Math for Investment Consultants

2nd Edition

Chapter 1: Algebra and Statistics

The best way to really understand the numbers that are used every day is to understand the mechanics behind the calculations. The best way to learn and internalize these mechanics is to put pencil to paper and calculate the numbers from scratch.

Students need to recognize formulas and know how to use them. Students also need to understand that mathematical formulas can be written in a variety of ways and with different notations. In the real world you will see the same measurement written different ways in different places. Therefore, in imitation of the real world, no attempt has been made to "standardize" the formulas that you will see in the program.

Notation

In algebra, letters of the alphabet, such as x, are variables used to represent numbers. Certain letters are most commonly used to represent special types of numbers.

r is used to represent rates of return.

W is used to represent weights.

i is used to represent an integer (0, 1, 2, 3, etc.).

Since there are more numbers than letters of the alphabet, and to make the equations concise, subscripts are used. For example, if we needed variables to represent the return achieved during each calendar year of a decade we might use $x_1, x_2, x_3, \ldots, x_{10}$.

n is used to represent how many numbers are in a set. In the example above, n would be 10. n may also be used when the number of values is not known.

The Summation Sign

The summation sign (Σ) is useful for ***compactly*** showing the sum of a series of values. A number of examples are used to show its uses. Consider the following data:

Year	Dividends per Share (D_i)
1	2
2	3
3	4

The sum of the annual dividends (D) is as follows:

(1) $$\sum_{i=1}^{3} D_i = D_1 + D_2 + D_3 = 2 + 3 + 4 = 9$$

The i at the bottom of the summation sign is called a *summation index*. The index value below the summation sign (i = 1) indicates the first D value in the series to be summed. The index value at the top of the summation sign (3) shows the last D value to be summed. In this particular example, the values of D starting with the first and ending with the third year are added together. If we want to add the second and the third values only, we would write the equation as follows:

(2) $$\sum_{i=2}^{3} D_i = D_2 + D_3 = 3 + 4 = 7$$

If we want to add the first and the second values only, we would write the equation as follows:

(3) $$\sum_{i=1}^{2} D_i = D_1 + D_2 = 2 + 3 = 5$$

If we want to square the first and the second values and add up the squared values, the equation would look like this:

(4) $$\sum_{i=1}^{2} D_i^2 = D_1^2 + D_2^2 = 2^2 + 3^2 = 13$$

To show the multiplication of the sum by a constant, we write:

(5) $$3 \times \sum_{i=1}^{3} D_i = 3(2 + 3 + 4) = 27$$

To show the division of the sum by a constant, we write:

(6) $$\frac{\sum_{i=1}^{3} D_i}{3} = \frac{(2 + 3 + 4)}{3} = 3$$

To raise the sum to a power, such as square, we write:

(7) $$\left(\sum_{i=1}^{3} D_i\right)^2 = (2 + 3 + 4)^2 = 81$$

When the number of values is not known, the unknown number can be represented by *n*, such as

(8) $$\sum_{i=1}^{n} D_i = D_1 + D_2 + D_3 + \ldots + D_n$$

Another important use of summation notation is in finding the mean.

The **mean** is defined as

$$\bar{X} = \frac{\sum x}{n}$$

For example, to find the mean of 12, 8, 7, 3, and 10, use the formula and substitute the values, as shown:

(9) $$\bar{X} = \frac{\sum x}{n} = \frac{12+8+7+3+10}{5} = \frac{40}{5} = 8$$

The notation $\sum \left(x - \bar{X}\right)^2$ means to perform the following steps, using the values from example (9):

- Step 1. Find the mean. (= 8)
- Step 2. Subtract the mean from each value. (12 − 8, 8 − 8, etc.)
- Step 3. Square the answers from each subtraction. (16, 0, etc.)
- Step 4. Find the sum. (16 + 0 + etc.)

This complex computation is part of finding the variance and standard deviation of a security or portfolio. We will learn the full computation in future sessions.

