Math for Investment Consultants

2nd Edition

Chapter 5: Performance Measurement

The reading material in the appendixes is optional, you will not be tested on it.

At this point it should be clear how extensive the calculations of devising a portfolio can be. Let's pull together some of what we have learned into the process of asset allocation.

Expected Return

The expected mean is the probability-weighted average of the possible outcomes of the random variable. The expected value of a random variable *X* is denoted *E*(*X*). The expected value looks at the future mean. The equation that summarizes expected value is

$$E(R) = P(R_1) \times R_1 + P(R_2) \times R_2 + \ldots + P(R_n) \times R_n = \sum_{i=1}^{n} P(R_i) \times R_i$$

Equation 5-1

Where, R_i is one of n possible outcomes of the random variable R.

Consider the following example:

Probability	Return (%)
0.25	5
0.35	15
0.40	25

The expected return is

$$E(R) = (0.25 \times 5) + (0.35 \times 15) + (0.40 \times 25) = 16.50\%$$

Expected Risk

Squaring the deviation of each occurrence from the mean and multiplying the result by its associated probability calculates the expected variance of a probability distribution. The sum of these values equals the variance of the distribution. The square root of the variance is then the standard deviation. The formula is

$$\sigma = \sqrt{p_1[R_1 - E(R)]^2 + p_2[R_2 - E(R)]^2 + \ldots + p_n[R_n - E(R)]^2}$$
$$= \sqrt{\sum_{i=1}^{n} p_i[R_i - E(R)]^2}$$

Equation 5-2

Where, $E(R)$ equals the expected or mean return, and R_i is the specific return outcome with probability p_i. Let us look at an example using the same return information from the previous question.

Return %	Probability	Probability Weighted Return	Differential Return from Mean	Probability Weighted Differential Return Squared
5	0.25	1.25	–11.50	33.06
15	0.35	5.25	–1.50	0.79
25	0.40	10.00	8.50	28.90
	10.00	16.50		62.75

The mean return is 16.50 percent, variance is 62.75, and the standard deviation is $\sqrt{62.75} = 7.92$

Portfolio Return

The expected return of a portfolio is the weighted average of the expected returns on the component securities.

$$E(R_p) = w_1 E(R_1) + w_2 E(R_2) + \dots + w_n E(R_n)$$

Equation 5-3

Where, w_n is the weight of an asset in a portfolio, and $E(R_n)$ is the expected return of that asset.

Examine a portfolio of three assets and assume the following risks and returns:

Table 5-1

Weights and Expected Returns		
Asset Class	Predicted Return %	Predicted Risk %
Stocks	10	15
Bonds	7	10
Cash	4	1

The portfolio consists of 55-percent stocks, 40-percent bonds, and 5-percent cash. Using the concept of weighting, the expected portfolio return is found to be 8.5 percent.

Table 5-2

Asset Class	Weight	Expected Return	Weight × Return
Stocks	0.55	10%	5.50 %
Bonds	0.40	7%	2.80%
Cash	0.05	4%	0.20%
		Sum:	8.50%

Portfolio Risk

Consider two securities, both with expected returns of 10 percent and standard deviations of 15 percent. Let us suppose that whenever the return of one is above its 10-percent average, producing for instance a 12-percent return, the other will be below 10 percent by the same amount (an 8-percent return). They are negatively correlated. If we divide our investment between the two securities, then together we have an investment with a 10-percent return and a standard deviation of zero. This is because together the two will always produce a 10-percent return. It should be evident that to calculate portfolio risk we need the correlation of the securities in the portfolio, and that the predicted risk of a portfolio can be *lower* than the predicted risk of assets that make up the portfolio.

Correlation to Diversify

The correlation between two securities can range from –1.0 to +1.0. A measure of –1.0 means that when one security's return is below its average, the return on the other will be above its average. They are perfectly inversely related in their movements. A measure of +1.0 means they move perfectly in sync relative to their averages. A measure of 0.0 means that the movement of one security relative to its average gives you no information about how the other moves relative to its average. The phrase "relative to the average" emphasizes that correlation measures the tendency of securities to move in the same direction relative to their averages—not the tendency to move the same amount. In the real world, intermediate- and long-term government bonds have a high degree of correlation because interest-rate changes cause their prices to go up and down together. The price changes are not of the same percentage—prices of long-term bonds change by a greater percentage.

The following table is called a correlation matrix and shows estimates of the correlations between every pair of our three asset classes.

Table 5-3

	Stocks	Bonds	Cash
Stocks	1.00	0.50	0.00
Bonds	0.50	1.00	0.05
Cash	0.00	0.05	1.00

The diagonal of the correlation matrix always is + 1 because everything is perfectly correlated with itself. The area below the diagonal is a mirror image of the area above the diagonal because corresponding cells (e.g., stocks to bonds and bonds to stocks) represent the same relationship. Often correlation matrixes show only the area below the diagonal. A matrix of 500 assets in a portfolio would be a 500 × 500, so when you are trying to find the variance of the market and are considering 2,000 or more securities, the matrix's size grows larger rapidly. If you were trying to build a global benchmark portfolio the size would be enormous. Large matrixes require large amounts of computing power to solve.

Standard Deviation of a Portfolio

The equation to calculate the standard deviation of a portfolio is not just a weighted average of the standard deviations of the assets. The process also must include the correlation.

$$\sigma_{portfolio} = \sqrt{\sum_{a=1}^{n}\sum_{b=1}^{n} w_a w_b \sigma_a \sigma_b \rho_{ab}}$$

Equation 5-4

Where, w is the weight of an asset, σ is the standard deviation of an asset, and ρ is the correlation of two assets.

