Implied and Realized Volatility in the Cross-Section of Equity Options

Manuel Ammann, David Skovmand, Michael Verhofen*
University of St. Gallen and Aarhus School of Business

Abstract

Using a complete sample of US equity options, we analyze patterns of implied volatility in the cross-section of equity options with respect to stock characteristics. We find that high-beta stocks, small stocks, stocks with a low-market-to-book ratio, and non-momentum stocks trade at higher implied volatilities after controlling for historical volatility. We find evidence that implied volatility overestimates realized volatility for low-beta stocks, small caps, low-market-to-book stocks, and stocks with no momentum and vice versa. However, we cannot reject the null hypothesis that implied volatility is an unbiased predictor of realized volatility in the cross section.

Keywords: Implied Volatility, Realized Volatility
JEL classification: G10

*Manuel Ammann (manuel.ammann@unisg.ch) is professor of finance at the University of St. Gallen, Switzerland, David Skovmand (davids@asb.dk) is assistant professor at the Aarhus School of Business, University of Aarhus and CREATES, and Michael Verhofen (michael.verhofen@unisg.ch) is lecturer in finance at the University of St. Gallen, Switzerland. We thank Sebastien Betermier, Peter Feldhütter, Thomas Gilbert, Sara Holland, Peter Tind Larsen, Miguel Palacios, Hari Phatak, Ryan Stever, and Stephan Süss for helpful comments.

1 Introduction

In this paper, we contribute to the existing literature by analyzing the relation between implied and realized volatility in the cross-section of equity options. While existing studies use small samples of option price data or only index options, we use a complete, survival bias free database of US equity options. Moreover, our focus is to analyze the relation between corporate fundamentals, implied and realized volatility while almost all existing studies focus on time-series models of volatility (e.g., GARCH).

Implied volatility can be regarded as the market's forecast of future volatility (Latané & Rendleman (1976), Poterba & Summers (1986), Day & Lewis (1988), Harvey & Whaley (1992), Sheikh (1989)). As for any forecast, the question arises how accurate this forecast is and whether this forecast is affected by any biases.

A number of authors analyze the accuracy of this forecast and the results have primarily shown a dependence on the analyzed underlying asset price process and the time period under investigation. The affirmative case for implied volatility can be found in Christensen & Prabhala (1998) and Jorion (1995), for example. The latter uses data from the foreign exchange market and finds that implied volatility is the best available forecast for realized volatility. The result, however, is subject to a number of biases. For example, he reports that implied volatility itself is too volatile. Day & Lewis (1992) use time-series models of volatility as a starting point and analyze the additional forecast contribution provided by implied volatility. Using S&P 100 options, the results suggests that implied volatility contains additional information over GARCH models. On the contrary, Canina & Figlewski (1993) analyze the forecasting performance of implied volatility. They find that implied volatility has no material correlation with realized volatility. Poon & Granger (2005) review 93 studies that conduct tests of volatility-forecasting methods on a wide range of financial asset returns. They find evidence that option-implied volatility is superior to time-series based forecasts. Within the group of time series models, they find that historical volatility is superior to GARCH and stochastic volatility models.

While most authors have analyzed index options, only very few have analyzed individual equity options. For example, Lamoureux & Lastrapes (1993) analyze the relation between implied volatility and realized volatility for 10 equity options between 1982 and 1984. They find that time series models (such as GARCH) can improve forecasts of future realized volatility. Bakshi & Kapadia (2003*a*) analyze the volatility risk premium embedded in index options. A portfolio long in call options and short in a delta-hedged portfolio of the underlying delivered a return below 0 in the sample period. Therefore, they argue that the volatility risk premium is negative. Similarly, Bakshi & Kapadia (2003*b*) argue that this finding also holds for 25 individual equity options albeit not as significantly as in the case of index options.

This study differs from existing studies in two ways. First, we use a complete, survival bias free sample of all US equity options over a time period of nine years (1997 to 2005) provided by OptionMetrics. Therefore, our sample size is much larger than the samples used in existing studies that usually focus on index options. Second, the focus of our study is on the relation between stocks characteristics, implied volatility and historical volatility. Since most existing studies focus on time series models of volatility (GARCH-type models), this is a perspective neglected so far.

We focus on stock characteristics (such as beta, size, market-to-book, and momentum) for a number of reasons. First, little is known about the interaction of stock characteristics, implied volatility, and historical volatility although there are good reasons why there might be a relation. For example, stocks with certain characteristics might be perceived more or less risky by market participants after controlling for historical volatility. Consequently, cross-sectional differences in the predictive power of implied volatility might be priced. Second, we seek an underlying economic explanation of volatility beyond pure statistical models. Time-series models of implied volatility might capture the dynamics of volatility, but usually neglect the economic foundations of volatility, e.g., why similar stocks might have different historical or implied volatilities.

Our main findings can be stated as follows. We find that high beta stocks, small stocks, stocks with a low market-to-book ratio, and non-momentum stocks trade at higher implied volatilities than justified by historical volatility. We find evidence that implied volatility overestimates future realized volatility for low beta stocks, small caps, low market-to-book, and stocks with no momentum. Implied volatility underestimates realized volatility for stocks with the opposite characteristics. However, we cannot reject the null hypothesis that implied volatility is an unbiased predictor of realized volatility in the cross section.

Section 2 starts by presenting the research design approach. Section 3 presents the empirical results. Section 4 concludes.

2 Research Design

2.1 Data

The option price data in this study are provided by OptionMetrics via WRDS. The raw data set consists of implied volatilities for all US equity options. In this study, we use implied volatilities for standardized call options with a maturity of 91 calendar days and a strike price equal to the forward price. The sample starts in January 1996 and ends in April 2006. Implied volatilities are computed as described in OptionMetrics (2005). Beside implied volatilities, return data and historical volatilities are retrieved from OptionMetrics. Historical volatility is also computed over a time period of 91 calendar days.

To form portfolios on size and market-to-book, the dataset was merged with Compustat data. To compute the market values, the data fields 24 (Price - Calendar Year - Close) and 25 (Common Shares Outstanding) are multiplied. To compute market-to-book-ratios, the information in data field 60 (Common Equity) was used. To compute the beta, the market risk premium as provided by Fama and French (via WRDS) is used.

2.2 Portfolio Formation

To explore the relation between historical and implied volatility, we use a portfolio formation approach as widely used in empirical asset pricing (see e.g., Fama & French (1993), Fama & French (1996)). In particular, we use two dimensional (5×5) sorted portfolios based on historical volatility and a second variable. We have selected the 5×5 sorting because it ensures that a sufficient number of stocks is included in any portfolio at any point in time. This research design enables us to assess cross-sectional effects, i.e., we can capture different patterns for stocks with specific characteristics. For example, we can explore whether small and large stocks with similar historical volatility trade at different implied volatility or have different predictive power.

