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CS1200 Fall 2018
Homework 4
Due: Monday 10/22/18

1. Using truth tables, prove the following arguments are valid.

(a)

P	Q	R	$P \to\, \sim Q$	$\sim R|Q$	R	$\sim P$
F	F	F	T	T	F	T
F	F	T	T	F	T	T
F	T	F	T	T	F	T
F	T	T	T	T	T	T
T	F	F	T	T	F	F
T	F	T	T	F	T	F
T	T	F	F	T	F	F
T	T	T	F	T	T	F

(b)

P	Q	R	$P \to Q$	$Q \to R$	$P \to R$
F	F	F	T	T	T
F	F	T	T	T	T
F	T	F	T	F	T
F	T	T	T	T	T
T	F	F	F	T	F
T	F	T	F	T	T
T	T	F	T	F	F
T	T	T	T	T	T

2. Use Python programs to verify the results in Problem 1.

(a) Python code for part a.

```
7    #2a
8    def h1(P, Q):
9        return P <= (not Q)
10
11   def h2(R, Q):
12       return (not R) or Q
13
14   def h3(R):
15       return R
16
17   def con(P):
18       return not P
19
20   print "|   P   |   Q   |   R   | P->~Q | ~R|Q  |   R   |  ~P   |"
21   print "========================================================="
22   for P in [False, True]:
23       for Q in [False, True]:
24           for R in [False, True]:
25               print "| %-5s | %-5s | %-5s | %-5s | %-5s | %-5s | %-5s |" \
26                   % (P, Q, R, h1(P, Q), h2(R, Q), h3(R), con(P))
```

(b) Python code for part b.

```python
29    #2b
30    def h1(P, Q):
31        return P <= Q
32
33    def h2(Q, R):
34        return Q <= R
35
36    def con(P, R):
37        return P <= R
38
39    print "|   P   |   Q   |   R   | P->Q  | Q->R  | P->R  |"
40    print "=================================================="
41    for P in [False, True]:
42        for Q in [False, True]:
43            for R in [False, True]:
44                print "| %-5s | %-5s | %-5s | %-5s | %-5s | %-5s |" \
45                    % (P, Q, R, h1(P, Q), h2(Q, R), con(P, R))
```

3. Derive the associated sets for the arguments in Problem 1.

(a) The set $[\sim Q, Q, R, P]$ is not satisfiable, so the argument is valid.

$$P \to \sim Q \qquad\qquad \sim P | \sim Q \qquad\qquad \sim Q$$
$$\sim R | Q \qquad\qquad\qquad \sim R | Q \qquad\qquad\quad Q$$
$$R \qquad\qquad\qquad\qquad R \qquad\qquad\qquad\quad R$$
$$\sim (\sim P) \qquad\qquad\qquad P \qquad\qquad\qquad\quad P$$

(b) The set $[Q, \sim Q, P, \sim R]$ is not satisfiable, so the argument is valid.

$$P \to Q \qquad\quad \sim P | Q \qquad\quad \sim P | Q \qquad\quad Q$$
$$Q \to R \qquad\quad \sim Q | R \qquad\quad \sim Q | R \qquad\quad \sim Q$$
$$\sim (P \to R) \quad \sim (\sim P | R) \quad\quad P \& \sim R \qquad\quad P$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \sim R$$

4. (a) Neither $[\sim P, \sim Q, P]$ or $[Q, \sim Q, P]$ are satisfiable so the argument is valid.

Start	A	A.1	A.2
$P \to Q$	$\sim P \mid Q$	$\sim P$	Q
$\sim Q$	$\sim Q$	$\sim Q$	$\sim Q$
$\sim P$	P	P	P

   i. $\sim P \mid Q$ splits A into A.1 and A.2.

(b) Neither B.1, B.2, C.1 or C.2 are satisfiable so the argument is valid.

Start	A	B	C
$P \to \sim Q$	$\sim P \mid \sim Q$	$\sim P$	$\sim Q$
$Q \mid R$	$Q \mid R$	$Q \mid R$	$Q \mid R$
P	P	P	P
R	$\sim R$	$\sim R$	$\sim R$

B.1	B.2	C.1	C.2
$\sim P$	$\sim P$	$\sim Q$	$\sim Q$
Q	R	Q	R
P	P	P	P
$\sim R$	$\sim R$	$\sim R$	$\sim R$

   i. $\sim P \mid \sim Q$ splits A into B and C.
   ii. $Q \mid R$ splits B into B.1 and B.2.
   iii. $Q \mid R$ splits C into C.1 and C.2.

