Monte Carlo Financial Models on U50

Group 7

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Github 連結: https://github.com/EvanLu0815/Final_monteCarloFinancialModel

Introduction

In this project, I am going to implement Black-Scholes Model on the payoff price of European vanilla option. In the Black-Scholes model, the price can be considered as geometric Brownian motion. Then we can obtain (1).

$$\mathrm{dS} = \mathrm{rSdt} + \sigma\mathrm{Sdz} \tag{1}$$

In (1), S is stock price, r is the fixed interest rate, σ is the constant volatility and z is a Wiener process. The analytical solution for the stochastic differential equation (1) is

$$S_{t+\Delta t} = S_t e^{(r-\frac{1}{2}\sigma^2)\Delta t+\sigma\epsilon\sqrt{\Delta t}} \tag{2}$$

In (2), $\epsilon \sim N(0,1)$ is the standard normal distribution.

The European vanilla option can be exercised only at expiration date. As a result, only the stock price at the expiration date can affect the payoff price. Then the payoff price can be implemented as (3).

$$P_{call} = max\{S_T - K, 0\} \tag{3}$$

In (3), T is the pre-set time, S_T is the stock price at expiration date and K is the strike price.

Algorithm

Black-Sholes model includes three main steps. Each time interval (0, T) is first divided into M steps, and denoted as t_0, t_1, …, t_M. Secondly, M standard normally distributed independent random numbers ϵ are generated. Finally, S_{t_M} is computed by (2) from S_{t_0}. After going through the three steps, one “path” is completed. In the model, it needs to compute N paths. The algorithm is shown below.

Algorithm 1 Black-Scholes model

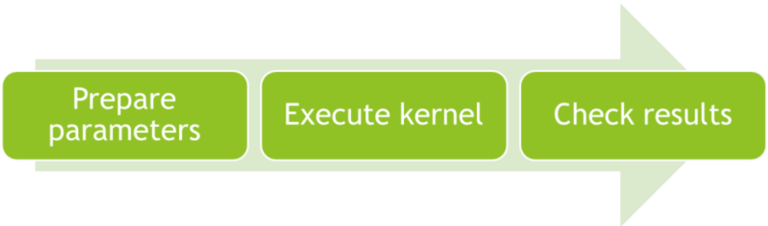
Input: parameters for the stock and option
Output: payoff price
  Initialization: Random number generators
  for $i = 1$ to N **do**
    for $k = 1$ to M **do**
      $U_1, U_2 \leftarrow MersenneTwist()$
      $\epsilon_1, \epsilon_2 \leftarrow BoxMuller(U_1, U_2)$
      $S_{t_{k+2}}, S_{t_{k+1}} \leftarrow Price(S_{t_k}, \epsilon_1, \epsilon_2)$
      $k += 2$
    end for
    $P_{option}[i] \leftarrow Option(S_t[], K)$
    i++
  end for
  return $P_{Call} = ave(P_{option})$

The random numbers ϵ is generated by Mersenne-Twister algorithm followed by Box-Muller transformation.

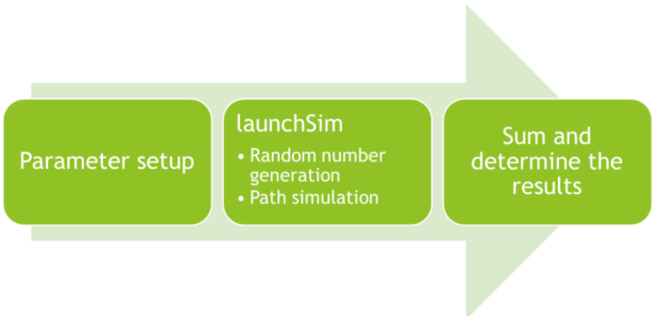
Hardware Implementation

In this project, I re-write the whole host program and adjust some of the pragma in order to make the project workable on the vitis 2021.2 and U50 platfrom.

System Structure



Kernel Execution Structure



1. Parameter setup

   The kernel is first setup the parameters. It uses sd to transfer all the float data into blackScholes model.

```
        static const int SIMS_PER_GROUP =32;
        stockData<float> sd(timeT,freeRate,volatility,initPrice,strikePrice);

        float call0, put0;
        blackScholes<SIMS_PER_GROUP,EuropeanOptionStatus<float>, float> bs0(sd, steps);
```

2. LaunchSim

   The kernel is then use launchSimulation in lauchSim.h to generate random number using box-Muller algorithm and use simulation offered by the class blackScholes to do path simulation. In launchSim, three functions are called, prng, BOX_MULLER, and simulation. I will introduce them in the following. Rng0、rng1 are first generate in launchSimulation to store the seed from prng. Prng is used to operate Mersenne twister algorithm, and BOX_MULLER is used to operate Box-Muller transformation. In launchSimulation, it use allocation to constrain the usage of multiplication. It can only use 1 multiplication in launchSimulation. Also, it use dataflow in launchSim to improve the throughput.

```
launchSimulation(call0, put0, g_id+0, bs0, sims*steps>>4, sims>>4);
```

```
void launchSimulation(DATA_T &pCall, DATA_T &pPut, int seed, Model &m, int numR, int sims){
#pragma HLS INLINE off
#pragma HLS ALLOCATION instances=mul limit=1 operation
        RNG<float> rng0(seed*SEED_STRIDE);
        RNG<float> rng1(seed*SEED_STRIDE + 1);
        launchSim(pCall, pPut, rng0, rng1, m, numR, sims);
}
```

```
void launchSim(DATA_T &pCall, DATA_T &pPut, RNG<DATA_T> &rng0,RNG<DATA_T> &rng1, Model &m, int numR, int sims){
#pragma HLS DATAFLOW
        hls::stream<unsigned int> s_num0;
        hls::stream<unsigned int> s_num1;
#pragma HLS STREAM variable=s_num0 depth=4 dim=1
#pragma HLS STREAM variable=s_num1 depth=4 dim=1
        hls::stream<DATA_T> sRNG0;
        hls::stream<DATA_T> sRNG1;
#pragma HLS STREAM variable=sRNG0 depth=4 dim=1
#pragma HLS STREAM variable=sRNG1 depth=4 dim=1

        prng(rng0, rng1, s_num0, s_num1, numR);

        RNG<DATA_T>::BOX_MULLER(s_num0, s_num1, sRNG0, sRNG1, numR>>1);

        m.simulation(sRNG0,sRNG1, sims, pCall, pPut);
}
```