Portfolio construction can be illustrated by using the size portfolio for 1997. At the end of 1996, we check which stocks have market value information available in Compustat as well as which stock options have implied and historical volatility data available in OptionMetrics. Using all stocks for which all information is available, we compute the breakpoints for historical volatility and market value. Then, we assign each stock into the corresponding portfolio. As is standard in asset pricing, we keep the portfolio constant over the year. We compute the average implied and historical volatility in each portfolio on a daily basis. At the end of 1997, the procedure is repeated to find the portfolio for 1998.

We drop the first year of data (1996) for the purpose of portfolio formation because we need the data in this year for the formation of the momentum portfolio and the beta portfolio. Moreover, historical volatilities in the first three months of 2006 are included to assess the accuracy of implied volatilities in the last three months of 2005. This gives us a time series of implied volatility for each portfolio in the time period between January 1997 and December 2005. For the historical volatility, data is available until March 2006.

Table 1 shows the breakpoints for beta, size, market-to-book, and momentum. Overall, the results are as expected. The breakpoints reflect the development of the US stock market during the sample period. For example, the market value breakpoints rise until 2000, drop between 2000 and 2003, and rise again after 2003. The market-to-book ratio shows the same pattern. For the momentum breakpoints, this evolution is reflected in the number of negative breakpoints. For example, only one breakpoint (Q1 to Q2) is negative in 1997, while three breakpoints are negative in 2003.

Table 2 shows the average number of stocks in each portfolio over the sample period. For the momentum premium and the market-to-book ratio, there is always a sufficiently high number of stocks in each portfolio (at least 30). However, a few low numbers can be observed. As expected, the combination high-beta, low-volatility (Q5 and Q1) and low-market value, low volatility gives somewhat low numbers (5 and13). But since we generally observe a sufficiently high number of stocks in most portfolios, the results are not likely to be negatively affected by the few small-number portfolios.

2.3 Spread Calculation

To capture the time series behavior of implied volatility in the cross-section, we construct four auxiliary time series. The construction of these variables is similar to the construction of risk premia in a number of asset pricing papers by Fama & French (1993) and Carhart (1997). They introduced four risk premia now widely used in asset pricing. The market risk premium (MRP) measures the excess return of the market portfolio over the risk-free rate, the small-minus-big (SMB) or size premium the return of small stocks over big stocks, the high-minus-low (HML) or value premium the return of stocks with a high book-to-market ratio in excess of stocks with a low book-to-market ratio, and the up-minus-down (UMD) or momentum premium the return of stocks with of a high momentum in excess of stocks with a low momentum.

In a similar manner, we construct four variables which measure the differences in implied volatilities for stocks with different characteristics. We refer to these variables as the market-risk spread (MRPS), the small-minus-big spread (SMBS), the high-minus-low spread (HMLS), and the up-minus-down spread (UMDS).

The market-risk spread (MRPS) measures the difference in implied volatilities between high beta stocks and low beta stocks, i.e.,

$$MRPS_t = \left[\frac{1}{B} \sum_{b=1}^{B} IV_{B,b,t}^{(beta)} \right] - \left[\frac{1}{B} \sum_{b=1}^{B} IV_{1,b,t}^{(beta)} \right] \quad , \tag{1}$$

where $IV_{a,b,t}^{(beta)}$ is the implied volatility at time t for the portfolio formed on beta and historical volatility where beta is in the ath quantile out of a total of A quantiles and the historical volatility is in the bth quantile out of a total of B quantiles. Since we use in this study 5×5 portfolios, A and B are equal to 5.

The small-minus-big spread (SMBS) measures the difference in implied volatilities between stocks with a low market value and stocks with a high market value, i.e.,

$$SMBS_t = \left[\frac{1}{B} \sum_{b=1}^{B} IV_{B,b,t}^{(size)} \right] - \left[\frac{1}{B} \sum_{b=1}^{B} IV_{1,b,t}^{(size)} \right] \quad , \tag{2}$$

where $IV_{a,b,t}^{(size)}$ is the implied volatility at time t for the portfolio formed on size and historical volatility, where the market value is in the ath quantile and the historical volatility is in the bth quantile.

The high–minus-low spread (HMLS) measures the difference in implied volatilities between stocks with a high and low book-to-market ratio, i.e.,

$$HMLS_t = \left[\frac{1}{B} \sum_{b=1}^{B} IV_{B,b,t}^{(btm)} \right] - \left[\frac{1}{B} \sum_{b=1}^{B} IV_{1,b,t}^{(btm)} \right] \quad , \tag{3}$$

where $IV_{a,b,t}^{(btm)}$ is the implied volatility at time t for the portfolio formed on the book-to-market ratio and historical volatility, where the book-to-market ratio is in the ath quantile and the historical volatility is in the bth quantile.

The up–minus-down spread (UMDS) measures the difference in implied volatilities between stocks with a high momentum and stocks with a low momentum, i.e.,

$$UMDS_t = \left[\frac{1}{B}\sum_{b=1}^{B} IV_{B,b,t}^{(mom)}\right] - \left[\frac{1}{B}\sum_{b=1}^{B} IV_{1,b,t}^{(mom)}\right] \quad , \tag{4}$$

where $IV_{a,b,t}^{(mom)}$ is the implied volatility at time t for the portfolio formed on momentum and historical volatility, where the momentum (one year return) is in the ath quantile and the historical volatility is in the bth quantile.

2.4 Forecasting Error

To assess the forecasting accuracy, we compute the mean relative error, MRE, as the average difference between realized volatility, HV, and implied volatility, IV, divided by the implied volatility

$$MRE_{a,b} = \frac{1}{T}\sum_{t=1}^{T} \frac{HV_{a,b,t+\tau}^{(x)} - IV_{a,b,t}^{(x)}}{IV_{a,b,t}^{(x)}} \quad , \tag{5}$$

where T denotes the sample size, τ the time to maturity of the options (91 calendar days), and (x) denotes the grouping criterion for the portfolio (e.g., mom for momentum).

The test statistic can be interpreted as follows. Values smaller than 0 mean that future realized volatility was smaller than implied volatility. Values larger than 0 mean that future realized volatility was higher than implied volatility.

Beside the mean relative error, we use a number of other approaches (such as the root mean squared error, forecasting regressions, qq-plots) to assess the predictive accuracy. However, as the MRE turned out to capture a high proportion of the information contained in the forecasting error, we report only the MRE.

Jorion (1995) suggests a regression-based approach the for evaluation of the performance of implied volatility. The predictive power of a volatility forecast can be estimated by regressing the realized volatility on forecast volatility:

$$HV_{a,b,t+\tau}^{(x)} = c_{0,a,b} + c_{1,a,b} \cdot IV_{a,b,t}^{(x)} + \varepsilon_{t,T} \tag{6}$$

where implied volatility at time t is used as the volatility forecast. If forecasts are perfect, we would expect the intercept, $c_{0,a,b}$, to be 0 and the slope coefficient, $c_{1,a,b}$, to be unity. Since overlapping periods introduce autocorrelations in the error terms, $\varepsilon_{t,T}$, we correct for this effect using the Newey & West (1987) correction with the appropriate number of lags.

