In this project, you will devise a numerical method to study the *wave equation*. The wave equation is one of the most important partial differential equations in engineering because it is used to model seismic waves, deformation of elastic rods (and related elastic media), motion of strings, and dynamics of acoustic waves (i.e., sound), to name just a few! In one spatial dimension (1D) , the simplest form that the wave equation takes is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \tag{1}$$

where c is a constant referred to as the wave speed. Note that there are many variants to (1) that can arise from having a wave speed c that depends on space, presence of a prescribed forcing function, etc. In addition, we know from class that to be able to determine a solution to the wave equation, one requires an appropriate set of initial conditions and boundary conditions that depend on the problem being chosen.

In this project, you will select a problem within a topic area where one of the 1D wave equation variants provides a meaningful model, determine which variant of the wave equation (and which initial and boundary conditions) is best suited to that problem, derive and implement a numerical method to solve that wave equation, and use this method to explore meaningfully chosen hypothetical situations/scientific questions associated with the problem you have chosen. Finally, you will write a report that clearly describes the problem, wave equation used, numerical method, and results that reflect insights you have obtained about the problem. Here is a further description of each of the tasks you should accomplish:

1. Select a problem that interests you within a topic area that can be represented meaningfully by the 1D wave equation. I suggest you stick to one topic from either (i) deformation of elastic rods, (ii) acoustics, or (iii) seismic waves. These three topics are conducive to interesting problems involving a 1D wave equation. There are other topics (e.g., the motion of a plucked string) that are modeled effectively by the wave equation but that require at least two spatial dimensions. *I suggest you do not pick a topic that requires a wave equation in 2D or higher, as adding this complexity is beyond the scope of this class*.

   (a) As part of selecting an interesting problem, you should determine which variant of the wave equation you should use (e.g., do you need to incorporate a spatially varying wave speed, and how does this modify equation (1) above?), what the appropriate boundary and initial conditions are, etc. Also, be thoughtful to select a problem that is conducive to meaningfully testing/validating your numerical method.

   (b) Please do not underestimate this part of the project. Reading through topics (i)–(iii), identifying a problem that interests you, and determining a meaningful wave equation for representing that problem (as well as the appropriate boundary and initial conditions) will take time and research. Make sure you give this part of the project the attention it deserves while allowing for enough time to complete the remaining parts as well.

2. Once you have selected a problem that drives your interest, develop a numerical method to solve this problem. You have significant freedom here. Your method may be a finite difference method, finite element method, or something not covered in class. Even if you do use a finite difference or finite element method, you need not use the specific methods described in class. Be sure that you are able to express your method using clear, precise mathematical expressions and textual descriptions, and that you are able to justify your choice of method through sound arguments.

3. Code up your method and demonstrate that your implementation exhibits the expected convergence rates. Also, test your method against meaningful validation problems that convincingly demonstrate that your results compare favorably against relevant external data sets and that you are using the correct values of Δx, Δt, etc.

4. Explore a scientific question/hypothetical scenario that really intrigues you about your problem. Again, there is significant flexibility here, just be sure to pick something that interests you and allows for a systematic study that you can convey clearly and compellingly.

   What do I mean by an intriguing scenario? You do not need to create a hypothetical scenario involving a fictitious world (though you are welcome to do so if it allows for a clear question/set of questions involving the wave equation to be investigated!). What you do need to do is create a clear narrative for what variant of the wave equation, initial and boundary conditions, and parameters you are considering, and have these choices be justified in understandable terms.

In your exploration of these scientific questions/hypothetical scenarios, you should create figures, tables, and other data that clearly and meaningfully connect to the question(s) you have laid out for your project.

5. Write a report that
   (a) Thoughtfully lays out how you have addressed points (1)-(4). Think of points (1)-(4) as the rubric by which I will be evaluating your report, so make sure that I can clearly see how you have addressed each point in your report.
   (b) Your report should be typed (*e.g.*, using LaTeXor Word) in a well-structured format, with equations and figures provided as appropriate. The tone should be professional and thoughtful; this is a formal report worth 35% of your semester grade. There is no page limit, *per se*; just make sure you have convincingly and clearly addressed points (1)-(4).

Remember that this is worth 35% of your grade, so I am looking for you to go well beyond what is expected from a homework submission, and take ownership of your work on this project. Going "beyond what is expected" will mean different things to different people. For some of you, that may mean exploring new numerical methods not considered in class. For others, it may mean taking a method used in class (and explaining it appropriately), and having the "beyond" part be an extensive exploration into your physical problem. Either option is great; just be sure to work hard and have fun!

A note on time investment. To enable you to fully engage with your project, there will be no new course content or additional HWs from here on out. As a hypothetical, let us say that I had not stopped course content, and you had to learn new material this next week and a half (4/24-5/3) as well as complete a new HW. Let's allocate 3 hours of "lecture time" for the new material, and say that the new HW would have taken 8 hours to complete. This means that for just a "normal" week, the total hours spent should be around 11. Likewise, you have an additional few days after next week to complete the project by its due date of May 8, so the total time is $\sim$22 hours for a "normal" 2 weeks of class-related time investment. Of course, this is not meant to be literal, but I really want to emphasize that I am trying to remove any other AE 370 work so that you can really prioritize this project, and I am expecting you to prioritize the project in a significant way, starting this week. This project is worth 35% of your grade, so please, please, treat it as such. I know this project is asking a lot: from identifying new problems in potentially unfamiliar topics to developing a numerical method for a new PDE, this is an ambitious undertaking. So work hard, allocate the requisite time for this project, use the resources available to you, and have a blast!

You will work in a small assigned group of students to complete this project. You may divvy up the work as you see fit: everyone can work equally on every part of the project or you can provide team leads for things like defining the problem statement, picking and justifying the method, writing the report, etc. ***Whatever of these options you take, be sure that every group member contributes meaningfully to the various topics (1)–(5) above. There will be a survey that each student will be asked to fill out after submitting the report, and if there is a consensus that one or more of the team members was not a significant contributor, there will be a substantial grade based penalty depending on the severity of the infraction.***

A note on the deadline. Because of the amount of time allocated to you to complete this project, as well as the fact that I can not assign the due date after the official final exam time for this class, there will be no extensions granted for this project. Please be aware of the time this project will take, and do not procrastinate working on this to the final week.

A sample document structure is:

1 Meaningfully worded introduction

Motivate the problem you chose, why it is worth studying, what premise/hypothetical scenario/scientific questions you are exploring.

2 The wave equation

Describe with clear language and appropriate mathematical precision which wave equation you are studying, what the initial and boundary conditions are, etc., and why these are the right fit for your problem.

3 The numerical method(s)

3.1 Description/overview

General description/derivation of the method and how you are implementing it.

3.2 Justification for this choice of method(s)

Use clear arguments to justify your choice of method.

3.3 Demonstration of correct implementation

Use convergence tests and comparisons with relevant external data/analytical results to convince the reader that you have correctly implemented your method, and that the results you present use a sufficiently small value of Δt and Δx.

4 Meaningfully named results section

The structure here will be up to your specific investigations, and should have logically designated subsections with clear descriptions of the parameters considered and well-reasoned explanations of the results (with compelling and easy-to-read figures and an appropriate associated discussion) that clearly tie back to your premise/hypothetical scenario/scientific questions.

5 Conclusions

Summarize your choice of method and key findings. Provide an outlook for follow-up investigations, extensions to the method, *etc.*

6 Appendix: code used