Yifan Wang

(1) Problem 13.8

A word is a 5-letter string using the characters a, b, c, . . . , z. How many words

(a) In all: $26*26*26*26*26 = 25^5$
(b) No repeated: $26*25*24*23*22$
(c) Begin with abc: $26*26 = 26^2$
(d) Begin with abc or end with xyz: $26*26+26*26 = 2*26^2$
(e) Begin with abc or end with cde: $26*26+26*26-1 = 2*26^2-1$

(2) Problem 13.44

There are 10 ice-cream sundae toppings from which you select 4. How many sundaes are possible if:

(a) You do not repeat a topping and the order in which the toppings are added does not matter to you?

$$\binom{10}{4} = 10!/(6!4!) = 210$$

(b) You do not repeat a topping and the order in which the toppings are added matters to you?

$$10*9*8*7 = 5040$$

(c) You may repeat toppings and the order in which the toppings are added does not matter to you?

$$\binom{10+4-1}{10-1} = 13!/(9!4!) = 715$$

(d) You may repeat toppings and the order in which the toppings are added matters to you?

$$10*10*10*10 = 10^4$$

<u>(3) Problem 13.50</u>

How many integer solutions are there to $x1 + x2 + x3 + x4 = 10$ if

(a) xi are positive?

It is same as inserting delimiters between 1111111111=10

$$\binom{9}{3} = 9!/(6!3!) = 84$$

(b) xi are non-negative?

It is same as inserting delimiters between 11111111111111=14

$$\binom{13}{3} = 13!/(10!3!) = 286$$

(c) $x1 \geq -3$, $x2 \geq -2$, $x3 \geq 1$, $x4 \geq 2$?

It is same as inserting delimiters between 1111111111111111=16

$$\binom{15}{3} = 15!/(12!3!) = 455$$

(4) Problem 13.51(a)

How many 20-bit binary strings contain 00. [Hint: Count strings not containing 00.]

Not containing 00

1 digit (2 cases): 0,1

2 digits (3 cases): 01,10,11

3 digits (5 cases): 010,011,101,110, 111

4 digits (8 cases): 0101,0110,0111,1010,1011,1101,1110,1111

Guess: the number of cases is a fibonacci sequence

Prove:

when counting cases for n+1 digits

We can add 1 to every binary string with n digits

We can also add 0 to every binary string that ends with 1

For binary with n digits that ends with 1, they are formed by adding 1 to the n-1 digit binary

So, #of n+1 digits binary strings

= #of n digit binary add 1 after + #of n digits binary string that can only add 0

= #of n digit binary strings + #of n-1 digit binary strings

It's a fibonacci sequence start with 2,3,5,8

By the fibonacci sequence, 20th fib number above is 17711

# of 20 digits binary strings don't contain 00 is 17711

# of 20 digits binary strings that contain 00 is 2^{20}-17711 = 1030865

(5) Problem 13.61

What are the coefficients of x^3, x^4, x^5, x^6, x^7 in the expansion of $(\sqrt{x} + 2x)^{10}$?

$= \binom{10}{0}[sqrt(x)]^{10}(2x)^0+\binom{10}{1}[sqrt(x)]^9(2x)^1+\binom{10}{2}[sqrt(x)]^8(2x)^2+...$

$...+\binom{10}{10}[sqrt(x)]^0(2x)^{10}$

x^3: not in the sequence, coeff = 0

x^4: not in the sequence, coeff = 0

x^5: in the first term of the sequence. coeff = $\binom{10}{0}$ * 1 = 1

x^6: in the third term, coeff = $\binom{10}{2}$ * 4 = 180

x^7: in the fifth term, coeff = $\binom{10}{4}$ * 16 = 3360

(6) Problem 14.5

In each case, determine the number of ways.

(a) 10 identical candies must be distributed among 4 children.

$\binom{13}{3} = 13!/(10!3!) = 286$

(b) A 15-letter sequences must be made up of 5 A's, 5B's and 5C's.

$\binom{15}{5} * \binom{10}{5} * \binom{5}{5}= 3303*252*1 = 756756$

(c) 10 identical rings must be placed on your 10 fingers.

$\binom{19}{9} = 19!/(10!9!) = 92378$

(d) 3 red, 3 green and 3 blue flags are to be arranged along the street for the parade.

$\binom{9}{3} * \binom{6}{3} * \binom{3}{3} = 84*20*1 = 1680$

<u>(7) Problem 14.14</u>

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Consider the binary strings consisting of 10 bits.

(a) How many contain fewer 1's than 0's?

$\frac{1}{2}\left(2^{10}-\binom{10}{5}\right) = 386$

(b) How many contain 5 or more consecutive 1's?

Every case has consecutive 1's in the middle, begin, end. Add up

have 5: $2^3*C(4,1)+2^4+2^4 = 64$

have 6: $2^2*C(3,1)+2^3+2^3 = 28$

have 7: $2^1*C(2,1)+2^2+2^2 = 12$

have 8: $1+2+2 = 5$

have 9: $1+1 = 2$

have 10: 1

<u>total = 64 + 28 + 12 + 5 + 2 + 1 = 112</u>

(c) How many contain 5 or more consecutive 0's?

Same as in (b) just change 1 to 0. So the answer is 112

(d) How many contain 5 or more consecutive 0's or 5 or more consecutive 1's?

The union of these two cases are 0000011111 and 1111100000

Result = 112+112-2 = 222