Yifan Wang, FOCS HW5, Oct 14 2019

DMC Problem 10.10

Use Euclid's GCD algorithm to compute gcd(356250895, 802137245) and express the GCD as an integer linear combination of the two numbers.

therom: gcd(m,n) = gcd(rem(n,m), m)

gcd(356250895, 802137245) let 356250895 be m, 802137245 be n.

=gcd(89635455, 356250895) 89635455=802137245-2*356250895=n-2m

=gcd(87344530,89635455) 87344530=356250895-3*89635455

=m-3n+6m=7m-3n

=gcd(2290925,87344530) 2290925=89635455-87344530

=n-2m-7m+3n=4n-9m

=gcd(289380,2290925) 289380=87344530-38*2290925

=7m-3n-152n+342m=349m-155n

=gcd(265265,289380) 265265=2290925-7*289380

=1089n-2452m

=gcd(24115,265265) 24115=289380-265265

=2801m-1244n

 $=\gcd(0,24115)$

According to the Euclid's Algorithm, the GCD is 24,115

Linear combination:

gcd(356250895, 802137245) = 2801*356250895-1244*802137245

DMC Problem 10.27(a)



gcd(6,15) = 3

(i)

(0,15)-(6,9)-(0,9)-(6,3)

(ii)

6 and 15 can only form result of their gcd which is 3, 4 is not multiple of 3, so not possible to measure.

(iii)

6 and 15 can only form result of their gcd which is 3, 5 is not multiple of 3, so not possible to measure.

DMC Problem 10.40(c)

Prove that $2^{70} + 3^{70}$ is divisible by 13.

$$2^2 \equiv -(3)^2 \pmod{13}$$

$$(2^2)^{35} \equiv (-(3)^2)^{35} \pmod{13}$$

$$2^{70} \equiv -3^{70} \pmod{13}$$

$$2^{70} + 3^{70} \equiv 0 \pmod{13}$$

same mod as 0, which means that it is divisible

DMC Problem 11.6

Give a graph satisfying the constraints or explain why it doesn't exist.

(a) The graph has 5 vertices each of degree 3.

Doesn't exist.

According to handshaking therom, total degrees must be 2 times of edges.

However, in this case, total degrees is 5*3=15 which is even number not 2's multiple.

(b) The graph has 4 vertices of degrees 1,2,3,4.

Doesn't exist.

4 vertices existing, the maximum degrees for a single vertives is 3. No way it can have degree of 4.

(c) The graph has 4 edges and vertices of degrees 1,2,3,4.

Doesn't exist.

By handshaking therom, total degrees must be 2 times of the number of edges.

In this case, the number of edges is 4. Total degrees should be 2*4=8.

The case has degrees of 10.

(d) The graph has 6 vertices of degrees 1,2,3,4,5,5.

Doesn't exist.

Since there are two 5 degrees and 6 vertices, there must be two vertices connecting all other 5 vertices. As the result, there's no way there existing a vertex has degree of 1, at least it should be 2.

DMC Problem 11.40

We show the 10 dominos using pairs of numbers in {0, 1, 2, 3} (0 is blank). We placed some of the dominoes in a ring so that touching dominos meet at the same number. The ring does not include all the 10 dominos.

(a) Can you place all the dominos in a ring?

It is impossible.

(b) How many dominos are there for [0, ..., n]?

Every number has n+1 matching domino. Thus, for a sequence of number 0 to n. We n+1 matching time for number 0 and 1 matching time for the last number. n+1 terms of element.

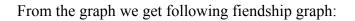
$$\frac{1}{2}(n+1+1)(n+1) = \frac{1}{2} n^2 + \frac{3}{2} n + n$$

(c) For which n can you place all the dominos in a ring?[Hints: Make each number a vertex]

To place a ring, we need each number repeat even times because each number need to have a right align same number and a left align same number. When having odd times, there are n+2 repeating times in the combination of domino cards which is still an odd number. So, when n is an even number, the domino is going to place a ring.

DMC Problem 12.73(l)

We show a friendship network. The vertices are people and the edges are the friendship links. Can the people be seated at a round table:



 $A=\{F,B,D,G\}$

 $B=\{A,D,G\}$

 $C=\{F,H\}$

 $D=\{A,B,I,H,G,F\}$

 $E=\{I,H,F\}$

 $F=\{A,C,D,E\}$

 $G=\{A,B,D,H\}$

 $H=\{C,D,E,G\}$

 $I=\{D,E\}$

(i) Harmoniously, so that every person has a friend to their left and their right?

A-G-H-C-F-E-I-D-B-A

(ii) Sadistically, so that every person has an enemy to their left and their right?

D-C-G-F-H-B-I-A-E-D