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# Effective Bid-Ask Spreads in Futures versus Futures Options

**Samarth Shah, B. Wade Brorsen, and Kim B. Anderson**

While considerable research has estimated liquidity costs of futures trading, little comparable research is available about options markets. This study determines effective bid-ask spreads in options and futures markets for Kansas City Board of Trade (KCBT) wheat. Effective bid-ask spreads are estimates of the actual liquidity cost of a round-trip order. Option liquidity costs are estimated using a new measure of effective spreads developed for options markets. Futures effective spreads are estimated using eight different measures developed in previous studies. Estimated effective bid-ask spreads of options contracts are at least double the effective bid-ask spreads of open-outcry futures contracts.

*Key words:* bid-ask spreads, execution cost, KCBT, liquidity cost, microstructure, options, wheat

## Introduction

Hedgers choosing between hedging in futures markets or hedging in futures options markets should consider differences in transaction costs,<sup>1</sup> which are mainly comprised of brokerage fees and liquidity costs. Liquidity cost is the difference in price paid by an urgent buyer and the price received by an urgent seller, and bid-ask spreads are the most frequently used measure of liquidity cost. For small orders, traders who desire to buy or sell a contract immediately will suffer an average markdown equal to half of the bid-ask spread. Knowledge of the magnitude of liquidity costs in both futures and options markets can help hedgers determine which one to use and help them choose between market and limit orders. Futures exchanges must also be aware of liquidity costs in order to evaluate new alternatives such as electronic trading. Moreover, estimates of liquidity costs can be deducted when simulating hedging strategies (e.g., Bertus, Godbey, and Hilliard, 2009) or speculative trading (e.g., Park and Irwin, 2010).

While considerable research estimates effective bid-ask spreads in commodity futures markets (e.g., Brorsen, 1989; Thompson and Waller, 1987; Thompson, Eales, and Seibold, 1993; Bryant and Haigh, 2004; Frank and Garcia, 2011a; Shah and Brorsen, 2011; Wang, Garcia, and Irwin, 2012), little comparable research is available for futures options markets. However, several studies have estimated bid-ask spreads of listed stock options (Baesel, Shows, and Thorp, 1983; Mayhew, 2002; Wei and Zheng, 2010). Even though liquidity costs could be an important factor in choosing between futures and futures options markets, we know of no research that has attempted to estimate and compare liquidity costs in both markets simultaneously. The purpose of this article is to estimate and compare liquidity costs in Kansas City Board of Trade (KCBT) wheat futures and options markets.

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Samarth Shah is a senior risk management analyst at Barri Financial Group, Houston, Texas; B. Wade Brorsen is a regents professor and the A.J. and Susan Jacques Chair and Kim B. Anderson is professor emeritus in the Department of Agricultural Economics at Oklahoma State University.

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<sup>1</sup> One concern expressed by producers is that options are overpriced because option sellers demand a risk premium. Urcola and Irwin (2011) find no risk premium for corn, wheat, or soybeans options.

Measures of liquidity costs developed for futures markets cannot be used for options markets because of the infrequency of option trades. We propose a new alternative measure of liquidity costs for options markets that predicts the hypothetical option premium in the absence of liquidity costs. The new estimate of effective spread is then twice the absolute value of the difference between actual and predicted premiums.

The factors influencing liquidity costs in options markets might be different than those in futures markets. For example, in futures markets, a market maker usually offsets a futures position by later taking the opposite position in the same contract, but market makers in options markets often have to offset their positions by constructing a hedge portfolio of futures and options positions. Therefore, the study also estimates the effects of factors expected to affect the magnitude of liquidity costs in options markets.

### Measures of Liquidity Costs

In open-outcry commodity futures markets, liquidity is primarily provided by floor traders who bid readily and offer a price for a specific contract. In order for the floor trader to profit from providing liquidity, the buy orders of off-floor traders are filled at a slightly higher price than their sell orders. Since trades typically occur at floor traders' bid or ask prices, the bid-ask spread is a measure of liquidity costs. There are two types of bid-ask spreads: quoted spreads and effective spreads. The quoted spread is the difference between floor traders' bid and ask prices. **The effective spread attempts to measure the liquidity cost that is actually paid.** Quoted spreads could overestimate liquidity costs if they miss negotiation or competition that occurs before a trade is made. Quoted spreads can also underestimate liquidity costs by missing the market impact of a large order. The present study focuses mainly on effective spreads because they represent the liquidity cost to hedgers.

Locke and Venkatesh (1997) estimate how much money is transferred from **off-the-floor** traders to floor traders, which is a very different question than that addressed here. We want to estimate how much an uninformed trader who always uses market orders will pay on average for liquidity services.<sup>2</sup> Similar to Glosten and Milgrom (1985), we assume that there are only three types of traders: (i) uninformed traders who use market orders,<sup>3</sup> (ii) liquidity providers (both scalpers and uninformed traders) who use limit orders, and (iii) informed traders. Liquidity providers gain from trading with uninformed traders but lose when trading with informed traders. Locke and Venkatesh's approach underestimates liquidity costs paid by uninformed traders using market orders.<sup>4</sup> Our interest is in helping wheat producers and marketing firms choose between futures and options markets and between market and limit orders. Kurov (2005) shows that common effective spread estimators like the Thompson and Waller (1987) measure closely match effective spreads for customer market orders.

