volatility excluded, when constructing the instrument. The results appear in the last line of Panel B. It is seen that the coefficient on implied volatility is higher than in the OLS case, and the Hausman (1975) test indicates that an EIV mechanism is at work. However, this analysis is subject to the criticism that in fact lagged realized volatility is significant in the implied volatility equation.

In sum, we have found that our study is not seriously affected by measurement problems. Our best specification obtains already in the OLS analysis, where implied volatility comes through as an unbiased and efficient forecast of subsequently realized return volatility.

3. SEPARATING PUT AND CALL OPTIONS

In this section we separately record an implied volatility for call options and an implied volatility for put options on the S&P 100 index. The implied volatilities are trade weighted averages of volatilities implied in options with all strike prices.

3.1 Descriptive statistics

In Table 3, we present a descriptive statistics for the raw call and put implied volatilities constructed according to Equation (1).

Comparing with the realized volatility from Table 1, we see that this is lower on average than both implied call volatility and implied put volatility. Furthermore, implied put volatility is slightly higher than implied call volatility. This is in good accordance with Harvey and Whaley (1991; 1992) who suggest that it

Table 3. Descriptive statistics

Statistic	impl. call vol.	impl. put vol.	log impl. call vol.	log impl. put vol.
Mean	0.1208	0.1265	–2.134	–2.084
Variance*100	0.0630	0.0572	4.044	3.471
Skewness	0.7744	0.5442	0.2863	0.1294
Kurtosis	3.2691	2.8281	2.8223	2.5714
Jarque-Bera	4.839	2.378	0.704	0.491

The table reports descriptive statistic for different time series of implied volatility for put and call options on the S&P 100 stock index. It is based on 47 monthly observations on each volatility series collected from Datastream and covers the period from April 1993 to February 1997.

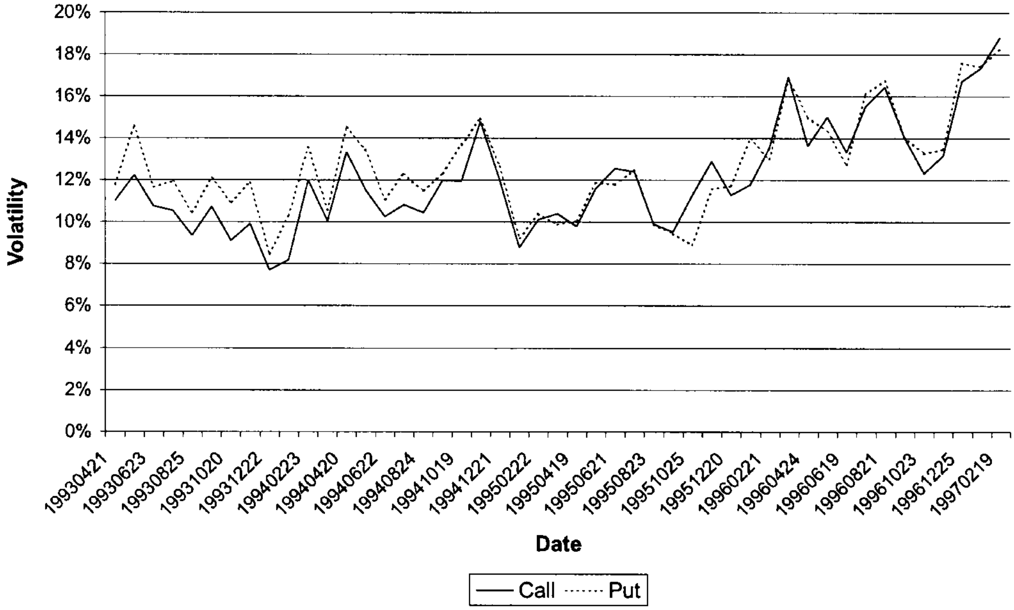

Fig. 2. Call and put implied volatility. Monthly implied call (put) volatility defined as the trade weighted average of OEX call (put) options of different moneyness. The date is shown on the *x*-axis according to the YYYYMMDD convention; April 1993 through February 1997

may be due to the fact that buying index puts is a convenient and relative inexpensive way to implement portfolio insurance. This leads to an excess buying pressure on index puts compared to index calls, and in turn a higher put implied volatility compared to those obtained from the corresponding calls. Existing empirical work on open interest data supports this explanation. Another reason is that American put options carry a higher early exercise premium than American call options.