We multiply the various weights times their probabilities times their correlations (from the matrix) and sum them up for the variance. In table 5-3 we have a 3 by 3 matrix so we have nine products to compute and sum. Using the numbers from tables 5-1, 5-2, and 5-3 we have:

Stocks and Stocks:	0.55 × 0.55 × 0.15 × 0.15 × **1.0**	= 0.006806
Stocks and Bonds:	0.55 × 0.40 × 0.15 × 0.10 × 0.5	= 0.00165
Stocks and Cash:	0.55 × 0.05 × 0.15 × 0.01 × 0.0	= 0
Bonds and Stocks:	0.40 × 0.55 × 0.10 × 0.15 × 0.5	= 0.00165
Bonds and Bonds:	0.40 × 0.40 × 0.10 × 0.10 × **1.0**	= 0.0016
Bonds and Cash:	0.40 × 0.05 × 0.10 × 0.01 × 0.05	= 0.000001
Cash and Stocks:	0.05 × 0.55 × 0.01 × 0.15 × 0.0	= 0
Cash and Bonds:	0.05 × 0.40 × 0.01 × 0.10 × 0.05	= 0.000001
Cash and Cash:	0.05 × 0.05 × 0.01 × 0.01 × **1.0**	= 0.00000025

Summing up all the values we have: 0.006806 + 0.00165 + 0 + 0.00165+ 0.0016 + 0.000001 + 0 + 0.000001 + 0.00000025 = 0.0117085 ≈ 0.0117.

The standard deviation of the portfolio is the square root of the variance:

$$\sigma_p = \sqrt{0.0117} = 0.1082 \text{ or } 10.82\%$$

If we had calculated a simple weighted-average standard deviation instead, we would have found $((0.55 \times 0.15) + (0.40 \times 0.10) + (0.05 \times 0.01)) = 12.30\%$. The correct value is smaller (10.82 percent). This difference between the 10.82 percent and 12.3 percent is the real benefit of diversification. There is less risk (10.82 percent) because of the less-than-perfect correlation between assets.

An alternative method to calculating the variance of a portfolio is

$$2 - \text{Stock Portfolio Variance} = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B COV_{(A,B)}$$ **Equation 5-5**

Where, w^2 is the weight of an asset squared, σ^2 is the variance of an asset, *COV* is the covariance of two assets.

$$3 - \text{Stock Portfolio Variance} = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + w_C^2\sigma_C^2 + 2w_A w_B COV_{(A,B)} + 2w_A w_C COV_{(A,C)} + 2w_B w_C COV_{(B,C)}$$ **Equation 5-6**

$$\sigma^2(R_p) = 0.55^2(0.15)^2 + 0.40^2(0.10)^2 + 0.05^2(0.01)^2 + 2(0.55)(0.40)(0.50)(0.15)(0.10) + 2(0.55)(0.05)(0)(0.15)(0.01) + 2(0.40)(0.05)(0.05)(0.10)(0.01)$$
$$= 0.0117085$$

Beta of a Portfolio

The beta coefficient for an *individual security* may be unstable over time and is not a good predictor of future movements in stock prices. Alternatively, the beta coefficient for a *portfolio of securities* is fairly stable over time because it averages out. A portfolio's historical beta coefficient can be used as an indicator of future portfolio volatility. The beta of a portfolio is the sum of the weighted betas of the individual securities. The equation to use is

$$\beta_p = w_1\beta_1 + w_2\beta_2 + \dots + w_n\beta_n$$ **Equation 5-7**

Where, w_n is the weight of asset *n*, and β_n is the beta of asset *n*.

Let us look at an example.

Holding (A)	Assets (B)	Weight Calculation (C) = (A) / (B)	Weight	Beta
1	$600,000	600,000 / 8,500,000	0.07	1.05
2	$1,000,000	1,000,000 / 8,500,000	0.12	2.70
3	$1,300,000	1,300,000 / 8,500,000	0.15	1.35
4	$1,600,000	1,600,000 / 8,500,000	0.19	0.40
5	$1,900,000	1,900,000 / 8,500,000	0.22	0.90
6	$2,100,000	2,100,000 / 8,500,000	0.25	0.15
Total:	**$8,500,000**		**1.00**	

$$\beta_p = 0.07(1.05) + 0.12(2.70) + 0.15(1.35) + 0.19(0.40) + 0.22(0.90) + 0.25(0.15) = 0.92$$

Coefficient of Determination (R-squared)

The coefficient of determination gives the variation in one variable explained by another, and is an important statistic in investments. No causality is claimed by the coefficient of determination. It is the job of the investor to interpret it.

We can compute the coefficient of determination by squaring the correlation coefficient between the dependent variable and independent variable. Thus, the coefficient of determination is known as R-squared. This method is used in linear regression with one independent variable. If there is more than one independent variable, we cannot square the correlation coefficient in order to measure the coefficient of determination.

Statistically, R-squared is the ratio of the portfolio's market-related (systematic) variance to its total variance. The formula is

$$R^2 = \frac{\beta^2 \sigma_m^2}{\sigma_p^2}$$

Equation 5-8

Where, R^2 is R-squared, β^2 is beta squared, σ_m^2 is the market-related variance, and σ_p^2 is the total portfolio variance.

R-squared gives us an indication of the level of diversification in a portfolio. A portfolio is well diversified if it has a high R-squared relative to an index. R-squared on a portfolio also gauges the reliability of alpha as an indicator of the manager's return and beta as an indicator of risk. If the R-squared is close to 1.00, alpha and beta statistics can be used to explain the return of the portfolio as a linear function of the market.

Performance Measure and Evaluation

When evaluating a manager, the consultant may be interested in whether the value added by a manager can be attributed to market timing or to security selection. To perform this analysis, the consultant must have established a normal asset allocation for the portfolio.

Consider the following:

Asset Class	Policy or Normal Weight	Actual Weight	Index Return
Stocks	57%	45%	6%
Bonds	37%	55%	3%
Cash	6%	0%	2%

The return on the managed portfolio was 5.1 percent

Setting a Benchmark

If the manager had been invested in index funds according to the policy weights, the portfolio return would have been 0.57(6) + 0.37(3) + 0.06(2) = 4.65%. This is the sum of the products of the policy weights with the index returns. This is the benchmark return. However, the manager did better than the benchmark. The manager achieved an added 0.45 percent of value (5.1% − 4.65%).

Attribution

How much of the manager's overperformance came from timing (active asset allocation) and how much came from security selection? If the manager had been invested according to the actual allocation using index funds, the return would have been 0.45(6) + 0.55(3) = 4.35%. This is the sum of the products of the actual weights and the index returns. So the timing decisions had reduced the return from 4.65 percent to 4.35 percent. The effect from timing is (4.35% − 4.65%) = –0.30%. However, overall the manager earned 5.1 percent, which is 0.45 percent above the benchmark. The lost return was made up and surpassed so there must have been a much larger security selection effect. Security selection must be +0.75% (0.45% + 0.30%).

