RICH SIMEON
EECS 725
HWK 1
2/15/17

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% EECS725 Homework 1

clear;
close all;
hold off;

% Universal constants
c = 3e8;              % speed of light (m/s)

% Problem constraints
f       = 1e9;        % carrier frequency (hz)
l_x     = 50 / 100;   % antenna array width (m)
l_y     = 10 / 100;   % antenna array height (m)
v_x     = 90;         % x speed (m/s)
h       = 1000;       % elevation (m)
offset  = -2000;      % target offset (m)
sigma   = 1;          % target rcs (m^2)

x = linspace(-2000,2000,100);   % position vectors (m)

%% Problem 1
% Compute radial distance
R = sqrt(h^2 + offset^2 + x.^2); % radial distance (m)

figure(1);
plot(x,R);
grid on;
axis([-2000 2000 2000 3000]);
title('Radial distance vs. x');
xlabel('x (m)');
ylabel('R (m)');

%% Problem 2
% Compute offset angle between aircraft vector and radial vector
gamma = acos(-x./R); % offset angle (radians)

figure(2);
plot(x,gamma*180/pi);
grid on;
axis([-2000 2000 40 140]);
title('Offset angle vs. x');
xlabel('x (m)');
ylabel('gamma (degrees)');

%% Problem 3
% Compute target elevation (theta) relative to dish orientation
theta = acos(h./R) - pi/4; % (radians) subtract 45 degrees for dish tilt
phi   = atan2(-offset,-x) - pi/2; % (radians) rotate phi -90 degrees to align to antenna
vector

lambda = c/f;            % wavelength (m)
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beta_xz = lambda / l_x; % 3dB BW (radians)
beta_yz = lambda / l_y; % 3dB BW (radians)
beta_phi = beta_xz;
beta_theta = beta_yz;
Aeff = l_x * l_y;       % effective antenna array area (m^2)
Go = ((4*pi)/(lambda^2))*Aeff; % max antenna gain
G = Go * (sin(2.773.*theta/beta_theta)./(2.773.*theta/beta_theta)).^2...
      .* (sin(2.773.*phi   /beta_phi)   ./(2.773.*phi   /beta_phi))   .^2;

figure(3);
plot(x,10*log10(G));
grid on;
title('Antenna gain vs. x');
xlabel('x (m)');
ylabel('gain (dB)');

%% Problem 4
% Compute power ratio of received to transmit power from target reflection
PrPt = (G.^2)*(lambda^2)*sigma ./ ( (4*pi)^3 * R.^4 );

figure(4);
plot(x,10*log10(PrPt));
grid on;
title('Power ratio (Pr/Pt) vs. x');
xlabel('x (m)');
ylabel('Pr/Pt (dB)');

%% Problem 5
% Compute Doppler shift
f_d = 2 * v_x * cos(gamma)/lambda; % alternative calculation

% Alternative calculation for Doppler shift calc (dR/dt)
v_r = v_x * x./R; % radial velocity (m/s) = (dR/dx)(dx/dt)
f_d2 = (2*v_x.*-x)./(lambda*R);

