FOCS HW2

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Problem 3.53

(a)

Domain of x should be in $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ For example, x=2, x^2=4.
2 is $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ at the same time.

(b)

Domain of x should be in $\mathbb{R}$
For example, x=$\sqrt{2}$, it's not integer, natural number, or rational.

(c)

Domain of x and y both should be in $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ Because if x in $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, it will be the same for y.

(d)

Domain of x should be in $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and y should be in $\mathbb{N}$
No matter what is x in, y is not negative. So have to be in $\mathbb{N}$.

Problem 4.9

(a)

Direct Proof:
$n^3 + 5 = (n^3 + 1) + 4 = (n + 1)(n^2 - n + 1) + 4.//$ Because $n^3 + 5$ is odd, 4 is even, $(n + 1)(n^2 - n + 1)$ is odd. Only odd multiply by odd is odd. So both $(n + 1)$ and $(n^2 - n + 1)$ are odd. Since n+1 is odd, n is even.

Contraposition Proof:
Assume n is odd, which means $n = 2 \cdot k + 1$.
$n^3 = (2 \cdot k + 1)^3 = 8k^3 + 12k^2 + 6k + 1$
So, $n^3 + 5 = 8k^3 + 12k^2 + 6k + 6 = 2 \cdot (4k^3 + 6k^2 + 3k + 3)$.
As the result, $n^3 + 5$ is even(not odd) number.
This contraposition is true, so the original statement is true.

(b)

Direct Proof:
If 3 is not dividible n, then n = 3k+1 or 3k+2.
$n^2 + 2 = (3k + 1)^2 + 2$ or $(3k + 2)^2 + 2 = 9k^2 + 6k + 3$ or $9k^2 + 12k + 6$
These two can be simplify as $3(3k^2 + 2k + 1)$ and $3(3k^2 + 4k + 2)$
which are both dividible by 3. So this "if then" relationship is true.

Contraposition Proof:
Assume $n^2 + 2$ is not divisible by 3. $n^2 + 2 = (n + 1)(n - 1) + 3$ since it is not divisible by 3 and 3 is divisible by 3, (n+1)(n-1) must not be divisible by 3. Neither (n+1) nor (n-1) can be divisible by 3. Because every 3 adjacent numbers should have a number that is divisible by 3 and (n+1)(n-1) are both not. n must be divisible by 3. This contraposition is true so that the original relationship is true.

Problem 4.15

(e)

(Direct Proof)
When n is odd, $n = 2k + 1$.
$n^2 + 3n + 4 = (2k + 1)^2 + 3(2k + 1) + 4 = 4k^2 + 4k + 1 + 6k + 3 + 4$.
This can be simplify to $4k^2 + 10k + 8 = 2(2k^2 + 5k + 4)$. Which is even.
When n is even, $n = 2k$.
$n^2 + 3n + 4 = (2k)^2 + 3(2k) + 4 = 4k^2 + 6k + 4 = 2(2k^2 + 3k + 2)$.
Which is also even. Since no matter if n is even or not as long as it is a integer, $n^2 + 3n + 4$ is always even. So, the statement is true.

(w)

(Contraposition Proof)
Assume min(a,b) $\geq$ 100. The smallest value of a and b we can get is a = 100, b = 100. In this situation, a $\cdot$ b = 10000 which is not $\leq$ 10000.
This contraposition is true. So the original statement is true as well.

Problem 4.26

(b)

To prove:
Contraposition
Prove that $\forall n$: P(n) is true, Q(n) is false.

To disprove:
Direct prove
Using truth table
Prove that $\exists n$: P(n) is false or when P(n) is true, Q(n) is true.

(d)

To prove:
Induction
Prove that $\forall n$: P(n) is false or $\forall n$: P(n) is true and $\forall x$: Q(x) is true.

To disprove:
Show for counter example
Prove that $\exists n$: P(n) is true and $\exists x$: Q(x) is false.

