CS 2200 S19 HW 01

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Due 2019-01-31 at 11:59 PM

Submit your assignment as a single PDF file with that includes all the answers to the individual questions, all pictures, all code, all code output, and anything else that you need to include to support your answers. This should all be well-organized and attractively laid out.

If you need a program that helps you put PDF files together into a single PDF file, try http://www.pdfsam.org/. The program there is open source and available for free. You can find a lot of material on recursion in my book, *Introduction to the Theory of Computation*, which is posted on Canvas.

1. (30 points) Read the handout about the Abacus and Roman Numerals that is in the HW01 folder on Canvas. It comes from `http://ds-wordpress.haverford.edu/bitbybit/wp-content/uploads/2012/07/` Abacus-and-Roman-Numerals.pdf. Please do not search for answers on the Internet. Read the handout and try to work out the answers. You can certainly do this. Use this as an opportunity to stretch your brain and understand the difficulties that your ancestors went through to build the world that you live in. Do all the problems in the handout. Each problem in the handout is worth 5 points. If you want to "draw" pictures of an abacus you can just use ASCII art, i.e., blanks, small "o", and the - character. For example the arrangement below represents the number 17.

```
o   o   o
o   o

          o
---------------
    o   o
        o


o
o   o
o   o   o
o   o   o
o   o   o
```

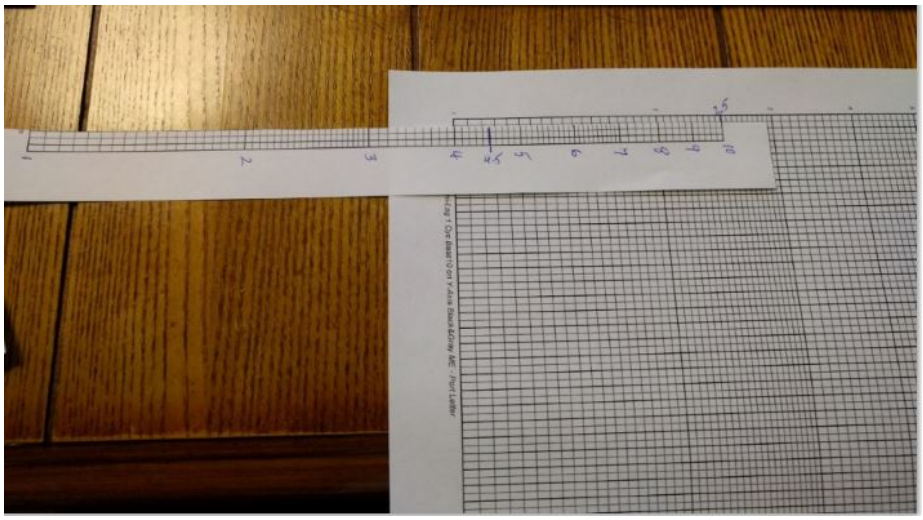

Figure 1: A Simple Paper Slide Rule

2. (10 points) Read the Wikipedia article on the slide rule. Print a copy of the semilog paper that is included in the HW01 Folder. Label the right edge and cut a strip off the right side of the paper. Put the cut-off piece next to the remainder and use it as a slide rule. The photos in Figures 1 and 2 below show how to compute the product of 2.5 and 4.5 which is 11.25

   Use your slide rule to compute the product of 75.68 and 127.5 as accurately as you can. Include a photo of your slide rule that looks like the photo above. Also, make sure to write your name on the slide rule so it is visible in the picture.

3. (10 points) Read the Wikipedia article on Napier's Bones. Create a set of Bones using cardboard or paper. Show how to compute the product of 9×2768 and produce a photo of your version of Napier's Bones with the correct result shown in the picture. Make sure that your name is visible in the photo.

4. (10 points) Read the handout called "50 Years of Army Computing" and answer the following questions:

   (a) (2 points) In what sense was ENIAC a personal computer?

   (b) (2 points) How much memory did ENIAC have?

   (c) (2 points) How common were subroutines in ENIAC programming?