Turning to the variances, realized volatility (Table 1) is almost twice as variable as both put and call implied volatility. Furthermore, implied put volatility is less volatile than implied call volatility. Figure 2 presents the time series of implied call volatility and implied put volatility.

Again, the distributions of both the implied volatilities are more skewed and leptokurtic for the raw volatility series than for the log-transformed series, so we analyse the log transformed volatility series. In the following we let i_t^c and i_t^p denote the natural logarithm of the call and put implied volatility, respectively.

3.2 Empirical results

We examine the relative information content of put and call implied volatility. We estimate the multiple regression:

$$h_t = \alpha_0 + \alpha_c i_t^c + \alpha_p i_t^p + \alpha_h h_{t-1} + e_t \tag{5}$$

This regression differs from previous studies in that we include implied put volatility. The OLS estimates are presented in Table 4. Several hypotheses can be tested within specification (5). Apart from the efficiency hypothesis ($\alpha_h = 0$) considered in the previous section, we may investigate the issue of which of the two implied volatilities is most informative about future volatility. Indeed, the regression can help identify the optimal weighting of the two implied volatilities for forecasting purposes. The optimal weighting may not coincide with the trade weighting used in the previous section. In the present context, a natural unbiasedness hypothesis is that $\alpha_c + \alpha_p = 1$.

From the first line of Table 4 we see that when all explanatory variables are included, the coefficients roughly add up to unity (precisely, to 0.96). The F-test (on 3 and 44 degrees of freedom) fails to reject the hypothesis that the sum is one. Only the coefficient on call implied volatility is significant at the 95% level in a one-sided test, with a t-statistic equal to 1.81. These results confirm the efficiency findings of the previous section. In addition, the results indicate that call implied volatility has more explanatory power than put implied volatility. It is noted that the point estimate of the coefficient on put implied volatility is positive, and it is conceivable that put implied volatility would be significant if

Table 4. Information content of implied volatility: OLS estimates

Dependent variable: h_t						
Intercept	i_t^c	i_t^p	h_{t-1}	Adj. R^2	DW	d
–0.3836	0.8722	0.3510	–0.2640	25.01%	2.011	–0.012
(–0.79)	(1.81)	(0.60)	(–0.98)			(–0.04)
–0.5149	1.0665[a]		–0.1853	26.10%	1.986	–0.023
(–1.19)	(2.98)		(–0.79)			(–0.09)
–0.5339		1.0497[b]	–0.1564	21.17%	2.065	–0.10
(–1.09)		(2.36)	(–0.58)			(–0.37)
–0.5598	0.7971	0.0750		25.08%	2.159	
(–1.24)	(1.63)	(0.15)				
–0.5818	0.8301[a]			26.71%	2.142	
(–1.38)	(4.21)					
–0.6324		0.8254[a]		22.33%	2.147	
(–1.38)		(3.77)				

[a]p-value $<$ 0:01; [b]p-value $\in$ [0.01; 0.05]; [c]autocorrelation

The table reports the OLS estimates of the specification: $h_t = \alpha_0 + \alpha_c i_t^c + \alpha_p i_t^p + \alpha_h h_{t-1} + e_t$. Here i_t^c and i_t^p denote the natural logarithm of the implied volatility calculated by Datastream for call and put options on the S&P 100 index respectively; and h_t denotes the natural logarithm of the ex-post realized daily return volatility of the S&P 100 index calculated over the remaining lives of the relevant options. Durbin's alternative, d, is reported as a supplement to the Durbin–Watson statistic DW in the regressions, where the lagged left-hand side variables are included as regressors. Data consist of 47 monthly observations on each volatility series constructed in a non-overlapping fashion, covering the period from April 1993 to February 1997. The numbers in parentheses denote asymptotic t-statistics.

we had a larger sample. Nonetheless, the optimal forecast does not appear to be a trade-weighted average, considering that puts are traded almost as frequently as calls. Indeed, the indication is that call implied volatility subsumes the information content of both put implied volatility and historical volatility, so we can also not assign alternative optimal weights on both call and put implied volatility.

