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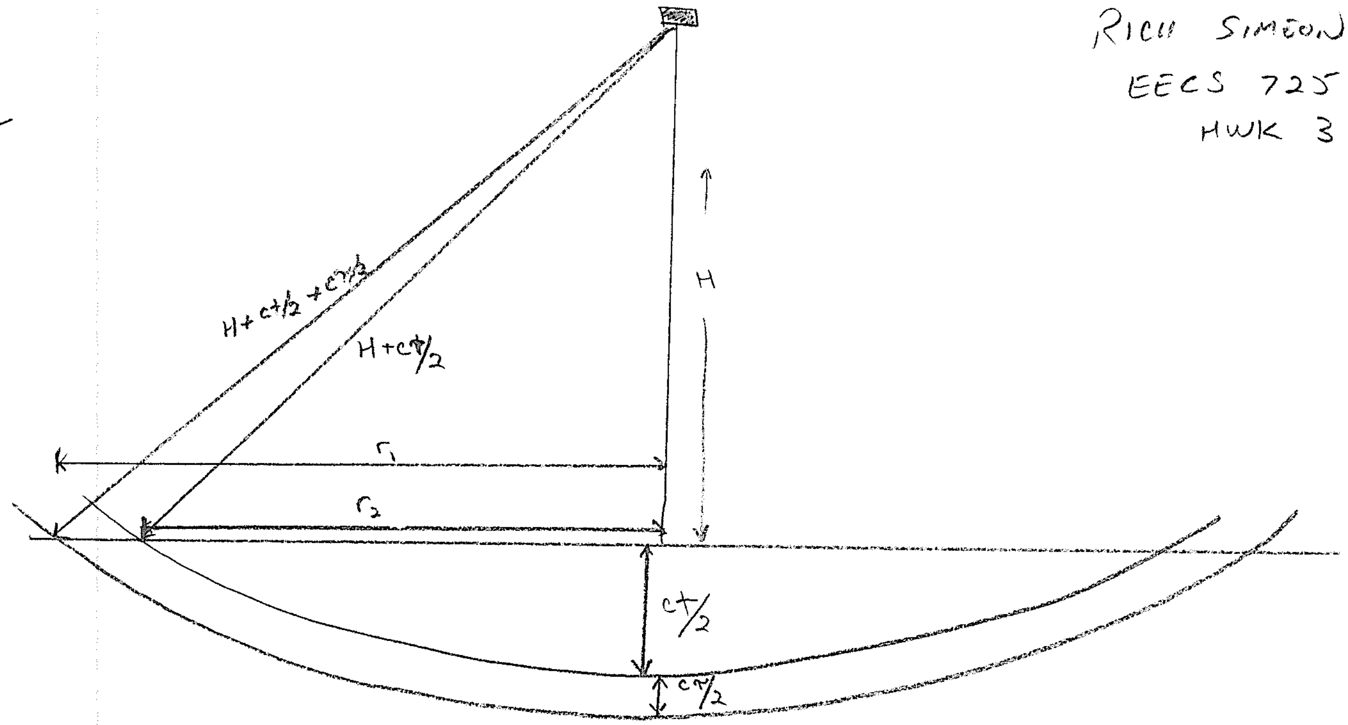
EECS 725
HWK 3