   (d) (2 points) What does BRL stand for?

   (e) (2 points) Why do punched cards need to be used in air conditioned areas?

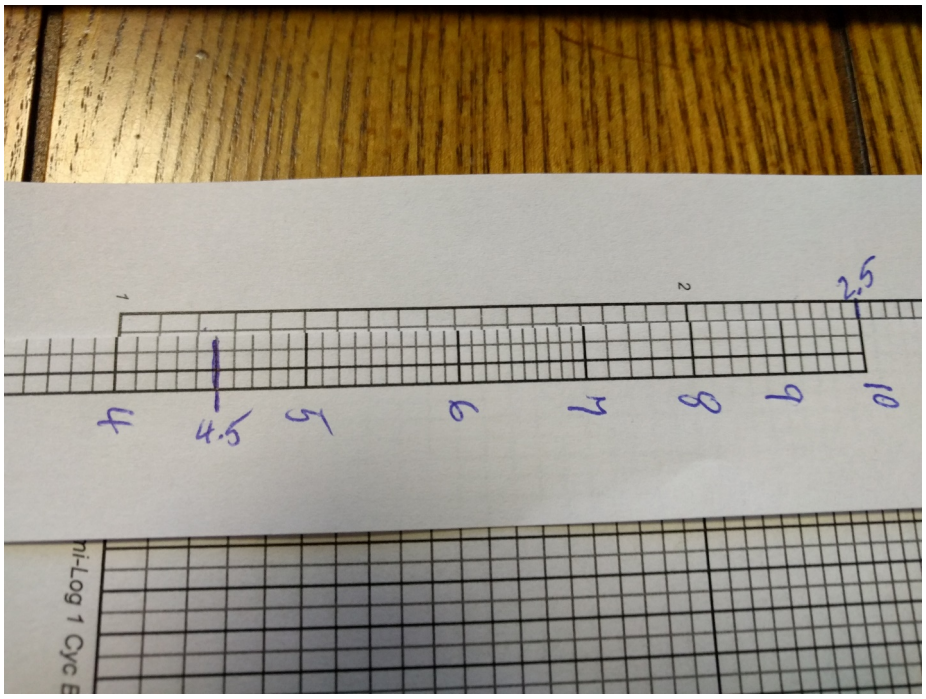

Figure 2: A Closeup of the Simple Paper Slide Rule

5. (20 points)

   (a) (15 points) Prove by induction on n, that every positive integer has a Fibonacci representation of the type discussed in class. If you can't complete the proof, you can earn at least 8 points by detailing the steps and setting up the proof correctly.

   (b) (5 points) Prove that in the Fibonacci representation of an integer the string '11' can be a subsequence of the representation only at the very end of the representation.

6. (20 points) Let fib_n be the Fibonacci numbers where $\text{fib}_0 = 0$ and $\text{fib}_1 = 1$. Also, let $\phi = \frac{1+\sqrt{5}}{2}$.

   (a) (10 points) Prove that the number of digits in the Fibonacci representation of all integers in range(fib_n,fib_{n+1}) is n.

   (b) (10 points) Write a Python program that computes the number of digits in the Fibonacci representation of any number and compares it to the functions $\log_2(Start) + 2$ and $\log_\phi(Start) + 2$. To be most useful, convert the floating point answers to integers in some useful way. If you do this, the first 11 rows of this table are shown below. Expand the table so that there are 30 numerical rows in addition to the header. Include the Python program in your Zip file both in the PDF file and as a separate .py file. Make a conjecture about the number of digits in the Fibonacci representation of a number as a function of n and include it in your PDF file.

Start	End	Num Digits	Log2(Start)+2	LogPhi(Start)+2
1	1	2	2.0	2.0
2	2	3	3.0	3.0
3	4	4	4.0	4.0
5	7	5	4.0	5.0
8	12	6	5.0	6.0
13	20	7	6.0	7.0
21	33	8	6.0	8.0
34	54	9	7.0	9.0
55	88	10	8.0	10.0
89	143	11	8.0	11.0
144	232	12	9.0	12.0