The 0.45-percent value added can be attributed to market timing (–0.30 percent) and security selection (0.75 percent). This is summarized as follows:

Benchmark Return = Policy Weights × Index Returns = **4.65%**

Value Added by Manager = Manager Return − Benchmark Return = **0.45%**

Manager Return Due to Allocation = Actual Weights × Index Returns = **4.35%**

Value Added by Manager = Value Added by Timing + Value Added by Security Selection = **0.45%**

Value Added by Timing = Manager Return Due to Allocation − Benchmark Return = **–0.30%**

Value Added by Security Selection = Value Added by Manager − Value Added by Timing = **0.75%**

This is a simple example of an attribution analysis.

Consider the following data about a mutual fund

Table 5-4

Asset Class	Fund Commitment	Normal Position	Fund Return for Class	Class Benchmark Return
Equity	0.70	0.60	7.28	5.81
Bonds	0.07	0.30	1.89	1.45
Cash	0.23	0.10	0.20	0.48

Calculate the following:

1. The normal position return
2. The value added for this fund
3. The added performance due to the asset mix (timing) decision
4. The added performance due to the asset selection decision

1. Normal Position Return = Σ[(Normal Position for Class) × (Class Benchmark Return)]

= Normal Position Equity × Benchmark Return Equity

+ Normal Position Bond × Benchmark Return Bonds

+ Normal Position Cash × Benchmark Return Cash

= (0.60)(5.81) + (0.30)(1.45) + (0.10)(0.48) = **3.97**

2. Value Added for This Fund = Σ[(Actual Return) – (Normal Position Return)]

Actual Return

= Fund Commitment Equity × Fund Return Equity

+ Fund Commitment Bonds × Fund Return Bonds

+ Fund Commitment Cash × Fund Return Cash

= (0.70)(7.28) + (0.07)(1.89) + (0.23)(0.20) = **5.27**

Value Added =

5.27 – 3.97 **= 1.30**

3. The added performance due to the asset mix (timing) decision

Value Added from Active Asset Allocation = Σ[Fund Weight for Class) – (Normal Position for Class)] × [(Class Benchmark Return) – (Normal Position Return)]

= (0.70 – 0.60) × (5.81 – 3.97)

+ (0.07 – 0.30) × (1.45 – 3.97)

+ (0.23 – 0.10) × (0.48 – 3.97)

= **0.31**

4. The added performance due to the asset selection decision

Value Added from Asset Selection Decision = Σ[Fund Weight for Class] × [(Fund Return for Class) – (Class Benchmark Return)]

= (0.70) × (7.28 – 5.81)

+ (0.07) × (1.89 – 1.45)

+ (0.23) × (0.20 – 0.48)

= **0.995**

Total Risk-Adjusted Measures

It is natural to try to represent performance with a single number that incorporates both the calculations of return and risk. This is risk-adjusted return. Since theory suggests that rate of return and risk are directly related, investors want to know if they are being compensated for the risk they have assumed. There are several ratios that attempt to do this.

Sharpe Ratio

The Sharpe ratio measures the total risk of the portfolio by including standard deviation instead of only the systematic risk (i.e., beta). It does not implicitly assume that a portfolio is well diversified. The foundation of the Sharpe ratio is modern portfolio theory (MPT), and the idea is the higher the Sharpe ratio, the closer the portfolio is to the mean–variance portfolio. This means that the Sharpe ratio standardizes the return in excess of the risk-free rate by the variability of the returns. The formula for the Sharpe ratio is as follows:

$$\text{Sharpe Ratio} = \frac{\text{average actual return} - \text{average riskfree return}}{\text{standard deviation of the portfolio}}$$

$$\text{Sharpe Ratio} = \frac{\overline{R}_p - \overline{R}_f}{\sigma_p}$$

Equation 5-9

The Sharpe ratio divides the excess return (actual return minus the risk-free return) of a portfolio by the standard deviation of the portfolio. Since standard deviation is in the denominator, the equation measures total risk. Technically, the Sharpe ratio should use the arithmetic average of the returns. In practice, the geometric mean frequently is used.

Consider that portfolio X earns a 13-percent return with a standard deviation of 30 percent, and portfolio Y earns 15 percent with a standard deviation of 20 percent. The risk-free rate is 5 percent. The Sharpe ratio is calculated for each portfolio as follows:

$$S_x = \frac{13-5}{30} = .27$$

$$S_y = \frac{15-5}{20} = .50$$

This indicates that portfolio Y outperformed portfolio X on a risk-adjusted basis.

Sortino Ratio

The Sortino ratio also is based on MPT and is closely related to the Sharpe ratio except that it uses downside risk. The Sortino ratio gives the same ranking as the Sharpe ratio of portfolio performance.

$$\text{Sortino Ratio} = \frac{\text{average actual return} - \text{average riskfree return}}{\text{downside semistandard deviation}}$$

$$\text{Sortino Ratio} = \frac{\overline{R}_p - \overline{R}_f}{\text{Downside } \sigma_p}$$

Equation 5-10

M-Squared

M-squared is a return measure that adjusts for total risk from the mix of funds or managers being evaluated. The weightings of the mix are selected to match the total risk of an index (e.g., S&P 500). The calculation is

$$M^2 = \left[1 - \frac{\sigma_{S\&P500}}{\sigma_{Fund}}\right] \times [T - \text{Bill Return}] + \left[\frac{\sigma_{S\&P500}}{\sigma_{Fund}}\right] \times [\text{Actual Return}]$$

Equation 5-11

Consider the following:

	Return	Standard Deviation
Portfolio	10%	15%
S&P 500	11%	17%

Also assume that the risk-free rate of return on a Treasury bill is 5 percent. M-squared can be found by:

$$M^2 = \left[1 - \frac{0.17}{0.15}\right] \times [0.05] + \left[\frac{0.17}{0.15}\right] \times [0.10]$$
$$= 0.1067 \text{ or } 10.67\%$$

This implies that the portfolio underperformed the market when adjusting for total risk. The portfolio's total risk-adjusted return of 10.67 percent is less than the S&P 500's return of 11 percent. M-squared is directly related to the Sharpe ratio. Therefore, a higher Sharpe ratio implies a higher M-squared, and a lower Sharpe ratio implies a lower M-squared.

