CS2500 Project 1
Merge Sort vs. Insertion Sort

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1 Motivation

Sorting is used very often in programs and being able to sort things quickly when needed is extremely important. There is no best sorting algorithm as certain algorithms are better in certain situations. Being able to analyse different algorithms to find the best algorithm for a given situation can be very useful.

This report will implement two algorithms in python known as merge sort and insertion sort. The algorithms will then be compared and contrasted by analysing their worst case, best case, and average case run times on varying data sets.

2 Background

Insertion sort is known to have a asymptotic complexity of $\Theta(n^2)$ while merge sort has a asymptotic complexity of $\Theta(n\lg n)$. Asymptotic complexity is the run time of an algorithm given a problem size n. As n grows increasingly large, an algorithm's run time will increase according to the algorithm's $\Theta()$.

$\Theta()$ is not the only measurement of asymptotic complexity. $\Omega()$ is used to measure an algoritm's best-case run time. Merge sort has a best-case complexity of $\Omega(n\lg n)$ while insertion sort's is $\Omega(n)$. Meaning that as n grows increasingly large, an algorithm's run time will perform at least as well as its $\Omega()$ times some constant.

Also, $O()$ is used to measure an algorithm's worst-case run time. Insertion sort is known to have a worst-case complexity of $O(n^2)$ while merge sort has a worst-case complexity of $O(n\lg n)$. Meaning that as n grows increasingly large, an algorithm's run time will perform worse then its $\Omega()$ times some constant.

Merge Sort
Merge sort is a sorting algorithm that sorts by continually splitting the set in half until all elements are in an set of their own, known as divide and conquer. It then merges the smaller sets by picking the smallest elements from each set, combining until the entire set is sorted.

Merge sort is often implemented using a technique called recursion. Recursion is the act of calling a function within the function its self on a smaller subset of the problem. Recursion breaks the problem into smaller problems until it can solve the smallest problem called the base case. It then combines the solutions to these smaller problems into the final solution.

Insertion Sort
Insertion sort is a sorting algorithm that iteratively sorts by swapping pairs of elements until the set is sorted. Insertion sort starts at the second element and compares it to the element to its left, if that element is greater it swaps the two elements and moves on to the next element, comparing it to all elements to its left until it finds one that is not greater than its self. Insertion sort is very similar to how you would sort a hand of cards.

3 Procedures

1. Develop a precondition, postcondition, and loop invariants for merge sort and insertion sort.
2. Show that the previous preconditions, postconditions, and loop invariants are correct.
3. Express merge sort and insertion sort using pseudocode.
4. Implement merge sort and insertion sort in python.
5. Implement preconditions, postconditions, and invariants using python assert statements to validate correctness.
6. Measure run times of the two algorithms to experimentally determine their run time complexity and compare to their expected run time complexity.
7. List problems encountered during development.
8. Develop and implement a testing plan.
9. Produce a conclusion addressing the efficacy of the methods used.

4 Pseudocode and pre/postconditions

Insertion Sort Pseudocode

```
INSERTION-SORT(A)
1  for j = 2 to A.length
2    key = A[j]
3
4    i = j - 1
5    while i>0 and A[i] > key
6        A[i+1] = A[i]
7        i = i - 1
8    A[i+1] = key
9
10 return A
```

Loop Invariant - Line 1
At the start of each iteration of the for loop on line 1 the subarray A[1..j-1] consists of the elements originally in A[1..j-1] but in sorted order.

Initialization
Before the for loop j=2 so the subarray A[1..j-1] consists of one element. This one element is sorted so the invariant holds.

Maintenance
The while loop moves A[j] around in the array until it is in sorted order. j is then incremented so the subarray A[1..j] consists of the elements originally in A[1..j] but in sorted order.

Termination
The condition causing the for loop to terminate is j > A.length. Because j increments by one each iteration we will have j = A.length + 1. Substituting A.length + 1 for j in the loop invariant gives us a sorted subarray of A[1..A.length] that consists of the elements originally in A[1..A.length] but in sorted order.

Merge Sort Pseudocode

```
MERGE-SORT(A)
if A.length <= 1
    return A
else
    m = A.length/2
    L = MERGE-SORT(A[1..m])
    R = MERGE-SORT(A[m+1..A.length])
    return MERGE(L, R)
```

```
MERGE(L, R)
1  S = []
2
3  while L.length > 0 and R.length > 0
4      if L[1] < R[1]
5          S = S + L[1]
6          L = L[2..L.length]
7      else
8          S = S + R[1]
9          R = R[2..R.length]
10
11 if L.length == 0
12     S = S + R
13 else if R.length == 0
14     S = S + L
15
16 return S
```

Loop Invariant - Line 3
At the start of each iteration of the while loop on line 3 the subarray S[1..S.length] consists of elements originally in L and R but in sorted order.