The next line in the table shows that if put implied volatility is excluded from the regression, call implied volatility still subsumes historical volatility, and the slope coefficient is quite close to unity. Interestingly, almost the same results obtain in the following line, with put implied volatility replacing call implied volatility. Thus, even the volatility information obtainable from put option prices suffices to subsume historical volatility in the forecasting of future index return volatility, and the slope coefficient is very close to unity. These findings considerably strengthen the results on the information content of implied volatility, relative to CP. The following line confirms that also when historical volatility is left out of the regression, call implied volatility dominates over put implied volatility. The F-test fails to reject that the two slopes add up to unity. The relatively low t-values reflect a clear multicollinearity problem, since the two implied volatility measures are highly correlated, as also evidenced by the similar results in the two foregoing regressions. When historical volatility is excluded, the correlation coefficient between the two implied volatility slopes is -0.91.

The final model appears in the second last line of the table. Only call implied volatility remains as an explanatory variable, based on the previous t-tests. Unbiasedness of call implied volatility ($\alpha_c = 1$) cannot be rejected, based on a simple t-test. Thus, the indication is that call implied volatility is an unbiased and efficient forecast of future volatility.

As a complement, the last line of the table shows that if put implied volatility replaces call implied volatility, the result is virtually identical. This result is of interest since it confirms the value of the information about future volatility contained in put option prices. However, the adjusted R^2 is now slightly lower, and the foregoing analysis shows that call implied volatility is the best forecast.

Although unbiasedness is not rejected in the models in the two last lines of the table, the point estimates of the slopes are less than unity, at 0.83. As in the previous section, we do consider the possibility that there is a measurement error problem in implied volatility. However, with two implied volatility measures, it is less clear-cut than in the previous section how to do the instrumentation if handling the EIV problem using an IV method. To analyse this issue, we next consider the relevant three-equation model explicitly as a simultaneous system (note that the 2SLS estimation appropriate for the EIV model in the previous section would equally be the correct procedure if alternatively implied volatility were considered an endogenous variable).

3.3 The three-equation system

We consider the following three-equation system:

$$h_t = \alpha_0 + \alpha_c i_t^c + \alpha_p i_t^p + \alpha_h h_{t-1} + e_t^h \tag{6}$$
$$i_t^c = \beta_0 + \beta_c i_{t-1}^c + \beta_p i_{t-1}^p + \beta_h h_{t-1} + e_t^c \tag{7}$$
$$i_t^p = \gamma_0 + \gamma_c i_{t-1}^c + \gamma_p i_{t-1}^p + \gamma_h h_{t-1} + e_t^p \tag{8}$$

As already noted, an EIV problem in the implied volatilities in the h-equation may be handled using an IV method, where the form of the instrumentation is based on the i^c and i^p equations. This is the 2SLS approach considered in the two-equation set-up of the previous section, where the two implied volatility measures entered as an aggregate. Alternatively, if we consider the above as a three-equation simultaneous system, accounting for the possible endogeneity of implied volatility in the first equation, then the same IV method can be used. Maximum likelihood provides an efficient alternative in this case. In the following, we consider the three-equation system, keeping in mind that the EIV interpretation and the simultaneity interpretation in this sense are observationally equivalent. Thus, we can test for whether at least one of these two effects is important in the data, but we cannot distinguish empirically between the two. With two implied volatility measures, new issues regarding the form of the last two equations arise. In the present paper, we conduct some of the discussion explicitly in the simultaneous equation jargon, while still keeping in mind the formal equivalence.