Market Risk-Adjusted Measures

Treynor Ratio

The Treynor ratio is similar to the Sharpe ratio, except it uses the beta of the portfolio in the denominator. This means that it is using systematic risk instead of total risk. This measure assumes the portfolio is fully diversified.

$$\text{Treynor Ratio} = \frac{\text{average actual return} - \text{average riskfree return}}{\text{beta of the portfolio}}$$

$$\text{Treynor Ratio} = \frac{\bar{R}_p - \bar{R}_f}{\beta_p}$$

Equation 5-12

Consider the following example to understand how to interpret the Treynor ratio. The S&P 500 earns 11 percent when a government T-bill earns 5 percent. Portfolio X earns a 13-percent return with a beta of 1.3, and the portfolio Y earns 15 percent with a beta of 2.0. Using the information, the Treynor ratio is calculated for the market portfolio and each portfolio as follows:

$$T_m = \frac{11-5}{1} = 6.00$$

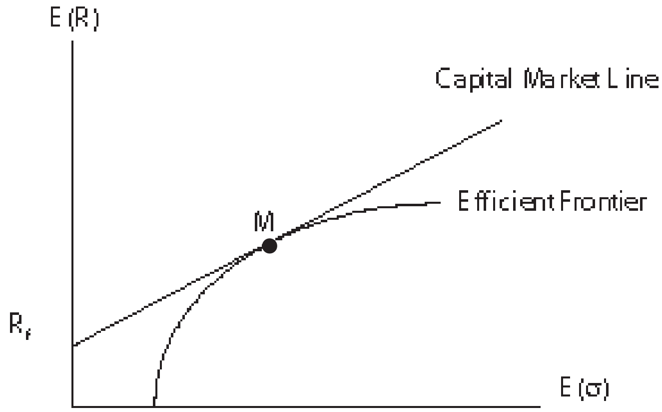
$$T_x = \frac{13-5}{1.3} = 6.15$$

$$T_y = \frac{15-5}{2} = 5.00$$

This indicates that portfolio X outperformed the market and portfolio Y on a risk-adjusted basis. However, the results do not indicate by how much each portfolio outperformed the market. You will notice that when calculating the Sharpe ratio earlier, the results were the opposite. That is, portfolio Y outperformed portfolio X when using Sharpe ratio. The difference in ranking occurs because the measure of risk is different.

Alpha (Jensen's Performance Index)

To explain Jensen's performance measure, it is necessary to first define alpha, but first let us take a step back. A further refinement that the capital asset pricing model (CAPM) brings to modern portfolio theory is the graph of expected return versus standard deviation. It is linear with standard deviation on the x-axis and expected return of the y-axis. The standard deviation of the portfolio measures the total market and nonmarket risk of the portfolio. Interestingly, this graph makes the case for passive investing in the market portfolio rather than an actively managed account and here is why:



As mentioned before, the standard deviation of the portfolio measures the total market and nonmarket risk of the portfolio. Portfolio optimizer software is available that uses CAPM to graphically represent portfolios along the risk-return trade-off curve. This curve is the efficient frontier. The capital market line (CML) represents the best combination of risky and risk-free assets. (Note the difference between the CML, which has an x-axis of σ_p, and the SML, which is β_p). Points below the CML offer too little return for the risk. Theoretically, no returns can be above the CML. The point at which the efficient frontier curve meets the straight line is the optimal market portfolio of risky assets. The market portfolio represents the ultimate or completely diversified portfolio. If, indeed, this is the optimal risky portfolio, why should an investor choose an active investment strategy?

The answer lies in the fact that actual portfolio returns may exceed the CAPM equation:

$$E\left(R_P\right) = R_F + \beta_P\left(R_M - R_F\right)$$

In other words, there is a residual tacked on to the expected return. This residual is termed alpha (α). It measures the nonmarket return associated with the portfolio. Positive alpha means that the manager has added value through his or her security selection and timing. Negative alphas indicate value subtracted. Alpha is calculated as:

Actual return = expected return + alpha

Actual return = $R_F + \beta_P (R_M - R_F) + \alpha$ **Equation 5-13**

Typically this is written as:

$R_P = \alpha + E(R_P)$

So, we solve for α:

$\alpha = R_p - [R_f + \beta_p (R_m - R_f)]$ **Equation 5-14**

Where, R_p is the actual return of the portfolio, R_f is the return on a risk-free asset, R_m is the return on the market, and β is the beta of an asset. The α (also referred to as Jensen's alpha) value indicates whether a portfolio manager is superior or inferior in market timing and stock selection.

For example, assume a return of 17 percent with a beta of 1.3 for manager X when the market return is 14.5 percent and the risk-free rate is 5 percent. Jensen's alpha is expressed as

$J_a = 17 - [5 + (14.5 - 5)1.3] = -0.35\%$

Non–Market Risk Measure

<u>Tracking Error</u>

Another statistic to consider is tracking error. Tracking error is the annualized standard deviation of monthly excess returns. You generally can assume the tracking error is normally distributed. To calculate an annual tracking error, multiply the observed tracking error by the square root of the number of periods in one year. If you use quarterly data, the annual tracking error is found by multiplying the observed tracking error by the square root of 4.

The following is a sample of quarterly returns (annualized) for portfolio A, portfolio B, and the index:

Portfolio A	Portfolio B	Index
3.5%	3.9%	4.5%
7.3%	7.1%	6.9%
6.5%	6.9%	7.4%
4.4%	4.3%	4.5%
5.3%	4.9%	5.3%

Which portfolio has the *least* tracking error to the index? The standard deviation of portfolio A is 1.53, for portfolio B is 1.49, and for the index is 1.36.

The portfolio with the least standard deviation ratio to the index has the least tracking error. The ratio of portfolio A to the index is 1.53/1.36 = 1.125. The ratio of portfolio B to the index is 1.49/1.36 = 1.096. Therefore, portfolio B exhibits the least tracking error to the index.

Information Ratio

The information ratio is the average excess return of a portfolio over a benchmark divided by the standard deviation of the excess returns. This measures the ability to select securities relative to a benchmark. It captures both the size of the excess return and the ability to do so consistently.

Information Ratio = excess return / tracking error **Equation 5-15**

Assume the following statistics for two small-cap portfolios:

Portfolio	Alpha (basis points)	Tracking Error (basis points)
1	345	607
2	455	935

Information ratio for portfolio 1 = 345/607 = 0.568

Information ratio for portfolio 2 = 455/935 = 0.487

The higher the information ratio, the more likely it is that a manager's performance is the result of skill rather than luck. A high information ratio will result from a positive alpha and low tracking error. However, the information ratio is of limited value in your decision if the information is based on relatively few years. It requires several years of gathering information for a high confidence ratio. If we do not know how many years the information is based on, we can state that if your risk profile is low, you might select portfolio 1; if high, you might select portfolio 2.