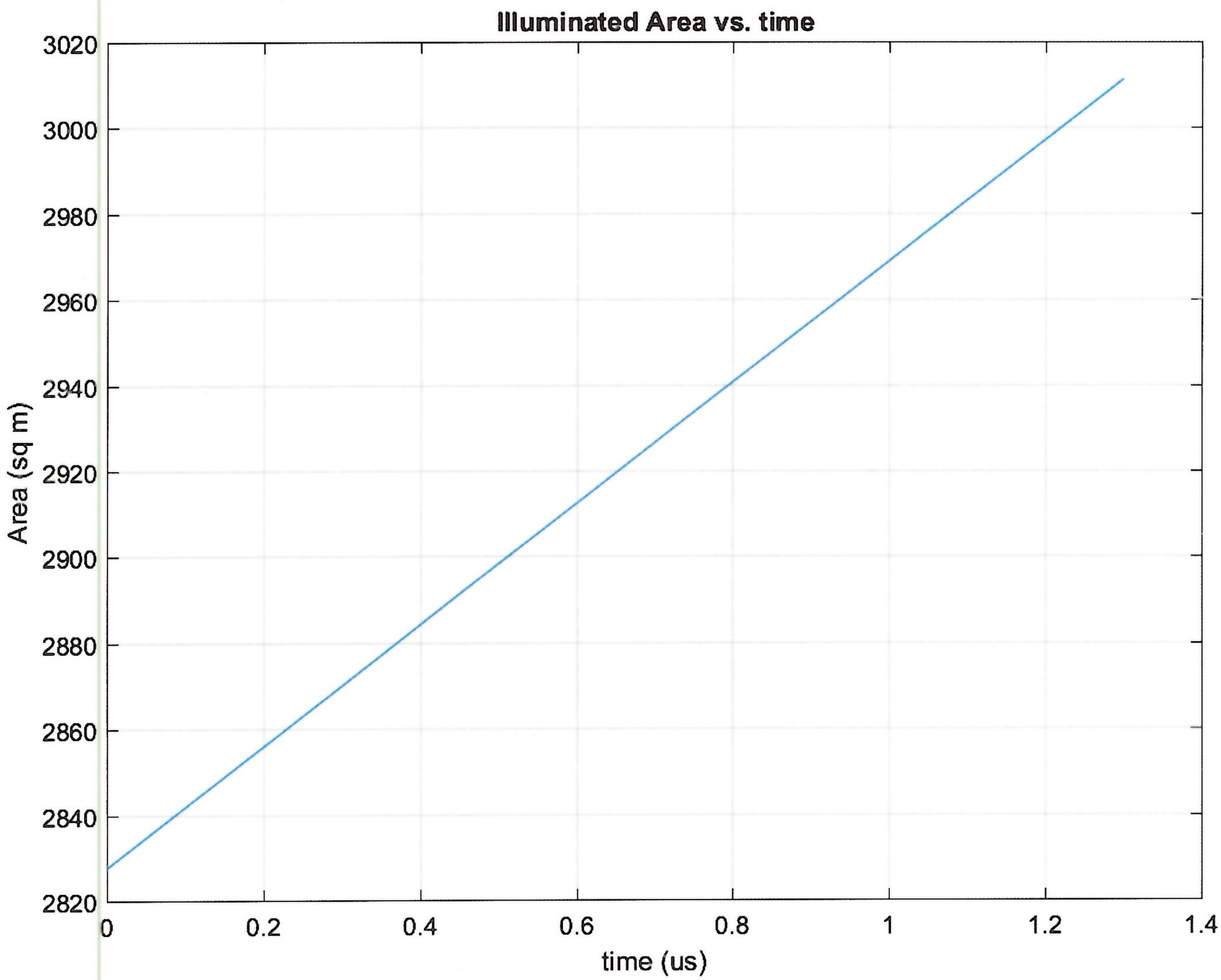
1



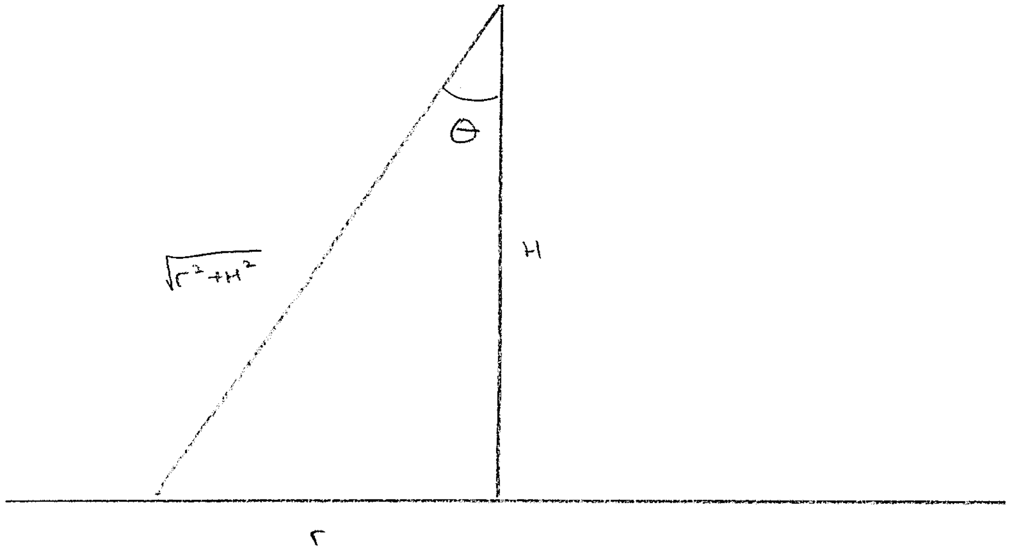
$$r_1^2 + H^2 = \left(H + \frac{ct}{2} + \frac{c\tau}{2}\right)^2 \Rightarrow r_1 = \sqrt{\left(H + \frac{c}{2}(t+\tau)\right)^2 - H^2}$$