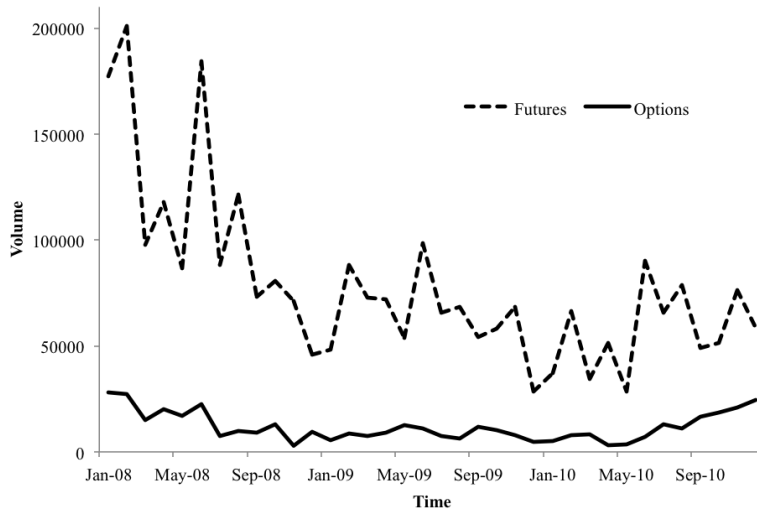
Previous research examining liquidity costs in futures markets finds that liquidity costs decrease as trading volume increases and increase as price variability increases (Thompson and Waller, 1987; Brorsen, 1989; Thompson, Eales, and Seibold, 1993; Bryant and Haigh, 2004; Frank and Garcia, 2011a,b). The volume effect implies that the supply of liquidity services is downward sloping (Brorsen, 1989). Scalpers benefit from economies of size, and these benefits are passed on in the form of lower liquidity costs. Trading volumes for the same commodity in options markets are considerably lower than those in futures markets (figure 1). Options markets are also expected to be less liquid than futures markets because, in addition to date of maturity, the option market is

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<sup>2</sup> A market order is an order for immediate execution at the best available price. A limit order is an order for a given price or better. A market order uses liquidity and a limit order provides liquidity.

<sup>3</sup> An uninformed trader has no ability to predict price direction. Uninformed traders are typically trading to reduce risk; noise traders are also uninformed.

<sup>4</sup> For relatively low-volume contracts like the one considered here, Locke and Venkatesh (1997) do not find as much difference between the two measures as they do with high volume contracts.



**Figure 1. Trading Volume of Open Outcry Wheat Futures and Options Contracts at KCBT in 2008-10**

segmented by puts, calls, and varying strike prices. Moreover, the market making in options is done largely by traders from firms specializing in trading options; thus there is potentially less competition in options markets.<sup>5</sup> Options are often offset with a futures transaction instead of another option transaction, and this process could prove more costly for option trading firms. Liquidity costs in the options market are therefore expected to be higher than those in the underlying futures market.

### Data

Futures trading at KCBT, the dominant futures market for hard red winter wheat, began in 1876. This wheat contract is the only contract actively traded at KCBT, and the minimum tick size is one-quarter cent per bushel for futures and one-eighth cent per bushel for options. KCBT introduced side-by-side electronic trading through GLOBEX on January 14, 2008. During the observation period, only 1.8% of option volume was on the electronic platform; all of our option transaction data are for the open-outcry market. We restrict our comparison to futures traded by open outcry so that the difference in liquidity costs is not due to trading method. Shah and Brorsen (2011) find that the KCBT wheat electronic market has even lower liquidity costs than the open-outcry market. Irwin and Sanders (2012) find that many agricultural futures are now traded primarily electronically. However, during the observation period the volume in the open-outcry market at KCBT remained strong.

KCBT employees overlook the trading pits from an area called “the pulpit.” As trading occurs, a “pit reporter” listens intently for prices shouted out by traders in the trading pit and relays them using a headset to a computer terminal operator known as the “data entry operator,” who enters the prices into a computer. The present study uses these intraday prices for hard red winter wheat open-outcry futures and option contracts. The dataset contains each trade price recorded for open-outcry wheat futures and options contracts from January 2008 to December 2010.<sup>6</sup> Calculations only include days with more than twenty futures transactions. Wheat futures contracts are the underlying assets for options contracts. At KCBT, wheat options contracts expire every month and futures contracts are traded with five expiration months: March, May, July, September, and December. Due to lack of volume in the other contracts, the present study only considers options contracts with the same five expiration months as futures contracts. The KCBT does not record bid and ask price

<sup>5</sup> Three traders do the bulk of the market making for options on the floor of the KCBT.

<sup>6</sup> Unlike the old time and sales data formerly available from the Chicago Board of Trade, zero price changes are included.

**Table 1. Descriptive Statistics of Hard Red Winter Wheat Futures and Options Contracts Traded at KCBT in 2008–10**

Contract	Options				Futures			
	Days	Trades per Day	Volume per Day	Volume per Trade	Days	Trades per Day	Volume per Day	Volume per Trade
2008								
March	36	16.61	470.92	28.35	37	179.22	3208.78	17.90
May	79	7.47	290.74	38.92	82	95.05	2001.04	21.05
July	119	35.24	557.17	15.81	133	167.23	2208.54	13.21
September	154	7.43	131.65	17.72	162	90.86	1163.20	12.80
December	173	5.03	165.00	32.80	221	76.95	1162.40	15.11
2009								
March	172	4.01	171.63	42.80	145	76.66	1016.15	13.26
May	110	3.14	70.16	22.34	89	69.55	1011.51	14.54
July	284	6.13	117.26	19.13	278	55.53	649.75	11.70
September	147	4.14	97.46	23.54	96	75.85	922.37	12.16
December	179	5.04	140.11	27.80	186	70.57	870.20	12.33
2010								
March	157	3.31	155.74	47.05	137	54.15	881.65	16.28
May	89	1.85	41.51	22.44	73	42.88	739.36	17.24
July	244	3.67	49.45	13.47	139	44.75	651.86	14.57
September	140	4.10	139.77	34.09	70	84.57	946.50	11.19
December	176	5.04	227.43	45.13	135	47.10	891.50	18.93

