

CS2200 Homework 2

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1. The two functions give the same result because $f(n)$ increments n until $n - 10$ is greater than 100 then prints n , while $g(n)$ just prints 91 or $n - 10$ which ever is greater.

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21 def g(n):
22     if n - 10 < 91:
23         return 91
24     else:
25         return n - 10

PS C:\Users\Evan\Documents\MST\CS2200\Homework 2> python .\hw02.py
f(1) = 91
f(-6) = 91
f(200) = 190
f(27) = 91
```

2. $A(3, 4)$ was the maximum value my computer was able to compute, for any greater values the Maximum Recursion Depth reached was given.
 $A(2, 2)$ produces the following chain of calls:
 $A(2, 2), A(2, 1), A(2, 0), A(1, 1), A(1, 0), A(0, 1), A(0, 2), A(1, 3), A(1, 2), A(1, 1), A(1, 0), A(0, 1), A(0, 2), A(0, 3), A(0, 4), A(1, 5), A(1, 4), A(1, 3), A(1, 2), A(1, 1), A(1, 0), A(0, 1), A(0, 2), A(0, 3), A(0, 4), A(0, 5), A(0, 6)$.

For a total of 27 calls.

3. Prove by induction that for any natural number n

$$\frac{1^2}{1 \cdot 3} + \frac{2^2}{3 \cdot 5} + \frac{3^2}{5 \cdot 7} + \dots + \frac{n^2}{(2n-1)(2n+1)} = \frac{n(n+1)}{2(2n+1)}$$

Define the problem.

If

$$f(n) = \frac{1^2}{1 \cdot 3} + \frac{2^2}{3 \cdot 5} + \frac{3^2}{5 \cdot 7} + \dots + \frac{n^2}{(2n-1)(2n+1)}$$

and

$$g(n) = \frac{n(n+1)}{2(2n+1)}$$

then $P(n)$ is true when $f(n) = g(n)$. Prove $P(n)$ is true for all natural numbers.

Stopping value and two more.

$$\begin{aligned} f(1) &= \frac{1^2}{1 \cdot 3} = \frac{1}{3} \\ g(1) &= \frac{1(1+1)}{2(2+1)} = \frac{1}{3} \\ f(2) &= \frac{1^2}{1 \cdot 3} + \frac{2^2}{3 \cdot 5} = \frac{1}{3} + \frac{4}{15} = \frac{8}{15} \\ g(2) &= \frac{2(2+1)}{2(2 \cdot 2+1)} = \frac{4}{6} = \frac{2}{3} \\ f(3) &= \frac{1^2}{1 \cdot 3} + \frac{2^2}{3 \cdot 5} + \frac{3^2}{5 \cdot 7} = \frac{1}{3} + \frac{4}{15} + \frac{9}{35} = \frac{67}{105} \\ g(3) &= \frac{3(3+1)}{2(2 \cdot 3+1)} = \frac{12}{14} = \frac{6}{7} \end{aligned}$$

Inductive Case.

$$\begin{aligned} f(n) &= f(n-1) + \frac{n^2}{(2n-1)(2n+1)} \\ &= g(n-1) + \frac{n^2}{(2n-1)(2n+1)} \\ &= \frac{(n-1)(n)}{2(2(n-1)+1)} + \frac{n^2}{(2n-1)(2n+1)} \\ &= \frac{n^2-n}{2(2n-1)} + \frac{n^2}{(2n-1)(2n+1)} \\ &= \frac{n^2-n}{2(2n-1)} \cdot \frac{2n+1}{2n+1} + \frac{n^2}{(2n-1)(2n+1)} \cdot \frac{2}{2} \\ &= \frac{(n^2-n)(2n+1)}{2(2n-1)(2n+1)} + \frac{2n^2}{2(2n-1)(2n+1)} \\ &= \frac{(n^2-n)(2n+1)+2n^2}{2(2n-1)(2n+1)} \\ &= \frac{n(2n^2+n-1)}{2(2n-1)(2n+1)} \\ &= \frac{n(2n-1)(n+1)}{2(2n-1)(2n+1)} \\ &= \frac{n(n+1)}{2(2n+1)} \\ &= g(n) \end{aligned}$$

Conclusion.

Because $P(n)$ is true for for all n we can conclude that

$$\frac{1^2}{1 \cdot 3} + \frac{2^2}{3 \cdot 5} + \frac{3^2}{5 \cdot 7} + \dots + \frac{n^2}{(2n-1)(2n+1)} = \frac{n(n+1)}{2(2n+1)}$$

4. Prove by induction that the sum of the cubes of 3 consecutive natural numbers is divisible by 9.

Define the problem.

The property $P(n)$ is true when

$$(n)^3 + (n+1)^3 + (n+2)^3 \% 9 == 0$$

Prove that $P(n)$ is true for all natural numbers n .

Stopping value and two more.

$$(1)^3 + (1+1)^3 + (1+2)^3 = 1 + 8 + 27 = 36 \% 9 == 0$$

$$(2)^3 + (2+1)^3 + (2+2)^3 = 8 + 27 + 64 = 99 \% 9 == 0$$

$$(3)^3 + (3+1)^3 + (3+2)^3 = 27 + 64 + 125 = 216 \% 9 == 0$$

Inductive Case.

Try $n+1$.

$$(n+1)^3 + (n+2)^3 + (n+3)^3 \% 9 == 0$$

$$(n+1)^3 + (n+2)^3 + (n+3)(n^2 + 6n + 9) \% 9 == 0$$

$$(n+1)^3 + (n+2)^3 + n^3 + 9n^2 + 27n + 27 \% 9 == 0$$

$$9n^2 + 27n + 27 \% 9 == 0$$

Because we assume $(n)^3 + (n+1)^3 + (n+2)^3$ is divisible by 9 and all other terms are also divisible by 9 we can say $P(n)$ is true for all n .

Conclusion.

Because $P(n)$ is true for all n we can conclude that the sum of the cubes of 3 consecutive natural numbers is divisible by 9.

5. Prove by induction that the version of insertion sort given in Lecture 4 is correct. In other words, prove that if you input a list of integers, you will output a sorted list of integers.

Define the problem.

Prove that Insertion sort sorts for any given array of integers.

Base Case.

The base case of insertion sort is an array of length zero, an empty array and this array is always sorted.

Inductive Case.

When insertion sort inserts a new element to the array, it inserts it in to a sorted array. So the array stays sorted until all elements are inserted.

Conclusion.

After showing the last two steps, we can conclude that insertion will sort an array of integers.