Prng:

```
void prng(RNG<DATA_T> &rng0,RNG<DATA_T> &rng1, hls::stream<unsigned int> &sRNG0,hls::stream<unsigned int> &sRNG1, int nums
        for(int i = 0 ; i < nums>>2;i++){
#pragma HLS PIPELINE
                unsigned int r0, r1, r2, r3;
                rng0.extract_number(&r0, &r1);
                rng1.extract_number(&r2, &r3);
                sRNG0.write(r0);
                sRNG0.write(r2);
                sRNG1.write(r1);
                sRNG1.write(r3);
        }
}
```

extract_number is used to prepare the seed for Box-Muller transformation. The INLINE pragma is used to break the function structure to decrease the use of hardware resources.

```
        void extract_number(uint* num1,uint* num2){
#pragma HLS INLINE
                uint id1=increase(1), idm=increase(RNG_MH), idm1=increase(RNG_MHI);

                uint x = this->seed,x1=this->mt_o[this->index],x2=this->mt_e[id1],
                                xm=this->mt_o[idm],xm1=this->mt_e[idm1];

                x = (x & upper_mask)+(x1 & lower_mask);
                uint xp = x >> 1;
                if ((x & 0x01) != 0)
                        xp ^= RNG_A;
                x = xm ^ xp;

                uint y = x;
                y ^= ((y >> RNG_U)&RNG_D);
                y ^= ((y << RNG_S)&RNG_B);
                y ^= ((y << RNG_T)&RNG_C);
                y ^= (y >> RNG_L);
                *num1 = y;
                mt_e[this->index]=x;

                x1 =( x1 & upper_mask) + (x2 & lower_mask);
                uint xt = x1 >> 1;
                if ((x1 &0x01) != 0)
                        xt ^= RNG_A;
                x1 = xm1 ^ xt;
```

```
                uint y1 = x1;
                y1 ^= ((y1 >> RNG_U)&RNG_D);
                y1 ^= ((y1 << RNG_S)&RNG_B);
                y1 ^= ((y1 << RNG_T)&RNG_C);
                y1 ^= (y1 >> RNG_L);
                *num2 = y1;
                mt_o[this->index]=x1;

                this->index=id1;
                this->seed=x2;
        }
```

BOX_MULLER:

In this function, pipeline is used to increase throughput.

```
        static void BOX_MULLER(hls::stream<uint>&num0,hls::stream<uint>&num1,
                        hls::stream<DATA_T>&data0,hls::stream<DATA_T>&data1, int rep){
                static const DATA_T _2PI= 2*3.14159265358979323846f;
                for(int i = 0; i<rep;i++){
#pragma HLS PIPELINE
                        DATA_T tp,tmp1,tmp2;
                        ap_ufixed<32,0> f_tmp1, f_tmp2;
                        f_tmp1(31, 0)=num0.read();
                        f_tmp2(31, 0)=num1.read();
                        tmp1 = f_tmp1.to_float();
                        tmp2 = f_tmp2.to_float();
                        tp=sqrtf(fmaxf(-2.0f*logf(tmp1),0.0f));
                        DATA_T g0=cosf(_2PI*tmp2)*tp;
                        DATA_T g1=sinf(_2PI*tmp2)*tp;
                        data0.write(g0);
                        data1.write(g1);
                }
        }
```

Simulation:

In simulation, path_sum and sum are called to complete the algorithm we have explained in algorithm section. The path_sim is used to divide the time interval, compute the stock price at expiration date by iteration, and the sum function is used to determine the final price. The first loop in path_sim is used to setup the parameter. The second for loop is used to compute the stock price at expiration date. The inner-most loop is used to update 2 stock price at once.

```
        void simulation(hls::stream<DATA_T>& s_RNG0,hls::stream<DATA_T>& s_RNG1, int sims,  DATA_T &pCall, DATA_T &pPut)
        {
                DATA_T call = 0, put = 0;
                hls::stream<DATA_T> prices;
#pragma HLS STREAM variable=prices depth=NUM_SIMS dim=1
#pragma HLS DATAFLOW
                path_sim(s_RNG0, s_RNG1, prices, sims);
                sum(prices, call, put, sims);
                pCall= call;
                pPut = put;
        }
```

```
        void path_sim(hls::stream<DATA_T>& s_RNG0,hls::stream<DATA_T>& s_RNG1, hls::stream<DATA_T>& prices, int sims){
                OptionStatus stockPrice[NUM_SIMS];
#pragma HLS ARRAY_PARTITION variable=stockPrice cyclic factor=2 dim=1
                for(int j=0;j<NUM_SIMS;j++)
#pragma HLS PIPELINE
                        stockPrice[j].init(data.price);

                for(int k=0;k<sims/NUM_SIMS;k++) {
                        for(int s=0; s <NUM_STEPS;s++){
                                for(int j=0;j<NUM_SIMS/2;j++) {
#pragma HLS PIPELINE
                                        DATA_T r0 = s_RNG0.read();
                                        DATA_T r1 = s_RNG1.read();
                                        update(stockPrice[j*2], r0);
                                        update(stockPrice[j*2+1], r1);
                                }
                        }
                        for(int j=0;j<NUM_SIMS;j++){
#pragma HLS DEPENDENCE variable=stockPrice array inter RAW false
#pragma HLS PIPELINE
                                DATA_T price = stockPrice[j].valid? stockPrice[j].price():data.strikePrice;
                                prices.write(price);
                                stockPrice[j].init(data.price);
                        }
                }
        }
```

```
        void update(OptionStatus &option, DATA_T r){
#pragma HLS INLINE
                const DATA_T Dt = data.timeT / (DATA_T)NUM_STEPS,
                                        Rdt = 1+data.freeRate*Dt,
                                        SqrtV = data.volatility * sqrtf(Dt);
                option.stockPrice *= Rdt +r *SqrtV;
                option.update();
        }
        void sum(hls::stream<DATA_T> &prices, DATA_T &call, DATA_T&put, int sims){
                for(int j=0;j<sims;j++) {
#pragma HLS PIPELINE
                        DATA_T price = prices.read();
                        call+=executeCall(price);
                        put+=executePut(price);
                }
        }
```

In sum function, the executeCall and executePut are called to determine which price result should be kept as shown in formula (3).

```
	DATA_T executeCall(DATA_T& price){
#pragma HLS INLINE
		if(price > data.strikePrice){
			return (price - data.strikePrice);
		}
		else
			return 0;
	}
	DATA_T executePut(DATA_T& price){
#pragma HLS INLINE
		if(price < data.strikePrice){
			return (data.strikePrice - price);
		}
		else
			return 0;
	}
};
```

3. Sum and determine the results

```
pCall[g_id] = (call0
               +call1
               +call2
               +call3
               +call4
               +call5
               +call6
               +call7
               +call8
               +call9
               +call10
               +call11
               +call12
               +call13
               +call14
               +call15
               )/sims;
```

```
pPut[g_id] =(put0
             +put1
             +put2
             +put3
             +put4
             +put5
             +put6
             +put7
             +put8
             +put9
             +put10
             +put11
             +put12
             +put13
             +put14
             +put15
             )/sims;
```

<u>Main function</u>

I use the host code from lab3 and adjust the kernel-related part.

The kernel interface is shown below. According to the kernel interface, the host function needs to prepare 2 axi master for pCall and pPut and 8 axi lite for the rest of parameters.

```
void blackEuro(float *pCall, float *pPut,   // call price and put price
            float timeT,                // time period of options
            float freeRate,         // interest rate of the riskless asset
            float volatility,           // volatility of the risky asset
            float initPrice,            // stock price at time 0
            float strikePrice,// strike price
            int steps,
            int sims,
            int g_id)
{
#pragma HLS INTERFACE m_axi port=pCall bundle=gmem
#pragma HLS INTERFACE s_axilite port=pCall bundle=control
#pragma HLS INTERFACE m_axi port=pPut bundle=gmem
#pragma HLS INTERFACE s_axilite port=pPut bundle=control
#pragma HLS INTERFACE s_axilite port=timeT bundle=control
#pragma HLS INTERFACE s_axilite port=freeRate bundle=control
#pragma HLS INTERFACE s_axilite port=volatility bundle=control
#pragma HLS INTERFACE s_axilite port=initPrice bundle=control
#pragma HLS INTERFACE s_axilite port=strikePrice bundle=control
#pragma HLS INTERFACE s_axilite port=steps bundle=control
#pragma HLS INTERFACE s_axilite port=sims bundle=control
#pragma HLS INTERFACE s_axilite port=g_id bundle=control
#pragma HLS INTERFACE s_axilite port=return bundle=control
```

The code below allocates memory for storing pCall and pPut and create write only buffer for them.

```
    cout << "All parameters are successfully generated. ";

    cout << "HOST-Info: Allocating memory for h_call    ... ";
    void *ptr=nullptr;
    if (posix_memalign(&ptr,4096,num_runs*sizeof(float))) {
        cout << endl << "HOST-Error: Out of Memory during memory allocation for h_call" << endl << endl;
        return EXIT_FAILURE;
    }
    cout << "h_call Allocated" << endl;
    h_call = reinterpret_cast<float*>(ptr);

    cout << endl;

    cout << "HOST-Info: Allocating memory for h_put    ... ";
    if (posix_memalign(&ptr,4096,num_runs*sizeof(float))) {
        cout << endl << "HOST-Error: Out of Memory during memory allocation for h_put" << endl << endl;
        return EXIT_FAILURE;
    }
    cout << "h_put Allocated" << endl;
    h_put = reinterpret_cast<float*>(ptr);
```

```
// ------------------------------------------------------------------
// Step 4.2: Create Buffers in Global Memory to store data
//              o) d_call     - stores h_call (W)
//              o) d_put      - stores h_put (W)
// ------------------------------------------------------------------
cl_mem d_call, d_put;

d_call = clCreateBuffer(Context, CL_MEM_WRITE_ONLY | CL_MEM_USE_HOST_PTR, num_runs * sizeof(float),\
                        h_call, &errCode);
if (errCode != CL_SUCCESS) {
    cout << endl << "Host-Error: Failed to allocate Global Memory for d_call" << endl << endl;
    return EXIT_FAILURE;
}

d_put = clCreateBuffer(Context, CL_MEM_WRITE_ONLY | CL_MEM_USE_HOST_PTR, num_runs * sizeof(float), \
                       h_put, &errCode);
if (errCode != CL_SUCCESS) {
    cout << endl << "Host-Error: Failed to allocate Global Memory for d_put" << endl << endl;
    return EXIT_FAILURE;
}
```