Initialization
Before the while loop S is initialized to the empty set []. The array is sorted because it is empty.

Maintenance
During the while loop, it either removes the first element from L or the first element from R (whichever is less) and adds it to S. Because L and R are sorted, elements are being added to S in sorted order so S stays sorted.

Termination
The condition causing the while loop to continue is L.length > 0 and R.length > 0. So, when either of those become false the while loop terminates, meaning, if either L or R are empty the while loop terminates. Because the while loop always removes the first element from L or R, whichever is smaller, either L.length or R.length will equal 0 eventually causing the loop to terminate.

5 Problems Encountered

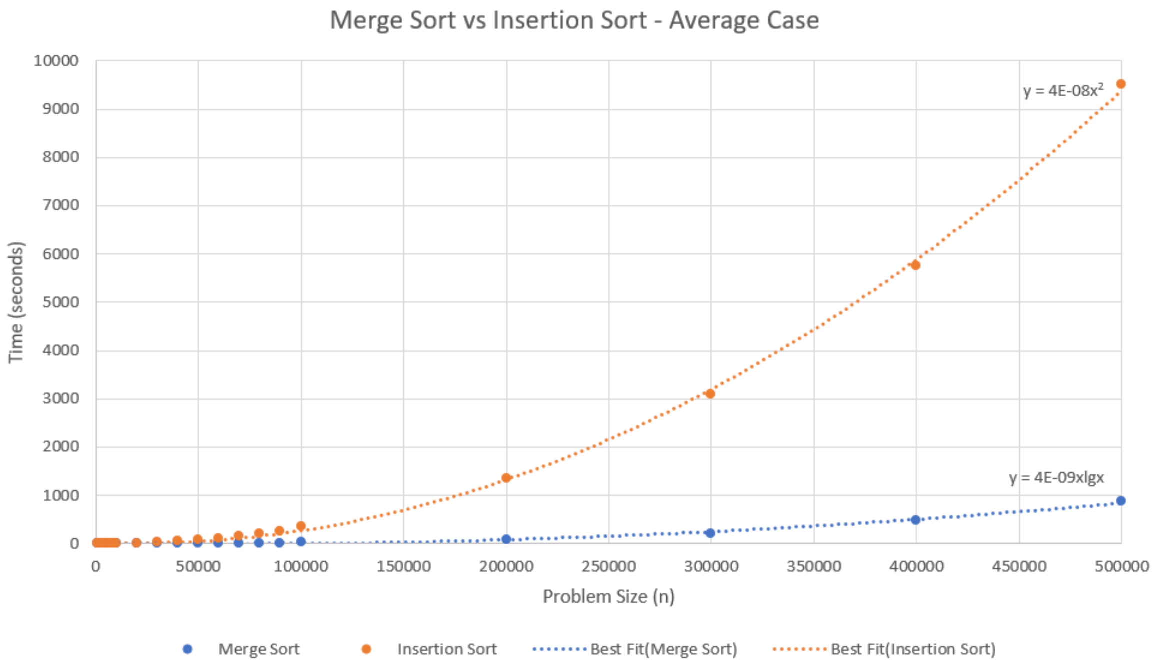
I had never used python classes before so I had to learn the structure and syntax of a python class. Another problem was converting from python 2.x to 3.x because I wanted to use a more recent version of python but was used to an older version.

