CS 1200 FS18 HW 2

Due Wednesday 9/19/18 at 11:59 PM

Please submit two files to Canvas (one is a PDF file and the other is the code .py file) :

1. A PDF file that contains all the answers to the individual questions, all pictures, all code, and all code output. This should all be well-organized. Points will be deducted for sloppy or disorganized work.
2. All the Python codes (.py file) (You may put all codes in one .py file).

If you need a program that helps you put PDF files together into a single PDF file, try http://www.pdfsam.org/. The program there is open source and available for free.

Note: Partial credit will be given on every problem.

1. (15 points) Compute the summations and productions.
   (a) $\sum_{m=0}^{4} \frac{1}{2^m}$
   (b) $\sum_{i=1}^{1} i(i+1)$
   (c) $\prod_{k=2}^{3} \left(1 - \frac{1}{k}\right)$
2. (25 points) See the following problems:
   (a) Write the summations in expanded form.

$$\sum_{k=0}^{n+1} \frac{1}{k!}.$$

   (b) Rewrite the following summation by separating off the final term

$$\sum_{i=1}^{n} (i^3 + i^2 + 1),$$

   (c) Write the following expression in a single summation notation.

$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2$$

(d) Transform the following summation by making the change of variable $j = i - 1$.

$$\sum_{i=1}^{n+1} \frac{(i-1)^2}{i+n}$$

3. (10 points) Write a recursive function in Python to compute

$$\sum_{i=1}^{n} (i^3 + i),$$

and list the output for n from 1 to 5.

4. (20 points) Prove by the Principle of Recursion that

$$\Sigma_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$$

Submit a listing of the program in Python you wrote for the proof and enough output to suggest that the equation is correct.

5. (15 points) Prove by induction that for all integer $n \geq 1$

$$1 + 6 + 11 + 16 + \cdots + (5n-4) = \frac{n(5n-3)}{2}.$$

6. (15 points) Prove by induction that for all natural numbers n

$$\Sigma_{i=0}^{n} \frac{i^2}{(2i-1)(2i+1)} = \frac{n(n+1)}{2(2n+1)}$$

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