figure(5)
plot(x,f_d);
hold on
plot(x,f_d2);
grid on;
title('Doppler shift vs. x');
xlabel('x (m)');
ylabel('Doppler shift (hz)');
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RICH SIMSON
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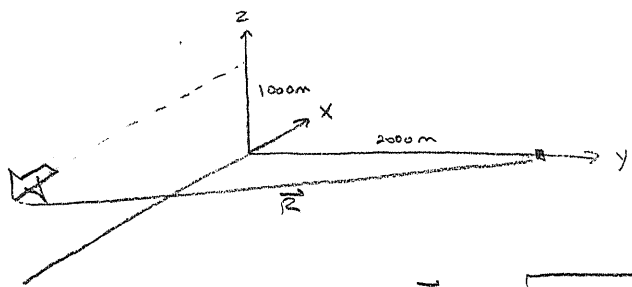
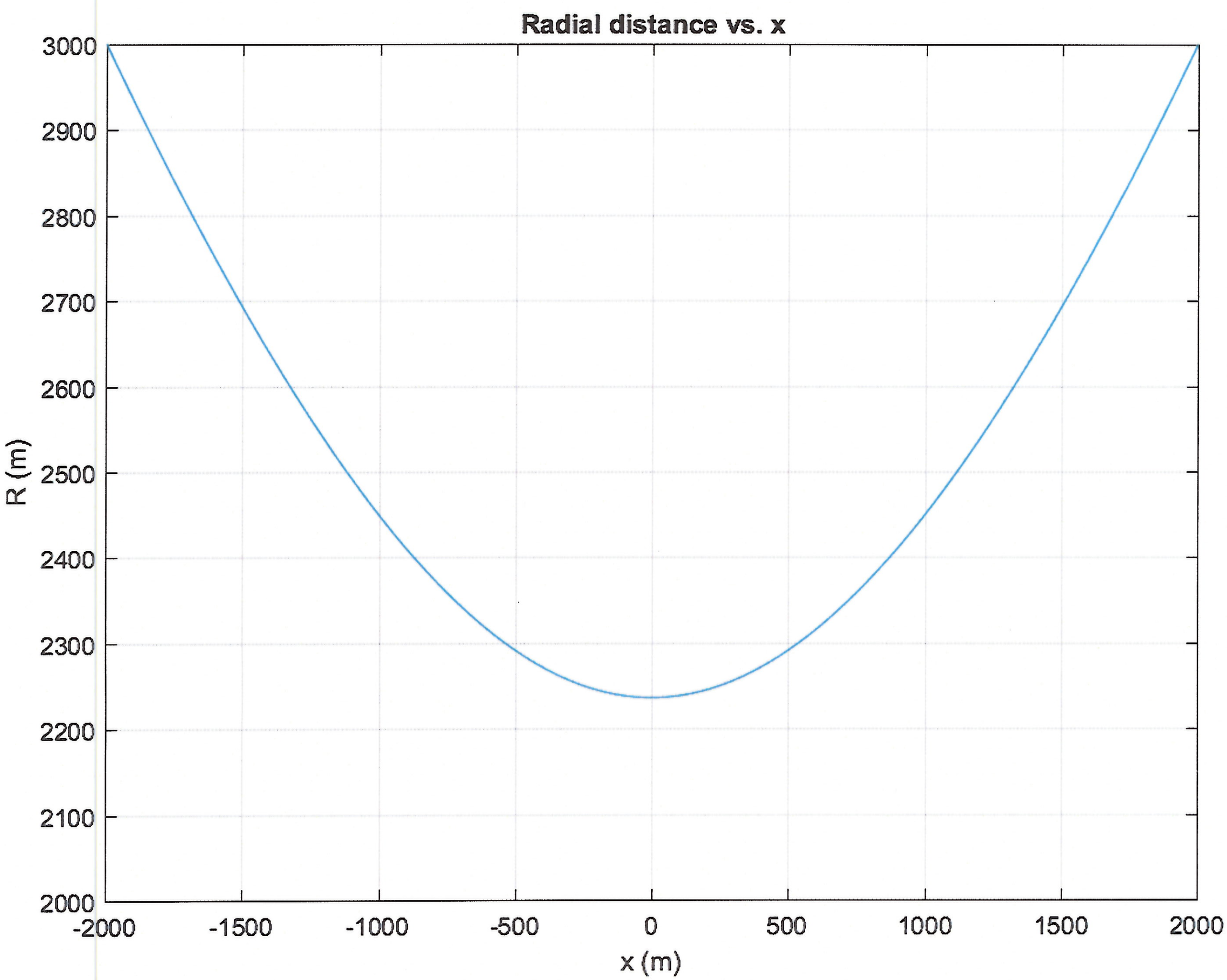
1



