CS2500 Project 3
0-1 Knapsack

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1 Motivation

The 0-1 knapsack is a classic optimization problem in computer science. Optimization problems can be seen in everyday life such as a CPU scheduler, a delivery route planner, or a car's brake controller. All of these problems have a solution but usually that solution is costly to generate. Developers instead create an algorithm that will generate an approximate solution but in much less time. Your trading accuracy for time. This report will analyse two algorithms and their approximate equivalent to determine the benefit of this trade off.

2 Background

In the 0-1 knapsack problem you are given a set of items each with a weight, w_i, and a value, v_i. The goal is to put a combination of items in a knapsack that has a capacity W that maximizes the profit of the items in the knapsack.

One solution to the 0-1 knapsack is implementing its recurrence directly into code using recursion. The recurrence is as follows:

$$f(i, W) = \begin{cases} \max\{f(i-1, W), f(i-1, W-w_i) + v_i\} \\ f(i-1, W) \end{cases}$$

with i being the number of items. Implemeting the recurrence directly in code will give you the solution but is very slow running in 2^n time because it recomputes subproblems many times over.

The recurrence can also be implemented iteratively. Using a 2d array you can store solutions to subproblems as you go so they only have to be computed once. This method will provide the same solution as the above recurrence but will run in linear time.

Another approach is the greedy method. The greedy method first sorts the items in decreasing order by its value per weight, v_i/w_i. The greedy algorithm proceeds to add the items to the knapsack unless the item won't fit or until the bag is full. Because this approach makes "greedy" choices, it is not a solution to the 0-1 knapsack but instead a close approximation. Even though he greedy algorithm runs in linear time making only one decision for each item, add the item or don't, it must first sort the items which runs in $n\lg n$ time.

3 Procedures

1. Express the three approaches in pseudocode.
2. Implement the three algorithms in c++.
3. Measure run times of the three algorithms to experimentally determine their run time complexity and compare to their expected run time complexity.
4. Develop and implement a testing plan.
5. List problems encountered during development.
6. Produce a conclusion addressing the efficacy of the methods used.

4 Pseudocode

Recursive Knapsack

```
knapSack(W, n)
if n == 0 or W == 0
    return 0
if wt[n − 1] > W
    return knapSack(W, wt, val, n − 1)
else
    return MAX{val[n − 1] + knapSack(W − wt[n − 1], wt, val, n − 1),
               knapSack(W, wt, val, n − 1)}
```

Iterative Knapsack

```
iterativeKnapSack(W, n)
new 2d array K[n + 1][W + 1]

for i = 1 to n
    for w = 1 to w
        if i == 0 or w == 0
            K[i][w] = 0
        else if wt[i − 1] <= w
            K[i][w] =MAX{val[i − 1] + K[i − 1][w − wt[i − 1]], K[i − 1][w]}
        else
            K[i][w] = K[i − 1][w]
return K[n][W]
```

Greedy Knapsack

```
greedyKnapSack(W, n)

sort(W, wt, val, n)
for i = 1 to n
    if wt_i <= W
        profit+ = val_i
        W- = wt_i
return profit
```

5 Testing Plan

All three algorithms were tested with the following testing plan:

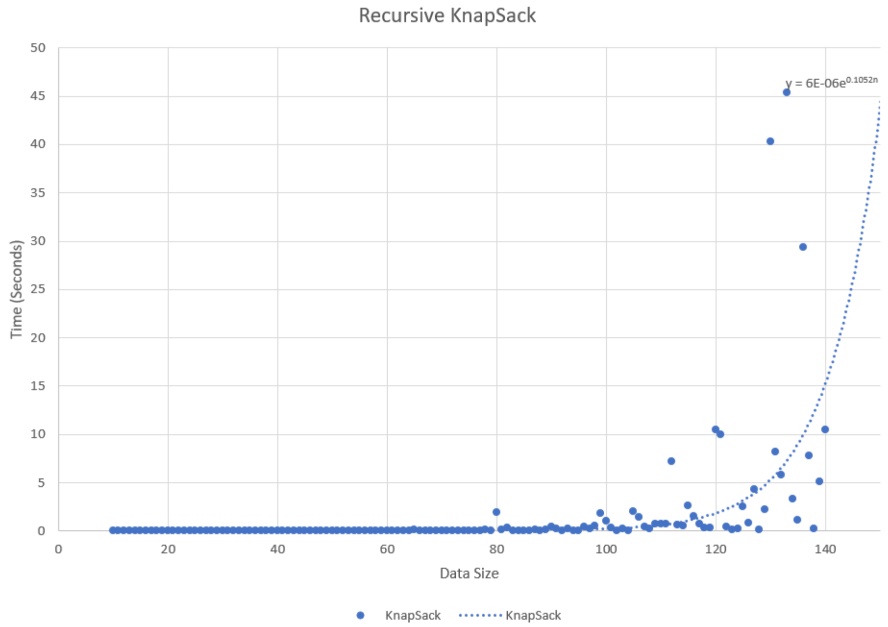
Input	**Expected Output**
Empty array, $n = 0$	Profit of 0
$W = 0$	Profit of