3 Empirical Results

3.1 Implied Volatility

In this section, we analyze the relation between implied volatility, IV_t, the historical volatility, HV_t, and the firm characteristics (market value, MV_t, the market-to-book ratio, MTB_t, the cumulative return over one year, MOM_t, and the beta of a stock, $BETA_t$). We focus on sample averages over the whole whole sample period.

Figure 1 shows the relation between historical volatility and the beta on implied volatility. Numerical results are displayed in Table 3. As expected, implied volatility is in general increasing in historical volatility, i.e., portfolios with higher historical volatility have also higher implied volatilities. Within each quantile, we find generally a positive relation between beta and implied volatility. Stock options with similar historical volatility trade at a higher implied volatility if the beta is higher. Market participants perceive stocks to be riskier if the stock is a high-beta stock. The differences are of considerable magnitude. For example, for portfolios in the first quantile of historical volatility, the average implied volatility is 26.12% for low beta stocks (Quantile 1) and 41.88% for high beta stocks (Quantile 5). However, for portfolios with higher historical volatility, we find a non-monotonic relation between size and implied volatility. Stocks with a very low beta (Q1) show a higher implied volatility than stocks with a beta in the center (e.g., in Q2 and Q3) for high volatility portfolios.

Figure 2 shows the relation between historical volatility and the market value on implied volatility. We find strong evidence for a size effect in implied volatility. Within each quantile of historical volatility, large stocks (market value in Q5) have a substantially lower implied volatility than small stocks (Q1) or medium-size (Q2-Q4) stocks after controlling for historical volatility. As shown in Table 3, the difference in implied volatility for large and small stocks is 6.77% (34.95%-28.19%) for stocks in the lowest quantile of historical volatility (Q1) and 21.67% (84.78%- 62.91%) for stocks in the highest quantile of historical volatility (Q5).

Figure 3 shows the relation between historical volatility and the market-to-book-ratio on implied volatility. In contrast to the clearly observable patterns for beta and market value on implied volatility, the market-to-book ratio has a different impact on implied volatility. For the portfolio in the quantile 2 to 5 formed on MTB, the implied volatility is very similar after controlling for historical volatility. However, stocks with a very low market-to-book ratio (Q1) have substantially higher implied volatility than all other stocks. As shown in Table 3, the difference in implied volatility for stocks with a very low (Q1) and low (Q2) market-to-book ratio is 3.51% (32.71%-29.20%) for stocks in the lowest quantile of historical volatility (Q1) and 4.27% (75.50%-71.21%) for stocks in the highest quantile of historical volatility (Q5).

Figure 4 shows the relation between historical volatility and the momentum on implied volatility. Similar to the findings for the market-to-book ratio, we find a higher implied volatility for stocks with a low momentum (Q1), i.e., stocks with an annual return in the lowest quantile, than for stocks with a higher momentum. For stocks in the first quantile of historical volatility (Q1), the difference of implied volatility between stocks with a momentum in Q1 and Q2 is 8.70% (40.00%-31.34%). For stocks with a high historical volatility (Q5), this difference is 8.79% (79.63%-70.84%) and therefore of similar magnitude. For the remaining portfolios, we find a smaller effect between momentum stocks in Q4 and Q5. Options on stocks with a very high momentum (Q5) trade at slightly higher implied volatility than options on stocks with a weaker momentum (Q4) after controlling for historical volatility.

Overall, the findings can be summarized as follows. We find that the equity options of high beta stocks, small stocks, low market-to-book stocks and non-momentum stocks trade at a higher implied volatility over the whole sample period. The findings are robust for different levels of historical volatility.

3.2 Time Series Behavior of Spreads

In this section, we analyze the time series behavior of differences in implied volatilities for various portfolios to check for robustness and time-variation of previous findings. For this purpose, we use the spreads introduced in section 2.3.

Figure 5 shows the time series of the market-risk spread (MRPS), the small-minus-big spread (SMBS), the high-minus-low spread (HMLS), and the up-minus-down spread (UMDS). MRPS measures the difference in implied volatility for high and low beta stocks, SMBS between small and big stocks, HMLS between stocks with a high and low book-to-market ratio, and UMDS between stocks with a high and low momentum. The time series average values are displayed in the title of each plot.

The MRPS is 11.21% on average, i.e., high-beta stocks have on average a 11.21% higher implied volatility than low-beta stocks. The spread is positive at almost all times. However, there is considerable fluctuation over time.

Between 2000 and 2002, the spreads is in general between 20% and 45%, in 2004, between 15% and 20%. In the remaining years, MRPS is between 0 and 10%.

The SMBS on average is 16.12%, i.e., small stocks have on average a 16.12% higher implied volatility than large stocks. Similar to MRPS, it is also positive most of the time, but in contrast to MRPS, it shows a higher stability.

The HMLS is -3.05% on average, i.e., stocks with a high book-to-market ratio trade on average at a 3.05% lower volatility than stocks with a low book-to-market ratio. This finding is surprising. In empirical asset pricing a number of authors document (e.g., Fama & French (1993), Fama & French (1998)) that stocks with a high book-to-market ratio have higher average returns than predicted by the CAPM. Our analysis shows that these stocks are perceived to be less risky than stocks with a low book-to-market ratio. In other words, option markets perceive stocks with high valuation ratios (low book-to-market) to be riskier than stocks with low valuation ratios.

The UMDS is on average -9.70%, i.e., stocks with a high positive momentum have on average a -9.70% lower volatility than stocks with a high negative momentum. In other words, stocks with a strong negative return in the previous year trade at substantially higher implied volatility. Except for 2003, the UMDS is negative in all years.

3.3 The Forecast Accuracy of Implied Volatility

In this section, we analyze the forecasting power of implied volatility. Table 4 shows the numerical results.

Figure 6 shows the mean relative error (MRE) for portfolios formed on historical volatility and beta. Negative values mean that realized volatility was lower than predicted and positive values mean than realized volatility was higher than expected. In the sample period, realized volatility was lower than expected for most portfolios. In particular, volatility was overestimated for stocks with a low beta (Q1 and Q2) and high historical volatility (Q4 and Q5). For stocks with a beta in the highest quantile, realized volatility was higher than implied volatility.

Figure 7 shows the MRE for portfolios formed on historical volatility and the market value. We find that implied volatility is a poor predictor for realized volatility for small caps. For almost all portfolios with a market value in Quantile 1 and Quantile 2, implied volatility overestimated future volatility. In contrast, for all medium and large caps (Q3 to Q5), historical volatility was higher than implied volatility.

In Figure 8, we show the MRE for portfolios formed on the market-to-book ratio and historical volatility. The analysis shows that realized volatility is higher than suggested by implied volatility for all portfolios in quantiles 4 and 5 for the market-to-book ratio. For all portfolios with a high valuation, realized volatility is higher than implied volatility. In contrast, for stocks with a low market-to-book ratio and high historical volatility, realized volatility is substantially lower than expected.

Figure 9 illustrates the relation between the mean relative error, momentum, and historical volatility. For the momentum portfolios, we find that implied volatility overestimates realized volatility in general. Only for stocks with a very strong momentum (Q5) and with a very low momentum (Q1), we find that realized volatility is higher than expected based on implied volatility.

