(1) Problem 18.20(a)

The independent random variables X and Y have the same PDF, P (k) = 2−k for k = 1, 2, . . . , ∞. For m, n ∈ N, compute these probabilities:

P[min(X, Y) ≤ m]

= P[X ≤ m] + P[Y ≤ m] - P[X ≤ m, Y ≤ m]

$$= \left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots + \frac{1}{2^m}\right) + \left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots + \frac{1}{2^m}\right) - \left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots + \frac{1}{2^m}\right)^2$$

$$= \left(1 - \frac{1}{2^m}\right) + \left(1 - \frac{1}{2^m}\right) - \left(1 - \frac{1}{2^m}\right)^2$$

$$= \left(1 - \frac{1}{2^m}\right)\left(1 + \frac{1}{2^m}\right)$$

$$= 1 - \frac{1}{4^m}$$

(2) Problem 18.33

Which random variables (measurements) are Binomial?

(l) Draw 10 cards from a shuffled deck and count the number of aces.

Not Binomial.

Not independent, drawing each card changes the # of cards left and the left aces. So the possibility of drawing aces changed after drawing each card.

(m) You have 10 shuffled decks. Draw one card from each deck and count the number of aces.

Is Binomial.

In each trail, the probability of drawing an ace is 4/45.Each trail does not change the probability of the next trail since it change deck.

(o) Toss 20 fair coins and re-toss (just once) all coins which flipped H. Count the number of:
(i) Coins showing heads at the end.

Is Binomial.

In each trail, we have 4 possibilities(HH,HT,TH,TT). The possibility is always ¼. Also, Flipping different coins can be considered as different trials. The trails are independent.

(ii) Heads tossed in the experiment.

Is Binomial.

In each trail, we have 4 possibilities(HH,HT,TH,TT). The possibility is always ¼. Also, Flipping different coins can be considered as different trials. The trails are independent.

(p) Your total winnings in n fair coin flips when you win $2 per H and lose $1 per T.

Is Binomial.

In each trail, the possibility of getting H's is 1/2.
Flipping coins at different time can be considered as different trails which is independent.

(q) A box has 50 bulbs in a random order, with 5 being defective. Of the first 5 bulbs, count the number defective.

Not Binomial.

Picking a bulb out of the box changes the number of bulbs and number of defective bulbs left in the box. So the possibility of drawing defective bulbs is not constant.

(3) Problem 19.11

A game costs \$x to play. You toss 4 fair coins. If you get more heads than tails, you win \$10 + x for a profit of \$10. Otherwise, you lose and get nothing back, so your loss is \$x. What is your expected profit?

All Possibilities:

{HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}

5 wins and 11 loses

E[X]

= 5/16 * (10+x) - 11/16 * x

= 25/8 - 3/8 * x

(4) Problem 19.35

A box has 1024 fair and 1 two-headed coin. You pick a coin randomly, make 10 flips and get all H.

(a) You flip the same coin you picked 100 times. What is the expected number of H?

E[X]

$= \frac{1}{2} * P[H \mid fair] * 100 + \frac{1}{2} * P[H \mid biased] * 100$

$= \frac{1}{2} * \frac{1}{2} * 100 + \frac{1}{2} * 1 * 100$

= 75

(b) You flip the same coin you picked until you get H. What is the expected number of flips you make?

$E[X] = \frac{1}{2} * \frac{1}{P[H \mid fair]} + \frac{1}{2} * \frac{1}{P[H \mid biased]} = \frac{1}{2} * 2 + \frac{1}{2} * 1 = \frac{3}{2}$

(5) Problem 19.54

A Martian couple has children until they have 2 males (sexes of children are independent). Compute the expected number of children the couple will have if, on Mars, males are:

$E[X]$

$= p * E[X|B] + (1-p)E[X|G]$

$= p * (1 + \frac{1}{p}) + (1-p) * (1 + E[X])$

$E[X] = \frac{2}{p}$

(a) Half as likely as females.

$P[B] = \frac{1}{2}P[G]$
$P[B] + P[G] = 1$
$P[B] = \frac{1}{3}$
$E[X] = 2 * 3 = 6$

(b) Just as likely as females.

$P[B] = P[G]$
$P[B] + P[G] = 1$
$P[B] = \frac{1}{2}$
$E[X] = 2 * 2 = 4$

(c) Twice as likely as females.

$P[B] = 2P[G]$
$P[B] + P[G] = 1$
$P[B] = \frac{2}{3}$
$E[X] = 2 * \frac{3}{2} = 3$

(6) Problem 20.11

Ten sailors have a night out on shore. They return drunk and sleep in random bunks. Compute:

(a) The probability that all sailors sleep in their own bunks.

$$P[X] = \frac{1}{10!}$$

(b) The probability that 1 sailor sleeps in the wrong bunk.

This is impossible since when 1 sailor sleep in the wrong bunk, there must be another sailor also in the wrong bunk.

(c) The probability that 2 sailors sleep in the wrong bunk.

$$P[X] = \frac{10 \; pick \; 2}{10!} = \frac{45}{10!}$$

(d) The expected number of sailors that sleep in their own bunk.

$$E[X] = \sum_{i=1}^{10} E[x_i] = \sum_{i=1}^{10} \frac{1}{10} = 1$$

(7) Problem 21.37 [Bonus]

100 people toss their hats up. The hats land randomly on heads. Let the random variables X be the number of people who get their hats back.

(a) Compute E[X] and var[X]. [Hint: Let Xi = 1 if person i gets their hat back and Xi = 0 otherwise. How is X related to the Xi? Are the Xi independent?]

Probability is 1/100, So E[X] = 1

$E[X^2] - E[X]^2 = 1$

(b) Give an upper bound on the probability that more than half the people get their hats back.

More than half people get the correct hats, so $a \geq 51$

$P(X \geq a) = \frac{E[X]}{a} = \frac{1}{51}$