$$r_2^2 + H^2 = \left(H + \frac{ct}{2}\right)^2 \Rightarrow r_2 = \sqrt{\left(H + \frac{ct}{2}\right)^2 - H^2}$$

$$\text{Area} = \pi r_1^2 - \pi r_2^2 = \pi\left(r_1^2 - r_2^2\right)$$

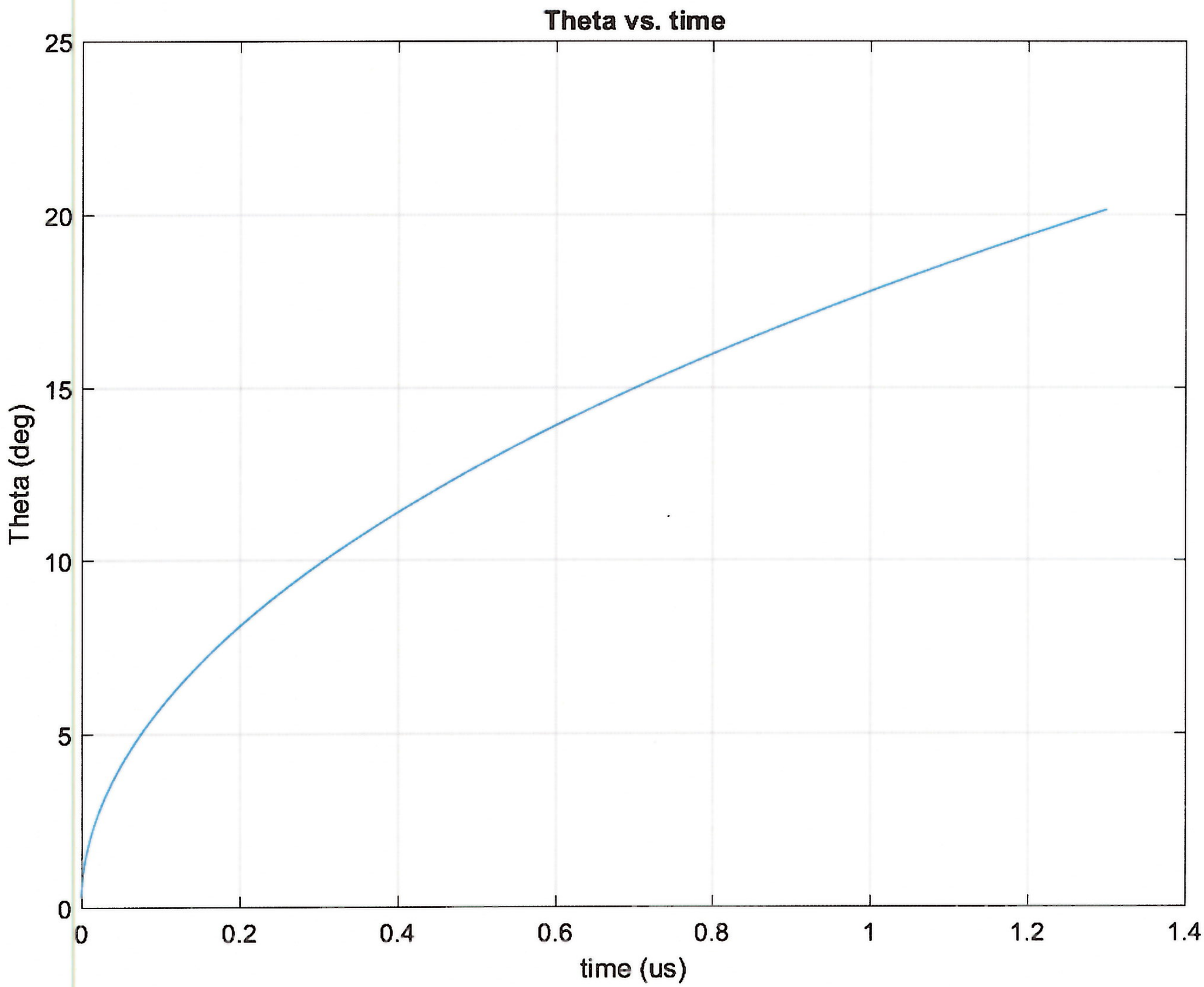

Illuminated Area vs. time
Area (sq m)
time (us)
3020
3000
2980
2960
2940
2920
2900
2880
2860
2840
2820
0
0.2
0.4
0.6
0.8
1
1.2
1.4