Table 5 and 6 show the estimated intercepts and slope coefficients of predictive regressions of realized volatility on implied volatility. For the intercept, we can reject the null hypothesis of an unbiased forecast ($c_0 = 0$) on an 95% level only in 4 out of 100 cases. For the slope coefficient, the results suggest that implied volatility seems to be a biased forecast for a few portfolios. We can reject the null hypothesis of a correct forecast ($c_1 = 1$) in 13 out of 100 cases. In particular, we can reject the null hypothesis for high-beta stocks with high volatility, for stocks with a low book-to-market ratio, and high-momentum stocks with high volatility.

On first sight the few rejections of the null in Table 5 and 6 appear to stand in opposition to the results in Table 4 and Figures 6- 9.[1] A positive intercept and a larger than 1 slope coefficient suggests that implied volatility underestimates realized volatility which is conflict with the negative signs in the corresponding buckets in Table 4. However all the statistically positive intercept coefficients in Table 5 are accompanied by a less than 1 slope coefficient in Table 6. Similarly the higher than one slope coefficients in Table 6 are accompanied by negative intercepts in Table 5. Therefore, underpricing need not be the case on an average percentage basis. Indeed the opposite is confirmed in Table 4.

Overall, we find considerable differences in the predictive power of implied volatility for future realized volatility in the sample period. Implied volatility overestimates realized volatility for low-beta stocks, small stocks, stocks with low market-to-book ratios, and stock with no momentum (in both directions). Implied volatility underestimates realized volatility for high-beta stocks, for large stocks, stocks with high market-to-book ratios

[1]We thank an anonymous referee for pointing this out

and stocks with a strong positive or negative momentum. However, we cannot reject the null hypothesis that implied volatility predicts realized volatility accurately in the cross-section except for very few cases.

3.4 Discussion of Results

As highlighted in the literature review, only very few studies have analyzed the behavior of implied volatility in the cross-section of equity options. Additionally, the setting of this study is different from other studies. Therefore, only a partial comparison of the findings is possible. For example, Bakshi & Kapadia (2003*b*) examine the relation between realized and implied volatility for 25 equity options. They find that Black-Scholes implied volatility is on average higher than the realized volatility, or - in other words - that there is a negative volatility risk premium. They argue that buyers of options may be willing to pay a premium in volatility because a long position in volatility helps hedge market risk due to an observed negative relation between market returns and market volatility. Adapting the argument by Bakshi & Kapadia (2003*b*) to our study, the results would imply that stocks for which implied volatility overestimates realized volatility (low beta stocks, small caps, low market-to-book, and stocks without a stock positive or negative momentum) should provide a better hedge against changes in market volatility.

This issue has recently been discussed by a number of authors. Ang, Hodrick, Xing & Zhang (2006) examine the pricing of aggregate volatility risk in the cross-section of stock returns and find that - consistent with expectations - that stocks with high sensitivities to innovations in aggregate volatility have low average returns. They find that size, book-to-market, and momentum effects can account for neither the low average returns earned by stocks with high exposure to systematic volatility risk nor for the low average returns of stocks with high idiosyncratic volatility. They argue that this is a puzzle requiring further investigation. Basically, our results go into the same direction, except that we use option price data.

4 Conclusion

In this paper, we analyze the relation between implied volatility, realized volatility and stock characteristics in the cross-section of equity options. Using a large dataset of US equity options, we form portfolios based on historical volatility and stock characteristics (such as beta, size, market-to-book, and momentum). We find a number of patterns that have not been documented up to now.

We find that high-beta stocks, small stocks, stocks with a low market-to-book ratio, and non-momentum stocks trade at higher implied volatilities after controlling for historical volatility. We find evidence that implied volatility overestimates realized volatility for low beta stocks, small caps, low market-to-book, and stocks without a stock positive or negative momentum. Implied volatility underestimates realized volatility for stocks with contrarian characteristics. However, we cannot reject the null hypothesis that implied volatility is an unbiased predictor of realized volatility in the cross section.

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Table 1: Breakpoints

The table shows the breakpoints for portfolio construction for each factor and each year. The table is interpreted as follows: A value of 0.48 for quantile 1 to quantile 2 (Q1 to Q2) and of 0.72 for Q2 to Q3 for beta in 1997 means that all stocks with a beta lower than 0.48 were assigned to portfolio 1, all stocks with a beta between 0.48 and 0.72 to portfolio 2, and so on. Market values are in million USD. The momentum breakpoint is the average daily continuously compounded return.

		Year								
		1997	1998	1999	2000	2001	2002	2003	2004	2005
Beta	Q1 to Q2	0.48	0.43	0.59	0.25	0.32	0.44	0.54	0.57	0.81
	Q2 to Q3	0.72	0.65	0.82	0.42	0.55	0.73	0.78	0.84	1.08
	Q3 to Q4	0.98	0.86	1.07	0.61	0.86	1.04	1.00	1.07	1.41
	Q4 to Q5	1.42	1.15	1.37	0.94	1.61	1.64	1.32	1.41	1.83
Market Value	Q1 to Q2	142.32	146.79	123.92	169.90	109.53	130.14	90.15	212.91	262.55
	Q2 to Q3	351.76	415.86	368.49	473.57	377.38	416.97	308.88	564.57	691.77
	Q3 to Q4	781.73	934.76	901.92	1169.78	1001.83	1029.20	838.13	1365.89	1596.23
	Q4 to Q5	2341.17	2798.52	2829.17	3518.05	3280.36	3178.77	2664.48	3950.69	4793.30
Market-to-Book	Q1 to Q2	1.56	1.64	1.17	1.14	0.89	1.07	0.84	1.42	1.52
	Q2 to Q3	2.20	2.38	1.93	1.94	1.64	1.74	1.36	2.04	2.16
	Q3 to Q4	3.17	3.33	2.97	3.49	2.59	2.57	1.95	2.89	3.02
	Q4 to Q5	5.17	5.24	5.41	7.72	4.68	4.23	3.16	4.67	4.57
Momentum	Q1 to Q2	-0.17	-0.21	-0.51	-0.35	-1.00	-0.45	-0.87	0.16	-0.15
	Q2 to Q3	0.07	0.09	-0.17	-0.06	-0.30	-0.11	-0.35	0.30	0.07
	Q3 to Q4	0.23	0.29	0.07	0.20	0.09	0.10	-0.11	0.47	0.20
	Q4 to Q5	0.42	0.48	0.34	0.69	0.39	0.38	0.10	0.76	0.37

Table 2: Average number of stocks in each portfolio

The table shows the average number of stocks in each portfolio over the whole sample period.