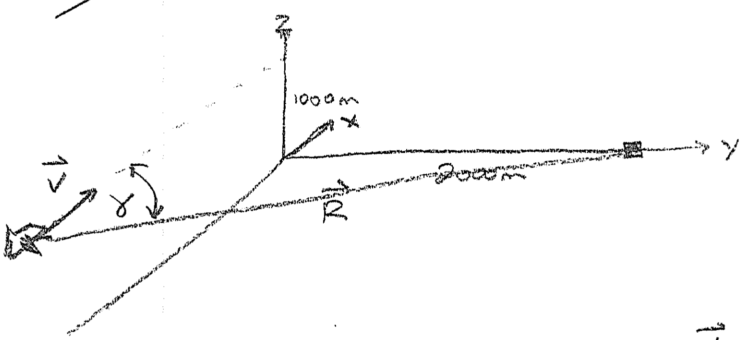
$$\|\vec{R}\| = \sqrt{1000^2 + 2000^2 + x^2}$$

$$= \sqrt{5\times10^6 + x^2}$$



Radial distance vs. x
3000
2900
2800
2700
2600
2500
2400
2300
2200
2100
2000
R (m)
-2000
-1500
-1000
-500
0
500
1000
1500
2000
x (m)

2



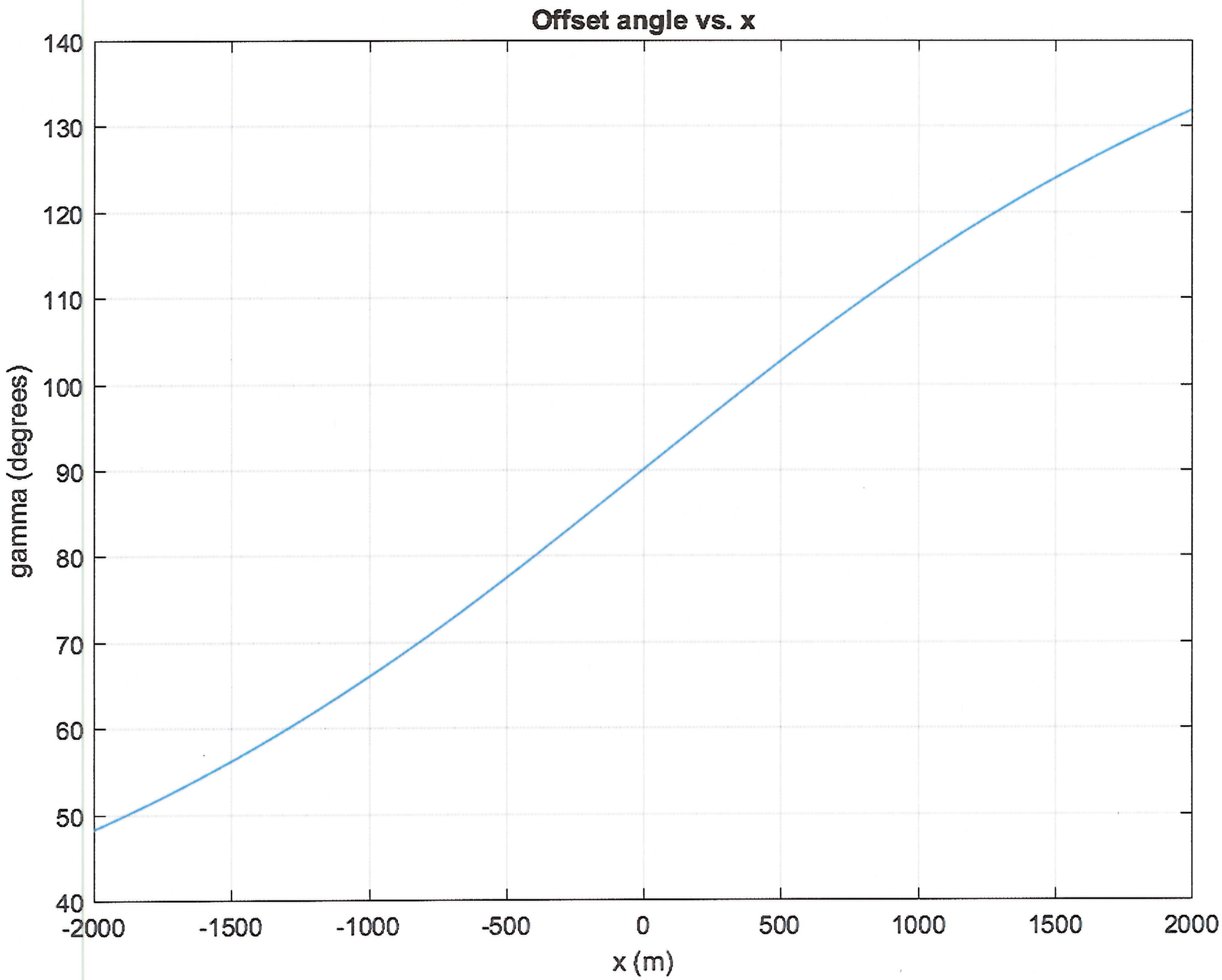
$$\cos\gamma = \frac{\vec{V}\cdot\vec{R}}{\|\vec{V}\|\|\vec{R}\|}$$

