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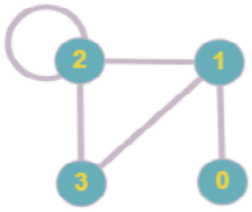
CS1200 Fall 2018
Homework 7
Due: Wednesday 12/5/18

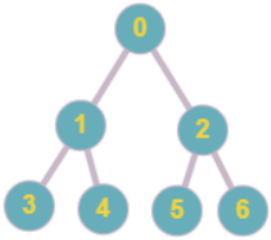
1. Either draw a graph with the specified properties or explain why no such graph exists.

   (a) This graph does not exist because the degree of the graph is odd.

   (b) Graph with four vertices of degrees 1,2,3 and 4.



   (c) Full binary tree with seven vertices.



   (d) This graph does not exist because the number of vertices - 1 equals the number of edges.

2. (a) A Euler Circuit for the graph does not exist because at least one vertex has an odd amount of connections.

   (b) A Euler Circuit for the graph is $v_1, v_2, v_3, v_4, v_5, v_2, v_5, v_4, v_1$.

   (c) A walk for the graph is A, B, D, E, A, D, C, A.

   (d) A Hamiltonian circuit for the graph is $v_0, v_7, v_6, v_2, v_5, v_4, v_3, v_1, v_0$.

3. (a) No there will be one person who can only shake two people hands.

   (b) The height of the given binary tree is 6.

   (c) Given the graph:

      i. Find adjacency matrices.

$$A \quad \begin{matrix} & B & & & \\ & 1 & 2 & 3 & 4 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 1 & 0 & 0 & 1 \end{matrix}$$

      ii. There are no distinct walks of length 2 from v_2 to v_3.

4. Find a system of pipelines to connect all the cities and yet minimize the total cost by using Kruskal's algorithm (Show the work step by step).

Iteration	**Edge Considered**	**Weight**	**Action**
1	Cheyenne, Denver	0.8	Added
2	Albuquerque, Amarillo	1.1	Added
3	Phoenix, Albuquerque	1.2	Added
4	Salt Lake City, Cheyenne	1.5	Added
5	Salt Lake City, Denver	1.6	Not Added
6	Denver, Amarillo	1.7	Added

5. (a) If any seven digits could be used to form a telephone number, how many seven-digit telephone numbers would not have any repeated digits?

$$\binom{10}{7} = \frac{10!}{7!(3!)} = 120$$

(b) How many seven-digit telephone numbers would have at least one repeated digit?

$$P(10,7) - \binom{10}{7} = 604,800 - 120 = 604,680$$

(c) What is the probability that a randomly chosen seven-digit telephone number would have at least one repeated digit?

$$\frac{P(10,7) - \binom{10}{7}}{P(10,7)} = \frac{604,680}{604,800} = 99.98\%$$

6. Suppose that in a certain state, all automobile license plates have four upper case letters followed by three digits.

(a) How many license plates could begin with M and end in 0?

$$P(26,3) * P(10,2) = \frac{26!}{(26-3)!} * \frac{10!}{(10-2)!} = 15,600 * 90 = 1,404,000$$

(b) How many license plates are possible in which all the letters and digits are distinct?

$$\binom{26}{4}\binom{10}{3} = \frac{26!}{4!(22!)} * \frac{10!}{3!(7!)} = 14,950 * 120 = 1,794,000$$

(c) How many license plates could begin with MO and have all letters and digits distinct?

$$\binom{24}{2}\binom{10}{3} = \frac{26!}{2!(22!)} * \frac{10!}{3!(7!)} = 276 * 120 = 33,120$$

7. A club is considering changing its bylaws. In an initial straw vote on the issue, 24 of the 40 members of the club favored the change and 16 did not. A committee of six is to be chosen from the 40 club members to devote further study to the issue.

   (a) How many committees of six can be formed from the club membership?

$$\binom{40}{6} = \frac{40!}{6!(36!)} = 3,838,380$$

   (b) How many of the committees will contain at least three club members who, in the preliminary survey, favored the change in the bylaws?

$$\binom{24}{3}\binom{16}{3} = \frac{24!}{3!(21!)} * \frac{16!}{3!(13!)} = 2,024 * 560 = 1,133,440$$

8. Write a program to calculate the binomial coefficients recursively.

```
6   #8
7   def C(n, k):
8       if (k == 0 or k == n):
9           return 1
10      else:
11          return C(n-1, k-1) + C(n-1, k)
12
13  print "C(13,5) =", C(13, 5)
```

$$C(13,5) = 1287$$

```
PS C:\Users\Evan\Documents\MST\CS1200\HW07> python .\hw07.py
C(13,5) = 1287
```