CS 1200 FS18 HW 06

Due Monday 11/26/18 at 11:59 PM

Please submit two files to Canvas (one is a PDF file and the other is the code .py file) :

1. A PDF file that contains all the answers to the individual questions, all pictures, all code, and all code output. This should all be well-organized. Points will be deducted for sloppy or disorganized work.
2. All the Python codes (.py file) (You may put all codes in one .py file).

If you need a program that helps you put PDF files together into a single PDF file, try http://www.pdfsam.org/. The program there is open source and available for free.

Note: Partial credit will be given on every problem.

HW6 Problems:

1. (10 pts) Let $A = \{2, 3, 4, 5, 6\}$ and $B = \{-5, 3, 7, 10, 15, 33, 37\}$. Let R be the relation on A and B defined by aRb iff a divides b.
   (a) List the pairs in R.
   (b) Represent the relation R as a 0,1-matrix.
2. (10 pts) Let R be the relation defined on $\mathbb{Z}$ as follows: for all m, n $\in \mathbb{Z}$, mRn iff m-n is odd. Determine whether R is reflexive, symmetric, antisymmetric or transitive.
3. (20 pts) Let $A = \{0, 1, 2, 3, 4\}$ and
   $$R = \{(0, 0), (0, 4), (1, 1), (1, 3), (2, 2), (3, 1), (3, 3), (4, 0), (4, 4)\}.$$
   (a) Is R reflexive? symmetric? antisymmetric? transitive? Is R an equivalence relation? a partial order? a total order? a well order? Justify your answers.
   (b)Use the Python functions defined in Section 12.7 to verify your answers. Submit a listing of the functions together with a run of the functions.
4. (15pts) Let $A = \{1, 2, 3, 4, \ldots, 20\}$. Let R be the relation
   $$\{(a, b)|\text{ a, b in A and } 0 = ((a - b) \text{ MOD } 4)\}$$
   (a) Draw the directed graph representing R. If R can be represented by an undirected graph, draw it?
   (b) Draw a 0,1-matrix to represent R with the rows and columns labeled.
   (c) Is the relation R an equivalence relation? a partial order? a total order?a Well order? Justify your answers.
5. (15pts) Let $A = \{0, 1, 2, 3, 4, 5\}$, let f: A$\rightarrow$A be given by $f(0) = 5, f(1) = 3, f(2) = 0, f(3) = 0,\ f(4) = 4$, and $f(5) = 1$.
   (a) Draw a bipartite graph representation of f.

(b) Determine if f is an injection, a surjection, or a bijection.

(c) Write a Python program to test whether the functions are injections, surjections or bijections.

6. (20pts) Let f:$\mathbb{Z} \to \mathbb{Z}$ be given by $f(n) = 3n + 2$ and g:$\mathbb{Z} \to \mathbb{Z}$ be given by $g(n) = 4n + 1$.
(a) Write the formulas for $g \circ f$ and $f \circ g$.
(b) Determine whether each of f, g, $g \circ f$ and $f \circ g$ are injections, surjections or bijections.
(c) Compute $(g \circ f)^{-1}(\{-5, -3, 0, 7, 9, 21, 22, 23, 45\})$ and $(f \circ g)^{-1}(\{-7, 0, 5, 7, 9, 17, 22, 41\})$.
(Note $(g \circ f)^{-1}$ and $(f \circ g)^{-1}$ denote the inverse image of $g \circ f$ and $f \circ g$ respectively.)

7. (10pts) Let $\mathbb{N}$ be the set of natural numbers and f:$\mathbb{N} \to \mathbb{N}$ be the function $f(n) = 5n + 4$. What is

$$f^{-1}(\{4, 5, 8, 9, 10, 11, 13, 14, 15, 24, 30\}?$$

(Note f^{-1} denotes the inverse image of f.)