Please read Appendix 5–1 "The Hitchhiker's Guide to Risk-adjusted Returns" and Appendix 5–2 "Taking Stock of Correlation Risk" for further discussion on measuring performance and risk and how to use these tools to benefit your clients.

Appendix 5–1

The Hitchhiker's Guide to Risk-adjusted Returns

By Rex Macey, CIMA®, CFA®

> In *The Hitchhiker's Guide to the Galaxy*, a computer—after working for 7.5 million years—finds that 42 is the answer to "life, the universe, and everything."

In *The Hitchhiker's Guide to the Galaxy*, a computer—after working for 7.5 million years—finds that 42 is the answer to "life, the universe, and everything." By its very nature, a risk-adjusted return (RAR) promises to encapsulate return and risk into a single performance measure. But just as simple answers to questions about the meaning of life tend to be elusive, no single measure succinctly describes investment performance. With that important caveat, here is a brief guide to risk-adjusted returns, including their uses and misuses.

The Sharpe Ratio

Due to its simplicity or to the pedigree of its Nobel Laureate creator, the Sharpe ratio is the most commonly used RAR. William Sharpe introduced his measure more than 40 years ago (Sharpe 1966). As it is used today, the Sharpe ratio is defined as the return of a portfolio over and above a risk-free rate divided by the standard deviation of the portfolio.

$$\text{Sharpe ratio} = \frac{r_{portfolio} - r_{risk\text{-}free}}{s_{portfolio}}$$

The Sharpe ratio and other RARs demand a return differential, which also is called an excess return (Sharpe 1994). An example will illustrate why using return alone, rather than a return differential, might lead to the choice of an inferior investment.

Consider investments A and B and a risk-free security with the following characteristics (see table 1):

While A offers more return per unit of risk (8/5 v. 7/5), B is the better investment, a fact signaled by its superior Sharpe ratio. This is because an investor placing one-half of his assets in B and the other half in the risk-free security would earn a return of 9.5 percent. The portfolio would have a standard deviation of 5 percent (because the standard deviation of the risk-free investment is zero). The blend of the second investment and the risk-free security offers more return for the same level of risk as A, making the second investment preferable.

Modigliani and Modigliani Ratio

Because the Sharpe ratio is the excess return of a portfolio divided by its standard deviation, the units are difficult to interpret. The newest score on the block, the Modigliani and Modigliani (MM) ratio, proposes a twist on the Sharpe ratio to simplify interpretation (Modigliani and Modigliani 1997). The MM ratio is the Sharpe ratio multiplied by the standard deviation of the market portfolio. The rankings of a set of portfolios will be identical whether ordered by Sharpe or MM. MM provides a value that is the excess return that investors would have achieved relative to the market (i.e., the benchmark portfolio) if they had assumed a level of risk equal to that of the market. The MM risk-adjusted performance arrives at the score by leveraging or deleveraging the portfolio so that its risk is identical to that of the market (i.e., benchmark) portfolio. It assumes that investors may borrow or lend at a risk-free (standard deviation = 0) rate.

The Information Ratio

The information ratio (IR) is a more general version of the Sharpe ratio. To use the IR one chooses a benchmark, such as the S&P 500 Index. Each period (e.g., month) the difference between the return of the portfolio and that of the benchmark is computed. The IR is the average of the differences divided by the standard deviation of the differences. In other words, it is the excess return divided by the tracking error.

The Sharpe, MM, and IR measures all use standard deviation to represent risk. Standard deviation measures the variability (i.e., uncertainty) of portfolio returns around an average. A criticism of standard deviation stems from the

TABLE 1

Investment	Return	Std. Dev	Ret/Std. Dev	Sharpe
Risk-free security	5%	0%		
A	8%	5%	8/5	0.6=(8–5)/5
B	14%	10%	7/5	0.9=(14–5)/10
½ B + ½ risk-free security	9.5%	5%	9.5/5	0.9=(9.5–5)/5

fact that observations above the mean contribute as much to the statistic as observations an equal distance below the mean. Critics point out that investors generally don't mind unexpectedly high returns; they mind unexpectedly low returns.

Because standard deviation measures total variability, it is appropriate for measuring total portfolio return as long as the distribution of returns is symmetric. If returns are symmetric, above and below average observations occur with the same frequency and magnitude. If returns are asymmetric, then it makes sense to focus on downside risk. One downside risk statistic, called semi-deviation, considers only the observations that are negative. Similarly, one can choose to focus on observations that fall below a target or required rate of return.

Sortino Ratio

The Sortino ratio resembles the Sharpe ratio, except that it uses downside risk in place of standard deviation. Use of this statistic instead of Sharpe makes sense when the distribution of returns is asymmetric.

RARs that use standard deviation are appropriate for well-diversified portfolios and for entire portfolios. A different measure should be used for component styles or subasset classes (e.g., emerging markets). The fathers of modern portfolio theory, Harry Markowitz, Sharpe, and others, recognized that investors should expect to be compensated for accepting only systematic (i.e., market) risk. They distinguish between systematic risk—that of the market, which cannot be diversified away—and unsystematic risk, which can be reduced through diversification. Thus, investors who take the extra risk of holding a concentrated portfolio can expect greater variability in returns (i.e., risk) but not higher average returns.

According to modern portfolio theory, investors should expect to be compensated according to the capital asset pricing model (CAPM).

$$R_{portfolio} = R_{risk\text{-}free} + \beta\left(R_{market} - R_{risk\text{-}free}\right)$$

The expected return of a portfolio should be the risk-free rate plus the excess return of the market multiplied by beta, which is the systematic risk of a portfolio. Within the framework of this model, beta is the measure of risk.

Imagine two emerging market investments with identical returns and standard deviations. While they may have the same Sharpe ratio, an investor would prefer the one with the lower correlation to the rest of his portfolio. Beta incorporates the correlation to the rest of the portfolio.