For simplicity, let $\vec{V} = 1\vec{x} + 0\vec{y} + 0\vec{z}$ (unit vector length)

$$\vec{V}\cdot\vec{R} = (1\cdot -x) + (0\cdot y) + (0\cdot z) = -x$$

$$\|\vec{V}\| = 1$$
$$\|\vec{R}\| = R$$

$$\gamma = \cos^{-1}\left(\frac{-x}{R}\right)$$


Offset angle vs. x
gamma (degrees)
x (m)
140
130
120
110
100
90
80
70
60
50
40
-2000
-1500
-1000
-500
0
500
1000
1500
2000

3

The $\frac{\sin(x)}{x}$ antenna radiation pattern is

$$G(\theta, \phi) = G_0 \left[\frac{\sin \frac{2.773\theta}{\beta_\theta}}{\frac{2.773\theta}{\beta_\theta}} \right]^2 \left[\frac{\sin \frac{2.773\phi}{\beta_\phi}}{\frac{2.773\phi}{\beta_\phi}} \right]^2$$

Let $\beta_\theta = \beta_{xz} = \frac{\lambda}{l_x} = \frac{30\text{cm}}{50\text{cm}} = 0.6 \text{ rad}$

$\beta_\phi = \beta_{yz} = \frac{\lambda}{l_y} = \frac{30\text{cm}}{10\text{cm}} = 3.0 \text{ rad}$, $\lambda = c/f = 3\times10^8 / 1\times10^9 = 30 \text{ cm}$

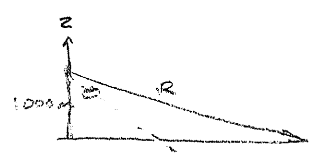
$G_0 = \frac{4\pi}{\lambda^2} \eta_1 A_{eff}$, $\eta_1 = 1$