The Multiplication Sign

The Greek letter Π is used to represent multiplication in the same way that Σ is used to represent summation.

$$\prod_{i=1}^{n} x_i = x_1 \times x_2 \times x_3 \times \ldots \times x_n$$

Common Mathematical Notations

- *s* End of series
- Σ Sum
- Π Multiplication
- $i + 1$ Beginning of series
- $\Sigma x / n$ Mean (= sum of *x* divided by number of periods)
- σ Standard deviation
- σ^2 Variance
- σ_{ij} Covariance of *i* and *j*
- ρ_{ij} Correlation of *i* and *j*
- β Beta
- α Alpha

Exponents

Oftentimes in investments, you are working with exponents. N to the fifth power is

$$N \times N \times N \times N \times N$$

The following are simple rules to keep in mind when working with exponents:

- Any number raised to the 0 power equals 1 (except 0)
  $n^0 = 1$
  $2^0 = 1$
- Any number raised to the power of 1 equals itself
  $n^1 = n$
  $2^1 = 2$
- Add the exponent when multiplying terms with the same base
  $n^2 \times n^3 = n^{2+3} = n^5$
  $2^2 \times 2^3 = 2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$
- A number with a negative exponent equals its reciprocal with a positive exponent

$$n^{-5} = \frac{1}{n^5}$$

$$2^{-5} = \frac{1}{2^5} = \frac{1}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{32}$$

When using a calculator, the exponent of a number *n* is commonly found using the y^x key.

Roots

Finding the square root of a number simply is the inverse operation of squaring the number. The square of a number is that number *times* itself.

Square of $n = n^2$

Square of $3 = 3 \times 3 = 3^2 = 9$

The root of any number is

$$\sqrt[n]{x} = x^{1/n}$$

$$\text{So, } \sqrt[5]{32} = 32^{1/5} = 2$$

That is, what number when multiplied by itself 5 times will result in 32. That number, of course, is 2.

When using a calculator, the *exponent* of $\frac{1}{n}$ is commonly calculated using the $1/x$ key.

Therefore, $x^{1/n}$ can be solved on a calculator by using both the $1/x$ key and the y^x key. You should review your own calculator's operating manual for its specific inputs. This principle is used when calculating the geometric mean and internal rate of return.

Order of Operation

If more than one operation exists, then we must use the following rules:

1. All operations that lie within parentheses must be done first.
2. Next, do any work with exponents.
3. Do all multiplication and division, moving from left to right.
4. Do all addition and subtraction, moving from left to right.

In the preceding example, the problem was solved by first calculating what was in the parentheses. If we did not do this, we would get a very different, and incorrect, answer.

Order of operation is important in investments and is used in standard deviation and covariance calculations.

Order of Operation	**Explanation**
$10 \times 2^2 + 7 \times (5 + 1 - 5) =$	First, calculate what is in parenthesis.
$10 \times 2^2 + 7 \times (1) =$	Next, calculate the exponent.
$10 \times 4 + 7 \times 1 =$	Working from left to right, do all multiplication and division.
$40 + 7 =$	Working from left to right, do all addition and subtraction.
47	Correct answer!

Isolating a Variable

When solving an equation, there always is an "unknown" variable for which you are solving. For instance, if we say that $2 + 3 = x$, x is the unknown. In this case, x is easily solved for by simply performing the addition of 2 and 3. But what if you said $2 + x = 5$? We can solve this easy example in our head, but in a more complicated situation this would be problematic.

To simplify the solution we need to get the x alone on one side of the equal sign. This process is called "isolating the variable." Let's see how to do this using our simple example:

$$2 + x = 5$$

To get the x by itself we need to get rid of the 2 on the left of the = sign. Wouldn't it be great if we could just take it away? We can, but only if we take it away from both sides of the equation to keep the equation balanced. So:

$$2 + x - 2 = 5 - 2$$

Here we have taken two from both sides. The rest is easy. The 2 – 2 on the left cancel each other out and all that is left is x.

$$x = 5 - 2$$

$$x = 3$$

The same process can work with multiplication and division. Consider the following:

$$25 \times x = 150$$

We can divide both sides by 25 to isolate the x.

$$\frac{25}{25} \times x = \frac{150}{25}$$
$$x = 6$$

If you have all of the information required for a formula except for one variable, the unknown variable often is found by the process of "isolating the unknown." The principle is used when solving for n periods or i interest in present or future value calculations.

Basis Point
A basis point is one hundredth of a percentage point (0.01%).

35 basis points = 0.35% = 0.0035 decimal

350 basis points = 3.50% = 0.035 decimal

Basis points often are used to measure changes in or differences between yields on fixed-income securities, since these often change by very small amounts. For example, a 100-basis-point move in yield for a bond is 1 percent.

A Note about Rounding
Rounding differences can account for slight differences in answers in many mathematical calculations. There is no optimal rounding policy. It can depend upon what you are measuring or what calculations you are doing. Because the math in this program is being used to internalize the concepts, and not to delve into the intricacies of mathematics, we will not be too concerned about the details of rounding. If you think you understand a concept, but are getting slightly different answers than the text, you still may be doing the calculation correctly. Check for rounding differences first to make certain that is not the cause of the discrepancies.