Treynor Ratio

Like the Sharpe ratio, the Treynor ratio divides the excess return of the portfolio by its risk. The difference is that the Treynor ratio uses beta to represent risk. It also is useful for ranking investments.

$$Treynor\ ratio = \frac{r_{portfolio} - r_{risk\text{-}free}}{\beta_{portfolio}}$$

Jensen's Measure

Also called Jensen's alpha, this statistic measures a portfolio's return in excess of that predicted by the CAPM.

$$Jensen's\ alpha = R_{portfolio} - \left[R_{risk\text{-}free} + \beta\left(R_{market} - R_{risk\text{-}free}\right)\right]$$

(The term in the square brackets is the return predicted by the CAPM so that the expression is the portfolio return less the CAPM prediction.)

While CAPM was enough to win a Nobel Memorial Prize for Sharpe, the model, particularly beta, has been criticized on practical terms. Fama and French found that beta did not explain differences in the returns of stocks. In their study of stocks on the NYSE, AMEX, and NASDAQ between 1963 and 1990, high beta portfolios did not outperform low beta portfolios, on average (Fama and French 1992). Entire papers, and probably books, could be written about beta. I simply will state that beta must be used with extreme care.

Fama-French Factor Models

Many professionals use what is known as the Fama-French three-factor model to compensate for the flaws associated with the single beta used in the Jensen measure. Whereas Jensen assumes that risk comes from one source—market exposure as represented by beta—Fama-French adds two factors.[1] One is an exposure to size, which they define as the market capitalization, and the other is an exposure to value, defined as book value divided by market value. In addition, if fixed income is included in the portfolio, they add two more factors: One measures the time to maturity of the portfolio's holdings and the other assesses default risk. The general idea is to measure the excess return of a portfolio relative to what would have been earned by a benchmark portfolio taking the same systematic risks.

$$\begin{aligned} &Fama\text{-}French\ alpha \\ &= R_{portfolio} - \Big[R_{risk\text{-}free} + \beta_{market}\left(R_{market} - R_{risk\text{-}free}\right) + \\ &\beta_{size}\left(R_{large} - R_{small}\right) + \beta_{value}\left(R_{value} - R_{growth}\right)\Big] \end{aligned}$$

Shapes Matter

Users of risk-adjusted returns should consider probability distributions, which define the likelihood of each possible event. For example, the probability that any one face of a die will appear when it is rolled is 0.1667 (1/6). This is called a uniform distribution. Distributions can come in many shapes. Most people are familiar with the normal distribution, which is shaped like a bell. This is a symmetric distribution. Security returns are not normally distributed. A common assumption is that the logarithm of relative stock prices (a fancy way of saying 1 + r, where r is a return, such as 8 percent) is normally distributed. This is the assumption of the Black-Scholes options-pricing model. While this assumption is pretty good (but not exact) for stocks, it should not be made with the distributions of many other securities and strategies. For example, neither the logarithm of returns on options nor the logarithm of returns on bonds are normally distributed.

Risk-adjusted returns capture two properties of investments that investors care about: return and risk. There are other potential properties of an investment, such as the symmetry in its returns. Investors like positive skewness. With positive skewness they are more likely to enjoy large positive returns than large negative returns (e.g., lottery tickets). Buying call options has positive skewness; writing calls has negative skewness. Options often are embedded in securities. For example, the prepayment option held by homeowners means that mortgage-backed bonds have embedded short calls.

Another property one should consider is kurtosis. A distribution has excess kurtosis if the probability of extreme events is higher than it would be under a normal distribution. Compared to a normal distribution, one with excess kurtosis would have fatter tails. Investors don't like kurtosis. It turns out that certain strategies often employed by hedge funds have excess kurtosis.

Because of these issues, simple RAR measures fail to adequately depict the performance of a portfolio. In addition, past performance may not be indicative of future results. If returns are not predictive, then it follows that RARs won't be. Generally, RARs are computed using past data. The future may differ so much from the past that the predictive value of an RAR is insignificant.

An assumption behind RARs is that investors can borrow or lend easily. Imagine an investor who is comfortable taking a market level of risk. She could index and receive market levels of return and risk. Also assume a low-risk investment, which if leveraged to the risk of the market, offers higher return than the market. The low-risk investment will have a more attractive RAR. However, if the investor is prohibited from leveraging, then the RAR is misleading. The higher-risk, lower-RAR index fund may be more attractive for that investor.

> "One must take care to consider appropriate risk measures and time periods, realizing that simple answers, like "42," may not adequately reflect the complexities of investing."

Ex-post risk-adjusted returns measure *observed* risk, not *assumed* risk. Just because risk was not detected does not mean it did not exist. Investors may assume risk without knowing it. I once had a client who won $1 million in a Pepsi promotion. My client would have won $1 *billion* if he had selected the same number between 0 and 999,999 picked by a chimpanzee. Pepsi was insured against paying $1 billion by Warren Buffet's Berkshire Hathaway. While Berkshire assumed the entire risk of losing $1 billion,[2] this risk was not observed in the company's financial statements. The important point is that the risk measured may be substantially less than the risk taken.

Summary

RARs appear to be fairly simple. Divide return by risk to arrive at return per unit of risk. But such measures may be misleading. Theoretically correct RARs normalize the varying risks of different investments, accounting for borrowing, lending, and shorting, before comparing returns. One must take care to consider appropriate risk measures and time periods, realizing that simple answers, like "42," may not adequately reflect the complexities of investing. I&W

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Endnotes

1 For details of the additional factors, see Ken French's Web site, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_bench_factor.html.

2 To be paid over 40 years.

Appendix 5–2

Taking Stock of Correlation Risk

BY DAVID KREIN

Correlation risk is a hot topic. After the equity bubble collapse revealed the severity of this risk within investor portfolios, volumes of research began highlighting a wide array of alternative asset classes, markets, and products. The arguments have been enormously persuasive, and investors since have shifted significant capital away from more-correlated exposures to less-correlated alternatives.

After all, modern portfolio theory no longer may have a sterling reputation, but every financial professional understands its implications: A portfolio of uncorrelated risk assets should have a higher risk-adjusted return than any individual asset within that portfolio.

The effect is so strong that the addition of an asset with uncorrelated risk can improve the risk-adjusted return even if the risk of the additional asset is greater than that of the portfolio itself.

Of course, the fact that an additional asset is risky is a necessary but insufficient criterion. The additional asset also must be *uncorrelated* with the assets already in the portfolio. If the additional asset has a high degree of correlation, then all bets are off.