		Historical Volatility				
		Q1	Q2	Q3	Q4	Q5
Beta	Q1	113	64	49	35	28
	Q2	127	103	73	49	34
	Q3	100	110	95	63	45
	Q4	42	81	105	102	84
	Q5	13	33	63	134	181
Market Value	Q1	5	7	21	47	81
	Q2	18	26	63	94	120
	Q3	48	68	95	102	87
	Q4	118	123	105	84	62
	Q5	209	173	109	64	35
Market-to-Book	Q1	50	52	70	71	78
	Q2	101	88	80	70	62
	Q3	103	96	79	76	62
	Q4	76	88	83	85	80
	Q5	59	68	76	84	98
Momentum	Q1	30	43	68	98	141
	Q2	86	91	90	84	66
	Q3	129	104	80	64	42
	Q4	106	94	81	63	47
	Q5	40	58	66	75	77

Table 3: Implied Volatility for Portfolios

The table shows the average implied volatilities in percent (for standardized options with a maturity of 91 days and a strike price equal to the forward price) over the sample period (daily data, 01/1997 - 12/2005) for portfolios formed on the historical volatility (91 days) at the end of each year and a second citerion. As second variable the beta (BETA), the market value (MV), the market-to-book ratio (MTB) and the return over a one year horizon (MOM) have been used. All stocks have been grouped into 20% quantiles for each criterion. Q1 to Q5 denotes the quantiles where Q1 is the portfolio in the lowest quantile and Q5 the portfolio in the highest quantile.

		Historical Volatility				
		Q1	Q2	Q3	Q4	Q5
Beta	Q1	26.12	35.38	43.18	55.85	71.32
	Q2	28.99	35.49	43.00	54.17	65.62
	Q3	30.42	36.57	45.36	55.92	69.46
	Q4	34.02	39.98	47.79	58.05	71.37
	Q5	41.88	48.49	54.17	64.35	75.08
Market Value	Q1	34.96	47.53	59.12	72.56	84.78
	Q2	33.38	46.85	55.04	65.38	76.61
	Q3	32.36	41.56	49.06	59.68	71.51
	Q4	29.77	37.93	45.75	56.32	68.67
	Q5	28.19	34.22	41.85	54.12	62.91
Market-to-Book	Q1	32.71	43.16	51.73	63.00	75.50
	Q2	29.20	38.15	46.60	59.14	71.21
	Q3	29.21	36.66	46.20	58.35	71.89
	Q4	30.18	36.20	46.39	60.05	73.41
	Q5	29.62	37.29	48.57	60.94	74.41
Momentum	Q1	40.04	49.12	56.79	68.65	79.63
	Q2	31.34	38.25	46.93	57.29	70.84
	Q3	28.06	35.06	43.37	54.77	68.95
	Q4	29.00	35.18	43.14	53.65	65.04
	Q5	31.82	40.08	46.55	58.08	69.20

Table 4: Mean relative prediction error

The table shows the mean relative error in percent, i.e., the difference of realized and implied volatility relative to implied volatility (for standardized options with a maturity of 91 days and a strike price equal to the forward price) over the sample period (daily data, 01/1997 - 12/2005) for portfolios formed on the historical volatility (91 days) at the end of each year and a second citerion. As second variable the beta (BETA), the market value (MV), the market-to-book ratio (MTB) and the return over a one year horizon (MOM) have been used. All stocks have been grouped into 20% quantiles for each criterion. Q1 to Q5 denotes the quantile where Q1 is the portfolio in the lowest quantile and Q5 the portfolio in the hightest quantile. Negative values mean that implied volatility overestimated realized volatility on average. Positive values correspond to an underestimation of realized volatility. * denotes significantly different from 0 on a 95% level and ** on a 99% level (t-test).

		Historical Volatility				
		Q1	Q2	Q3	Q4	Q5
Beta Q1	Q1	-5.60**	-3.91**	-6.38**	-9.02**	-8.64**
	Q2	-2.23**	-4.13**	-4.13**	-6.53**	-8.05**
	Q3	2.35**	-0.45	-1.01**	-4.03**	-5.70**
	Q4	5.24**	4.42**	2.06**	0.03	-4.90**
	Q5	12.40**	11.77**	8.48**	5.68**	3.43**
Market Value	Q1	-4.81**	-8.40**	-5.97**	-6.44**	-7.54**
	Q2	-4.97**	1.26**	-0.44	-2.44**	-3.83**
	Q3	-0.29	1.05**	0.96**	0.93**	2.12**
	Q4	-1.70**	-1.51**	1.90**	2.30**	2.17**
	Q5	1.17**	2.52**	2.18**	3.74**	3.28**
Market-to-Book	Q1	-1.01**	0.12	-2.10**	-4.60**	-4.60**
	Q2	0.01	-0.37	-0.49	-2.54**	-4.19**
	Q3	0.14	-0.45	-0.12	-0.08	-1.48**
	Q4	0.47	2.92**	3.11**	2.08**	0.90**
	Q5	2.17**	3.03**	3.02**	3.43**	1.69**
Momentum	Q1	15.11**	13.79**	7.76**	3.75**	0.74 *
	Q2	1.26**	-0.64	-1.52**	-2.47**	-3.91**
	Q3	-2.35**	-2.17**	-2.92**	-3.03**	-3.69**
	Q4	-1.41**	-2.10**	-1.10**	-1.56**	-2.30**
	Q5	5.57**	2.70**	1.16**	-0.06	-1.90**

Table 5: Predictive regressions: intercept

The table shows the intercepts of predictive regressions (realized volatility is regressed on a constant and on lagged implied volatility). Moreover, the table shows the average factor loadings for each quantile. Standard errors have been estimated with Newey-West. * denotes significantly different from 0 on a 95% level and ** on a 99% level.

		Historical Volatility					
		Q1	Q2	Q3	Q4	Q5	Mean
Beta	Q1	-0.01	-0.02	0.03	0.08*	0.06	0.03
	Q2	-0.00	0.01	-0.00	0.03	0.08	0.02
	Q3	-0.00	-0.02	0.00	-0.02	0.00	-0.01
	Q4	0.01	-0.03	-0.01	-0.05	-0.08	-0.03
	Q5	0.01	0.06	-0.05	-0.10	-0.13	-0.04
	Mean	0.00	0.00	-0.00	-0.01	-0.01	
Market Value	Q1	0.07*	0.00	0.04	0.04	-0.00	0.03
	Q2	-0.03	0.04	-0.00	-0.02	-0.06	-0.01
	Q3	0.02	0.02	0.01	-0.06	-0.07	-0.02
	Q4	-0.00	-0.00	-0.02	-0.07	-0.07	-0.03
	Q5	-0.00	-0.01	-0.02	-0.05	-0.09	-0.04
	Mean	0.01	0.01	0.00	-0.03	-0.06	
Market-to-Book	Q1	0.02	-0.09	-0.06	-0.13	-0.18	-0.09
	Q2	-0.03	-0.03	-0.05	-0.05	-0.10	-0.05
	Q3	-0.00	0.00	0.01	-0.07	-0.08	-0.03
	Q4	0.03*	0.02	0.02	0.01	-0.02	0.01
	Q5	0.07	0.06*	0.05	0.04	0.01	0.04
	Mean	0.02	-0.01	-0.01	-0.04	-0.08	
Momentum	Q1	-0.00	0.01	0.03	-0.00	0.02	0.01
	Q2	0.02	0.02	-0.01	-0.02	0.01	0.01
	Q3	-0.00	0.01	-0.02	-0.02	-0.09	-0.02
	Q4	-0.01	-0.02	-0.01	-0.06	-0.12	-0.04
	Q5	-0.00	-0.02	-0.05	-0.12	-0.16	-0.07
	Mean	0.00	0.00	-0.01	-0.04	-0.07	

Table 6: Predictive regressions: slope coefficients

The table shows the slope coefficients of predictive regressions (realized volatility is regressed on a constant and on lagged implied volatility). Moreover, the table shows the average factor loadings for each quantile. Standard errors have been estimated with Newey-West. * denotes significantly different from 1 on a 95% level and ** on a 99% level.