The code below prepares the rest of the parameters and set kernel arguments.

```
cout << "HOST-Info: generate parameters    ... ";
cl_float initprice = 100;            // -s
cl_float strikeprice = 110;             // -k
cl_float rate = 0.05;           // -r
cl_float volatility = 0.2;  // -v
cl_float time = 1.0;                 // -t
int num_runs=16;
cl_uint sims = 1024, steps=1024;
```

```
int Nb_Of_Mem_Events = 4,
    Nb_Of_Exe_Events = num_runs;

cl_event Mem_op_event[Nb_Of_Mem_Events],
         K_exe_event[Nb_Of_Exe_Events];

#ifdef ALL_MESSAGES
cout << endl;
cout << "HOST-Info: ============================================================= " << endl;
cout << "HOST-Info: (Step 5) Run Application                                     " << endl;
cout << "HOST-Info: ============================================================= " << endl;
#endif
```

```
// ----------------------------------------
// Step 5.1: Set Kernel Arguments
// ----------------------------------------
#ifdef ALL_MESSAGES
cout << "HOST-Info: Setting Kernel arguments ..." << endl;
#endif
errCode  = false;

errCode |= clSetKernelArg(K_blackEuro,  0, sizeof(cl_mem), &d_call);
errCode |= clSetKernelArg(K_blackEuro,  1, sizeof(cl_mem), &d_put);
errCode |= clSetKernelArg(K_blackEuro,  2, sizeof(cl_float), &time);
errCode |= clSetKernelArg(K_blackEuro,  3, sizeof(cl_float), &rate);
errCode |= clSetKernelArg(K_blackEuro,  4, sizeof(cl_float), &volatility);
errCode |= clSetKernelArg(K_blackEuro,  5, sizeof(cl_float), &initprice);
errCode |= clSetKernelArg(K_blackEuro,  6, sizeof(cl_float), &strikeprice);
errCode |= clSetKernelArg(K_blackEuro,  7, sizeof(cl_uint), &steps);
errCode |= clSetKernelArg(K_blackEuro,  8, sizeof(cl_uint), &sims);

if (errCode != CL_SUCCESS) {
    cout << endl << "Host-ERROR: Failed to set Kernel arguments from 0 to 8" << endl << endl;
    return EXIT_FAILURE;
}
```

```
errCode = clEnqueueMigrateMemObjects(Command_Queue, 1, &d_call, CL_MIGRATE_MEM_OBJECT_CONTENT_UNDEFINED, \
0, NULL, &Mem_op_event[0]);
if (errCode != CL_SUCCESS) {
    cout << endl << "Host-Error: Failed Migrate d_call without migrating content" << endl << endl;
    return EXIT_FAILURE;
}

// ------------------------------------------------------

errCode = clEnqueueMigrateMemObjects(Command_Queue, 1, &d_put, CL_MIGRATE_MEM_OBJECT_CONTENT_UNDEFINED, \
0, NULL, &Mem_op_event[1]);
if (errCode != CL_SUCCESS) {
    cout << endl << "Host-Error: Failed Migrate d_put without migrating content" << endl << endl;
    return EXIT_FAILURE;
}
```

The code below is submitting kernel for execution

```
// ----------------------------------------
// Step 5.3: Submit Kernels for Execution
// ----------------------------------------

cout << "HOST-Info: Submitting Kernel K_blackEuro ..." << endl;
errCode  = false;
for (int i=0; i < num_runs; i++) {
    errCode |= clSetKernelArg(K_blackEuro,  9, sizeof(int), &i);
    errCode |= clEnqueueTask(Command_Queue, K_blackEuro, 0, NULL, &K_exe_event[i]);
}
if (errCode != CL_SUCCESS) {
    cout << endl << "HOST-Error: Failed to submit K_blackEuro" << endl << endl;
    return EXIT_FAILURE;
}
```

<u>Kernel Function</u>

HLS allocation:

The origin pragma has wrong syntax:

```
#pragma HLS ALLOCATION instances=mul limit=1 operation
#pragma HLS ALLOCATION instances=fmul limit=1 operation
#pragma HLS ALLOCATION instances=fexp limit=1 operation
#pragma HLS ALLOCATION instances=fdiv limit=1 operation
```

```
WARNING: [HLS 207-5551] unexpected pragma argument 'instances', expects function/operation
(/users/course/2022S/HLS17000000/g110061562/final/Vitis_V3/blackEuro_kernels/src/launchSim.h:39:24)
WARNING: [HLS 207-5551] unexpected pragma argument 'instances', expects function/operation
(/users/course/2022S/HLS17000000/g110061562/final/Vitis_V3/blackEuro_kernels/src/blackEuro.cpp:57:24)
WARNING: [HLS 207-5551] unexpected pragma argument 'instances', expects function/operation
(/users/course/2022S/HLS17000000/g110061562/final/Vitis_V3/blackEuro_kernels/src/blackEuro.cpp:58:24)
WARNING: [HLS 207-5551] unexpected pragma argument 'instances', expects function/operation
(/users/course/2022S/HLS17000000/g110061562/final/Vitis_V3/blackEuro_kernels/src/blackEuro.cpp:59:24)
WARNING: [HLS 207-5551] unexpected pragma argument 'instances', expects function/operation
(/users/course/2022S/HLS17000000/g110061562/final/Vitis_V3/blackEuro_kernels/src/blackEuro.cpp:60:24)
```