$A_{eff} = 50\times10 \text{ cm}^2 = 500 \text{ cm}^2$

$= \frac{4\pi}{(0.3)^2}(1)(0.05) = 6.9813$

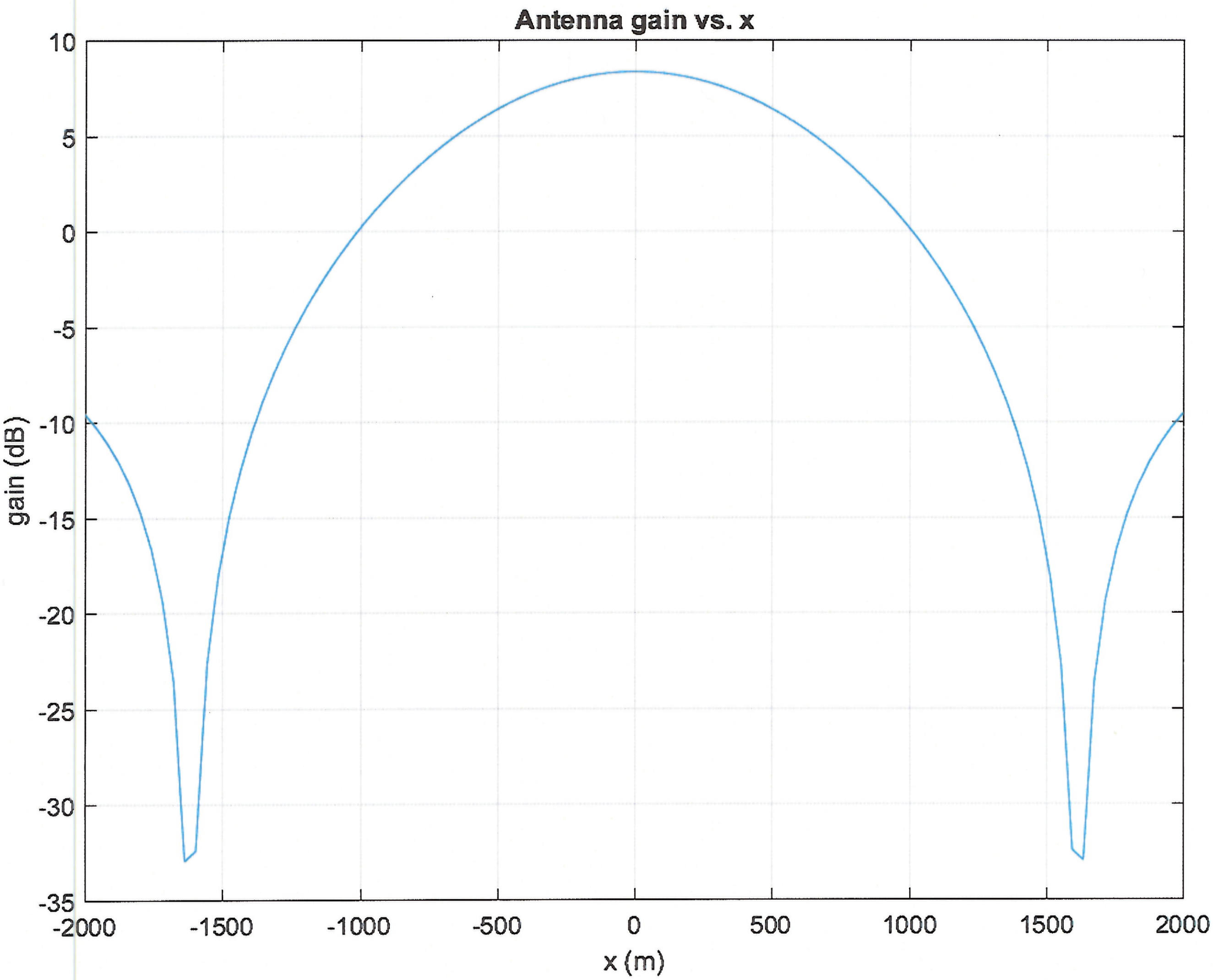
$\phi = \tan^{-1}\left(\frac{2000\text{ m}}{-x}\right) - \pi/2$ rotates ϕ 90° clockwise to align to antenna vector

$\theta = \cos^{-1}\left(\frac{1000}{R}\right) - \frac{\pi}{4}$



accounts for antenna dish tilt downwards (135° from zenith)

$$G(\theta, \phi) = 6.9813 \left[\frac{\sin 4.6217\theta}{4.6217\theta} \right]^2 \left[\frac{\sin 0.9243\phi}{0.9243\phi} \right]^2$$



Antenna gain vs. x
gain (dB)
x (m)
10
5
0
-5
-10
-15
-20
-25
-30
-35
-2000
-1500
-1000
-500
0
500
1000
1500
2000