5. Translate the argument into symbolic form and use the truth tree method to decide if the argument is valid.

> $(M\&C) \to (\sim S|H)$
> $(\sim C|S) \to (\sim M\& \sim H)$
> $(\sim M \to C)|(\sim S \to H)$
> _____
> $(M|S) \to (C\& \sim H)$

$(M\&C) \to (\sim S|H)$ $\sim (M\&C)|(\sim S|H)$ $\sim M|\sim C|\sim S|H$
$(\sim C|S) \to (\sim M\& \sim H)$ $\sim (\sim C|S))|(\sim M\& \sim H)$ $(C\& \sim S)|(\sim M\& \sim H)$
$(\sim M \to C)|(\sim S \to H)$ $(\sim (\sim M)|C)|(\sim (\sim S)|H)$ $M|C|S|H$
$\sim ((M|S) \to (C\& \sim H))$ $\sim (\sim (M|S)|(C\& \sim H))$ $\sim ((\sim M\& \sim S)|(C\& \sim H))$

$\sim M|\sim C|\sim S|H$ $\sim M|\sim C|\sim S|H$ $\sim M|\sim C|\sim S|H$
$(C\& \sim S)|(\sim M\& \sim H)$ $(C\& \sim S)|(\sim M\& \sim H)$ $(C\& \sim S)|(\sim M\& \sim H)$
$M|C|S|H$ $M|C|S|H$ $M|C|S|H$
$\sim (\sim M\& \sim S)\& \sim (C\& \sim H))$ $(M|S)\&(\sim C|H)$ $(M|S)$
                                                                   $(\sim C|H)$

1.	$\sim M|\sim C|\sim S|H$		1.	$\sim C*$		1.	$\sim C$
2.	$(C\& \sim S)	(\sim M\& \sim H)$		2.	$\sim M\& \sim H$	2a.	$\sim M$
3.	$M|C|S|H$		3.	$M|C|S|H$	2b.	$\sim H$
4.	$M|S$		4.	$M|S$	3.	S
5.	$\sim C	H$		5a.	$\sim C$	4.	S
						5a.	$\sim C$

_____

1.	$\sim M|\sim C|\sim S|H$		1.	$H*$		1.	H
2.	$(C\& \sim S)	(\sim M\& \sim H)$		2.	$C\& \sim S$	2a.	C
3.	$M|C|S|H$		3.	$H*$	2b.	$\sim S$
4.	$M|S$		4.	$M|S$	3.	H
5.	$\sim C	H$		5b.	H	4.	$M*$
						5a.	H

The argument is not valid. $[\sim M, \sim C, S, \sim H]$ and $[M, C, \sim S, H]$ are counter examples.

6. Write Python functions to solve the problem.

```python
123    #6
124    people = ["Clark", "Daw", "Fuller"]
125    def distinct(L):
126        if len(L) < 2:
127            return True
128        else:
129            j = L.pop()
130        if j in L:
131            return False
132        return distinct(L)
133
134    def workedFor(p1, p2):
135        global carpenter, painter, plumber
136        if (p1 == painter) and (p2 == carpenter):
137            return True
138        if (p1 == carpenter) and (p2 == plumber):
139            return True
140        return False
141
142    def notHeardOf(p1, p2):
143        return (p1 == "Fuller") and (p2 == "Daw")
144
145    def earnsMore(p1, p2):
146        global carpenter, painter, plumber
147        if p1 == p2:
148            return False
149        if p2 == plumber:
150            return False
151        if p2 == "Daw":
152            return False
153        return True
154
155    def solve():
156        global carpenter, painter, plumber
157        for carpenter in people:
158            for painter in people:
159                for plumber in people:
160                    if distinct([carpenter, painter, plumber]):
161                        sol = workedFor(painter, carpenter)
162                        sol = sol and workedFor(carpenter,plumber)
163                        sol = sol and earnsMore(plumber,painter)
164                        sol = sol and notHeardOf(painter, plumber)
165                        if sol:
166                            print "Carpenter = %s, Painter = %s, Plumber = %s" \
167                            % (carpenter, painter, plumber)
168
169    solve()
```

```
C:\Users\Evan\Documents\MST\CS1200\HW04>python hw04.py
Carpenter = Clark  Painter = Fuller  Plumber = Daw
```

5

7. Indicate whether each of the following predicates is true or false, and give
   a brief justification for each answer.

Predicate	True or False?	Why?
$\forall A\ \forall B$ PRED(A,B)	False	If A = 0 and B = 1, then the predicate is false.
$\forall A\ \exists B$ PRED(A,B)	False	There is no value of B that makes the predicate true for all values of A.
$\exists A\ \forall B$ PRED(A,B)	False	There is no value of A that makes the predicate true for all values of B.
$\exists A\ \exists B$ PRED(A,B)	True	If A = 1 and B = -1, then the predicate is true.