The adjusted pragma:

```
#pragma HLS ALLOCATION operation instances=mul limit=1
#pragma HLS ALLOCATION operation instances=fmul limit=1
#pragma HLS ALLOCATION operation instances=fexp limit=1
#pragma HLS ALLOCATION operation instances=fdiv limit=1
```

Stream:

The origin pragma:

```
#pragma HLS STREAM variable=prices depth=NUM_SIMS dim=1
```

```
WARNING: [HLS 207-5505] the 'dim' option to 'Stream' pragma is not supported and will be ignored
(/users/course/2022S/HLS17000000/g110061562/final/Vitis_V3/blackEuro_kernels/src/blackScholes.h:49:56)
WARNING: [HLS 207-5505] the 'dim' option to 'Stream' pragma is not supported and will be ignored
(/users/course/2022S/HLS17000000/g110061562/final/Vitis_V3/blackEuro_kernels/src/launchSim.h:23:49)
WARNING: [HLS 207-5505] the 'dim' option to 'Stream' pragma is not supported and will be ignored
(/users/course/2022S/HLS17000000/g110061562/final/Vitis_V3/blackEuro_kernels/src/launchSim.h:24:49)
WARNING: [HLS 207-5505] the 'dim' option to 'Stream' pragma is not supported and will be ignored
(/users/course/2022S/HLS17000000/g110061562/final/Vitis_V3/blackEuro_kernels/src/launchSim.h:27:48)
WARNING: [HLS 207-5505] the 'dim' option to 'Stream' pragma is not supported and will be ignored
(/users/course/2022S/HLS17000000/g110061562/final/Vitis_V3/blackEuro_kernels/src/launchSim.h:28:48)
```

The adjusted pragma:

```
#pragma HLS STREAM variable=prices depth=NUM_SIMS
```

Function extract:

The origin pragma:

```
        {
#pragma HLS FUNCTION_EXTRACT
#pragma HLS DATAFLOW
                launchSimulation(call0, put0, g_id+0, bs0, sims*steps>>4, sims>>4);
                launchSimulation(call1, put1, g_id+1, bs1, sims*steps>>4, sims>>4);
                launchSimulation(call2, put2, g_id+2, bs2, sims*steps>>4, sims>>4);
                launchSimulation(call3, put3, g_id+3, bs3, sims*steps>>4, sims>>4);
                launchSimulation(call4, put4, g_id+4, bs4, sims*steps>>4, sims>>4);
                launchSimulation(call5, put5, g_id+5, bs5, sims*steps>>4, sims>>4);
                launchSimulation(call6, put6, g_id+6, bs6, sims*steps>>4, sims>>4);
                launchSimulation(call7, put7, g_id+7, bs7, sims*steps>>4, sims>>4);
                launchSimulation(call8, put8, g_id+8, bs8, sims*steps>>4, sims>>4);
                launchSimulation(call9, put9, g_id+9, bs9, sims*steps>>4, sims>>4);
                launchSimulation(call10, put10, g_id+10, bs10, sims*steps>>4, sims>>4);
                launchSimulation(call11, put11, g_id+11, bs11, sims*steps>>4, sims>>4);
                launchSimulation(call12, put12, g_id+12, bs12, sims*steps>>4, sims>>4);
                launchSimulation(call13, put13, g_id+13, bs13, sims*steps>>4, sims>>4);
                launchSimulation(call14, put14, g_id+14, bs14, sims*steps>>4, sims>>4);
                launchSimulation(call15, put15, g_id+15, bs15, sims*steps>>4, sims>>4);
        }
```

```
WARNING: [HLS 207-5566] unknown HLS pragma ignored
(/users/course/2022S/HLS17000000/g110061562/final/Vitis_V3/blackEuro_kernels/src/blackEuro.cpp:97:9)
WARNING: [HLS 207-5554] Only for-loops and functions support the dataflow
(/users/course/2022S/HLS17000000/g110061562/final/Vitis_V3/blackEuro_kernels/src/blackEuro.cpp:98:9)
```