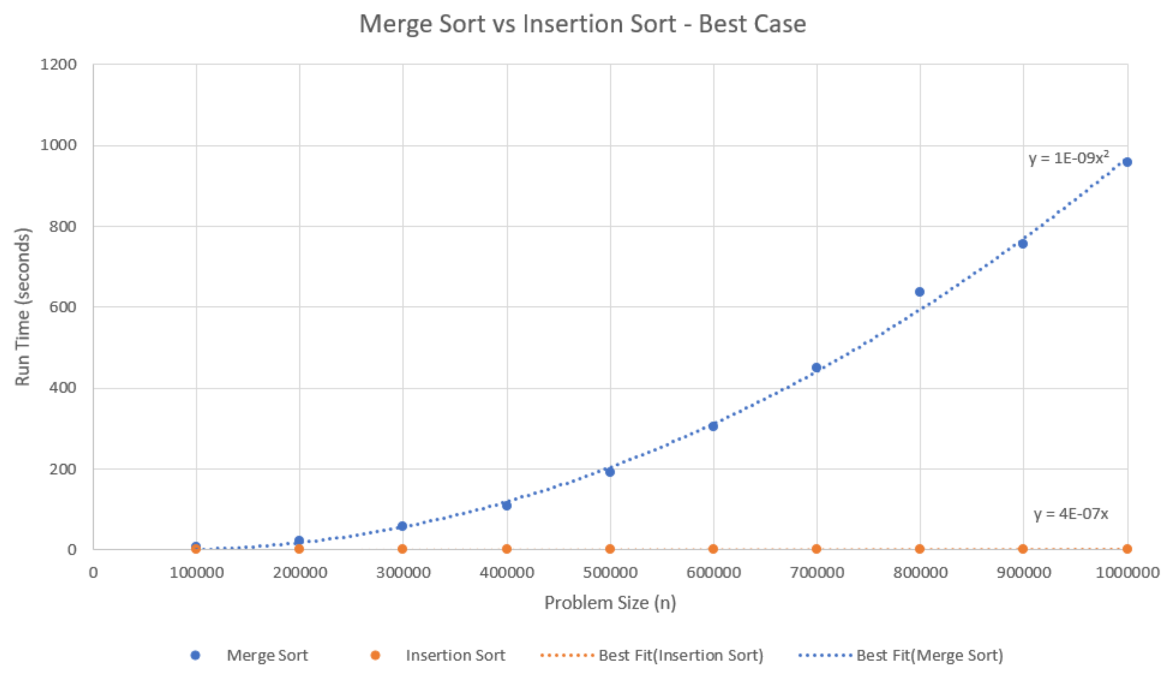
6 Testing Plan

I plan on testing each algorithm with varying lengths of sorted and unsorted arrays to represent best-case and average-case run time complexities. As well as any edge cases such as a zero length array, wrong input types, or arrays of wrong types.

Input	Expected Output	Output Recieved
Zero length array	Zero length array	Zero length array
Wrong input type	Error	Error caught by precondition checks
Array of wrong type variables	Error	Error caught by precondition checks

7 Performance Results





As shown in the first graph, when sorting a random array of integers both Merge sort and Insertion sort follow their expected asymptotic complexities of $n \lg n$ and n^2 respectively. Sorting a random array of integers represents the average-case time complexity. As well in the second graph both algorithms follow their expected best-case complexities of $n \lg n$ and n respectively. Sorting a already sorted array represents the best-case complexity of the algorithms.

8 Conclusion

In this report we showed that merge sort and insertion sort both follow their expected run time complexity measurements. First we showed each algorithm's loop invariants and pre/postconditions then proved they were correct. The algorithms were then implemented in a python class to experimentally prove their run time complexities.