Notes: The 2008 data have fewer days because the data began in January 2008. The days vary between futures and options because options data are limited to days with at least one trade and futures data are limited to days with twenty or more trades. The futures data include only open-outcry trades.

for open-outcry wheat futures markets, but it provides irregular time-stamped bid or ask prices for its open-outcry wheat options market. Only bid and ask quotes observed simultaneously are used to estimate observed bid-ask spreads in wheat options markets. However, because there are few instances where bid and ask quotes occur concurrently, the data only provide 1,934 observations with which to calculate average quoted bid-ask spreads. There is a possibility of selectivity bias in using the bid-ask quotes since these are only reported when no trade occurs. If a trade is less likely to happen given a wide bid-ask spread, bid-ask spreads would overestimate liquidity costs. These are quoted spreads, and so they can differ from effective spreads. Our interest here is in effective spreads.

The measures of liquidity cost are calculated daily, and weighted averages are obtained using the number of transactions as weights. Table 1 presents descriptive statistics of the futures and option contracts. Prices were unusually volatile during this period and may have incurred structural change due to an influx of money from index funds (Auerlich, Hoffman, and Plato, 2009). For the risk-free rate of interest, we use the interest rate on three-month U.S. Treasury Bills (U.S. Department of the Treasury, 2008–2010).

### Option-Effective Spread Measure

Often bid-ask quotes in the trading pit are not recorded; bid-ask spreads must therefore be estimated based on the available transaction data. A bid-ask spread is the difference between the ask price and the bid price observed at the same time. The quoted bid-ask spread can provide an estimate of

liquidity cost:

$$(1) \quad \text{quoted spread}_t = \text{ask}_t - \text{bid}_t,$$

where  $t$  is trading time. The available bid-ask quotes that occur at the same time are used to calculate quoted bid-ask spreads as a way of confirming our new measure of effective spreads.

Various estimators of effective bid-ask spreads have been developed. Existing spread estimators have primarily used either the covariance of successive price changes or averages of absolute price changes. These measures require high-frequency data. Price changes can have positive autocorrelation when transactions are infrequent, which causes measures based on the covariance to be undefined. Similarly, price changes used by price-change measures will be composed largely of changes in equilibrium prices when markets are thin. Generally, agricultural futures markets have enough observations for the measures to perform effectively, but the options markets of agricultural commodities are scarcely traded by comparison. Due to the small number of daily transactions in the options markets, the spread estimators cannot estimate options liquidity costs as well as they do in the futures markets. Hence, a new measure of bid-ask spread is required to estimate liquidity costs in thin markets. We propose a new measure to estimate effective spreads in options markets.

When the data are available, effective bid-ask spreads are often calculated as twice the absolute value of the difference between the actual execution price and the midpoint of the quoted spread immediately before the transaction (e.g., Wu, Krehbiel, and Brorsen, 2011). Such calculations require quoted bid-ask spreads to be observed as frequently as every second or every half second. The midpoint of the quoted spread is the price of the security if there were no liquidity costs. Because the midpoint of the quoted spread is not observed in our data, we must estimate it. We use Black's (1976) valuation model for options on futures, which uses predicted implied volatilities to estimate the midpoint of the quoted spread and assumes a lognormal distribution and no riskless arbitrage. In practice, the lognormal distribution does not hold, so implied volatilities estimated by the Black model vary by strike price. Let  $\pi_t$  be the observed price of an option at time  $t$ . Let  $\hat{\pi}(\hat{v}_t)$  be the estimated midpoint of the quoted spread based on the Black model using  $\hat{v}_t$  as the estimated volatility. Then, liquidity costs incurred by a trader for a round trip trade can be estimated as:

$$(2) \quad 2 \times E[|\pi_t - \hat{\pi}(\hat{v}_t)|].$$

Black (1976) defines the price of a futures option as a function of five variables: strike price ( $K$ ), risk free interest rate ( $r$ ), time to expiration ( $T$ ), underlying futures price ( $F_t$ ), and volatility of the underlying futures price ( $v_t$ ). The strike price, risk free interest rate, time to expiration, and underlying futures price are observed directly, but volatility of the underlying futures price is not. The Black formula using the estimated volatility is:

$$(3) \quad \begin{aligned} \hat{\pi}(\hat{v}_t) &= e^{-rT} [F_t N(d_1) - KN(d_2)] \text{ for a call option,} \\ \hat{\pi}(\hat{v}_t) &= e^{-rT} [KN(d_2) - F_t N(d_1)] \text{ for a put option,} \end{aligned}$$

where:

$$(4) \quad \begin{aligned} d_1 &= \left[ \ln\left(\frac{F_t}{K}\right) + \frac{\hat{v}_t^2 T}{2} \right] / (\hat{v}_t \sqrt{T}), \\ d_2 &= \left[ \ln\left(\frac{F_t}{K}\right) + \frac{\hat{v}_t^2 T}{2} \right] / (\hat{v}_t \sqrt{T}). \end{aligned}$$