2



Let $r =$ average of r_1 & r_2

$$\Theta = \sin^{-1}\left(\frac{(r_1 + r_2/2)}{\sqrt{\left(\frac{r_1+r_2}{2}\right)^2 + H^2}}\right)$$


Theta vs. time
Theta (deg)
time (us)
0
5
10
15
20
25
0.2
0.4
0.6
0.8
1
1.2
1.4

3

Using $$\frac{P_r}{P_t} = \frac{\lambda^2 G^2 \sigma^\circ(\theta)\, \Delta A}{(4\pi)^3 R^4}$$

Let ΔA be the area calculated in problem 1

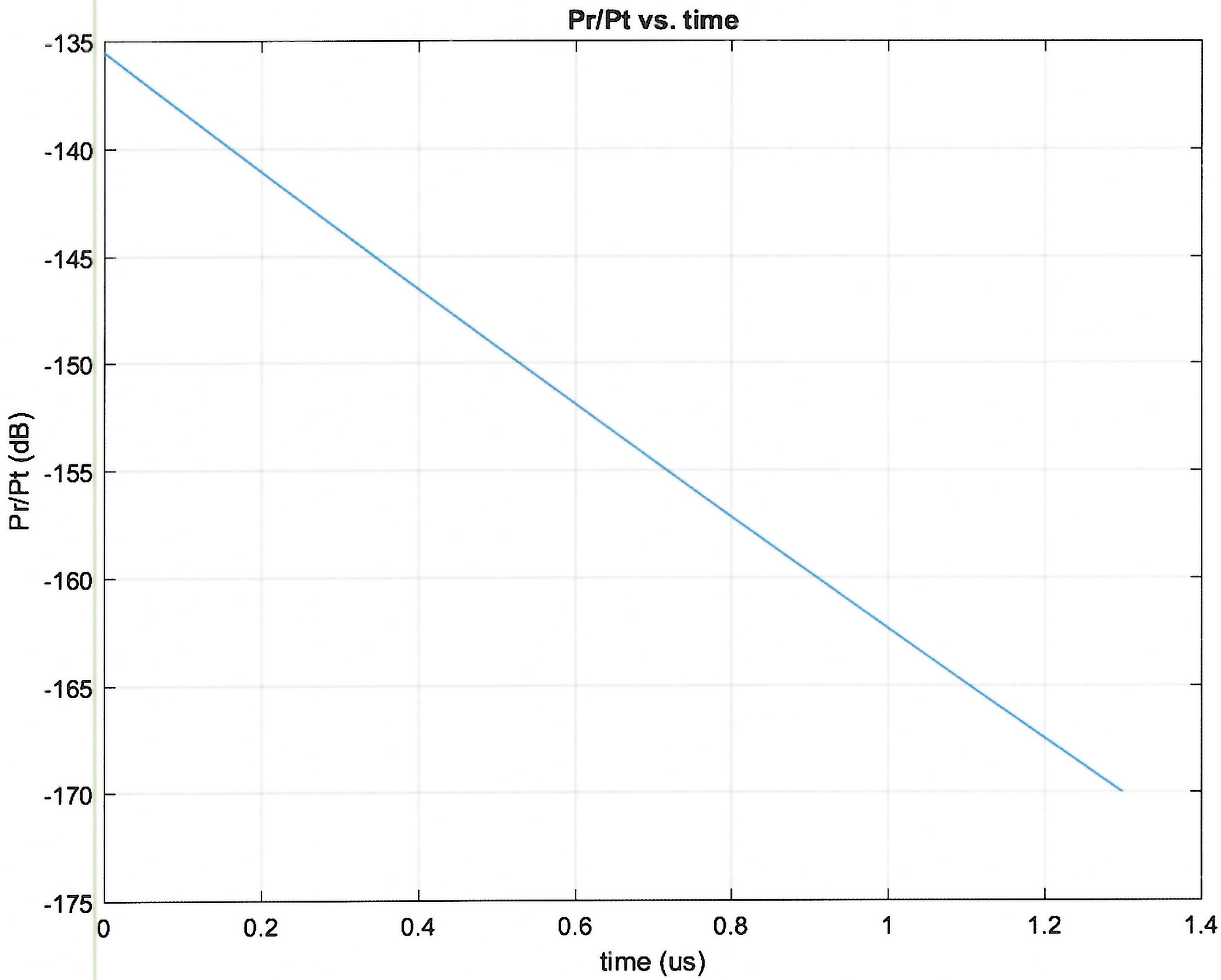
$\sigma^\circ(\theta) = \sigma^\circ(0)\cos^9(\theta)$ exponent 9 indicates a smooth surface