The adjusted pragma:

```
//  {
//#pragma HLS FUNCTION_EXTRACT
//#pragma HLS DATAFLOW
        launchSimulation(call0, put0, g_id+0, bs0, sims*steps>>4, sims>>4);
        launchSimulation(call1, put1, g_id+1, bs1, sims*steps>>4, sims>>4);
        launchSimulation(call2, put2, g_id+2, bs2, sims*steps>>4, sims>>4);
        launchSimulation(call3, put3, g_id+3, bs3, sims*steps>>4, sims>>4);
        launchSimulation(call4, put4, g_id+4, bs4, sims*steps>>4, sims>>4);
        launchSimulation(call5, put5, g_id+5, bs5, sims*steps>>4, sims>>4);
        launchSimulation(call6, put6, g_id+6, bs6, sims*steps>>4, sims>>4);
        launchSimulation(call7, put7, g_id+7, bs7, sims*steps>>4, sims>>4);
        launchSimulation(call8, put8, g_id+8, bs8, sims*steps>>4, sims>>4);
        launchSimulation(call9, put9, g_id+9, bs9, sims*steps>>4, sims>>4);
        launchSimulation(call10, put10, g_id+10, bs10, sims*steps>>4, sims>>4);
        launchSimulation(call11, put11, g_id+11, bs11, sims*steps>>4, sims>>4);
        launchSimulation(call12, put12, g_id+12, bs12, sims*steps>>4, sims>>4);
        launchSimulation(call13, put13, g_id+13, bs13, sims*steps>>4, sims>>4);
        launchSimulation(call14, put14, g_id+14, bs14, sims*steps>>4, sims>>4);
        launchSimulation(call15, put15, g_id+15, bs15, sims*steps>>4, sims>>4);
//  }
```

```
    template<typename DATA_T, typename Model>
void launchSim(DATA_T &pCall, DATA_T &pPut, RNG<DATA_T> &rng0,RNG<DATA_T> &rng1, Model &m, int numR, int sims){
#pragma HLS function_instantiate variable=rng0
#pragma HLS DATAFLOW
```

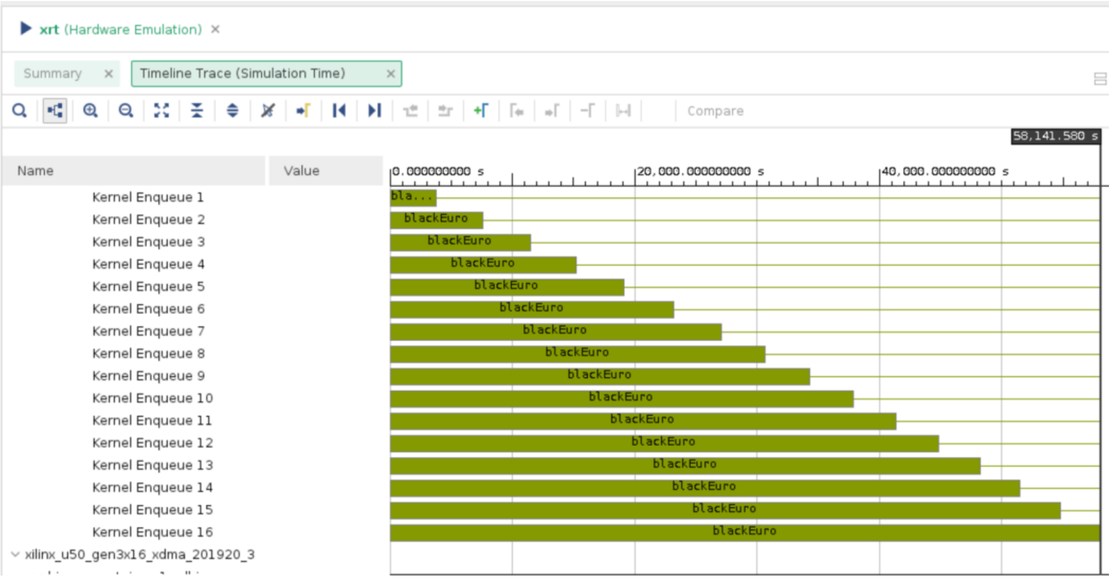
I add function_instantiate to make 16 launchSimulation run in parallel.

Results

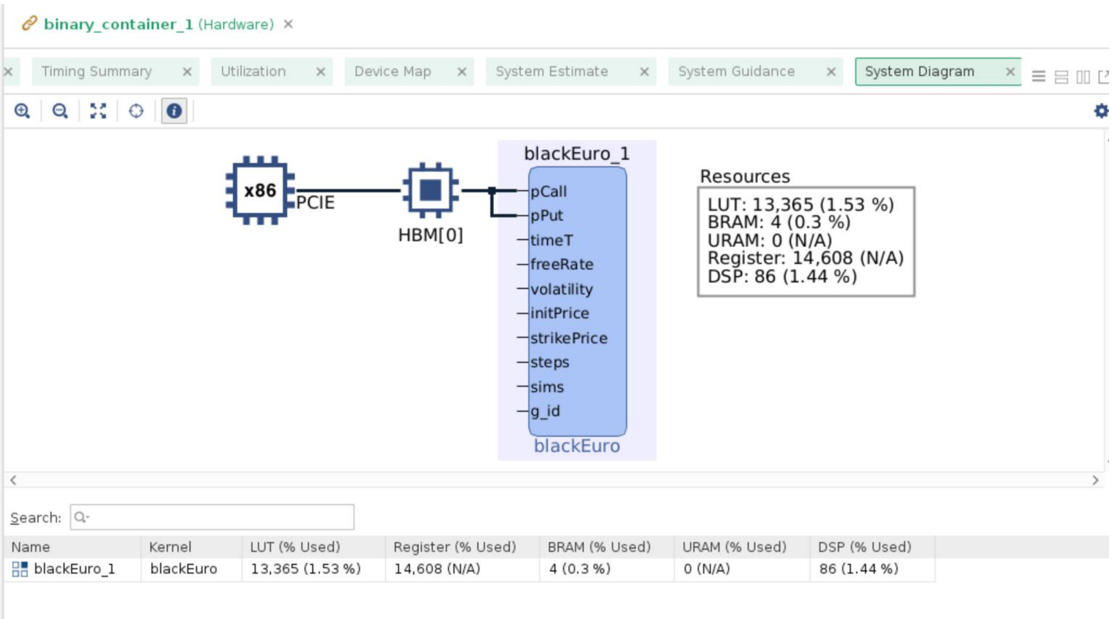
```
HOST_Info: Waiting for application to be completed ...
The 1st call price is 6.26604
The 1st put price is 10.8582
The 2nd call price is 6.4935
The 2nd put price is 10.7894
The 3th call price is 6.45818
The 3th put price is 10.7916
The 4th call price is 6.36644
The 4th put price is 11.173
The 5th call price is 6.3803
The 5th put price is 11.2333
The 6th call price is 6.42416
The 6th put price is 11.3428
The 7th call price is 6.49111
The 7th put price is 11.2489
The 8th call price is 6.46597
The 8th put price is 11.3446
The 9th call price is 6.28576
The 9th put price is 11.4276
The 10th call price is 6.26392
The 10th put price is 11.5545
The 11th call price is 1.34525e-43
The 11th put price is 1.66313e+22
The 12th call price is 6.31635
The 12th put price is 11.4937
The 13th call price is 6.40219
```