Both (7) and (8) are of economic interest in their own right. First, they give us a way of forecasting future implied call and put volatility and thereby a way of forecasting future option prices. Second, they can be used to test whether implied volatility itself is related to past volatility. Up to now, we have tested if implied volatility can predict future realized volatility, but if option prices (implied volatilities) contain volatility information, implied volatility should not only predict future realized volatility but also depend endogenously on past volatility. This is so, especially since we have found that past and future volatility are positively related.

Thus, the implied volatilities may be considered as endogenous variables in the first equation, or they may be afflicted with EIV problems. Either situation calls for a remedy in the estimation. We start out with a full information maximum likelihood (FIML) estimation of the entire system.

The results are presented in Table 5, Panel A. The first line shows the results for the h-equation. We see that the three slopes still add up to roughly one. However, perhaps somewhat surprisingly, the coefficient on call implied volatility has dropped dramatically in magnitude and is no longer significant comparing to the OLS results. While none of the three slopes is significant, put implied volatility now appears to be the most informative forecast, with the largest t-statistic and a slope very close to unity. This indicates that there is possibly even more value in the volatility information contained in put prices than is suggested by the OLS results.

The next two lines show the results for the two implied volatility equations. An interesting pattern immediately emerges. The implied volatility may be explained by its own history and past realized volatility. The history of the opposite implied volatility measure enters insignificantly in both cases. The slopes on own history and past realized volatility add to one, approximately.

Table 5. FIML estimates

FIML estimates						
Panel A:						
Dependent	Intercept	i_t^c	i_t^p	h_{t-1}	Adj. R^2	DW
h_t	−0.3088	0.3489	0.9678	−0.3037	14.79%	2.048
	(−0.36)	(0.36)	(0.66)	(−0.59)		
Dependent	Intercept	i_{t-1}^c	i_{t-1}^p	h_{t-1}	Adj. R^2	DW
i_t^c	−0.3807[b]	0.5496[b]	−0.2135	0.4319[a]	78.24%	2.044
	(−2.34)	(3.36)	(−1.26)	(8.60)		
i_t^p	−0.4479[a]	0.0468	0.2223	0.4529[a]	79.42%	2.045
	(−3.06)	(0.32)	(1.46)	(10.00)		
Panel B:						
Dependent	Intercept	i_t^c	i_t^p	h_{t-1}	Adj. R^2	DW
h_t	−0.2306	0.1376	1.2731	−0.3491	14.79%	2.041
	(−0.29)	(0.11)	(0.78)	(−0.75)		
Dependent	Intercept	i_{t-1}^c	i_{t-1}^p	h_{t-1}	Adj. R^2	DW
i_t^c	−0.2999	0.3850[a]		0.4262[a]	78.01%	1.900
	(−1.98)	(4.99)		(8.50)		
i_t^p	−0.3980[a]		0.3014[a]	0.4464[a]	79.85%	2.134
	(−2.85)		(5.25)	(10.26)		
Panel C:						
Dependent	Intercept	i_t^c	i_t^p	h_{t-1}	Adj. R^2	DW
h_t	−0.6840	0.2533	0.5413		16.17%	2.129
	(−1.38)	(0.20)	(0.40)			
h_t	−0.6719		0.8634[a]		17.11%	2.128
	(−1.35)		(3.38)			
Panel D:						
Dependent	Intercept	i_t^c	i_t^p	h_{t-1}	Adj. R^2	DW
h_t	−0.7410	0.7555[a]			18.06%	2.072
	(−1.58)	(3.45)				
Dependent	Intercept	i_{t-1}^c	i_{t-1}^p	h_{t-1}	Adj. R^2	DW
i_t^c	−0.3039[b]	0.3811[a]		0.4281[a]	78.01%	1.893
	(−2.01)	(4.97)		(8.57)		
i_t^p	−0.4031[a]		0.2970[a]	0.4482[a]	79.85%	2.126
	(−2.89)		(4.21)	(10.34)		