$\sigma^\circ(0) = 0.1$

Let G = Gaussian beampattern

$$G(\theta) = G_0 \exp\left[-2.773\left(\left(\frac{\theta}{\beta_\theta}\right)^2 + \left(\frac{\phi}{\beta_\phi}\right)^2\right)\right] \quad , \quad \theta = \phi \text{ (Symmetric)}$$

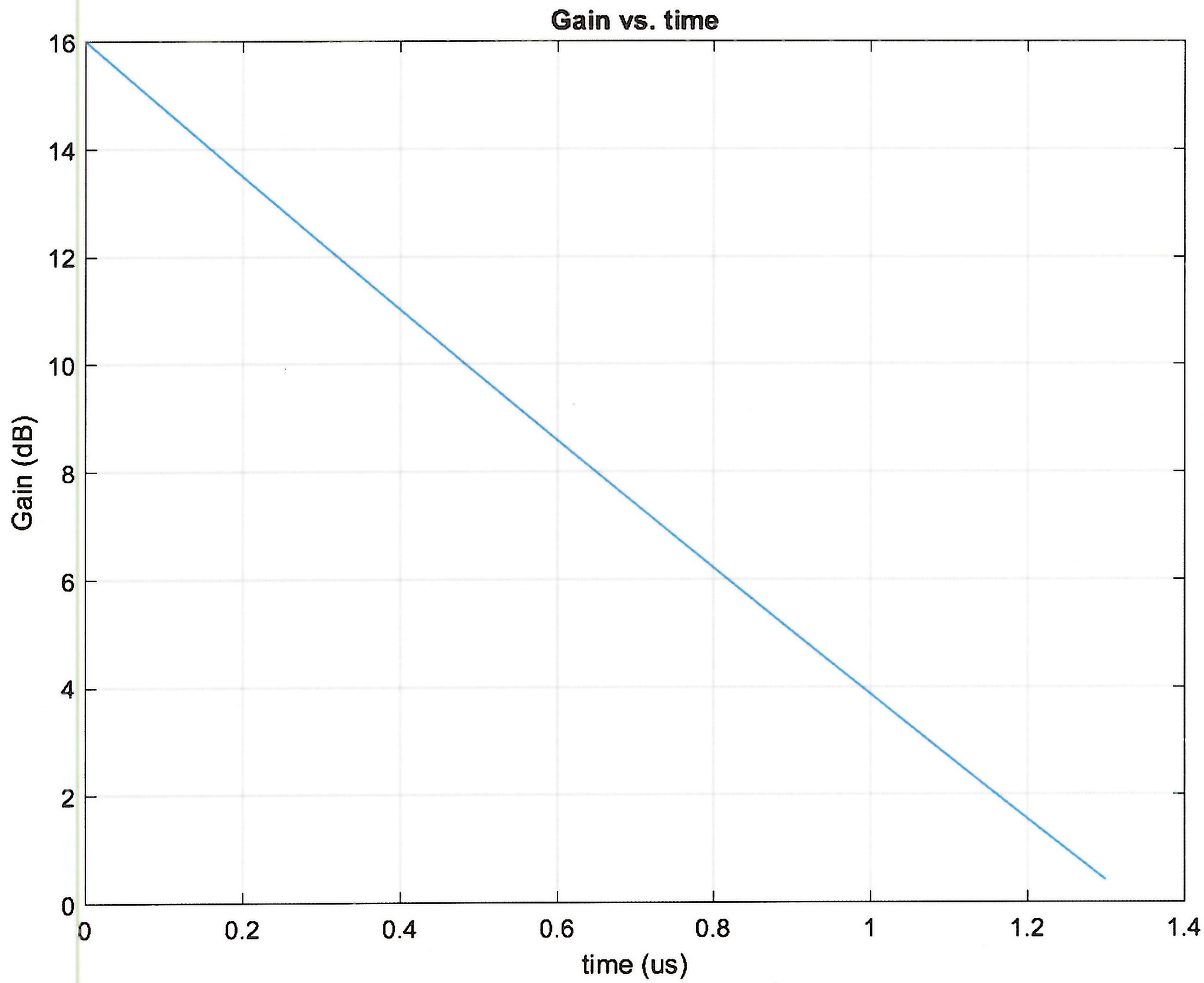
$\beta = 25^\circ = 0.43633$ rad $= \beta_\theta = \beta_\phi$ (Symmetric)