		Historical Volatility					
		Q1	Q2	Q3	Q4	Q5	Mean
Beta	Q1	0.98	1.01	0.85	0.76	0.82	0.88
	Q2	0.98	0.94	0.96	0.87	0.79	0.91
	Q3	1.04	1.04	0.98	1.00	0.94	1.00
	Q4	1.03	1.13	1.04	1.09	1.06	1.07
	Q5	1.09	0.99	1.18*	1.22**	1.22*	1.14
	Mean	1.02	1.02	1.00	0.99	0.97	0.00
Market Value	Q1	0.72	0.91	0.87	0.87	0.92	0.86
	Q2	1.05	0.92	1.00	1.00	1.04	1.00
	Q3	0.94	0.96	0.99	1.10	1.12	1.02
	Q4	1.00	0.99	1.06	1.16*	1.13	1.07
	Q5	1.02	1.07	1.07	1.14	1.18*	1.10
	Mean	0.95	0.97	1.00	1.06	1.08	0.00
Market-to-Book	Q1	1.10	1.35**	1.19*	1.24**	1.25**	1.22
	Q2	1.10	1.08	1.09	1.07	1.11*	1.09
	Q3	0.98	0.97	0.94	1.11	1.08	1.02
	Q4	0.87	0.91	0.94	0.97	1.01	0.94
	Q5	0.83	0.86	0.91	0.93	0.97	0.90
	Mean	0.98	1.03	1.01	1.06	1.08	0.00
Momentum	Q1	0.99	0.98	0.92	0.96	0.93	0.96
	Q2	0.92	0.94	1.01	1.01	0.94	0.97
	Q3	1.01	0.96	1.05	1.04	1.11	1.03
	Q4	1.04	1.09	1.06	1.12	1.18*	1.10
	Q5	1.04	1.07	1.13	1.24**	1.24*	1.14
	Mean	1.00	1.01	1.03	1.07	1.08	

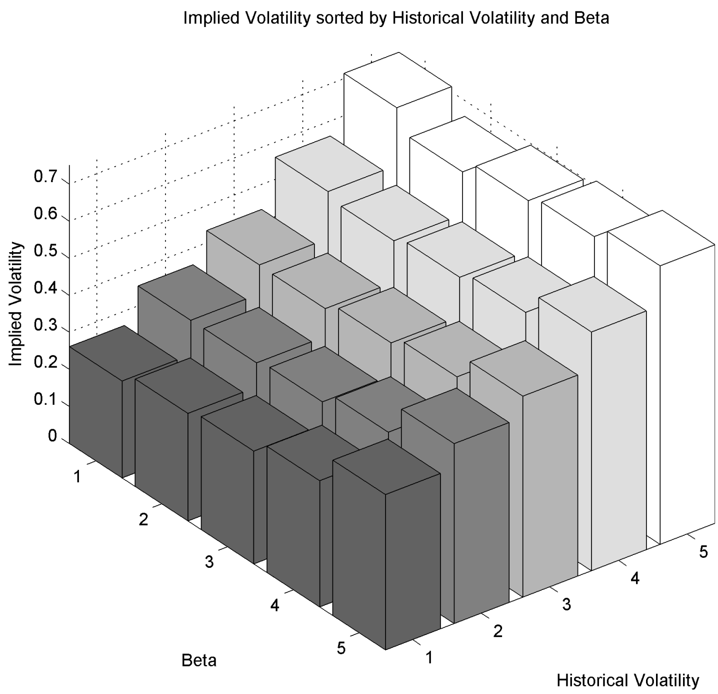

Figure 1: Relation between historical volatility and beta on implied volatility

The Figure shows the average implied volatilities over the sample period (daily data, 01/1997 - 12/2005) for portfolios formed annually based on the beta (OLS regression of returns on CRSP market return based on daily data over one year) and on the historical volatility (91 days) at the end of each year.

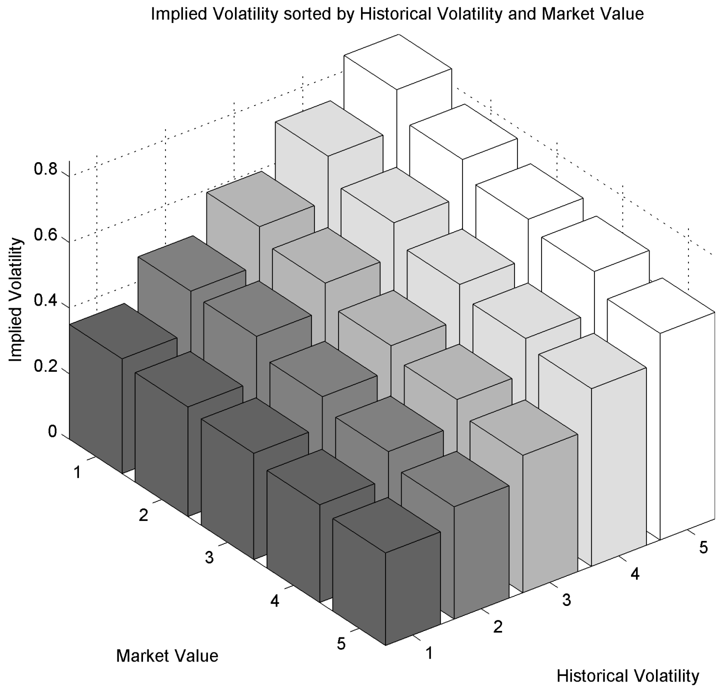

Figure 2: Relation between historical volatility and market value on implied volatility

The Figure shows the average implied volatilities over the sample period (daily data, 01/1997 - 12/2005) for portfolios formed annually based on the market value of equity (as provided by Compustat) and on the historical volatility (91 days) at the end of each year.