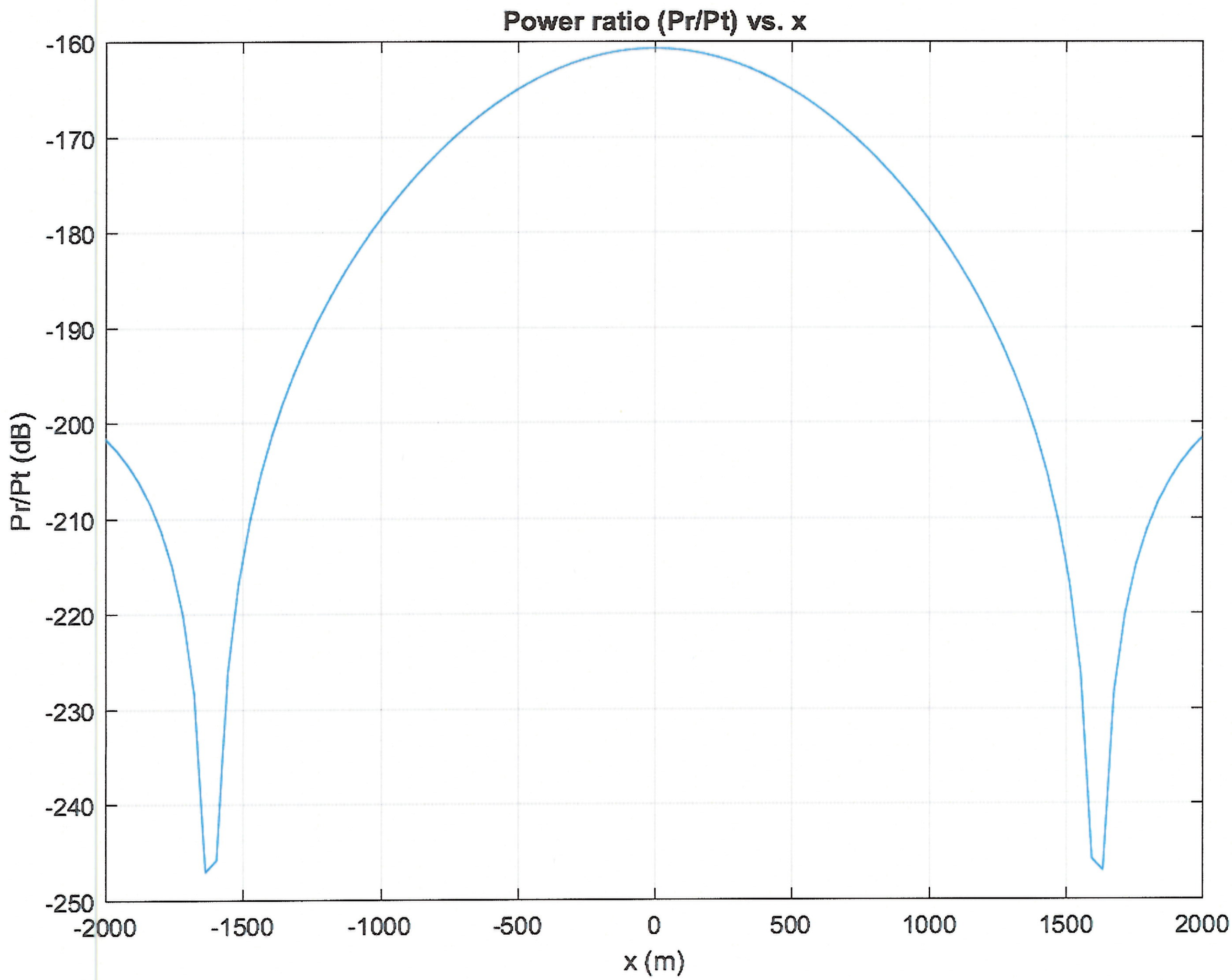
4 Radar range equation

$$\frac{P_r}{P_t} = \frac{G^2 \lambda^2 \sigma L}{(4\pi)^3 R^4} \quad , \quad \sigma = 1\ m^2$$

$L = 1$

$\lambda = c/f = 0.3\ m$

G as given in problem 3 (SINC radiation pattern)



Power ratio (Pr/Pt) vs. x
-160
-170
-180
-190
-200
-210
-220
-230
-240
-250
Pr/Pt (dB)
-2000
-1500
-1000
-500
0
500
1000
1500
2000
x (m)

5

Doppler shift is given by

$$f_d = \frac{2v_x \cos(\gamma)}{\lambda}$$

Alternatively, $f_d = \frac{2v_R}{\lambda}$, $V_R = \frac{dR}{dt} = \frac{dR}{dx}\frac{dx}{dt}$

