RICH SIMSON
EECS 725
HWK 2

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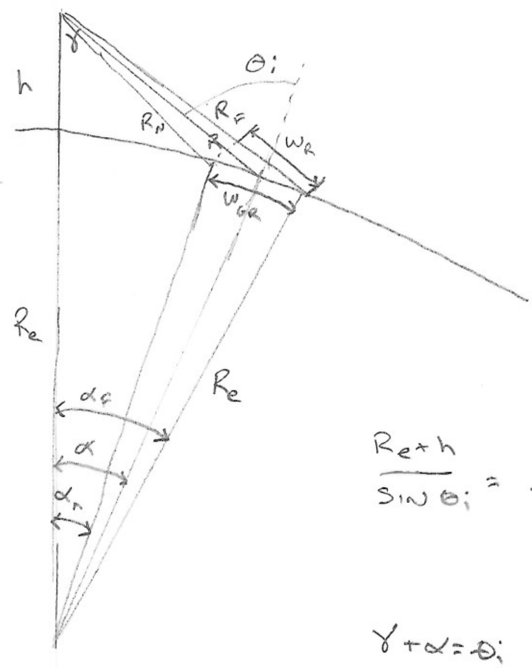

$h = 330$ km

$R_e = 6378.145$ km

$W_{gr} = 50$ km

$\theta_i = 37°$

$f = 1$ Ghz

$l = 5$ m

$\tau = 50\ \mu s$

$$\frac{R_e+h}{\sin\theta_i} = \frac{R_e}{\sin\gamma} \Rightarrow \sin\gamma = \frac{R_e}{R_e+h}\sin\theta_i$$

$$\Rightarrow \gamma = \sin^{-1}\left(\frac{R_e}{R_e+h}\sin\theta_i\right) = 34.90°$$

$$\gamma + \alpha = \theta_i \Rightarrow \alpha = \theta_i - \gamma = 37 - 34.90° = 2.10°$$

$$W_{gr} = 50\text{ km} = (\alpha_F - \alpha_n) R_e \Rightarrow \alpha_F - \alpha_n = 7.84\times10^{-3}\text{ rad}$$

$$R = \sqrt{R_e^2 + (R_e+h)^2 - 2R_e(R_e+h)\cos\alpha} = 407.59\text{ km}$$

$$R_N = \sqrt{R_e^2 + (R_e+h)^2 - 2R_e(R_e+h)\cos\left(\alpha - \frac{\alpha_F - \alpha_n}{2}\right)} = 393.09\text{ km}$$

$$R_F = \sqrt{R_e^2 + (R_e+h)^2 - 2R_e(R_e+h)\cos\left(\alpha + \frac{\alpha_F - \alpha_n}{2}\right)} = 423.14\text{ km}$$

$$T_N = \frac{2R_N}{c} = 2.621\text{ ms} \qquad T_F = \frac{2R_F}{c} = 2.821\text{ ms}$$

$$PRF_{max} \leq \frac{1}{2\tau + T_F - T_N} = 3326.6\text{ hz}$$ (due to swath width)

$$v = \sqrt{\mu/(R_e+h)} = 7.708\text{ km/s}$$

$$PRF_{min} = \frac{2v}{l} = \frac{2(7708)}{5} = 3083.4\text{ hz}$$ (due to Doppler constraints)

Using $PRF_{min} \leq \frac{N-1}{T_N - \tau} < PRF < \frac{N}{T_F + \tau} \leq PRF_{max}$, N integers,

Only $N=9$ satisfies PRF_{min} (3083.4 hz) and PRF_{max} (3326.6 hz) constraints

SO $$\frac{8}{2.6 - 0.05\text{ ms}} = \boxed{3112.1\text{ hz} < PRF \leq 3134.9\text{ hz}} = \frac{9}{2.8 + 0.05\text{ ms}}$$

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Normally, swath width is determined by $\beta = \frac{\lambda}{l}$, which are functions of frequency (λ) and geometry (l). However, if swath width is unchanged (even with a change in λ), PRF ranges should stay the same.

For PRF_{min}, we know that it is $\underbrace{\text{independent}}$ of λ:

$$PRF_{min} = 2B_{dop} = 2\frac{v}{\lambda}\beta \approx 2\frac{v}{\cancel{\lambda}}\left(\frac{\cancel{\lambda}}{l}\right) = 2\frac{v}{l} \qquad \text{(no dependence on } \lambda)$$

For PRF_{max}, pulse duration τ and echo times T_{near}/T_{far} are dependencies which do not depend on λ (again, assuming beamwidth remains constant)

$$PRF_{max} \leq \frac{1}{2\tau + T_{far} - T_{near}} \qquad \text{(no dependence on } \lambda)$$

Similarly, for multiple pulses in the air,

$$\frac{N-1}{T_{near} - \tau} < PRF < \frac{N}{T_{far} + \tau} \qquad \text{(no dependence on } \lambda)$$

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$$PRF_{max} \leq \frac{1}{2\tau + T_F - T_N}, \quad T = \frac{2R}{c}$$

$$\leq \frac{1}{2\tau + \frac{2R_F}{c} - \frac{2R_N}{c}} = \frac{1}{2\tau + \frac{2}{c}(R_F - R_N)}, \quad R_F - R_N = \text{Swath width (slant range)}$$

As slant range swath width increases, PRF_{max} decreases

Let $PRF_{max} = 3112.1$ hz (from answer for PRF_{min} in problem 1)

$$PRF_{max}\left(2\tau + \frac{2}{c}(R_F - R_N)\right) = 1$$

$$\frac{2}{c}(R_F - R_N) = \frac{1}{PRF_{max}} - 2\tau$$

$$R_F - R_N = \frac{c}{2}\left(\frac{1}{PRF_{max}} - 2\tau\right) = 33{,}199 \text{ km} = w_r$$

$$w_{gr} = \frac{w_r}{\sin\theta_i} = \frac{33.199 \times 10^3}{\sin 37^\circ} = 55.16 \text{ km} \quad \text{max swath width}$$