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CS1200 Fall 2018
Homework 2
Due: Wednesday 9/19/18

1. Compute the summations and productions.

   (a) $\sum_{m=0}^{4} \frac{1}{2^m} = \frac{31}{16}$

   (b) $\sum_{i=1}^{1} i(i+1) = 2$

   (c) $\prod_{k=2}^{3} (1 - \frac{1}{k}) = \frac{1}{3}$

2. (a) Write the summation in expanded form.

$$\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{(n+1)!}$$

   (b) Rewrite the following summation by separating off the final term.

$$\sum_{i=1}^{n}(i^3 + i^2) + \sum_{i=1}^{n} 1$$

   (c) Write the following expression in a single summation notation.

$$\sum_{i=0}^{6}(-1)^i(i+1)^2$$

   (d) Transform the following summation by making the change of variable j = i - 1.

$$\sum_{j=0}^{n+1} \frac{j^2}{j+n+1}$$

3. Write a recursive function in Python to compute and list the output for n from 1 to 5.

```python
#3
def sum(n):
    if n <=1:
        return (n**3 + n)
    else:
        return (n**3 + n) + sum(n-1)

for n in range(1, 6):
    print sum(n)
```

```
C:\Users\Evan\Documents\MST Student Drive\CS1200\HW02>python hw02.py
2
12
42
110
240
```

4. Prove by the Principle of Recursion that

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$$

1. Define the problem.
   The domain for the problem is the natural numbers 1...∞.

$$f(n) = \sum_{i=1}^{n} i(i+1)$$

$$g(n) = \frac{n(n+1)(n+2)}{3}$$

   Prove $f(n) = g(n)$ for all of the domain.
   P(n) is true if $f(n) = g(n)$.

2. Check stopping values and two more.
   $f(1) = 1(1+1) = 2$
   $g(1) = \frac{1(1+1)(1+2)}{3} = 2$
   $f(2) = 1(1+1) + 2(2+1) = 8$
   $g(2) = \frac{2(2+1)(2+2)}{3} = 8$
   $f(3) = 1(1+1) + 2(2+1) + 3(3+1) = 20$
   $g(3) = \frac{3(3+1)(3+2)}{3} = 20$

3. Prove recursion stays in D.
   The recursive relation is $f(n) = f(n-1) + n(n+1)$. Recursion is only called if $n \geq 1$ so in all cases $n-1 \geq 0$. So, the call is made with a value in D.

4. Prove that recursion halts.
Using the counting strategy, you can see that n decreases from n to $n-1$, so recursion halts.

5. Check that P is inherited recursively.
Assume $P(n-1)$ is true.
$f(n) = f(n-1) + n(n+1)$
$= g(n-1) + n(n+1)$
$= \frac{(n-1)(n)(n+1)}{3} + n(n+1)$
$= \frac{(n-1)(n)(n+1)}{3} + \frac{3n(n+1)}{3}$
$= \frac{(n-1)(n)(n+1)+3n(n+1)}{3}$
$= \frac{(n^3-n)+(3n^2+3n)}{3}$
$= \frac{n(n^2+3n+2)}{3}$
$= \frac{n(n+1)(n+2)}{3}$
$= g(n)$

6. Conclusion
After verifying steps 1-5, we can conclude that $P(n)$ is true for all elements in D. Thus, $f(n) = g(n)$ for all real numbers.

```python
#4
def f(n):
    if n <= 1:
        return n*(n+1)
    else:
        return n*(n+1) + f(n-1)

def g(n):
    return n*(n+1)*(n+2)/3


for n in range(1, 11):
    print 'n = %s, f(n) = %s, g(n) = %s' % (n, f(n), g(n))
```

```
C:\Users\Evan\Documents\MST Student Drive\CS1200\HW02>python hw02.py
n = 1, f(n) = 2, g(n) = 2
n = 2, f(n) = 8, g(n) = 8
n = 3, f(n) = 20, g(n) = 20
n = 4, f(n) = 40, g(n) = 40
n = 5, f(n) = 70, g(n) = 70
n = 6, f(n) = 112, g(n) = 112
n = 7, f(n) = 168, g(n) = 168
n = 8, f(n) = 240, g(n) = 240
n = 9, f(n) = 330, g(n) = 330
n = 10, f(n) = 440, g(n) = 440
```