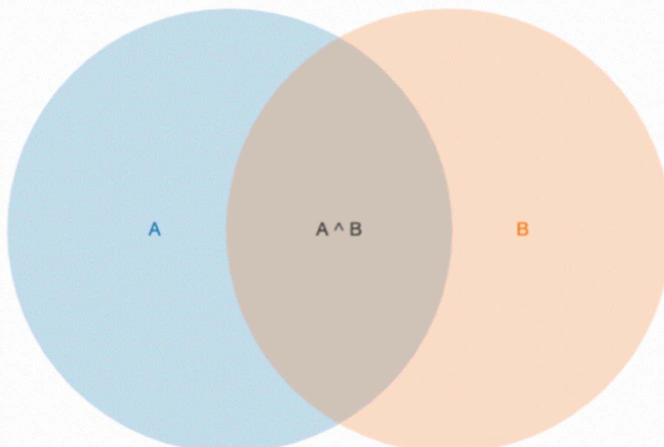
(f)

To prove:
Show for counter example
Prove that $\exists n$: P(n) is false or $\exists n$: P(n) is true and $\exists x$: Q(x) is true.

To disprove:
Show for general object
Prove that $\forall n$: P(n) is true and $\forall x$: Q(x) is false.

Problem 4.36

(j)



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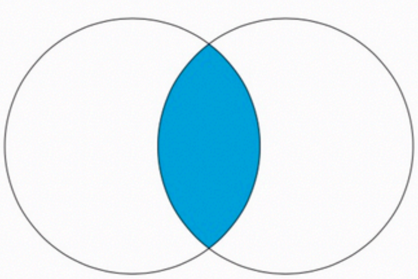

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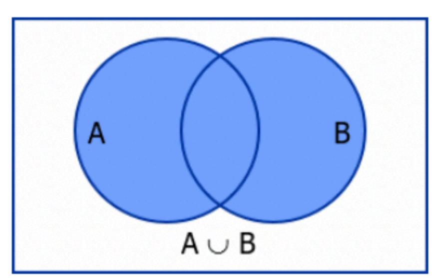

Problem 4.45

(b)

The solution is (ii).
Proof of (ii):
f(n)-1 = (n+3)/(n+1)-1 = ((n+3)-(n+1))/(n+1) = 1/(n+1)
When ε >0, n_ε=(2/ε-1)-(2/ε-1)%1+1
2/ε-1 < (2/ε-1)-(2/ε-1)
f(n_ε)-1 = 2/n_ε < 2/(2/ε-1+1) = ε
f(n_ε)-1 < n_ε, which means that the statement is true

Disproof of (i):
f(n) = (n+3)/(n+1) = (n+1+2)/(n+1) = 2/(n+1)+1 < 2/(1+1)+1 = 2
when C = 3, $\forall n \in \mathbb{N}$, f(n) < C which is not what (i) states.

Disproof of (iii):
f(n)-2 = (n+3)/(n+1)-2 = ((n+3)-(2n+2))/(n+1) = (-n+1)/(n+1)
—f(n)-2— = (n-1)/(n+1)
Let's set ε = 1/3, when n>2, (n-1)/(n+1) = 1-2/(n+1) > 1/3
This is not what (iii) states.

Problem 5.7

(f)

Let P(n): (1-1/2)(1-1/3)(1-1/4)...(1-1/n)=1/n
When n = 2, 1-1/2 = 1/2
Let's see if for n $\geq$ 2, P(n) $\rightarrow$ P(n+1)
(1-1/2)(1-1/3)(1-1/4)...(1-1/n)(1-1/(n+1))
= (1/n)(1-1/(n+1))
= (1/n)-(1/(n(n+1)))
= ((n+1)/(n(n+1)))-(1/(n(n+1)))
= n/(n(n+1))
= 1/(n+1)
We can see that by induction P(n) $\rightarrow$ P(n+1) is true when n $\geq$ 2