

**Problem 1**

1.  $\{ x \text{ is even } \&\& y == x + 1 \}$
2.  $\{ 1 \leq x \leq 3 \}$
3.  $\{ x > 0 \&\& y > 0 \}$
4.  $\{ x \text{ is divisible by } 50 \}$
5. None
6.  $\text{abs}(\text{result} * \text{result} - x) \leq 0.000001$

## **Problem 2**

1. Valid
2. Valid
3. Invalid;

After adding 1 to i and subtracting 1 from j, i+j should stay the same.

So, the postcondition should be  $\{i+j \neq 0\}$

4. Invalid;

the postcondition should be  $\{ m == x \ \&\& \ x > y \} \parallel \{ m == y \ \&\& \ y \leq x \}$

**Problem 3**

1. Valid
2. Invalid

#### **Problem 4**

1.  $\{ x > 0 \}$

$x = 10;$

$\{ x == 10 \}$

$y = 20 - x;$

$\{ x == 10 \ \&\& \ y == 10 \}$

$z = y + 4;$

$\{ x == 10 \ \&\& \ y == 10 \ \&\& \ z == 14 \}$

$y = 0;$

$\{ x == 10 \ \&\& \ y == 0 \ \&\& \ z == 14 \}$

2.  $\{ |x| > 11 \}$

$x = -x;$

$\{ x > 11 \ \parallel \ x < -11 \}$

$x = x * x;$

$\{ x > 121 \}$

$x = x + 1;$

$\{ x > 122 \}$

3.  $\{ |x| < 5 \}$

if ( $x > 0$ ) {

$\{ 0 < x < 5 \}$

$y = x + 2;$

$\{ 2 < y < 7 \}$

} else {

$\{ -5 < x \leq 0 \}$

$y = x - 1;$

$\{ -6 < y \leq -1 \}$

}

$\{ (0 < x < 5 \ \&\& \ 2 < y < 7) \ \parallel \ (-5 < x \leq 0 \ \&\& \ -6 < y \leq -1) \}$

### **Problem 5**

1.  $\{ \text{wp}(y > -2 * x, x = -5) = (y > 10) \}$   
 $x = -5;$   
 $\{ \text{wp}(z = 2 * x + y, z > 0) = (2 * x + y > 0) = (y > -2 * x) \}$   
 $z = 2 * x + y;$   
 $\{ z > 0 \}$
  
2.  $\{ \text{wp}(\text{if}(x > 0) x = x + 6; \text{ else, } x = 4 - x)$   
 $= ((x > 1 \ \&\& \ x > 0) \parallel (x < -3 \ \&\& \ x \leq 0)) \}$   
 $= (x > 1 \parallel x < -3)$   
 $\text{if } (x > 0) \{$   
 $\quad \{ \text{wp}(x = x + 6, x > 7) = (x > 1) \}$   
 $\quad x = x + 6;$   
 $\text{else } \{$   
 $\quad \{ \text{wp}(x = 4 - x, x > 7) = (x < -3) \}$   
 $\quad x = 4 - x;$   
 $\}$   
 $\{ x > 7 \}$
  
3.  $\{ \text{wp}(\text{if } (x > 4), x = x - 3; \text{ else, } (\text{if } (x < -4), x = x + 3; \text{ else, } x = x + 1))$   
 $= ((x > 3 \ \&\& \ x > 4) \parallel (x > -1 \ \&\& \ -4 \leq x \leq 4)) \}$   
 $= (x > 4) \parallel (-1 < x \leq 4)$   
 $\text{if } (x > 4) \{$   
 $\quad \{ \text{wp}(x = x - 3, x > 0) = (x > 3) \}$   
 $\quad x = x - 3;$   
 $\}$  else {  
 $\{ \text{wp}(\text{if}(x < -4), x = x + 3; \text{ else, } x = x + 1) = (x > -3 \ \&\& \ x < -4) \parallel (x > -1 \ \&\& \ x \geq -4)$   
 $= ([\text{false}] \parallel (x > -1 \ \&\& \ x \geq -4))$   
 $= (x > -1 \ \&\& \ x \geq -4) \}$   
 $\text{if } (x < -4) \{$   
 $\quad \{ \text{wp}(x = x + 3, x > 0) = (x > -3) \}$   
 $\quad x = x + 3;$

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    } else {
        { wp( $x = x + 1, x > 0$ ) = ( $x > -1$ ) }
         $x = x + 1$ ;
    }
}
{  $x > 0$  }

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4. { wp( $x = y + 2, x > 2 * y - 1$ ) = ( $y + 2 > 2 * y - 1$ ) = ( $y < 3$ ) }  
 $x = y + 2$ ;  
 { wp( $z = x + 1, z > 2 * y$ ) = ( $x + 1 > 2 * y$ ) = ( $x > 2 * y - 1$ ) }  
 $z = x + 1$ ;  
 {  $z > 2 * y$  }

5. { wp(if ( $x \geq 0$ )  $z = x$ ; else,  $z = x + 1$ )  
 = ( $(x \neq 0 \ \&\& \ x \geq 0) \parallel (x \neq -1 \ \&\& \ x < 0)$ )  
 = ( $x > 0 \parallel x < -1$ ) }  
 if ( $x \geq 0$ )  
 { wp( $z = x, z \neq 0$ ) = ( $x \neq 0$ ) }  
 $z = x$ ;  
 else  
 { wp( $z = x + 1, z \neq 0$ ) = ( $x \neq -1$ ) }  
 $z = x + 1$ ;  
 {  $z \neq 0$  }

### **Problem 6**

1.      $\{ x < 2 \}$   
           $\{wp(z=x-1, z<0) = (x<1)\}$   
           $z = x - 1;$   
           $\{wp(w=-z, w>0) = (z<0)\}$   
           $w = -z;$   
           $\{wp(w=w+1, w>1) = (w>0)\}$   
           $w = w + 1;$   
           $\{w > 1\}$

Sufficient or Insufficient: **Insufficient**, because the  $x<1$  is stronger than  $x<2$ . So, the precondition given cannot guarantee the postcondition.

2.      $\{ (x == y \ \&\& \ y > 0) \parallel (y \neq x) \}$   
           $\{wp(\text{if}(x==y), x=-1; \text{else}, x=y-1)$   
           $= ((x==y \ \&\& \ y > -1) \parallel (x \neq y \ \&\& \ [\text{True}]))$   
           $= ((x==y \ \&\& \ y > -1) \parallel (x \neq y))\}$   
           $\text{if } (x == y)$   
               $\{wp(x=-1, x<y) = (y>-1)\}$   
               $x = -1;$   
           $\text{else}$   
               $\{wp(x=y-1, x<y) = (y-1<y) = (-1<0) = [\text{True}]\}$   
               $x = y - 1;$   
           $\{ x < y \}$

Sufficient or Insufficient: **Sufficient**