5. Prove by induction that for all integers $n \geq 1$

$$1 + 6 + 11 + 16 + ... + (5n - 4) = \frac{n(5n-3)}{2}$$

1. Define the problem.
   - Domain is $1...\infty$.
   - Prove $f(n) = g(n)$ for all of D.

   $$f(n) = 1 + 6 + 11 + 16 + ... + (5n - 4)$$

   $$g(n) = \frac{n(5n-3)}{2}$$

   - $P(n)$ is true if $f(n) = g(n)$.
2. Check stopping values and two more.
   $f(1) = 1$
   $g(1) = \frac{1(1*5-3)}{2} = 1$
   $f(2) = 1 + 6 = 7$
   $g(2) = \frac{2(2*5-3)}{2} = \frac{2(7)}{2} = 7$
   $f(3) = 1 + 6 + 11 = 18$
   $g(3) = \frac{3(3*5-3)}{2} = \frac{36}{2} = 18$
3. Inductive Case.
   $f(n) = f(n-1) + (5n-4)$
   $= g(n-1) + (5n-4)$
   $= \frac{(n-1)(5(n-1)-3)}{2} + (5n-4)$
   $= \frac{(n-1)(5n-5-3)}{2} + \frac{2(5n-4)}{2}$
   $= \frac{(n-1)(5n-8)+(10n-8)}{2}$
   $= \frac{5n^2-5n-8n+8+10n-8}{2}$
   $= \frac{5n^2-3n}{2}$
   $= \frac{n(5n-3)}{2}$
   $= g(n)$
4. Conclusion.
   $P(n)$ is true for all of n in D. Therefore

   $$1 + 6 + 11 + 16 + ... + (5n - 4) = \frac{n(5n-3)}{2}$$

   is true for all of n in D.

6. Prove by induction that for all natural numbers n

$$\sum_{i=0}^{n} \frac{i^2}{(2i-1)(2i+1)} = \frac{n(n+1)}{2(2n+1)}$$

1. Define the problem.
   - Domain is all natural numbers or $0 \ldots \infty$.

   $$f(n) = \sum_{i=0}^{n} \frac{i^2}{(2i-1)(2i+1)}$$

   $$g(n) = \frac{n(n+1)}{2(2n+1)}$$

   - Prove $f(n) = g(n)$ for all of D.
   - $P(n)$ is true if $f(n) = g(n)$.
2. Check stopping values and two more.
   $f(0) = \frac{0}{(2*0-1)(2*0+1)} = 0$
   $g(0) = \frac{0(0+1)}{2(2*0+1)} = 0$
   $f(1) = \frac{1^2}{(2*1-1)(2*1+1)} = \frac{1}{3}$
   $g(1) = \frac{1(1+1)}{2(2*1+1)} = \frac{1}{3}$
   $f(2) = 0 + \frac{1}{3} + \frac{2^2}{(2*2-1)(2*2+1)} = \frac{1}{3} + \frac{4}{15} = \frac{9}{15} = \frac{3}{5}$
   $g(2) = \frac{2(2+1)}{2(2*2+1)} = \frac{6}{10} = \frac{3}{5}$
3. Inductive Case.

   $$\begin{aligned}
   f(n) &= f(n-1) + \tfrac{n^2}{(2n-1)(2n+1)} \\
   &= g(n-1) + \tfrac{n^2}{(2n-1)(2n+1)} \\
   &= \tfrac{(n-1)(n)}{2(2(n-1)+1)} + \tfrac{n^2}{(2n-1)(2n+1)} \\
   &= \tfrac{n(n-1)}{2(2n-1)} + \tfrac{n^2}{(2n-1)(2n+1)} \\
   &= \tfrac{n(n-1)(2n+1)}{2(2n-1)(2n+1)} + \tfrac{2n^2}{2(2n-1)(2n+1)} \\
   &= \tfrac{n(n-1)(2n+1)+2n^2}{2(2n-1)(2n+1)} \\
   &= \tfrac{2n^3-n^2-n+2n^2}{2(2n-1)(2n+1)} \\
   &= \tfrac{2n^3+n^2-n}{2(2n-1)(2n+1)} \\
   &= \tfrac{n(2n^2+n-1)}{2(2n-1)(2n+1)} \\
   &= \tfrac{n(2n-1)(n+1)}{2(2n-1)(2n+1)} \\
   &= \tfrac{n(n+1)}{2(2n+1)} \\
   &= g(n)
   \end{aligned}$$

4. Conclusion.
   $P(n)$ is true for all of n in D. Therefore

   $$\sum_{i=0}^{n} \frac{i^2}{(2i-1)(2i+1)} = \frac{n(n+1)}{2(2n+1)}$$

   is true for all of n in D.