6 Problems Encountered

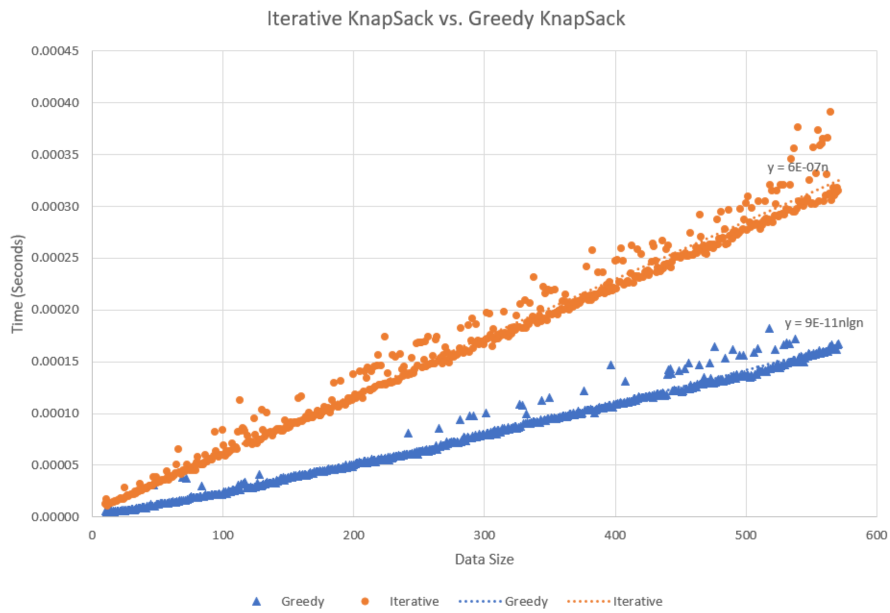
One problem encountered was that the iterative knapsack algorithm hit a segmentation fault error when the problem size was greater than about 500. I narrowed this problem down to an error int the method used to allocate the memory used for the 2d array. Also my recursive knapsack algorithm was not finding the best solution until I realised there was an error in my recursive case.

7 Performance Results

After testing each algorithm with varying sets of data, all three algorithms followed their expected run time complexity. The recursive knapsack algorithm started to show its expected runtime of 2^n at around $n \approx 100$.



The greedy approach actually ended up having a lower average run time than the iterative version even though the iterative version has a better runtime complexity of n vs. the greedy method's $n\lg n$. Both algorithms started showing their runtime complexities at around $n \approx 10$.



8 Conclusion

Both the recursive and iterative versions of the 0-1 knapsack will provide the correct solution to the given data set with the iterative version doing it much faster because of it's linear complexity. However, the greedy algorithm beat the iterative algorithm despite it's worse expected runtime complexity. I suspect this is because even though the greedy algorithm needs to sort the items the amount of items being sorted is relatively small so it doesn't contribute much to the overall runtime. Also the iterative algorithm used in testing dynamically allocates a 2d array to store the solutions to the subproblems while the greedy version doesn't allocate any new memory.

In conclusion, if you want the solution to the 0-1 knapsack you should use the iterative version because it is the fastest way to always get the solution. But if you are okay with a small margin of error, the greedy version will generate an approximate solution faster than the iterative version will generate the full solution.

