1. **(15 points)** Give an algebraic proof that a straight line in the world projects onto a straight line in the image. In particular

   (a) Write the parametric equation of a line in three-space.

   (b) Use the simplest form of the perspective projection camera from the start of the Lecture 5 notes to project points on the line into the image coordinate system. This will give you equations for the pixel locations x and y in terms of t. Note that t will be in the denominator.

   (c) Combine the two equations to remove t and rearrange the result to show that it is in fact a line. You should get the *implicit form* of the line.

   (d) Finally, under what circumstances is the line a point? Show this algebraically.

(a) $x(t) = x_1 t + x_0$
$y(t) = y_1 t + y_0$
$z(t) = z_1 t + z_0$

(b) $u = \frac{fx}{z} = \frac{f(x_1 t + x_0)}{z_1 t + z_0}$

$v = \frac{fy}{z} = \frac{f(y_1 t + y_0)}{z_1 t + z_0}$

(c) rearrange equation for u: $t = \frac{fx_0 - uz_0}{uz_1 - fx_1}$

rearrange equation for v: $t = \frac{fy_0 - vz_0}{vz_1 - fy_1}$

set $\frac{fx_0 - uz_0}{uz_1 - fx_1} = \frac{fy_0 - vz_0}{vz_1 - fy_1}$

we get

$(y_0 z_1 - y_1 z_0)u - (x_0 z_1 - x_1 z_0)v + f(x_0 y_1 - x_1 y_0) = 0$

(d) The line will present as a point on the image when the line go through the origin of camera coordinate. When this happens:
$x(t) = 0$
$y(t) = 0$
$z(t) = z_1 t + z_0$
For this set of 3D line equation, $u = \frac{fx}{z} = 0$ and $v = \frac{fy}{z} = 0$ which means it was projected to the origin as a point