Pr/Pt vs. time
-135
-140
-145
-150
-155
-160
-165
-170
-175
Pr/Pt (dB)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
time (us)

4/ Based on the P_r/P_t plot in problem 3, the dynamic range of the receiver should be at least 35 dB (ratio was from -135 dB to -170 dB),

- The gain of the antenna as a function of θ contributed the most; it varied from 16 dB down to 0 dB (16 dB range). Because P_r/P_t is a function of G^2, G contributed $16 \times 2 = 32$ dB of the dynamic range

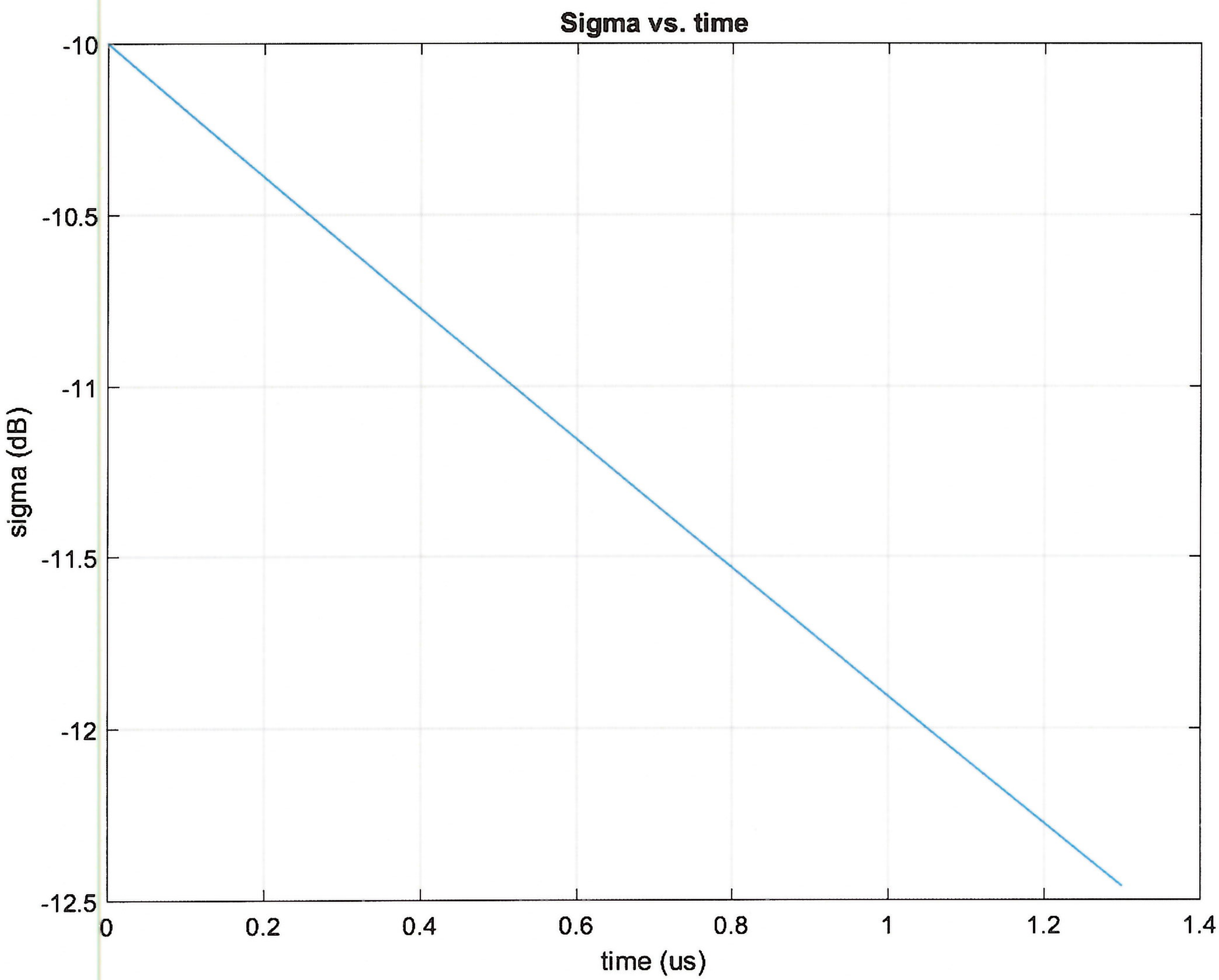


(from prob 1)

- Area increased by a factor of $10 \log_{10}\left(\frac{3010}{2830}\right) = 0.26$ dB, a negligable contribution to dynamic range

- R^4 increased by a factor of $10 \times 4 \times \log_{10}\left(\frac{3195}{3000}\right) = 1.08$ dB, also a negligable amount

σ^o varied by 2 dB over the time interval, so it did not have a big impact on dynamic range (see below)



```
% EECS725 Homework 3

clear;
close all;

% Universal constants
c           = 3e8;       % speed of light (m/s)

% Problem constraints
h           = 3e3;       % height (m)
lambda      = 10e-2;     % radar wavelength (m)
beta        = 25*pi/180; % 3dB beam width of antenna (Gaussian beam shape)
Go_dB       = 16;        % antenna peak gain (dB)
tau         = 1e-9;      % pulse duration (s)
sigma_o_0   = 0.1;       % backscattering coefficient

Go          = 10^(Go_dB/10); % antenna gain (linear)

% Simulation parameters
t_max       = 1.3e-6;    % max simulation time
N           = 1000;      % simulation granularity
t           = linspace(0,t_max,N); % time

%% Start simulation

t0 = h/c; % time that front part of transmitted pulse hits the earth
