CS 2200 Spring 2019 HW 02

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Due Thursday, February 14, 2019, 11:59PM

Submit your HW as a SINGLE PDF file that includes your answers, your programs, and their output. Also include any data files if you used them. Please make sure that your PDF is well-organized so we can quickly find all the pieces of each problem that belong together.

PROBLEMS

Some of the problems come from the *Introduction to Computability* text by George Markowsky, which is available on CANVAS. These problems come with page numbers, so you can look in the relevant parts of the text.

1. (20 points) Do Problem 7 from Chapter 1: Consider a recursive function defined from the integers into the integers by:

```
def f(n):
   if n > 100:
      return n-10
   else:
      return f(f(n+11))
```

   Determine the following values: f(1), f(-6), f(200), f(27). Then give a simpler, non-recursive rule for f. Are you sure that the two rules produce the same result? If you are, give a reason why you are sure.

2. (20 points) Do Problem 9 from Chapter 1: The Ackermann function, defined below, provides a good test of the recursive capabilities of your system. Implement this function in Python and test your system by trying to generate the values of A(x,y) as x and y both range from 0 to 5. You won't be able to compute A(5,5) so just note the place where your system begins to take an inordinate amount of time. Watch for the computer hanging up. At first glance it might not be obvious that A(x,y) always halts for all values of x and y. To better understand how the Ackermann function works, display the chain of calls that A(2,2) makes and include this chain in your homework. State clearly the number of calls that A(2,2) makes to A.

```
def A(x,y):
   if x == 0:
      return y+1
   elif y == 0:
      return A(x-1,1)
   else:
      return A(x-1,A(x,y-1))
```

3. (20 points) Do Problem 2 from Chapter 3: Prove by induction that for any natural number n

$$\frac{1^2}{1{\cdot}3} + \frac{2^2}{3{\cdot}5} + \frac{3^2}{5{\cdot}7} + \ldots + \frac{n^2}{(2n-1)\,(2n+1)} = \frac{n(n+1)}{2(2n+1)} \tag{1}$$

4. (20 points) Do Problem 3 from Chapter 3: Prove by induction that the sum of the cubes of 3 consecutive natural numbers is divisible by 9.

5. (20 points) Prove by induction that the version of insertion sort given in Lecture 04 is correct. In orther words, prove that if you input a list of integers, you will output a sorted list of integers.