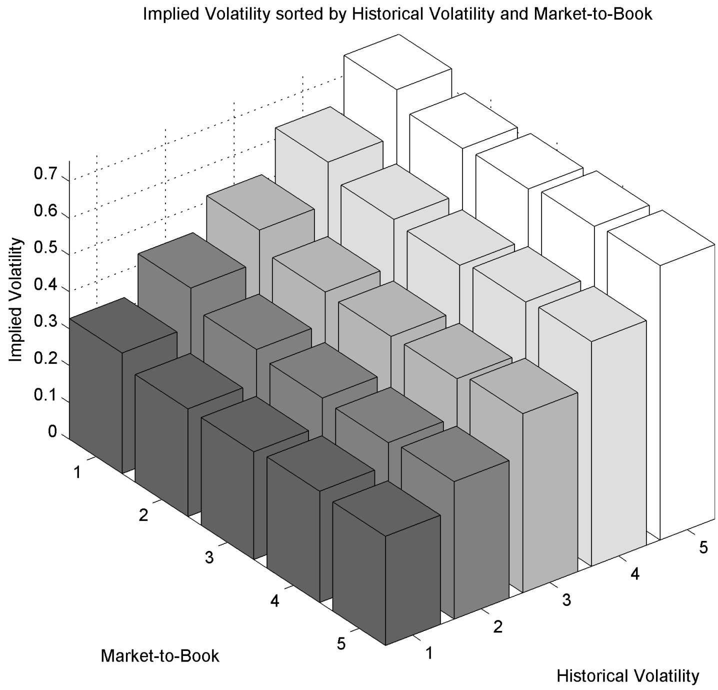

Figure 3: Relation between historical volatility and market-to-book ratio on implied volatility

The Figure shows the average implied volatilities over the sample period (daily data, 01/1997 - 12/2005) for portfolios formed annually based on the market-to-book ratio (as provided by Compustat) and on the historical volatility (91 days) at the end of each year.

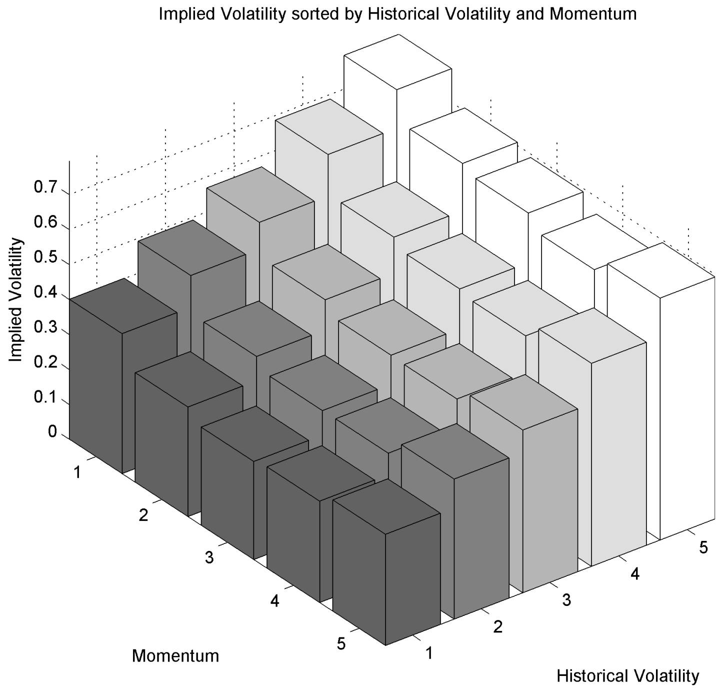

Figure 4: Relation between historical volatility and the momentum on implied volatility

The Figure shows the average implied volatilities over the sample period (daily data, 01/1997 - 12/2005) for portfolios formed annually based on the total return (as provided by OptionMetrics) and on the historical volatility (91 days) at the end of each year.

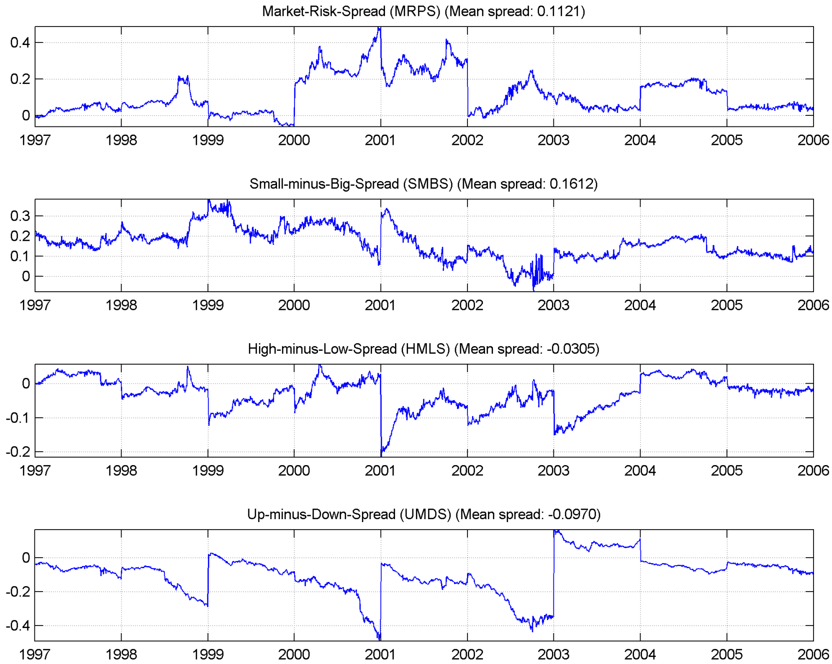

Figure 5: Time series of spreads

The Figure shows the calculated spreads over the sample period. The market-risk spread (MRPS) or market spread measures the difference in implied volatilities between high and low beta stocks. The small-minus-big spread (SMBS) or size spread measures the difference in implied volatilities between stocks with a small and big market value. The high-minus-low spread (HMLS) or value spread measures the difference in implied volatilities between stocks with a high and a low book-to-market ratio. The up-minus-down spread (UMDS) or momentum spread measures the difference in implied volatilities between stocks with a high and a low momentum.

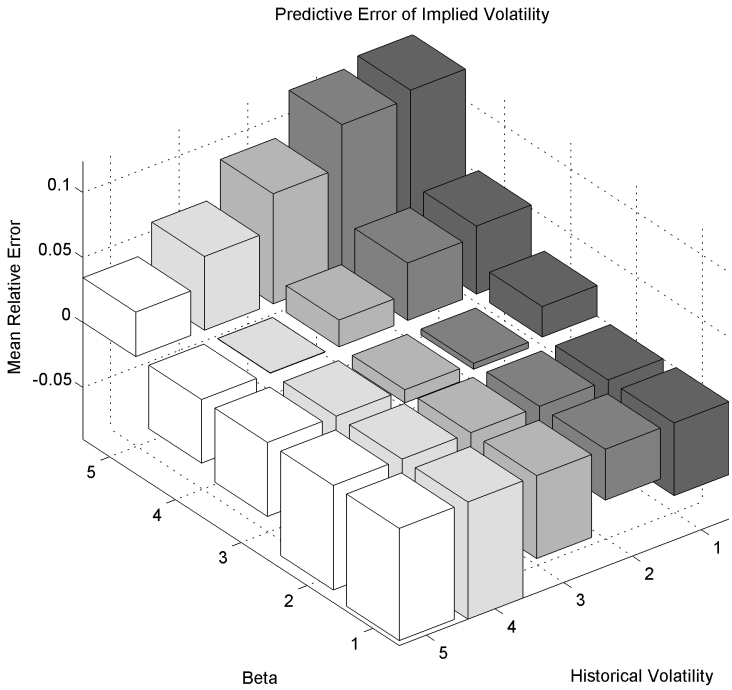

Figure 6: Relation between historical volatility and beta on the prediction error
The Figure shows the mean relative error, i.e., the difference of realized and implied volatility relative to implied volatility, over the sample period (daily data, 01/1997 - 12/2005) for portfolios formed annually based on the beta (OLS regression of returns on CRSP market return based on daily data over one year) and on the historical volatility (91 days) at the end of each year. Negative values mean that implied volatility overestimated realized volatility on average. Positive values correspond to an underestimation of realized volatility.

