<u>(1) Problem 15.39(q)</u>

Give the probability space for each of these "experiments."

Each pair from {Adam, Barb, Charlie, Doris} randomly decides whether or not to be friends.

A for Adam, B for Barb, C for Charlie, D for Doris

Ω = {AB, AC, AB, BC, BD, CD}

(2) Problem 16.4

Two worlds have 1 million birds each. World 1 has 100 black ravens and the rest are other birds. World 2 has 1000 black ravens, 1 white raven and the rest are other birds. You enter a randomly picked world.

(a) What are the chances that all ravens are black in your world?

P(all reavens balck) = P(World 1) = ½

(b) You see a random bird and it's a black raven. Now, what are the chances that all ravens are black in your world?
(In this case, observing a black raven decreases your belief that "all ravens are black." See also Problem 3.55)

P[all balck raven|random black raven]

$$= \frac{P(all\ black\ raven \cap random\ black\ raven)}{P(random\ black\ raven)}$$

$$= \frac{(1/2) * \frac{100}{1\ miliion}}{(1/2)\frac{100}{1\ million} + (1/2)\frac{1000}{1\ million}}$$

$$= \frac{1}{11}$$

(3) Problem 16.37

Five out of 100 coins are two-headed. You randomly pick a coin and flip it “fairly” twice (each side is equally probable). What is the probability to get

(a) 2 heads

(5/100) + (95/100) * ½ * ½ = 28.75%

(b) 2 tails

(95/100) * ½ * ½ = 23.75%

(c) matching tosses?

P(matching tosses) = P(2 head or 2 tails) = 23.75% + 28.75% = 52.5%

(4) Problem 16.40

Baniaz has two kids. What are the chances both are girls in each of the situations below.

(a) Baniaz confirms that one of her children is a girl.

$$P[\{GG\}|\{GG,GB,BG\}] = \frac{P[\{GG\}\cap\{GG,GB,BG\}]}{P[\{GG,GB,BG\}]} = \frac{1/4}{1/4+1/4+1/4} = \frac{1}{3}$$

(b) Baniaz confirms one of her children is a girl named Leilitoon (a rare name, assuming names are randomly picked).

Assume the probability of name Leilitoon is X

$$\frac{(1/4)X^2+(1/4)X(1-X)+(1/4)X(1-X)}{(1/4)X^2+(1/4)X(1-X)+(1/4)X(1-X)+(1/4)X+(1/4)X} = \frac{2X-X^2}{4X-X^2} = \frac{1}{2}$$

(c) Baniaz confirms one of her children is a girl who was born on a Sunday.

The probability of Sunday is 1/7

The X in part b should be 1/7

$$P(\text{Sunday}) = \frac{2*1/7-(1/7)^2}{4*(1/7)-(1/7)^2} = \frac{13}{27}$$

(5) Problem 17.9

On a standard 8 × 8 chessboard (alternating black and white squares), label the rows and columns 1, . . . , 8. You pick a square at random. Are these events independent.

Definition of independent: P[A|B] = P[A]

(a) A = {white square}; B = {black square}.

P(white|balck) = 0

P(white) = ½

P(white|balck) ≠ P(white)

Not independent

(b) A = {even row}; B = {even column}.

P(even row|even col) = ½

P(even row) = ½

P(even row|even col) = P(even row)

Independent

(c) A = {white square}; B = {even column}.

P(white|even col) = ½

P(white) = ½

P(white|even col) = P(white)

Independent

(6) Problem 17.28

You toss a 100-sided die (die faces are 1, . . . , 100) 5 times. Compute the probability to roll some number more than once

P(roll number more than once)
= 1 - P(not repeated values for 5 times)
= 1 - $(99/100)^4(98/100)^3(97/100)^2(96/100)^1$
= 9.65 %