State of the Market

Unfortunately, several markets and asset classes that historically have had low correlation have become highly correlated in recent months and years. Analysts and economists have developed a variety of theories to explain the phenomenon of rising correlation, including the convergence of financial markets, instant communication of ideas, declining asset price volatility, and global liquidity.

Regardless, this evolution has dramatic implications for investors who seek to manage an appropriately diversified portfolio. A strategic asset allocation policy established with low-correlation expectations now functions in a high-correlation environment, and it may offer diminished diversification value relative to the less-correlated historical performance observed over a longer time frame.

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Looking Beyond Correlation

Correlation itself, however, is a severely limited metric for diversification and cannot contribute to a meaningful dialogue about financial market relationships. Furthermore, it is a poor foundation for asset allocation decisions, portfolio analysis, risk management, and product development.

It may seem strange to discredit the tool that establishes a common language for this category of risk, but let us examine the following key observations and considerations:

- Correlation is a linear estimation of the relationship between two variables. Nonlinear relationships, outlying observations, and external factors will significantly distort the results.
- Correlation assumes that the variables being considered are both normally distributed and that the combination also is normally distributed. Skew, kurtosis, and higher-order moments cannot be taken into account.
- Correlation assumes that the volatility of each variable does not change. If the volatility increases, the correlation will decrease regardless of the relationship.
- Zero correlation does not imply that two variables are independent. Similarly, knowing that two variables are independent does not mean that their correlation is zero. (They may be influenced by an unobserved third variable.)
- Correlation cannot provide sufficient information, intuitive or

otherwise, on the relative magnitude of risk.
- Most importantly, investors do not directly "experience" correlation.

But If Not Correlation, Then What?

"Spread" and "spread volatility" best describe the risk associated with combining two assets into a single portfolio or adding a new asset class to an existing portfolio. Since they are experienced directly by diversified investors, spread and spread volatility more thoroughly illustrate the nature and magnitude of multi-asset risk than correlation alone.

Let's define "spread" as a measure of the relative performance of two variables, whether they are asset classes, indexes, sectors, stocks, or specific products.

Let's define "spread volatility" (also known as "spread dispersion," or simply "dispersion") as the variability of the spread. Generally, it will be calculated as the standard deviation of the spread's distribution over time.

Assembling Spread and Dispersion Data

As an example, consider the relationship between the Standard & Poor's 500 (SPX) and the Russell 2000 (RTY) indexes from January 1, 1988, through March 31, 2007. Figure 1 shows the two indexes over time.

Figure 2 shows the rolling correlation of three-month returns for SPX and RTY. Each data point shows the correlation of three-month periods of returns for SPX and RTY observed over rolling 24, 36, and 60 month-end to month-end periods. For example, the December 2006 value for the rolling 60-month correlation of three-month returns is 90.5 percent; this is derived from the set of three-month returns observed on the last trading day of December 2006 (calculated as month-end September 2006 to month-end December 2006) and in each of the preceding 59 months (August–November 2006, July–October 2006, etc.).

FIGURE 1 Index Level

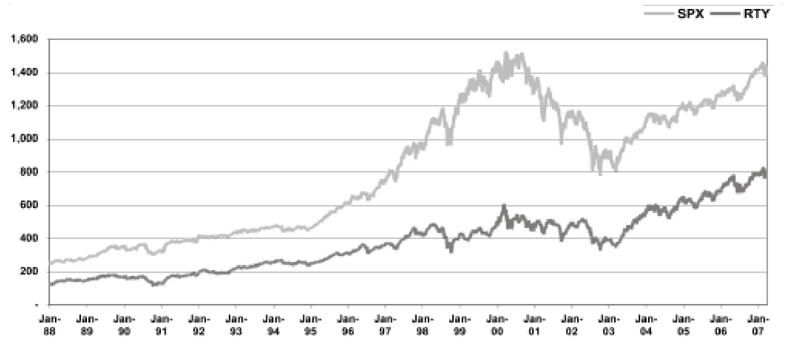

FIGURE 2 Rolling Correlation of 3-Month Returns (SPX & RTY)

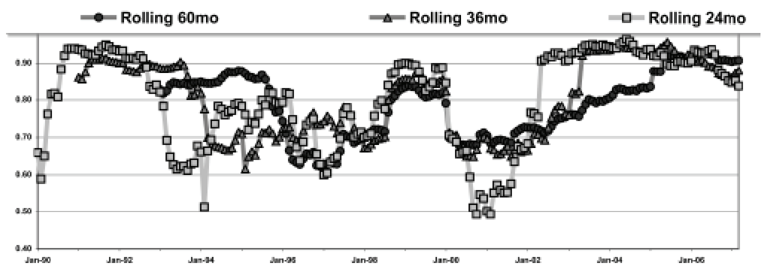

FIGURE 3 Rolling 91-Day Performance

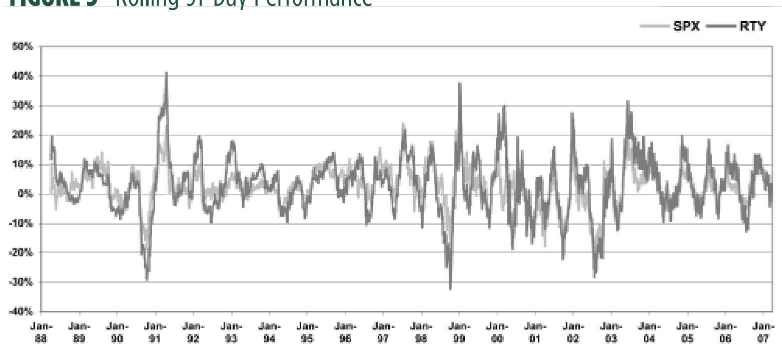

FIGURE 4 Rolling 91-Day Performance Spread

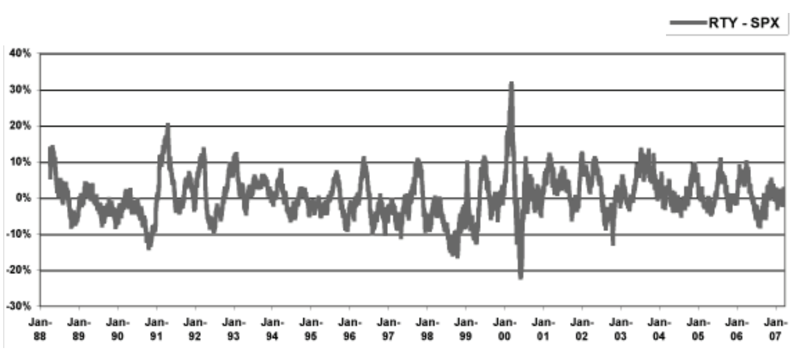

Figure 2 allows us to discuss whether the correlation of three-month returns is high or low and whether it is rising or falling. But it provides little if any actionable information for a diversified investor. For example, it doesn't show which index had the better performance, the magnitude of that relative performance, the risk of each underlying, or the frequency of outliers. In short, correlation does not give us much insight into the relationship between these two indexes.