$$\frac{dx}{dt} = 90 \text{ m/s}$$

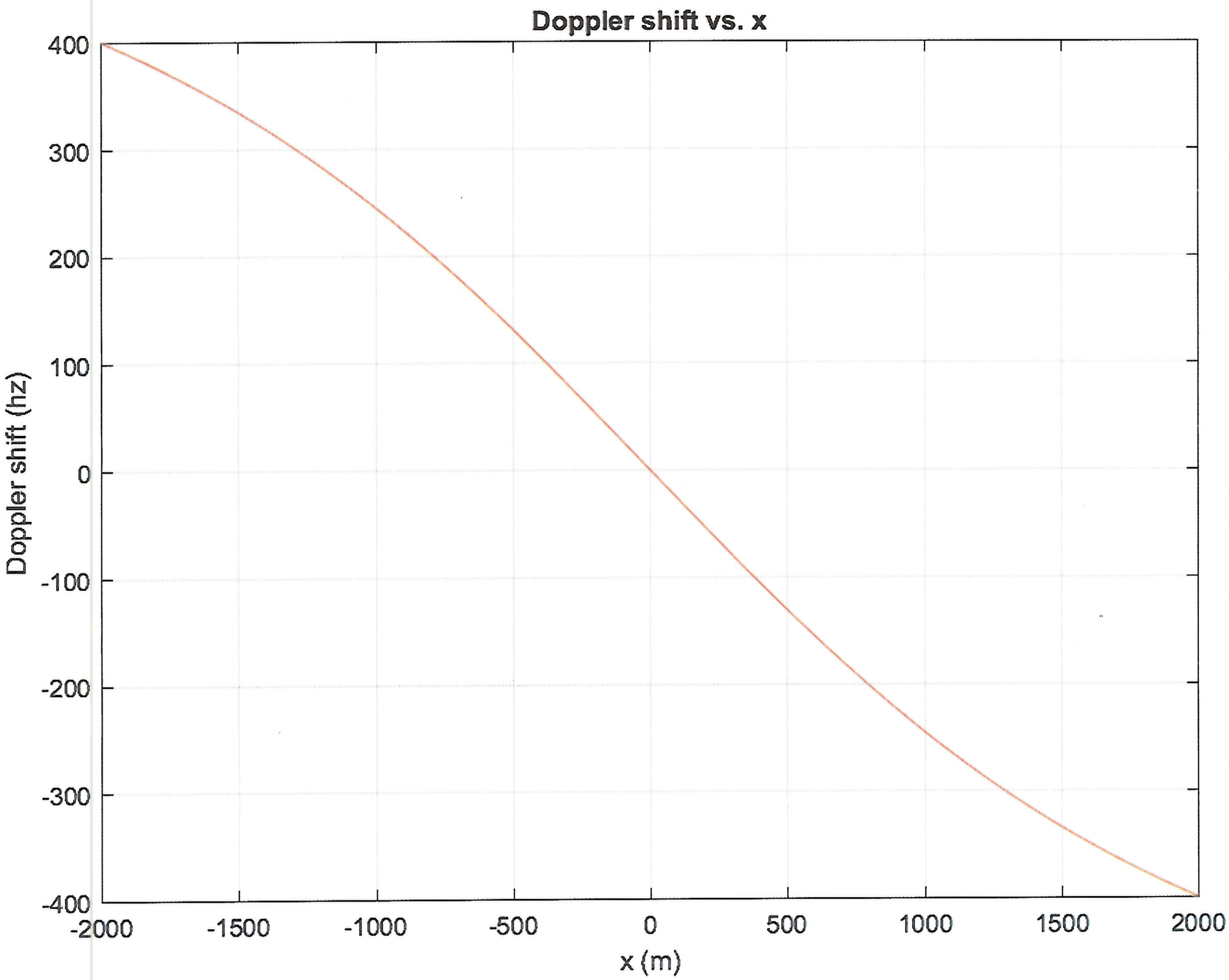
$$\frac{dR}{dx} = (5\times10^6 + x^2)^{1/2} \, d_x$$

$$= \frac{1}{2}(5\times10^6 + x^2)^{-1/2} \, 2x$$

$$= x(5\times10^6 + x^2)^{-1/2}$$

$$= \frac{x}{R}$$

$$f_d = \frac{2V_x x}{\lambda R} \qquad \Rightarrow V_R = V_x \frac{x}{R}$$


Doppler shift vs. x
400
300
200
100
0
-100
-200
-300
-400
Doppler shift (hz)
-2000
-1500
-1000
-500
0
500
1000
1500
2000
x (m)