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DMC Problem 10.10

Use Euclid's GCD algorithm to compute gcd(356250895, 802137245) and express the GCD as an integer linear combination of the two numbers.

therom: gcd(m,n) = gcd(rem(n,m), m)

gcd(356250895, 802137245)	let 356250895 be m, 802137245 be n.
=gcd(89635455, 356250895)	89635455=802137245-2*356250895=n-2m
=gcd(87344530,89635455)	87344530=356250895-3*89635455 =m-3n+6m=7m-3n
=gcd(2290925,87344530)	2290925=89635455-87344530 =n-2m-7m+3n=4n-9m
=gcd(289380,2290925)	289380=87344530-38*2290925 =7m-3n-152n+342m=349m-155n
=gcd(265265,289380)	265265=2290925-7*289380 =1089n-2452m
=gcd(24115,265265)	24115=289380-265265 =2801m-1244n
=gcd(0,24115)	

According to the Euclid's Algorithm, the GCD is <u>24,115</u>

Linear combination:

gcd(356250895, 802137245) = 2801*356250895-1244*802137245

DMC Problem 10.27(a)

Using 6 and 15 gallon jugs, measure (i) 3 gallons (ii) 4 gallons (iii) 5 gallons.

gcd(6,15) = 3

(i)

(0,15)-(6,9)-(0,9)-(6,3)

(ii)

6 and 15 can only form result of their gcd which is 3, 4 is not multiple of 3, so not possible to measure.

(iii)

6 and 15 can only form result of their gcd which is 3, 5 is not multiple of 3, so not possible to measure.

DMC Problem 10.40(c)

Prove that $2^{70} + 3^{70}$ is divisible by 13.

$2^2 \equiv -(3)^2 \pmod{13}$

$(2^2)^{35} \equiv (-(3)^2)^{35} \pmod{13}$

$2^{70} \equiv -3^{70} \pmod{13}$

$2^{70}+3^{70} \equiv 0 \pmod{13}$

same mod as 0, which means that it is divisible

DMC Problem 11.6

Give a graph satisfying the constraints or explain why it doesn’t exist.

(a) The graph has 5 vertices each of degree 3.

Doesn’t exist.

According to handshaking therom, total degrees must be 2 times of edges.

However, in this case, total degrees is 5*3=15 which is even number not 2’s multiple.

(b) The graph has 4 vertices of degrees 1,2,3,4.

Doesn’t exist.

4 vertices existing, the maximum degrees for a single vertives is 3. No way it can have degree of 4.

(c) The graph has 4 edges and vertices of degrees 1,2,3,4.

Doesn’t exist.

By handshaking therom, total degrees must be 2 times of the number of edges.

In this case, the number of edges is 4. Total degrees should be 2*4=8.

The case has degrees of 10.

(d) The graph has 6 vertices of degrees 1,2,3,4,5,5.

Doesn’t exist.

Since there are two 5 degrees and 6 vertices, there must be two vertices connecting all other 5 vertices. As the result, there’s no way there existing a vertex has degree of 1, at least it should be 2.

DMC Problem 11.40

We show the 10 dominos using pairs of numbers in {0, 1, 2, 3} (0 is blank). We placed some of the dominoes in a ring so that touching dominos meet at the same number. The ring does not include all the 10 dominos.

(a) Can you place all the dominos in a ring?

It is impossible.

(b) How many dominos are there for [0, . . . , n]?

Every number has n+1 matching domino. Thus, for a sequence of number 0 to n. We n+1 matching time for number 0 and 1 matching time for the last number. n+1 terms of element.

$\frac{1}{2}(n+1+1)(n+1) = \frac{1}{2} n^2+3/2\ n+n$

(c) For which n can you place all the dominos in a ring?[Hints: Make each number a vertex]

To place a ring, we need each number repeat even times because each number need to have a right align same number and a left align same number. When having odd times, there are n+2 repeating times in the combination of domino cards which is still an odd number. So, when n is an even number, the domino is going to place a ring.

DMC Problem 12.73(l)

We show a friendship network. The vertices are people and the edges are the friendship links. Can the people be seated at a round table:

From the graph we get following fiendship graph:

A={F,B,D,G}

B={A,D,G}

C={F,H}

D={A,B,I,H,G,F}

E={I,H,F}

F={A,C,D,E}

G={A,B,D,H}

H={C,D,E,G}

I={D,E}

(i) Harmoniously, so that every person has a friend to their left and their right?

A-G-H-C-F-E-I-D-B-A

(ii) Sadistically, so that every person has an enemy to their left and their right?

D-C-G-F-H-B-I-A-E-D