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CS1200 Fall 2018
Homework 3
Due: Friday 10/05/18

1. Show that $\rightarrow$ does not have the associative or commutative laws.

   (a) $P \rightarrow (Q \rightarrow R)$ and $(P \rightarrow Q) \rightarrow R$ have different truth tables.

P	Q	R	$P \rightarrow (Q \rightarrow R)$	$(P \rightarrow Q) \rightarrow R$
F	F	F	T	F
F	F	T	T	T
F	T	F	T	F
F	T	T	T	T
T	F	F	T	T
T	F	T	T	T
T	T	F	F	F
T	T	T	T	T

   (b) $P \rightarrow Q$ and $Q \rightarrow P$ have different truth tables.

P	Q	$P \rightarrow Q$	$Q \rightarrow P$
F	F	T	T
F	T	T	F
T	F	F	T
T	T	T	T

2. Verify the second DeMorgan's Law $\sim (P|Q) =\sim P\& \sim Q$ manually using truth table.

P	Q	$\sim P$	$\sim Q$	$\sim (P\|Q)$	$\sim P\& \sim Q$
F	F	T	T	T	T
F	T	T	F	F	F
T	F	F	T	F	F
T	T	F	F	F	F

```
6   #2
7   def ls(P, Q):
8       return not (P or Q)
9
10  def rs(P, Q):
11      return (not P) and (not Q)
12
13  print "|   P   |   Q   | ~(P|Q) | ~P&~Q |"
14  print "---------------------------------"
15  for p in [False, True]:
16      for q in [False, True]:
17          print "| %s | %s | %s | %s |" % (p, q, ls(p, q), rs(p, q))
```

```
C:\Users\Evan\Documents\MST\CS1200\HW03>python hw03.py
|   P   |   Q   | ~(P|Q) | ~P&~Q |
---------------------------------
| False | False | True | True |
| False | True | False | False |
| True | False | False | False |
| True | True | False | False |
```

3. (a) Manually construct a truth table for $\sim (P\&Q \rightarrow R\&S)$

P	Q	R	S	$P\&Q$	$R\&S$	$\rightarrow$	$\sim (P\&Q \rightarrow R\&S)$
F	F	F	F	F	F	T	F
F	F	F	T	F	F	T	F
F	F	T	F	F	F	T	F
F	F	T	T	F	T	T	F
F	T	F	F	F	F	T	F
F	T	F	T	F	F	T	F
F	T	T	F	F	F	T	F
F	T	T	T	F	T	T	F
T	F	F	F	F	F	T	F
T	F	F	T	F	F	T	F
T	F	T	F	F	F	T	F
T	F	T	T	F	T	T	F
T	T	F	F	T	F	F	T
T	T	F	T	T	F	F	T
T	T	T	F	T	F	F	T
T	T	T	T	T	T	T	F

(b) Find the disjunctive normal form of $\sim (P\&Q \rightarrow R\&S)$.

$$(P\&Q\& \sim R)|(P\&Q\& \sim S)$$

4. Let $G(A, B, C)$ be the function:

$$B\&A|C\&C \leftarrow B\&B \rightarrow B! = A|C$$

(a) Completely parenthesize the above expression for G.

$$(((((B\&A)|(C\&C)) \leftarrow (B\&B)) \rightarrow B)! = (A|C))$$

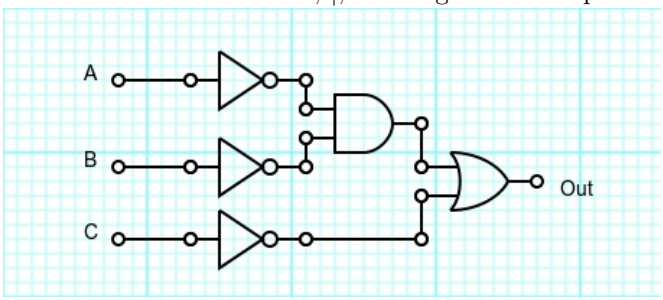
(b) Create a truth table for G.

A	B	C	$B\&A$	$\|$	$C\&C$	$\leftarrow$	$B\&B$	$\rightarrow B$	$!=$	$A\|C$	G
F	F	F	F	F	F	T	F	F	F	F	F
F	F	T	F	T	T	T	F	F	T	T	T
F	T	F	F	F	F	F	T	T	T	F	T
F	T	T	F	T	T	T	T	T	F	T	F
T	F	F	F	F	F	T	F	F	T	T	T
T	F	T	F	T	T	T	F	F	T	T	T
T	T	F	T	T	F	T	T	T	F	T	F
T	T	T	T	T	T	T	T	T	F	T	F

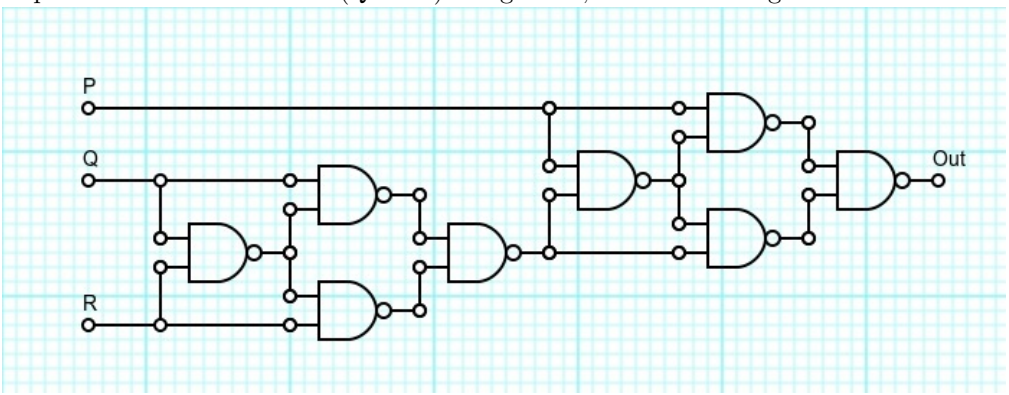
(c) Express G in disjunctive normal form.

$$(\sim A\& \sim C)| \sim B$$

(d) Draw a circuit that uses &, |, and $\sim$ gates to compute G.



5. Represent the function $P! = (Q! = R)$ using NOT, AND and OR gates.



6. Design a circuit for three switches that turns a light on only if at least two of the three switches are on.