Physics 174 - Spring 2021: Homework 1

Due: Monday, March 29, 2:30pm

Question 1: In class we wrote the Navier-Stokes equation as:

$$\rho\vec{g} - \vec{\nabla}P + \text{ add'n driving terms } = \rho(\vec{u}\cdot\vec{\nabla})\vec{u} + \eta\nabla^2\vec{u}$$

where the final term $\vec{F}_{drag} = \eta\nabla^2\vec{u}$ represents the viscous drag.

When we talk about an organism swimming, the driving force comes from the organism's self-propulsion. Pressure gradients are generally not relevant and the gravitational force is largely cancelled out by buoyancy. For flow at low Reynold's number, we can get a pretty good estimate of $\vec{F}_{drag}$ from dimensional analysis. At low Re the inertial term is not significant, meaning that ρ does not play a significant role in the equation of motion. The physics of the problem is therefore captured by our characteristic velocity, u; our characteristic length scale, L; and the viscosity, η.

a) using dimensional analysis, what is the drag force (defined up to a constant)

b) For a bacterium moving through water what would you use for the characteristic length scale and velocity. Numbers are not expected, just a connection between the equations and the motion/physical properties of the bacterium (which you can assume is approximately spherical). This part of the question is only here to set you up for question 2, it should be a very straightforward response so please don't spend time overthinking it.

Question 2: A familiar experience from swimming at finite Reynolds' numbers is the ability to coast. We can push off from the side of the pool and travel a finite distance without having to expend additional energy. Here we will explore what "coasting" is like at very low Reynolds' number.
The drag force for a sphere at low Reynolds' number was calculated by Stokes and is given by $F_D = -6\pi\eta a\vec{u}$, where a is the diameter of the sphere and $\vec{u}$ is the velocity of the sphere relative to the fluid. You can find a derivation of Stokes formula for the drag force here (You do not need to follow this link it is just for enrichment if interested): https://www.math.nyu.edu/faculty/childres/chpseven.PDF

Bacteria have flagella that they use to propel themselves. Imagine a bacterium is moving at velocity, $\vec{u}$ when it stops propelling itself.

Approximately how far can the bacterium coast (travel without propulsion)? You can use the approximation we made in class that $u \approx 10$ BL/sec (body lengths per second) and approximate the bacterium as a sphere of 1μm diameter. (Hint: you will have to reintroduce the inertial term into the equation of motion to answer this question.)

Question 3: a) A 1 μm gold sphere and a 2 μm silver sphere sediment (fall under gravity) in a viscous fluid (you can assume it is in the low Reynolds' number regime). Which one sediments faster? You can make use of the Stokes' formula for the force due to viscous drag on a sphere, from question 2.

b) Molecular biologists routinely make use of centrifugation (spinning solutions containing protein, at rates as high as 10^5 rpm) to separate proteins and other macromolecules by size. Find an expression for the drift velocity of a protein, in a solvent of viscosity η and density, ρ undergoing centrifugation, with an angular velocity ω at a distance R from the axis of rotation. Treat the protein as a sphere of radius, R; density, ρ_P. Hint: to solve this problem you will need to set up the equation of motion equating viscous drag to a term proportional to the centriptal acceleration. It is important to note in this equation that the relevant mass that is being accelerated is not the total mass but the relative mass from the greater density of the protein to water (this is analogous to Archimedes principle).