Appendix A - Source Code

```
/*
* Author: Evan Wilcox
* File: main.cpp Date: 4/4/2019
* Class: CS 2500 Sec. 1
* Instructor : Bruce McMillin
* Main file for Project 3
*/

#include <iostream>
#include <fstream>
#include "knapSack.h"
#include <time.h>
using namespace std;


int main()
{
  ofstream fout;
  fout.open("out.csv");

  const int MAX = 100000;

  int val[MAX];
  int wt[MAX];

  Item arr[MAX];
  int W = 50;

  clock_t t;
  fout << "N," << "KnapSack," << "Greedy," << "Iterative," << endl;

  int n;
  for(n = 10; n < MAX; n++)
  {
    for(int i = 0; i < n; i++)
    {
      int v = (rand() % 100) + 51;
      int w = (rand() % 50) + 1;

      val[i] = v;
      wt[i] = w;

      arr[i].m_val = v;
      arr[i].m_wt = w;
    }

    fout << n << ",";
    cout << n << endl;
```

```
    t = clock();
    knapSack(W, wt, val, n);
    fout << (float)(clock()-t)/CLOCKS_PER_SEC << ",";

    t = clock();
    greedyKnapSack(W, arr, n);
    fout << (float)(clock()-t)/CLOCKS_PER_SEC << ",";

    t = clock();
    iterativeKnapSack(W, wt, val, n);
    fout << (float)(clock()-t)/CLOCKS_PER_SEC;

    fout << endl;
  }

  fout.close();

  return 0;
}

/*
* Author: Evan Wilcox
* File: knapSack.h Date: 4/4/2019
* Class: CS 2500 Sec. 1
* Instructor : Bruce McMillin
* Header file for knapSack.cpp
*/

#ifndef KNAPSACK_H
#define KNAPSACK_H

using namespace std;

/*
* Struct: Item
* A pairing of an item's weight and value.
*/
struct Item
{
  int m_val;
  int m_wt;

  Item(int val, int wt) : m_val(val), m_wt(wt){}
  Item() : m_val(0), m_wt(0){}
};

/*
* Function: compare
* Description: function used to compare two items by their m_val/m_wt
```

```
* Pre: a.m_wt != 0 and b.m_wt != 0
* Post: an int is returned
* Param a : an item
* Param b : an item
* Return:  1 if a's m_val/m_wt is less than b's,
          -1 if b's m_val/m_wt is greater than a's,
           0 if they are equal
*/
int compare(const void * a, const void * b);

/*
* Function: greedyKnapSack
* Description: a greedy implementation of the 0-1 knapsack
* Pre: W >= 0, arr[i].m_wt > 0, n >= 0
* Post: arr is sorted by arr[i].m_val/arr[i].m_wt ratio
* Param W : capacity remaining in the knapsack
* Param arr : array of items to try and put in the knapsack
* Param n : index of item in arr to try and put in the knapsack
* Return: greedy approximate to 0-1 knapsack problem
*/
int greedyKnapSack(int W, struct Item arr[], int n);

/*
* Function: knapSack
* Description: recursive implementation of the 0-1 knapsack
* Pre: W >= 0, wt[i] > 0, n >= 0
* Post: none
* Param W : capacity remaining in the knapsack
* Param wt : array of weights of items
* Param val : array of values of items
* Param n : index of item in wt/val to try and put in the knapsack
* Return: solution to knapsack problem
*/
int knapSack(int W, int wt[], int val[], int n);

/*
* Function: iterativeKnapSack
* Description: iterative implementation of the 0-1 knapsack
* Pre: W >= 0, wt[i] > 0, n >= 0
* Post: none
* Param W : capacity remaining in the knapsack
* Param wt : array of weights of items
* Param val : array of values of items
* Param n : index of item in wt/val to try and put in the knapsack
* Return: solution to knapsack problem
*/
int iterativeKnapSack(int W, int wt[], int val[], int n);

/*
* Function: max
```

```
* Description: A simple max function
* Pre: none
* Post: an int is returned
* Param a : an int
* Param b : an int
* Return: a if a > b, otherwise b
*/
int max(int a, int b);

#endif

/*
* Author: Evan Wilcox
* File: knapSack.cpp Date: 4/4/2019
* Class: CS 2500 Sec. 1
* Instructor : Bruce McMillin
* Implementation file for three solutions to the 0-1 knapSack
*/

#include "knapSack.h"


int compare(const void * a, const void * b)
{
  double r1 = (double)(*(Item*)a).m_val / (*(Item*)a).m_wt;
  double r2 = (double)(*(Item*)b).m_val / (*(Item*)b).m_wt;

  if( r1 <  r2 )
  {
    return 1;
  }
  else if( r1 >  r2 )
  {
    return -1;
  }
  else
  {
    return 0;
  }
}


int greedyKnapSack(int W, struct Item arr[], int n)
{
  qsort(arr, n, sizeof(arr[0]), compare);

  int curWeight = 0;
  double finalvalue = 0.0;

  for(int i = 0; i < n; i++)
```

```
  {
    if(curWeight + arr[i].m_wt <= W)
    {
      curWeight += arr[i].m_wt;
      finalvalue += arr[i].m_val;
    }
  }

  return finalvalue;
}


int knapSack(int W, int wt[], int val[], int n)
{
  if(n == 0 || W == 0)
  {
    return 0;
  }

  if(wt[n-1] > W)
  {
    return knapSack(W, wt, val, n-1);
  }
  else
  {
    return max(val[n-1] + knapSack(W-wt[n-1], wt, val, n-1), knapSack(W, wt, val, n-1));
  }
}


int iterativeKnapSack(int W, int wt[], int val[], int n)
{
  int i, w;

  int** K = new int*[n+1];
  for(int j = 0; j < n+1 ; j++)
  {
    K[j] = new int[W+1];
  }

  for(i = 0; i <= n ; i++)
  {
    for (w = 0; w <= W ; w++)
    {
      if(i == 0 || w == 0)
      {
        K[i][w] = 0;
      }
      else if(wt[i-1] <= w)
      {
```

```
        K[i][w] = max(val[i-1] + K[i-1][w-wt[i-1]], K[i-1][w]);
      }
      else
      {
        K[i][w] = K[i-1][w];
      }
    }
  }

  for(int j = 0; j < n+1 ; j++)
  {
    delete K[j];
  }
  delete K;

  return K[n][W];
}


int max(int a, int b)
{
  return (a > b) ? a : b;
}
```