Analyzing spread and dispersion data, however, allows a more thorough analysis.

Figure 3 shows rolling performance data for the two indexes. We've used a daily (rather than monthly) rolling calculation, and a 91-day metric better standardizes the subsequent analysis.

We immediately notice that SPX and RTY track each other fairly well in figure 3. By itself, this would suggest a significant degree of correlation, although the correlation strength and variability would not be obvious without further analysis.

Figure 4 shows the spread between the two sets of rolling returns, i.e., the rolling 91-day performance of RTY minus the corresponding rolling 91-day performance of SPX.

Figure 4 provides evidence that the relationship between SPX and RTY is not constant. The spread varies over time in both magnitude and velocity and contains numerous outlying observations. However, each of these aspects tends to be more muted than for either of the individual underlying indexes—just as modern portfolio theory would have it.

Furthermore, we observe an aspect of the relationship that cannot be extracted from the correlation analysis alone: This spread tends to mean-revert to zero. Of course, this likely will hold true for any pair of equity indexes that have similar long-term expected returns. In the case of SPX and RTY, mean reversion may play out over a period of years, but it certainly will get there. (If it were not the case, one of the two indexes eventually would dominate 100 percent of the equity marketplace.)

FIGURE 5 Rolling 91-Day Performance Spread

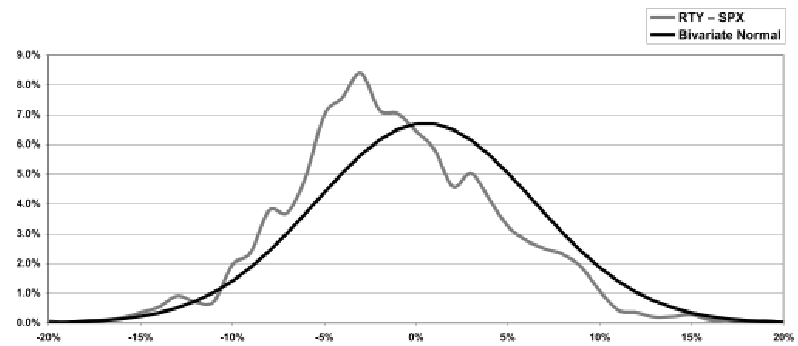

Figure 5 shows the distribution of rolling 91-day spread observations over the full period.

Figure 5 also shows the normal distribution that most closely approximates this realized dataset. Its standard deviation is approximately 6 percent—this is the metric that best captures the degree of diversification risk for the pair of underlying indexes over this time frame. Is this value high? Is this value low? Does the distribution show evidence of skew, kurtosis, or other dimensions of risk? A more thorough analysis is beyond the immediate scope of this article, but it would begin to provide the fuller picture of how the relationship between these two indexes impacts the portfolio's diversification.

Product Construction

Remember that all of the data thus far are backward-looking. Relying solely on this data for investment guidance implies that certain historical relationships will persist. It would be challenging to support or justify such simplicity in a complex and rapidly evolving marketplace.

Investors certainly would benefit from having access to the informational building blocks of dispersion—forward-looking financial products that allow for market-based discovery of dispersion. Yet no dispersion indexes, no benchmarks, no exchanges for observing dispersion expectations, and few securities exist for the pricing of dispersion risk. Innovation has been underwhelming at best as this substantial yet obvious piece remains missing from the marketplace.

In the over-the-counter marketplace, however, institutions are witnessing the creation of a dispersion options market.

Dispersion Options

Generally speaking, the price of a dispersion option—an option with a payoff tied to the spread between two underlying instruments—is driven by dispersion expectations. If the dispersion is expected to be wide (i.e., the spread has high variability), the option premium would be large. If the dispersion is expected to be narrow (i.e., the spread has low variability), the option premium would be small.

For example, consider a hypothetical diversified investor with a two-asset portfolio consisting of SPX and RTY. This investor's performance will be the average performance of the two indexes (the baseline performance), which is consistent with the objective of maximizing risk-adjusted returns via diversification.

If the investor believed, however, that RTY likely would outperform SPX this year, the investor could elect to overweight their capital allocation to RTY at the expense of SPX. In effect, the investor has a view on dispersion and will adjust the portfolio weights accordingly.

But the investor also could instead take a similar position utilizing dispersion options. Specifically, the investor could combine a 100-percent allocation to SPX with a call option on the spread between RTY and SPX.

The net performance of SPX and this dispersion option is economically equivalent to receiving the performance of the better-performing index, whether SPX or RTY, less the dispersion option premium.

If SPX outperforms RTY, then the investor holds 100 percent of SPX (the better performing index) and a worthless dispersion option. If RTY outperforms SPX, then the investor holds 100 percent SPX (the lesser performing index) plus a dispersion option that pays RTY's performance over SPX; the net performance is equal to 100 percent of RTY. In either case, the investor's performance will be the better performing index (less the dispersion option premium).

For target investments that have low spread-variability expectations, dispersion options are a relatively cost-effective approach to diversification, offer protection against cross-market dislocations, and deliver an attractive risk-reward alternative to traditional direct investments.

Further, if the realized dispersion is greater than the dispersion expectation embedded in the option price, then this portfolio's return will be greater than that of the baseline portfolio. If the realized dispersion is less than the dispersion expectation embedded in the option price, then this portfolio's return will be less than that of the baseline portfolio.

Such a substitution strategy has obvious diversification, risk management, and performance implications. For target investments that have low spread-variability expectations, dispersion options are a relatively cost-effective approach to diversification, offer protection against cross-market dislocation and tail risk, and deliver an attractive risk-reward alternative to traditional direct investments. The capabilities to develop effective solutions tailored to the needs of a specific investor will only continue to grow over time. M

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