[a]p-value < 0.01; [b]p-value $\in [0.01; 0.05]$; [c]autocorrelation

The table reports the FIML estimates of the simultaneous equation model: $h_t = \alpha_0 + \alpha_c i_t^c + \alpha_p i_t^p + \alpha_h h_{t-1} + e_t^h$, $i_t^c = \beta_0 + \beta_c i_{t-1}^c + \beta_p i_{t-1}^p + \beta_h h_{t-1} + e_t^c$, $i_t^p = \gamma_0 + \gamma_c i_{t-1}^c + \gamma_p i_{t-1}^p + \gamma_h h_{t-1} + e_t^p$. Here i_t^c and i_t^p denote the natural logarithm of the implied volatility calculated by Datastream for call and put options on the S&P 100 index respectively; and h_t denotes the natural logarithm of the ex-post realized daily return volatility of the S&P 100 index calculated over the remaining lives of the relevant options. Data consist of 46 monthly observations on each volatility series constructed in a non-overlapping fashion, covering the period from April 1993 to February 1997. The numbers in parentheses denote asymptotic t-statistics.

Furthermore, the adjusted R^2s are relatively high. This shows that each implied volatility measure may be forecasted quite precisely using a parsimonious two-variable model. This has implications for the prediction of future prices of put and call options, separately. In addition, the findings confirm the hypothesis that implied volatility indeed depends endogenously on variables in the market's information set, which may be seen as an additional rationality condition, beyond unbiasedness and efficiency in the first equation.

Panel B shows the results from an instrumentation. Here, the lagged values of the opposite implied volatility measures are excluded from the second and third equation. Call implied volatility is now even weaker than in the first equation, thus reinforcing the results from Panel A. The two implied volatility equations are now in their final form: their respectively lagged value and past realized volatility are significant in both equations, as are the intercepts.

Keeping the specification of the two implied volatility equations unchanged, we next reduce the *h*-equation. Past realized volatility has a negative sign and an insignificant *t*-statistic in Panels A and B. Furthermore, we expect from the previous section and from CP that this is the weakest explanatory variable. The change in the first equation resulting from dropping past realized volatility can be seen in the first line of Panel C. Compared to the Panel B results, the coefficient on call implied volatility is now slightly higher, and that on put implied volatility is lower. Both remain insignificant, which may be due to multicollinearity. The result of eliminating call implied volatility appears in the last line of Panel C. Now, put implied volatility is significantly greater than zero, and not significantly less than one.

In spite of the fact that the results so far make it difficult to choose between call and put implied volatility, or to make an optimal combination of the two, we expect that call implied volatility is in fact at least as valuable a forecast as put implied volatility. This expectation is based among others on the OLS results. Therefore, we report as our final model in Panel D the system where put implied volatility has been dropped from the first equation. It is seen that the *t*-statistic for call implied volatility is slightly higher than that for put implied volatility from the corresponding regression in Panel C. The estimation of the other two equations does not come out much different from that in Panel B. Any slight difference that may be detected is due to the fact that the error correlation matrix is not restricted to be diagonal in the FIML estimation. With a diagonal matrix, the system would be recursive, so that OLS would suffice for the two implied volatility equations. In this case, the method would reduce to 2SLS for the realized volatility equation.

In the final model, the slope on call implied volatility is not much different from that in the corresponding OLS regression, nor is the *t*-statistic (see the second last line of Table 4). We conclude from this analysis that the OLS results are quite reliable. There is no strong evidence that they are adversely affected by either EIV problems or simultaneity bias. In particular, the conclusion remains that call implied volatility is an unbiased and efficient forecast of future volatility. Along the way, we have picked up further indications that there is in addition valuable volatility information present in put option prices.

