

**(1) [10] DMC Problem 25.8(f)**

- $S \rightarrow A0A$
- $A \rightarrow 0A1|1A0|AA|0A|\epsilon$

**(2) [20] DMC Problem 26.5(d)**

(i)

1. If read \*, move right and read  $\omega$
2. If read marked 1, halt and REJECT; jump and mark every other unmarked 0
3. Return to \* and check if the first unmarked 0 is the last digit
  - a. If is the last digit, halt and ACCEPT
  - b. If not the last digit, go back to step 2
4. If there is no unmarked 0 left, halt and REJECT

(ii)

1.  $\{q_0\}\{*\} \rightarrow \{q_1\}\{*\}\{R\}$
2.  $\{q_1\}\{1\} \rightarrow \{E\}\{1\}\{S\}$  ---- If 1, halt and REJECT  
 $\{q_1\}\{\check{0}\} \rightarrow \{q_1\}\{\check{0}\}\{R\}$  ---- Skip all marked 0's  
 $\{q_1\}\{0\} \rightarrow \{q_2\}\{\check{0}\}\{R\}$  ---- Mark every other unmarked 0, go to  $q_2$   
 $\{q_1\}\{\sqcup\} \rightarrow \{q_3\}\{\sqcup\}\{L\}$  ---- If reach the end, start over  
 $\{q_2\}\{1\} \rightarrow \{E\}\{1\}\{S\}$  ---- If 1, halt and REJECT  
 $\{q_2\}\{\check{0}\} \rightarrow \{q_2\}\{\check{0}\}\{R\}$  ---- Skip marked 0's  
 $\{q_2\}\{0\} \rightarrow \{q_1\}\{0\}\{R\}$  ---- Mark every other unmarked 0  
 $\{q_2\}\{\sqcup\} \rightarrow \{q_3\}\{\sqcup\}\{L\}$  ---- If reach the end, start over, go to  $q_3$   
 $\{q_3\}\{\check{0}\} \rightarrow \{q_3\}\{\check{0}\}\{L\}$  ---- Return to \*  
 $\{q_3\}\{0\} \rightarrow \{q_3\}\{0\}\{L\}$  ---- Return to \*  
 $\{q_3\}\{*\} \rightarrow \{q_4\}\{*\}\{R\}$  ---- Next stage: whether to halt
3.  $\{q_4\}\{\check{0}\} \rightarrow \{q_4\}\{\check{0}\}\{R\}$  ---- Skip marked 0's, prepare to check REJECT  
 $\{q_4\}\{\sqcup\} \rightarrow \{E\}\{\sqcup\}\{S\}$  ---- Halt and Reject
4.  $\{q_4\}\{0\} \rightarrow \{q_5\}\{0\}\{R\}$  ---- Prepare to check to ACCEPT or to continue  
 $\{q_5\}\{\sqcup\} \rightarrow \{A\}\{\sqcup\}\{A\}$  ---- Halt and ACCEPT  
 $\{q_5\}\{\check{0}\} \rightarrow \{q_6\}\{\check{0}\}\{L\}$  ---- Prepare to continue (return \* in the next state)

$\{q_5\}\{0\} \rightarrow \{q_6\}\{0\}\{L\}$  ---- Prepare to continue (return \* in the next state)

$\{q_5\}\{\checkmark 0\} \rightarrow \{q_6\}\{\checkmark 0\}\{L\}$  ---- Return to \*

$\{q_6\}\{0\} \rightarrow \{q_6\}\{0\}\{L\}$  ---- Return to \*

$\{q_6\}\{*\} \rightarrow \{q_0\}\{*\}\{R\}$  ---- Continue

(iii)

$\{q_0\}\{*\} \rightarrow \{q_1\}\{*\}\{R\}$

$\{q_1\}\{1\} \rightarrow \{E\}\{1\}\{S\}$

$\{q_1\}\{\checkmark 0\} \rightarrow \{q_1\}\{\checkmark 0\}\{R\}$

$\{q_1\}\{0\} \rightarrow \{q_2\}\{\checkmark 0\}\{R\}$

$\{q_1\}\{\sqcup\} \rightarrow \{q_3\}\{\sqcup\}\{L\}$

$\{q_2\}\{1\} \rightarrow \{E\}\{1\}\{S\} \{q_2\}$

$\{\checkmark 0\} \rightarrow \{q_2\}\{\checkmark 0\}\{R\} \{q_2\}$

$\{0\} \rightarrow \{q_1\}\{0\}\{R\}$

$\{q_2\}\{\sqcup\} \rightarrow \{q_3\}\{\sqcup\}\{L\}$

$\{q_3\}\{\checkmark 0\} \rightarrow \{q_3\}\{\checkmark 0\}\{L\}$

$\{q_3\}\{0\} \rightarrow \{q_3\}\{0\}\{L\}$

$\{q_3\}\{*\} \rightarrow \{q_4\}\{*\}\{R\}$

$\{q_4\}\{\checkmark 0\} \rightarrow \{q_4\}\{\checkmark 0\}\{R\}$

$\{q_4\}\{\sqcup\} \rightarrow \{E\}\{\sqcup\}\{S\}$

$\{q_4\}\{0\} \rightarrow \{q_5\}\{0\}\{R\}$

$\{q_5\}\{\sqcup\} \rightarrow \{A\}\{\sqcup\}\{A\}$

$\{q_5\}\{\checkmark 0\} \rightarrow \{q_6\}\{\checkmark 0\}\{L\}$

$\{q_5\}\{0\} \rightarrow \{q_6\}\{0\}\{L\}$

$\{q_5\}\{\checkmark 0\} \rightarrow \{q_6\}\{\checkmark 0\}\{L\}$

$\{q_6\}\{0\} \rightarrow \{q_6\}\{0\}\{L\}$

$\{q_6\}\{*\} \rightarrow \{q_0\}\{*\}\{R\}$

**(3) [20] DMC Problem 26.8(f)**

(1) Step 0: Go to \*

(2) Step 1: remember and mark the first unmark bit we meet after \*.

- If we meet an empty place, halting accept
- If we meet an unmark bit, go to step 2

(3) Step 2: Go right until we meet an empty place. Then, go left until meeting an unmark bit.

- If this bit is as same as the bit we meet in the step 1, mark it and go back step 0
- If this bit is not as same as the bit we meet in the steep 1 or meet an empty place or meet \*, halting reject.

**(4) [10] DMC Problem 27.4(b)**

Given a program that count the number of  $n$  that satisfies both  $n$  and  $n+2$  are primes

If there is an ultimate-debugger, then the debugger would tell if the program would halt.

If halt, then finite, the conjecture is false

If cannot halt, the infinite, the conjecture is true

**(5) [10] DMC Problem 27.20**

- (a) B is decidable
- (b) Unsure
- (c) Unsure
- (d) A is decidable

**(6) [10] DMC Problem 27.45**

(a)  $3 \cdot d_1 + 4 \cdot d_2$

(b)  $d_1 \cdot 2 + d_3 + d_4 \cdot 2 + d_2 \cdot 2$

**(7) [10] DMC Problem 27.46**

Input:  $a_1b_1a_2b_2a_3b_3\dots a_nb_n$

(1) Check if  $a_i$  and  $b_i$  have the same length

(2) Return to \*

(3) Keep moving right until find the first unmarked  $a_i$ , then compare with  $b_i$ .

If move to  $\sqcup$ , ACCEPT.

If find either a or b, REJECT

If we compare successfully, mark them and return to step 2.