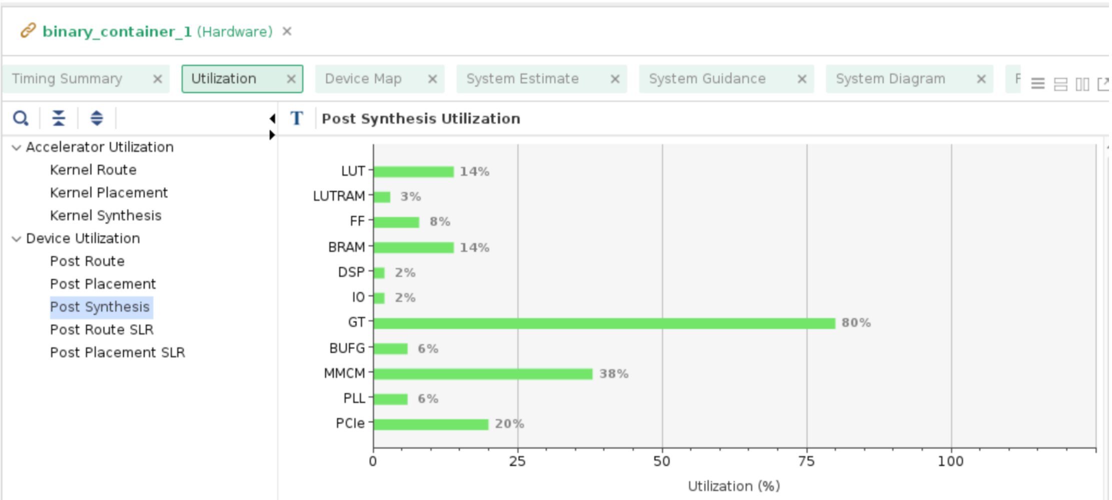
```
The 14th call price is 0
The 14th put price is 7.18483e+22
The 15th call price is 0
The 15th put price is 0
The 16th call price is 6.26251
The 16th put price is 11.8452
the sum of call price is: 82.8764
the sum of put price is: 8.84795e+22
the call price is: 4.92716      the difference with the reference value is 592.716%
the put price is: 5.26027e+21   the difference with the reference value is 1185.82%
```

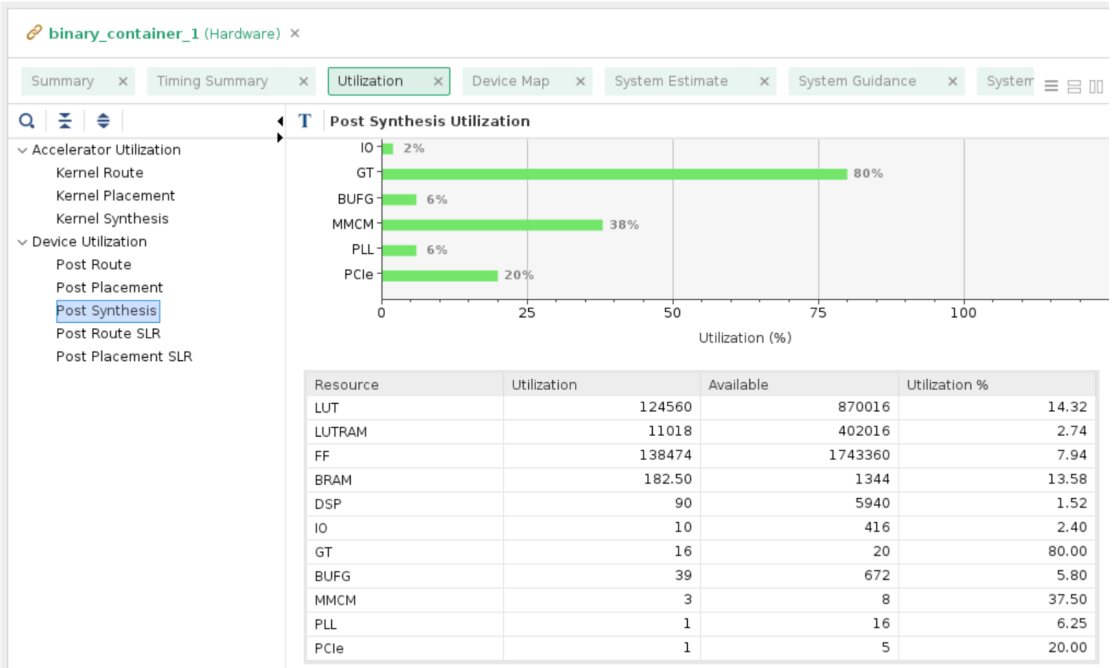
Hardware emulation timeline trace:



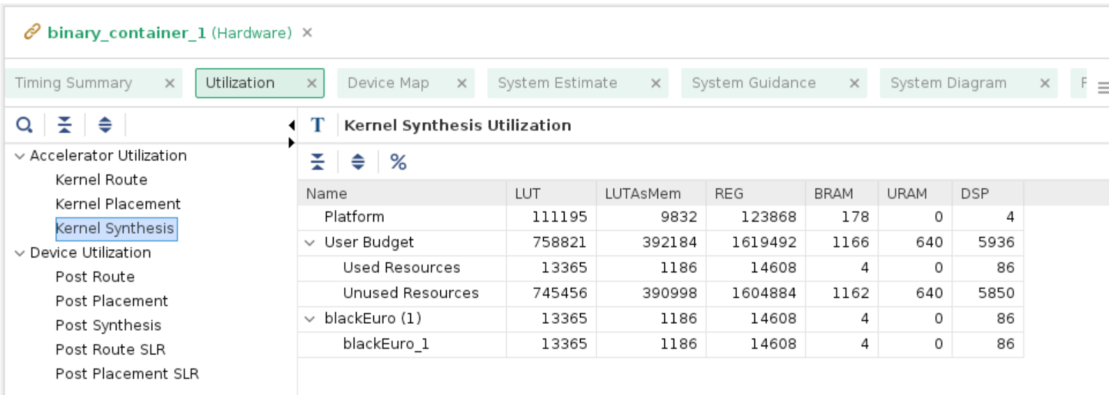
Hardware utilization:



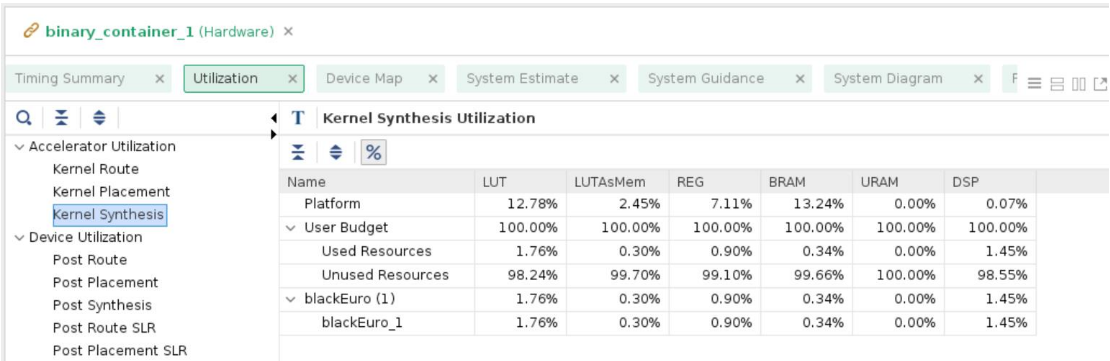




Resource	Utilization	Available	Utilization %
LUT	124560	870016	14.32
LUTRAM	11018	402016	2.74
FF	138474	1743360	7.94
BRAM	182.50	1344	13.58
DSP	90	5940	1.52
IO	10	416	2.40
GT	16	20	80.00
BUFG	39	672	5.80
MMCM	3	8	37.50
PLL	1	16	6.25
PCIe	1	5	20.00



Name	LUT	LUTAsMem	REG	BRAM	URAM	DSP
Platform	111195	9832	123868	178	0	4
User Budget	758821	392184	1619492	1166	640	5936
Used Resources	13365	1186	14608	4	0	86
Unused Resources	745456	390998	1604884	1162	640	5850
blackEuro (1)	13365	1186	14608	4	0	86
blackEuro_1	13365	1186	14608	4	0	86



Name	LUT	LUTAsMem	REG	BRAM	URAM	DSP
Platform	12.78%	2.45%	7.11%	13.24%	0.00%	0.07%
User Budget	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Used Resources	1.76%	0.30%	0.90%	0.34%	0.00%	1.45%
Unused Resources	98.24%	99.70%	99.10%	99.66%	100.00%	98.55%
blackEuro (1)	1.76%	0.30%	0.90%	0.34%	0.00%	1.45%
blackEuro_1	1.76%	0.30%	0.90%	0.34%	0.00%	1.45%

According to the results and the timeline trace above, we can observe that the utilization of hardware is very low. The kernel can only use one multiplication, division, exponent hardware. If we can remove this constrain and make 16 kernels to work in parallel. We can improve the timing. And, 16 kernels share the same buffer to store the result. Each kernel should wait for the previous kernels complete their writing. If we can make 16 buffers for each kernel, we will improve the performance further.

Reference

L. Ma, F. B. Muslim and L. Lavagno, "High Performance and Low Power Monte Carlo Methods to Option Pricing Models via High Level Design and Synthesis," 2016 European Modelling Symposium (EMS), 2016, pp. 157-162, doi: 10.1109/EMS.2016.036.

https://docs.xilinx.com/r/en-US/ug1399-vitis-hls/pragma-HLS-inline

https://docs.xilinx.com/r/en-US/ug1399-vitis-hls/pragma-HLS-allocation

https://www.boledu.org/textbooks/hls-textbook/io-interface/axi-lite

https://github.com/dr-liangma/FinancialModels_AmazonF1