As a robustness check, we re-estimate the final model using simple 2SLS, as in the previous section. The results appear in Table 6, Panel A. There is no

Table 6. 2SLS and 3SLS estimates

Panel A: 2SLS estimates						
Dependent	Intercept	i_t^c	i_y^p	h_{t-1}	Adj. R^2	DW
h_t	−0.7740	0.7400[a]			18.06%	2.057
	(−1.62)	(3.30)				
Dependent	Intercept	i_{t-1}^c	i_{t-1}^p	h_{t-1}	Adj. R^2	DW
i_t^c	−0.3193	0.3713[a]		0.4305[a]	78.01%	1.872
	(−1.96)	(4.31)		(8.15)		
i_t^p	−0.4524[a]		0.2643[a]	0.4563[a]	79.85%	2.058
	(−3.00)		(3.34)	(10.01)		
Panel B: 3SLS estimates						
Dependent	Intercept	i_t^c	i_t^p	h_{t-1}	Adj. R^2	DW
h_t	−0.7755	0.7393[a]			18.06%	2.056
	(−1.62)	(3.30)				
Dependent	Intercept	i_{t-1}^c	i_{t-1}^p	h_{t-1}	Adj. R^2	DW
i_t^c	−0.3061	0.3801[a]		0.42814[a]	78.01%	1.891
	(−1.96)	(4.79)		(8.28)		
i_t^p	-0.4056[a]		0.2958[a]	0.4482[a]	79.85%	2.123
	(−2.81)		(4.05)	(10.02)		
Panel C: 3SLS estimates for volatility levels						
Dependent	Intercept	σ_t^c	σ_t^p	σ_{t-1}^h	Adj. R^2	DW
σ_t^h	0.0280	0.5932[a]			18.78%	2.095
	(1.28)	(3.34)				
Dependent	Intercept	σ_{t-1}^c	σ_{t-1}^p	σ_{t-1}^h	Adj. R^2	DW
σ_t^c	0.0202[b]	0.4013[a]		0.5340[a]	79.22%	1.789
	(2.31)	(5.05)		(8.33)		
σ_t^p	0.0288[a]		0.3248[a]	0.5739[a]	82.66%	1.865
	(3.54)		(4.74)	(10.48)		
Panel D: No intercept						
Dependent	Intercept	σ_t^c	σ_t^p	σ_{t-1}^h	Adj. R^2	DW
σ_t^h		0.8169[a]			93.28%	2.300
		(25.02)				

[a]p-value < 0.01; [b]p-value $\in [0.01; 0.05]$; [c]autocorrelation

Panel A and Panel B report respectively the 2SLS and 3SLS estimates of the simultaneous equation model: $h_t = \gamma_0 + \alpha_c i_t^c + e_t^h$, $i_t^c = \beta_0 + \beta_c i_{t-1}^c + \beta_h h_{t-1} + e_t^c$, $i^p t = \gamma_0 + \gamma_p i_{t-1}^p + \gamma_h h_{t-1} + e_t^p$ Panel C reports 3SLS estimates of the simultaneous equation model: $\sigma_t^h = \alpha_0 + \sigma_c \alpha_t^c + e_t^h$; $\sigma_t^c = \beta_0 + \beta_c\ \sigma_{t-1}^c + \beta_h \sigma_{t-1}^h + e_t^c$; $\sigma_t^p = \gamma_0 + \gamma_p \sigma_{t-1}^p + \gamma_h \sigma_{t-1}^h + e_t^p$. Where σ_t^c and σ_t^p denotes the implied volatility for call and put options on S&P 100 index respectively. σ_t^h is the ex-post realized daily return volatility of theS&P100 index calculated over the remaining life of the options. Data consist of 46 monthly observations on each volatility series constructed in a non-overlapping fashion, covering the period from April 1993 to February 1997. The numbers in parentheses denote asymptotic t-statistics.

appreciable difference between the 2SLS and FIML results. Note that 2SLS would suffice as a correction for the EIV problem, while efficient estimation under full simultaneity (including a non-diagonal error correlation structure) requires FIML, or at least three-stage least squares (3SLS). Thus, the similarity between the OLS and 2SLS results indicate that there is no EIV problem, and the similarity between the 2SLS and FIML results indicate that there is no simultaneity bias. Of course, even 2SLS produces consistent (albeit inefficient) estimates under full simultaneity, so while the first comparison can be made the basis for a Hausman (1975) test, the second is more heuristic.