We use a linear regression to estimate the implied volatility to use in the Black model. Black's formula is for European options, while the KCBT options are American options, which can be exercised before expiration. Our approach lets the early-exercise premium be captured in the predicted implied volatilities. An alternative would be to calculate implied volatilities based on an

**Table 2. Example Parameter Estimates of Regression on the Natural Logarithm of Implied Volatility**

Variable	Estimate	SE
Intercept	−1.93***	0.037
Moneyiness (cents/bushel)	0.0005***	< 0.0001
Calls	0.079***	0.002
Standard error of the regression	0.011	-

Notes: Dummy variables for trading day have a significant effect on volatility (F-statistic = 60.73), but are not presented in the above table due to space considerations. Moneyiness is futures minus strike for a call option and strike minus futures for a put option. Triple asterisks (\*\*\*) indicate significance at the 1% level.

American option formula using a binomial option-pricing model. American option formulas are available for futures options in Ramaswamy and Sundaresan (1985) and Shastri and Tandon (1986). Interest rates were extremely low during 2008–2010 (as low as 0.1%), and thus the early-exercise premium was low enough during the data period that there is little difference between American and European options. However, later researchers using this method during a period of higher interest rates may want to use American option formulas. Note that the same general procedure can also be used with pricing models based on jumps or nonnormal distributions.

To create the data for the linear regression, we calculate implied volatility using actual premiums by solving the Black model in equation (2) for  $v_t$  using the Newton-Raphson method. Implied volatility can vary depending on how far the option is in or out of the money (moneyiness), time to maturity of the option, and option type (Derman, Kani, and Zou, 1996; Cont and da Fonseca, 2002). The well-known volatility smile results in at-the-money options generally having lower implied volatilities compared to in- and out-of-the-money options. A volatility smirk, which is common in agricultural markets, results in volatility being a linear function of moneyiness. The nonnormality of implied distributions of agricultural options has been confirmed by Bozic and Fortenbery (2010) and Ji and Brorsen (2009). Similarly, different option maturities also affect the predicted implied volatilities. We estimate a regression to filter out the effects of the above-discussed factors and use the predicted volatilities from this model to estimate what the option premium would be if liquidity costs were zero. A different regression is estimated for each of the fifteen different maturities:

(5) 
$$\ln(v_t) = \beta_0 + \beta_1 M_t + \beta_2 D_t + \sum_{i=1}^N \beta_i T_{it} + e_t,$$

where  $v_t$  is implied volatility at time  $t$ ,  $M_t$  is moneyiness (futures minus strike for a call option and strike minus futures for a put option) of the option at time  $t$ ,  $T_{it}$  are dummy variables for trading day  $i$ ,  $D_t$  is a dummy variable for type of option, which is 1 for call options and 0 for put options. Note that  $t$  provides an ordering of all option transactions and so the clock time between observations is not constant. Bozic and Fortenbery (2010) find that most of the nonnormality in the implied distribution of agricultural option premiums is due to skewness. They further show that this skewness can be approximately captured with a linear function of price versus moneyiness, as we have done here. Table 2 presents a representative estimate. Since the estimate of the futures price volatility depends on the above regression, the estimation error of the regression is a source of error in the measure of liquidity costs. As shown in table 2, the standard error of this example regression is small.

A potential source of error comes from the staleness of the underlying futures price used in the Black model.<sup>7</sup> The Black model assumes that the underlying futures price is observed at the same time as the option price. In practice, the underlying futures price is generally not available at the

<sup>7</sup> The most recent futures price is stale if the true equilibrium futures price has changed since the most recent futures transaction.

exact time of the option transaction, so we use the most recently transacted futures price.<sup>8</sup> Hence, if the equilibrium price of the underlying futures contract changes in the interim, the staleness of the futures price could affect the estimate of the true option price and consequently affect the proposed measure of liquidity cost. The effect of this staleness is removed by implementing a new technique to adjust for the staleness and estimating the following regression:

$$(6) \quad |\pi_t - \hat{\pi}(\hat{v}_t)| = \alpha_0 + \alpha_1 S_t + \epsilon_t,$$

where  $S_t$  is the length of time between the observed option transaction and the most recent underlying futures price. Different regressions are estimated for each maturity month and, as with all calculations, no observations across days were included. If staleness of the futures price is zero,  $\alpha_0$  represents the absolute difference between observed and predicted option premiums.<sup>9</sup> Thus, the liquidity costs can be calculated as twice the expected value of equation (6) when  $S_t$  is zero:

$$(7) \quad \text{liquidity cost} = 2 \times \alpha_0.$$

The estimates of liquidity costs from the new measure are biased downward because an in-sample estimate of volatility was used,<sup>10</sup> but the estimates are consistent (sample size must increase for each day because adding days is not enough to achieve consistency).

Liquidity costs in the options market estimated using the new measure are compared with liquidity costs from the futures market. Several different measures of effective bid-ask spreads have been developed and applied to futures market data, and the properties and limitations of these measures have been studied comprehensively in the literature. The different measures produce different estimates of liquidity costs for the same market due to different underlying assumptions (Bryant and Haigh, 2004; Frank and Garcia, 2011a,b). To make a comprehensive comparison of liquidity costs in options and futures markets, we use eight different measures to estimate liquidity costs in the futures market.