t1 = t0 + (c*tau/2); % time that back part of transmitted pulse hits the earth

% Compute radii for leading and trailing edge of wave pulse
r2 = sqrt((h + c.*t./2).^2 - h^2); % (m)
r1 = sqrt((h + (c.*t./2) + (c*tau/2)).^2 - h^2);% (m)
r2(1) = 0; % trailing edge has not contacted ground yet, so first time unit is zero
R = sqrt(((r1+r2)/2).^2 + h^2);

% Illuminated area (annulus)
area = pi*(r1.^2 - r2.^2); % (m)

% Theta (angle of center of annulus with respect to nadir)
theta = asin( ((r1+r2)./2) ./ sqrt(((r1+r2)/2).^2 + h^2) );% (rad)

% Antenna gain at theta
G = Go * exp(-2.773 * ((theta/beta).^2 + (theta/beta).^2));% (linear)

% Terrain backscaterring coefficient at theta
sigma_o = sigma_o_0 * (cos(theta).^9); % (linear)

% Distance from annulus center back to transmitter
R = sqrt( h^2 + ((r1+r2)/2).^2 ); % (m)

% Pr/Pt
PrPt = (lambda^2 * G.^2 .* area .* sigma_o) ./ ((4*pi)^3 * R.^4);% (linear)
```

```
%% Plots---------------------

figure(1)
plot(t*1e6,area);
title('Illuminated Area vs. time');
xlabel('time (us)');
ylabel('Area (sq m)');
grid on;

figure(2)
plot(t*1e6,theta*180/pi);
title('Theta vs. time');
xlabel('time (us)');
ylabel('Theta (deg)');
grid on;

figure(3)
plot(t*1e6,10*log10(PrPt));
title('Pr/Pt vs. time');
xlabel('time (us)');
ylabel('Pr/Pt (dB)');
grid on;

figure(4)
plot(t*1e6,10*log10(G));
title('Gain vs. time');
grid on;
xlabel('time (us)');
ylabel('Gain (dB)');
% Contribution is squared, so multiply dB spread by 2

figure(5)
plot(t*1e6,10*log10(sigma_o));
title('Sigma vs. time');
xlabel('time (us)');
ylabel('sigma (dB)');
grid on;

figure(6)
plot(t*1e6,r1);
hold on;
plot(t*1e6,r2);
title('r1 and r2 vs. time');
xlabel('time (us)');
ylabel('Radius (m)');
grid on;
```