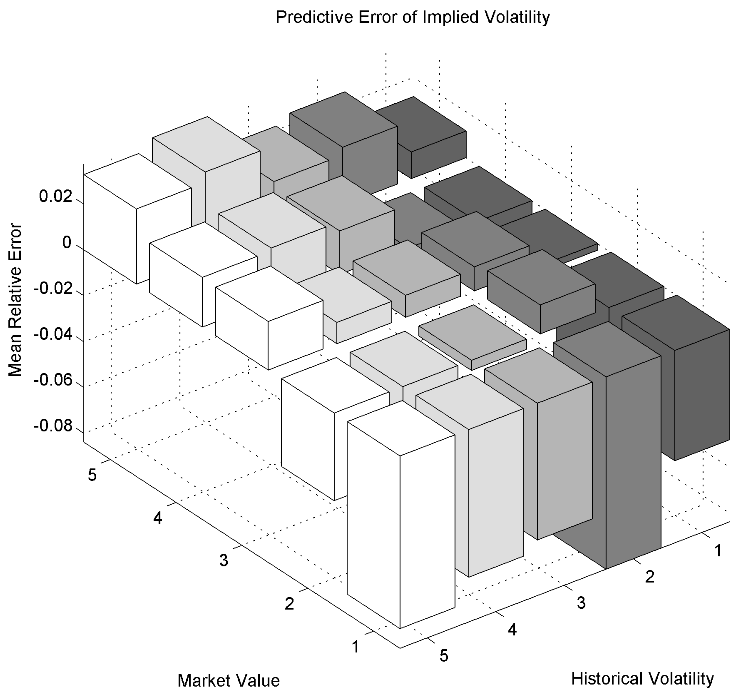

Figure 7: Relation between historical volatility and market value on the prediction error

The Figure shows the mean relative error, i.e., the difference of realized and implied volatility relative to implied volatility, over the sample period (daily data, 01/1997 - 12/2005) for portfolios formed annually based on the market value of equity (as provided by Compustat) and on the historical volatility (91 days) at the end of each year. Negative values mean that implied volatility overestimated realized volatility on average. Positive values correspond to an underestimation of realized volatility.

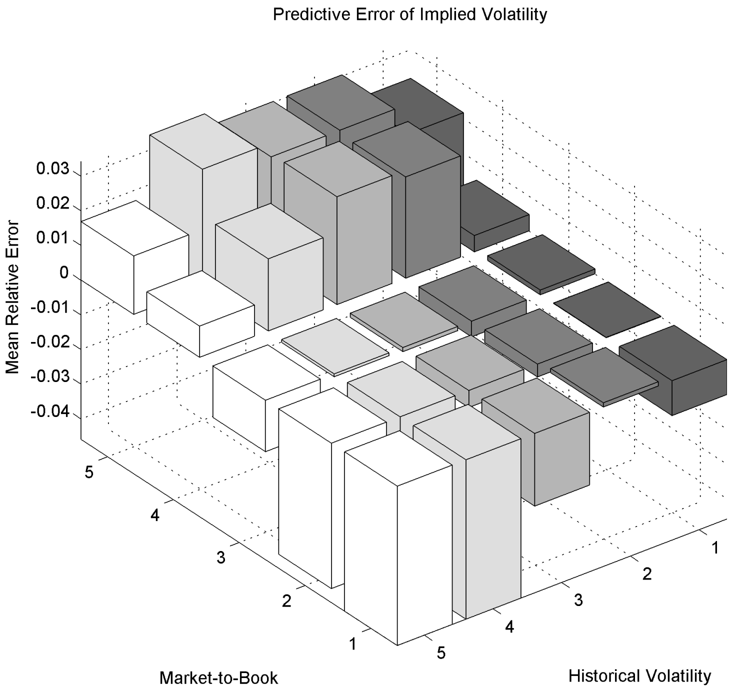

Figure 8: Relation between historical volatility and market-to-book-ratio on the prediction error

The Figure shows the mean relative error, i.e., the difference of realized and implied volatility relative to implied volatility, over the sample period (daily data, 01/1997 - 12/2005) for portfolios formed annually based on the market-to-book ratio (as provided by Compustat) and on the historical volatility (91 days) at the end of each year. Negative values mean that implied volatility overestimated realized volatility on average. Positive values correspond to an underestimation of realized volatility.

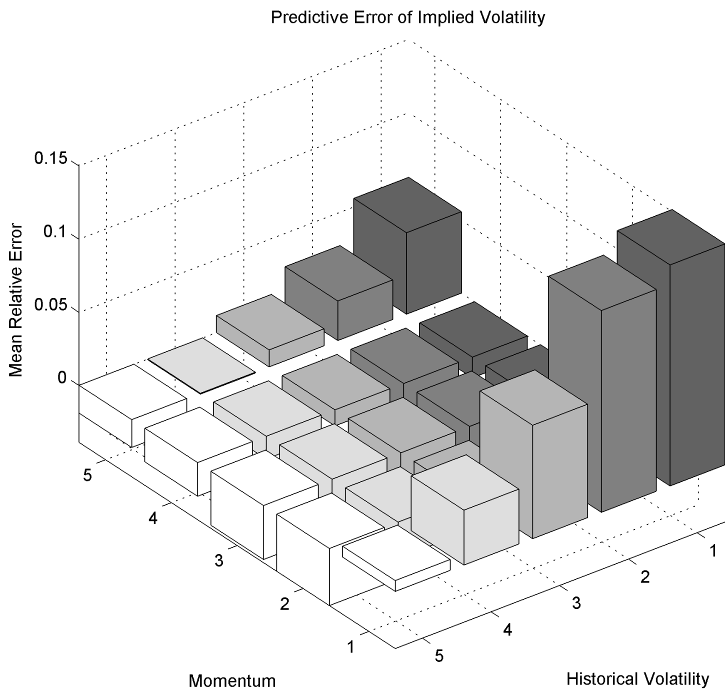

Figure 9: Relation between historical volatility and the momentum on the prediction error
The Figure shows the mean relative error, i.e., the difference of realized and implied volatility relative to implied volatility, over the sample period (daily data, 01/1997 - 12/2005) for portfolios formed annually based on the total return (as provided by OptionMetrics) and on the historical volatility (91 days) at the end of each year. Negative values mean that implied volatility overestimated realized volatility on average. Positive values correspond to an underestimation of realized volatility.