### Futures Market Effective Spread Measures

Measures of futures effective spreads use either an average of the absolute value of price changes or the covariance of successive price changes. Thompson and Waller (1987) suggest using the average absolute value of nonzero price changes as a direct measure of the average execution cost. The Thompson and Waller measure (TWM) is:

$$(8) \quad TWM = \frac{1}{T} \sum_{t=1}^T |\Delta F_t|,$$

where  $\Delta F_t$  are series of nonzero price changes. The Thompson and Waller measure assumes that the bid-ask spread is the main determining factor of price changes and ignores true price changes. Thompson and Waller (1987) apply this measure to estimate liquidity costs in coffee and cocoa futures markets and Thompson, Eales, and Seibold (1993) use it to compare liquidity costs between

<sup>8</sup> Since the futures price used will have been executed at a bid or ask price, this extra noise is another potential source of error in estimating the option premium with no liquidity cost. Since most options traded are out of the money, this error is not large. One possible alternative to using the most recent futures price is to use the average of the last two trades, which Ederington and Lee (1995) call pseudo-equilibrium prices.

<sup>9</sup> Note that most estimates of  $\alpha_1$  were small and so the adjustment for staleness did not have a large effect on the results. Dropping observations when the most recent futures observation is several minutes previous, as previous research has sometimes done, would give similar answers. We require the futures price to not be more than five minutes earlier than the corresponding options price. Also, as long as the error is never more than half the bid-ask spread, the error does not create bias.

<sup>10</sup> An alternative would be to use historical volatility or the previous day's estimated volatility, but both of these would increase noise and bias estimates upward. We elected to use the more conservative measure since our conclusions are based on effective spreads being larger for options than for futures.



two similar markets. Ma, Peterson, and Sears (1992) use the TWM to study intra-day patterns in spreads and the determinants of spreads for various Chicago Board of Trade (CBOT) contracts. The Thompson and Waller measure (TWM), as argued by Smith and Whaley (1994), contains real price changes along with bid-ask spreads. In an attempt to filter out the real price changes in the TWM measure, CFTC uses only nonzero price changes that are in the opposite direction of the previous change. Hence, the CFTC measure is the average of the absolute value of nonzero, opposite-sign, absolute price changes. For comparison, we include a third price change measure (ABS) that is the absolute value of all price changes in order to see how much difference excluding the zero price changes makes.

Roll (1984) develops an estimator of effective bid-ask spreads based on the covariance of successive price changes. According to Roll, if markets are efficient and successive transactions have an equal probability of being sale or purchase, then the covariance between price changes is negative and directly related to the size of the bid-ask spread. Roll's measure (RM) is:

$$(9) \quad RM = 2\sqrt{-\text{cov}(\Delta F_t, \Delta F_{t-1})},$$

where  $\Delta F_t$  is change in price at time  $t$ . Roll's measure assumes that each transaction has an equal probability of being a buy or sell order, which is generally inappropriate for futures markets (Bryant and Haigh, 2004). Choi, Salandro, and Shastri (1988) extend Roll's measure by relaxing the assumption of equal probability. They define their measure (CSS) as:

$$(10) \quad CSS = \frac{\sqrt{-\text{cov}(\Delta F_t, \Delta F_{t-1})}}{1 - \delta},$$

where  $\delta$  is the conditional probability that the next transaction type (bid or ask) is the same as the current transaction type. If  $\delta = 0.5$  the CSS measure reduces to Roll's measure. With positive correlation in transaction type ( $\delta > 0.5$ ), the estimates produced by Roll's measure are downward-biased estimates of the effective bid-ask spread (Choi, Salandro, and Shastri, 1988). Chu, Ding, and Pyun (1996) further extend Roll's measure by considering the direction at  $t - 1$  and  $t + 1$ . The Chu, Ding, and Pyun measure (CDP) is:

$$(11) \quad CDP = \sqrt{\frac{-\text{cov}(\Delta F_t, \Delta F_{t-1})}{(1 - \delta)(1 - \alpha)}},$$

where  $\alpha$  is the conditional probability that the previous transaction type is the same as the current transaction type. When  $\alpha = \delta$ , the CDP measure reduces to the CSS measure, and when  $\alpha = \delta = 0.5$  it reduces to Roll's measure. To estimate the probabilities  $\alpha$  and  $\delta$ , the transaction types are classified as bid or ask using the tick test suggested by Lee and Ready (1991).<sup>11</sup>

Hasbrouck (2004) developed a Bayesian estimator of bid-ask spreads based on Roll's model:

$$(12) \quad F_t = m_t + cq_t,$$

where  $m_t$  is the efficient price,<sup>12</sup>  $F_t$  is the observed transaction price,  $c$  is the half bid-ask spread, and  $q_t = \{+1 \text{ for a buy order, } -1 \text{ for a sell order}\}$  is the trade direction indicator such that the ask price is  $a_t = m_t + c$  and the bid price is  $b_t = m_t - c$ . The bid-ask spread, the difference between  $a_t$  and  $b_t$ , is  $2c$ . The assumption is that  $m_t$  follows a random walk; that is,  $m_t = m_{t-1} + u_t$ , where  $u_t$  is identically and independently normally distributed with mean 0 and variance  $\sigma_u^2$ . Taking first differences of equation (12) yields:

$$(13) \quad \Delta F_t = c\Delta q_t + u_t \quad u_t \sim N(0, \sigma_u^2),$$

<sup>11</sup> The tick test first categorizes each trade as an uptick, a downtick, a zero-uptick, or a zero-downtick. An uptick is a trade at a higher price, a downtick at a lower price, and a zero-tick at the same price as the previous trade. A zero-uptick is a zero tick when the price change before it was an uptick (a zero-downtick is defined analogously). Upticks and zero-upticks are categorized as buys, and all other trades are categorized as sells (Lee and Ready, 1991, p. 735).