Appendix A - Source Code

```
'''
# @file   proj1.py
# @author Evan Wilcox, CS2500, Section 1A
# @brief  Short program used for experimentally testing merge sort and insertion
#         sort's expected asymptotic complexity while testing the Sort class's
#         functionality.
'''

from Sort import Sort
import time, random

sort = Sort()
sort.setDebug(False)

# Used for testing average-case run time.
# Sorts a random array of integers between 0 and n.
for x in [1, 10, 100, 1000, 10000, 100000, 1000000, 10000000]:
    for y in range(1, 10):
        n = x*y
        A = []
        for i in range(0, n):
            A.append(random.randint(0, n))

        B = A
        print("N = ", n)
        start = time.time()
        sort.MergeSort(A)
        print("Merge Sort     = ", time.time()-start)
        start = time.time()
        sort.InsertionSort(B)
        print("Insertion Sort = ", time.time()-start, "\n")

# Used for testing best-case run time.
# Sorts a sorted array of integers between 0 to n.
for x in [1, 10, 100, 1000, 10000, 100000, 1000000, 10000000]:
    for y in range(1, 10):
        n = x*y
        A = []
        for i in range(0, n):
            A.append(i)

        print("N = ", n)
        start = time.time()
        sort.MergeSort(A)
        print("Merge Sort     = ", time.time()-start)
        start = time.time()
        sort.InsertionSort(A)
        print("Insertion Sort = ", time.time()-start, "\n")
```

```
'''
# @file   Sort.py
# @author Evan Wilcox, CS2500, Section 1A
# @brief  A class wrapper for Merge Sort and Insertion Sort.
'''

'''
# @class Sort
# @brief A class wrapper for Merge Sort and Insertion Sort.
'''
class Sort:
    '''
    # @fn    __init__
    # @brief Class initializer.
    # @pre   No pre-condition.
    # @post  A Sort object is created.
    '''
    def __init__(self):
        self.debug = False      # indicates whether debug mode is enabled.

    '''
    # @fn    setDebug
    # @brief Sets the debug mode on or off.
    # @pre   val must be a boolean
    # @post  The new debug mode is set.
    # @param val new intended debug mode
    '''
    def setDebug(self, val):
        # Precondition
        if self.debug:
            assert type(val) == bool

        self.debug = val

    '''
    # @fn     MergeSort
    # @brief  Sorts the given array using Merge Sort.
    # @pre    A must be an array of integers.
    # @post   A is sorted.
    # @param  A an array containing integers.
    # @return A sorted array of integers.
    '''
    def MergeSort(self, A):
        # Precondition
        if self.debug:
            assert type(A) == list
            for i in range(len(A)):
                assert type(A[i]) == int
```

```
    if len(A) <= 1:
        return A
    else:
        m = int(len(A)/2)
        L = self.MergeSort(A[0:m])
        R = self.MergeSort(A[m:])

        # Postcondition
        if self.debug:
            assert self.checkSort(self.merge(L, R))

        return self.merge(L, R)

'''
# @fn     merge
# @brief  Merges two sorted arrays in to one sorted array.
# @pre    Both L and R must be arrays of integers in sorted order.
# @post   L and R are merged in to one sorted array.
# @param  L An array of sorted integers.
# @param  R An array of sorted integers.
# @return An array of sorted integers.
'''
def merge(self, L, R):
    # Preconditions
    if self.debug:
        assert type(L) == list
        assert type(R) == list

        for i in range(len(L)):
            assert type(L[i]) == int

        for i in range(len(R)):
            assert type(R[i]) == int

        assert self.checkSort(L)
        assert self.checkSort(R)

    S = []
    '''
    # loop description while loop that merges two arrays
    # invariant:      S consists of elements from L and R in sorted order
    # proof:
    # initialization:
    #    S is initialized to [] so it is sorted.
    #
    # maintenance:
    #   The first element is removed from L or R and added to
    #   S. Because L and R are sorted S stays sorted.
```

```
    #
    # termination:
    #   Because an element is always removed from L or R either
    #   L or R size will equal 0 so the while loop will terminate.
    '''
    while len(L) > 0 and len(R) > 0:
        # Invariant
        if self.debug:
            assert self.checkSort(S)

        if L[0] < R[0]:
            S.append(L[0])
            L = L[1:]
        else:
            S.append(R[0])
            R = R[1:]

    if len(L) == 0:
        S = S + R
    elif len(R) == 0:
        S = S + L

    # Postcondition
    if self.debug:
        assert self.checkSort(S)

    return S


'''
# @fn     InsertionSort
# @brief  Sorts the given array using Insertion Sort.
# @pre    A must be an array of integers.
# @post   A is sorted.
# @param  A an array containing integers.
# @return A sorted array of integers.
'''
def InsertionSort(self, A):
    # Preconditions
    if self.debug:
        assert type(A) == list
        for i in range(len(A)):
            assert type(A[i]) == int
    '''
    # loop description iterates through elements in the array
    # invariant:       A[1..j-1] consists of elements originally in A[1..j-1]
    #                  but in sorted order.
    # proof:
    # initialization:
```

```
    #    Before the loop j=2 so the subarray A[1..j-1] contains 1 element.
    #
    # maintenance:
    #   The while loop moves A[j] around until it is in sorted order. j is incremented
    #   so the subarray A[1..j] is in sorted order.
    #
    # termination:
    #   The terminating condition is j > A.length, at this time j = A.length + 1.
    #   Subsitute that in to the invariant and the subarry A[1..A.length] contains
    #   elements originally from A[1..A.length] but in sorted order.
    '''
    for j in range(1, len(A)):
        # Invariant
        if self.debug:
            assert self.checkSort(A[0:j])

        key = A[j]
        i = j - 1

        while i >= 0 and A[i] > key:
            if self.debug:
                assert self.checkSort(A[0:i])

            A[i + 1] = A[i]
            i = i - 1
        A[i + 1] = key

    # Postcondition
    if self.debug:
        assert self.checkSort(A)

    return A


'''
# @fn     checkSort
# @brief  Checks if the given array A is in sorted order.
# @pre    A must be an array on integers.
# @post   A boolean is returned.
# @param  A an array of integers.
# @return Boolean indicating if the passed array is sorted in
          increasing order.
'''
def checkSort(self, A):
    # Preconditions
    if self.debug:
        assert type(A) == list
        for i in range(len(A)):
            assert type(A[i]) == int
```

```
for i in range(0, len(A)-1):
    if A[i] > A[i+1]:
        return False

return True
```