Panel B shows the results of the corresponding 3SLS estimation. Again, there is no big change. The estimated correlation coefficients are 0.13 between the realized volatility equation error and the call implied volatility equation error, 0.14 between the realized and put implied equation errors, and 0.63 between the two implied equations. These correlation estimates have approximate standard errors of 0.21, so only the correlation between the two implied volatility equations is significant. This means that the system remains recursive in a generalized sense. In particular, it is recursive between the two-equations implied volatility block and the realized equation. This confirms that there should be no simultaneity bias, and that call implied volatility indeed is an efficient volatility forecast. In particular, the error term in the realized volatility equation contains no significant trace of variables in the market's information set (such as the two implied volatility equation errors).

Finally, we re-estimate the model in levels, i.e. without applying the log transform to the volatility series. The results appear in Panel C and are similar to those in the previous panels. In the realized volatility equation, it is relevant to consider the sharpening of the unbiasedness hypothesis that the intercept should vanish. This is valid in the levels regression, although not in the previous regressions for the logarithms. It is seen that this hypothesis cannot be rejected, based on the *t*-statistic (1.28). Re-estimating the system with the first equation forced through the origin produces the results in Panel D. Only the first equation is reported, since the other two again are similar to those estimated in the original system. It is seen that the coefficient on the call implied volatility level rises, to 0.82 in this specification, and with this reduction in the number of right-hand side variables, the adjusted R^2 is now very high.

All the results in Table 6 confirm that the conclusions based on the previous two tables are valid. Thus, call implied volatility is an efficient forecast of subsequently realized index return volatility, and it even appears unbiased, in some of our specifications.

4. CONCLUSION

In this paper we have studied whether volatility implied in OEX option prices can predict future realized index return volatility. We have confirmed the results of Christensen and Prabhala (1998), namely, that implied volatility is an unbiased and efficient forecast of future volatility, and subsumes the information content of historical volatility. In particular, we obtain these results in a more recent period, using a trade weighted average of implied volatilities from options that are both in-the-money and out-of-the-money (CP focused on at-the-money call options). As an extension, we separated out the analysis of the volatilities implied in put and call options. We found that implied put volatility on average is slightly greater than implied call volatility, possibly because buying put index options is a relatively cheap and convenient way of implementing portfolio insurance.

The main difference between previous studies on the S&P 100 index options and the sample used in this paper is that our estimated variances of the volatility series (levels) seem to be three to four times smaller. The picture is a little less dramatic for the log-transformed series, but still our variances are only about half as big as those reported by CP. We attribute this difference to the difference in data and sampling periods. Our data are more up-to-date and our results thereby suggest that option and stock prices may have become less volatile in recent years after a period with high volatilities following the stock market crash in October 1987.

The previous literature has only considered the information content of call option prices. We run a horse race between implied call, implied put, and historical return volatility. Our OLS results indicate that call implied volatility is a better volatility forecast than put implied volatility. We also perform a FIML analysis, which firmly establishes that valuable volatility information is contained in put option prices. This is so even though the option pricing formula used to back out this volatility estimate is stylistic and suppresses some of the features that might be expected to matter for puts, such as dividends, American features, and portfolio insurance inducing a price pressure.[7] Based on our empirical findings, we may hypothesize that these effects (which are of opposite signs) roughly cancel out.

ACKNOWLEDGEMENTS

We gratefully acknowledge comments and suggestions from Thomas J. Nielsen and two anonymous referees as well as the participants at the Second Annual International Conference on Money, Investment and Risk, The Nottingham Trent University, UK 1999. Research support was received from the Centre of Analytical Finance (CAF) Aarhus, and the Danish Social Science Research Council.

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