<sup>12</sup> The efficient price is estimated by  $\hat{\pi}(\hat{v}_t)$  in equation (2).

where  $c$ , the half bid-ask spread, is the estimated coefficient in the model. Equation (13) includes two parameters,  $c$  and  $\sigma_u^2$ , and  $T$  latent data values,  $\mathbf{q} = \{q_1, q_2, \dots, q_T\}$ ; the regression is estimated using Bayesian methods. We define  $\mathbf{F}$  as a vector of the observed transaction prices, such that the full posterior over parameters and latent data is summarized by the distribution function  $\Phi(c, \sigma_u^2, \mathbf{q}|\mathbf{F})$ . Since the closed form representation of the distribution function does not exist, it is characterized by simulation using Markov Chain Monte Carlo (MCMC) simulation and the Gibbs sampling method.

As described in Hasbrouck (2004), the Gibbs sampler is an iterative procedure. Initially, the parameters and latent data are set to any values (subject only to feasibility). We denote these initial values as  $\{c^{[0]}, \sigma_u^{2[0]}, \mathbf{q}^{[0]}\}$ , where  $c^{[0]} = .01$  and  $\sigma_u^{2[0]} = .001$ . The steps in the first sweep  $j = 1$  are:

1. Draw  $\mathbf{q}^{[1]}$  from  $\Phi(\mathbf{q}|c^{[0]}, \sigma_u^{2[0]}, \mathbf{F})$ .
2. Draw  $\sigma_u^{2[1]}$  from  $\Phi(\sigma_u^2|c^{[0]}, \mathbf{q}^{[1]}, \mathbf{F})$ .
3. Draw  $c^{[1]}$  from  $\Phi(c|\sigma_u^{2[1]}, \mathbf{q}^{[1]}, \mathbf{F})$ .

The sample values of  $c$ ,  $\mathbf{q}$ , and  $\sigma_u^2$  are drawn from their full conditional distribution based on observed transaction prices. That is, all parameters and latent data except for the component being drawn are taken as given. The next iteration starts with a draw of  $\mathbf{q}^{[2]}$  conditional on  $\sigma_u^{2[1]}$ ,  $c^{[1]}$ , and  $\mathbf{F}$ . Repeating this  $n$  times, a sequence of draws  $\{c^{[j]}, \sigma_u^{2[j]}, \mathbf{q}^{[j]}\}$  for  $j = 1, \dots, n$  is generated. The Gibbs principle ensures that after a sufficient number of samples, the sample distribution converges to  $\Phi(c, \sigma_u^2, \mathbf{q}|\mathbf{F})$ . In the Hasbrouck measure (HAS), a truncated normal prior is used for  $c$ , producing a conditional distribution of  $c$  that is truncated and restricted to positive values:

$$(14) \quad c|\mathbf{F} \sim N^+(\mu_c^{post}, \Omega_c^{post}),$$

where  $\mu_c^{post} = Dd$ ,  $\Omega_c^{post} = \sigma_u^2(\Delta\mathbf{q}'\Delta\mathbf{q})^{-1}$ ,  $D^{-1} = \Delta\mathbf{q}'(\sigma_u^2)^{-1}\Delta\mathbf{q} + (\Omega_c^{prior})^{-1}$ , and  $d = \Delta\mathbf{q}'(\sigma_u^2)^{-1}\Delta\mathbf{F} + (\Omega_c^{prior})^{-1} + \mu_c^{prior}$ . The positive normal distribution of  $c$  imposes a nonnegativity restriction on bid-ask spreads. In the HAS measure, the truncation of the distribution of  $c$  influences the mean and variance of the bid-ask spread estimates. To circumvent this, Frank and Garcia (2011a) modify the HAS measure by using a normal distribution as the prior for  $c$  and using absolute values of price changes and trade direction. The conditional distribution of  $c$  for the Frank and Garcia measure (FGM) is:

$$(15) \quad c|\mathbf{F} \sim N(\mu_c^{post}, \Omega_c^{post}),$$

where  $\mu_c^{post} = Dd$ ,  $\Omega_c^{post} = \sigma_u^2(|\Delta\mathbf{q}|'|\Delta\mathbf{q}|)^{-1}$ ,  $D^{-1} = |\Delta\mathbf{q}|'(\sigma_u^2)^{-1}|\Delta\mathbf{q}| + (\Omega_c^{prior})^{-1}$ , and  $d = |\Delta\mathbf{q}|'(\sigma_u^2)^{-1}|\Delta\mathbf{F}| + (\Omega_c^{prior})^{-1} + \mu_c^{prior}$ . The conditional distributions of  $\sigma_u^2$  and  $\mathbf{q}$  for both HAS and FGM measures are:

$$(16) \quad \sigma_u^2|\mathbf{F} \sim IG(\alpha^{post}, \beta^{post}),$$

where  $\alpha^{post} = \alpha^{prior} + t/2$ ,  $\beta^{post} = \beta^{prior} + \Sigma u_t^2/2$ , with  $\alpha^{prior} = \beta^{prior} = 10^{-12}$  and:

$$(17) \quad q_t^{post}|\mathbf{F} \sim \text{Bernoulli}(p_{buy}),$$

where  $p_{buy} = e^{(4c p_t)/(\sigma_u^2)}(e^{2c(m_{t-1}+m_{t+1})/(\sigma_u^2)} + e^{(4c F_t)/(\sigma_u^2)})$  is the probability that  $q_t = +1$ . The prior parameters for  $c$  are  $\mu_c^{prior} = 0$  and  $\Omega_c^{prior} = 10^6$ , while the prior for  $q_t$  is  $q_t^{prior} \sim \text{Bernoulli}(1/2)$ . We run 2,000 sweeps with the Gibbs sampler to estimate the full posterior  $F(c, \sigma_u^2, \mathbf{q}|\mathbf{F})$ . To reduce dependency on the initial values, the first 400 (20%) are considered burn in and are discarded. The posterior mean of  $c$  is obtained as the mean of the remaining 1,600 simulated values of  $c$ . The posterior mean is multiplied by two to get the estimate of the bid-ask spread.

To test hypotheses about factors influencing option liquidity costs, we estimate the following regression equation:

$$(18) \quad y_{kt} = \beta_0 + \beta_1 M_{kt} + \beta_2 T_{kt} + \beta_3 D_{kt} + \beta_4 Z_{kt} + \beta_5 N_{kt} + \vartheta_k + \varepsilon_{kt},$$

**Table 3. Measures of Bid-Ask Spreads (cents/bushel) in Hard Red Winter Wheat Futures and Options Contracts, 2008–10**

Market	Measure	Liquidity Costs	Std Error
Options	BAS	3.59	0.23
	BASyz	4.30	0.26
	New Measure	4.33	0.14
Futures	RM	1.16	0.10
	CSS	1.49	0.14
	CDP	1.23	0.11
	ABS	1.10	0.09
	TWM	1.38	0.13
	CFTC	1.16	0.12
	HAS	0.85	0.02
	FGM	1.64	0.06

*Notes:* BAS = observed bid-ask spread, BASyz = observed bid-ask spread including pre-open quotes, RM = Roll's measure, CSS = Choi, Salandro and Shastri measure, CDP = Chu, Ding and Pyun measure, ABS = Absolute price changes, TWM = Thompson and Waller measure, CFTC = Commodity Futures Trading Commission measure, HAS = Hasbrouck measure, and FGM = Frank and Garcia measure. Measures reported are weighted averages of daily estimates. For the futures measures, only days with at least twenty trades are included.

where  $y_{kt}$  is the new measure of liquidity cost for contract  $k$  at time  $t$  ( $|\pi_t - \hat{\pi}(\hat{v}_t)|$ ),  $M_{kt}$  is the moneyness of the option,  $T_{kt}$  is time to maturity in days,  $D_{kt}$  is a dummy variable that is 1 for a call option and 0 for a put option,  $Z_{kt}$  is daily average trade size, and  $N_{kt}$  is daily number of trades. The daily average trade size and number of trades vary by strike, contract maturity, type of option, and day. The error terms  $k$ ,  $\vartheta_k$ , and  $\varepsilon_{kt}$  are assumed to be independently distributed normal with mean zero and variances  $\sigma_v^2$  and  $\sigma_e^2$ , respectively. The parameters are estimated with restricted maximum likelihood due to the inclusion of the contract random effect. The contract random effect can capture any systematic differences associated with contract month. The regression is based on 12,008 observations.

## Results

The aggregate estimates of liquidity costs in KCBT wheat futures and options markets during the sample period of 2008–10 are presented in table 3. Using the new measure in the wheat options market, the effective spread is an estimated 4.33 cents per bushel. The average quoted bid-ask spread is 4.30 cents per bushel (3.59 cents when pre-open quotes are excluded). The quoted bid-ask spreads can be biased measures and are from a small nonrepresentative sample, but they still offer some confirmation of the new measure.

The effective spreads in futures markets estimated using eight different measures range from 0.85 to 1.64 cents per bushel. The option market clearly has much higher liquidity costs regardless of which measure is used. The weighted average Roll's measure and Thompson and Waller measure for the wheat futures market are 1.16 cents per bushel and 1.38 cents per bushel. Thompson, Eales, and Seibold (1993) also estimate the same two measures for selected 1985 KCBT wheat futures contracts. Their estimates of average absolute deviations are 0.26–0.29 cents per bushel for highly traded contracts, but are about double these values for lightly traded contracts such as the March contract during March or the September contract in February. The higher values may be explained by more volatile markets during our study period or the electronic market siphoning away many of the smaller transactions.<sup>13</sup>

<sup>13</sup> Note that in an earlier version (Shah, Brorsen, and Anderson, 2009), we used only the July 2007 KCBT wheat contract, where bid-ask spreads were considerably lower but option bid-ask spreads were still two to three times as large as futures bid-ask spreads.

The volumes traded in wheat futures contracts are considerably higher than the volumes in options contracts. Figure 1 shows monthly volumes in KCBT wheat open-outcry futures and options contracts in 2008–10. Table 1 presents average daily volume and average volume per trade in the two markets. The daily volumes for the futures market are markedly higher than the daily volumes in the option market for all contracts traded during 2008–10. Previous studies of liquidity costs in commodity markets have found that volume has a negative impact on liquidity costs (Thompson and Waller, 1987; Thompson, Eales, and Seibold, 1993; Bryant and Haigh, 2004; Shah and Brorsen, 2011). Thus, lower volumes in option markets may explain their higher liquidity costs. In futures, liquidity costs decrease with volume because lower volumes imply more risk of holding contracts, resulting in higher liquidity costs. The reasoning in options may be different, as market makers in options often have to offset their options position in the futures market rather than through an opposite position in the options market. Another possible explanation for higher liquidity costs in option markets is higher volume per trade. For all the contracts in the sample period, the average volume per trade in the options markets is 28.76 contracts, compared to 14.82 contracts for the futures market. Higher volume per trade could indicate higher risk in holding the larger number of contracts or higher transaction costs of creating a delta hedge portfolio.<sup>14</sup>

Our results show that options liquidity costs are more than double that of futures liquidity costs; this finding holds regardless of which measure of liquidity cost is used. However, the liquidity costs calculated here assume a round turn in both futures and options markets. An option that is held to expiration and is allowed to expire worthless would only have half of the estimated liquidity cost. Thus, the difference in liquidity cost would be less if the options were always held until expiration, but options would still have larger liquidity costs. Also, most futures trading is now done electronically; Shah and Brorsen (2011) show that electronic trading has even lower liquidity costs than the open-outcry market, creating an even larger advantage for electronic futures over open-outcry options.

The Hasbrouck (2004) measure is the smallest. Frank and Garcia (2011a) also find the Hasbrouck measure to be the smallest of the four measures that they calculate. Our measures are much more consistent with each other than the measures in Frank and Garcia (2011a). Our use of weighted averages and the price clustering on the KCBT (Shah and Brorsen, 2011) might contribute to the narrow range of our futures liquidity cost measures.

To determine the relationship among liquidity costs of option contracts and factors expected to influence liquidity costs, we estimate the model in equation (18) using restricted maximum likelihood. The new measure is used as the dependent variable in the model; table 4 presents these results. Days to maturity has a significant positive impact on liquidity costs, which may be due to the higher value of an option contract that is farther from maturity. Options liquidity costs increase with moneyness and as option premiums increase. The type of option has a significant impact on liquidity costs, with call options having greater liquidity costs than put options. Possibly because prices rose during the study period, the average value of the call options was \$1.00 per bushel while the average value of the put options was \$0.48 per bushel. As with the previous two variables, liquidity costs rise as the value of the option goes up.

The effect of average trade size on liquidity costs is not significant, but the number of trades has a strong negative effect. Frank and Garcia (2011a) find that futures effective spreads go down with volume and up with average trade size. The insignificance of average trade size here suggests that the factors determining liquidity costs in options markets may differ from the factors determining liquidity costs in futures markets.

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<sup>14</sup> In the options literature, delta is the change in value of a derivative or a portfolio with a change in the futures price. Options firms try to create portfolios of options and futures contracts that have a delta near zero in order to reduce their risk.

**Table 4. Estimated Parameters of Factors Affecting Effective Bid-Ask Spreads (cents/bushel) in Wheat Option Contracts at KCBT with the New Measure as the Dependent Variable**

Variable	Estimate	SE
Intercept	2.60	1.46
Moneyness (cents/bushel)	0.007***	0.0005
Days to expiration	0.035***	0.001
Calls	1.73***	0.13
Trade size	0.00015	0.0013
Number of trades	-0.295***	0.017

Notes: Triple asterisks (\*\*\*) indicate significance at the 1% level.

### Summary and Conclusions

Available measures for estimating the effective bid-ask spread are not appropriate for options markets because these markets are very thin. This study presents a new measure for estimating effective bid-ask spreads using transaction data from option markets. This measure uses the Black (1976) model with predicted implied volatilities and option transaction prices. Although this measure is an approximation, its accuracy is suggested by the fact that available bid-ask quotes are close to estimates created using the new measure.

This study estimates and compares liquidity costs in futures and options markets simultaneously using intraday prices of wheat futures and options contracts traded on the KCBT during 2008–10. The estimated liquidity cost in the wheat option market is 4.33 cents per bushel and the quoted bid-ask spread in the option market is 4.30 cents per bushel. Eight other measures estimate liquidity costs in the open-outcry wheat futures market ranging from 0.85 cents per bushel to 1.64 cents per bushel.

In general, liquidity costs for options increase with factors that cause option premiums to increase, including days to expiration, moneyness of the option, and the option being a call option. These positive effects are likely due to the factors also being positively correlated with option premiums. Additionally, liquidity costs increased with number of trades, but average trade size was insignificant.

Only open-outcry futures are considered here, which allows us to compare futures and options traded in the same manner. Shah and Brorsen (2011) find that the KCBT wheat electronic futures market has even lower liquidity costs than the open outcry market (at least for small trades). Electronic markets now exist for options, but electronic trading of options has not shown the growth of electronic trading of futures (Irwin and Sanders, 2012). Producers are typically small traders and so they could get even lower liquidity cost in the KCBT electronic futures market, which adds to the advantage of futures over options.

Option contracts are often suggested as an alternative to futures contracts because buying an option avoids margin calls and allows the buyer to benefit from favorable price movements. The higher liquidity cost of options is only part of the decision whether to hedge with futures or with options.<sup>15</sup> While limit orders have a risk of leaving orders unfilled, using limit orders rather than market orders is one possible way for hedgers to reduce liquidity costs when trading options. Certainly the higher liquidity costs of options should be considered when choosing between hedging with futures and hedging with options.

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<sup>15</sup> The choice problem of selecting between hedging with futures and hedging with options is a discrete-choice expected utility maximization, much like a partial budget with a trade-off between risk reduction and costs (see Brorsen, Buck, and Kointz, 1998, for an example). While a few cents per bushel may not appear to be a large amount, it could be enough to change choices if futures and options are close substitutes.

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