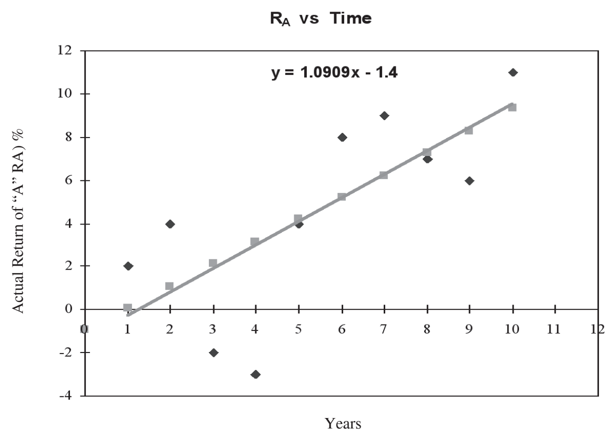
Graphing

Many concepts of modern portfolio theory can be illustrated using graphs. The first step is to review graphing on an *x*-axis / *y*-axis two-dimensional graph. A given set of returns for investor A is

Years	Return of A(%)
1	2
2	4
3	–2
4	–3
5	4
6	8
7	9
8	7
9	6
10	11

If we were to graph the percent return of investment A on the *y*-axis versus "time in years" on the *x*-axis, we would consider the ordered pairs (years, return) = (x, y):

(1, 2), (2, 4), (3, –2), (4, –3), (5, 4), (6, 8), (7, 9), (8, 7), (9, 6), (10, 11).



If we view the graph, the straight line represents the best fit between the points. It represents a kind of average and is called a regression line. The regression line passes through a point with coordinates equal to the mean of the independent and dependent variables.

The linear equation (straight line) is expressed mathematically by:

$y = mx + b$ **Equation 1-1**

Where,

m = the ***slope*** of the line (the change in the rise divided by the change in the run).

b = the ***y-intercept***, the place on the graph where the line intercepts the y-axis.

The estimated slope coefficient for the regression line describes the change in *y* for a given change in *x*. Depending on the relationship between the regression variables, it can be positive, negative, or zero. The intercept term, b, is the point where the line intersects with the *y*-axis at $x = 0$. The *y* value is often called the ***dependent variable***, since it depends on which *x* is input into the equation. Therefore, *x* is the ***independent variabl***e. The slope and intercept are important, as we will find in future sessions, because a combination of the risk-free asset and a risky asset produces a linear risk/return line with an intercept of the risk-free asset's return and a slope equal to the security market line (SML).

Consider a straight line where $y = 2x - 1$. The ordered pairs (*x*, *y*) are provided in the following table:

$y = (2)x - 1$	
x	y
0	–1
1	1
2	3
3	5
4	7
5	9

To calculate the slope of the line (*m*):

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in Rise}}{\text{Change in Run}}$$

Equation 1-2

The *x*'s and y's in the formula indicate that there are first and second values for the *x* variable, and first and second values for the *y* variable. Let's illustrate by taking two ordered pairs from the table, say (1, 1) and (4, 7). Remember that an ordered pair is (x, y). What is important in the slope equation is that we subtract the *x*'s and *y*'s in the correct order. To create the equation we will assign:

$x_1 = 1, x_2 = 4$

$y_1 = 1, y_2 = 7$

So,

$$m = \frac{7-1}{4-1} = \frac{6}{3} = 2 = m = \text{slope}$$

Suppose in our example we had picked the order pairs of (2, 3) and (5, 9). The slope still is 2:

$$m = \frac{9-3}{5-2} = \frac{6}{3} = 2 = m = \text{slope}$$

Central tendencies are single values used to characterize a set of data values. There are three commonly used measures of central tendencies: mode, median, and mean.

Mode

The mode may be thought of as the data item that appears most frequently in a given set of numbers. For instance, if you have the following set of returns: 5 percent, 8 percent, 7 percent, 5 percent, then 5 percent would be the mode. In larger data sets there may be more than one mode, as in: 3 percent, 8 percent, 3 percent, 7 percent, 3 percent, 24 percent, 28 percent, 24 percent, 26 percent. Here 3 percent and 24 percent would be called local modes and 3 percent would be the overall mode (most frequent).

Median

Another measure of central tendency is the *median*. The median is used to find the value from which one-half the data will be higher and one-half will be lower. This is the middle value of an ordered sequence. As an example, examine the following returns of nine money managers:

Manager	Return
1	33%
2	31%
3	27%
4	24%
5	18%
6	13%
7	9%
8	7%
9	5%

To find the median, the returns are arranged by the numerical values (lowest value to highest value or vice versa). The median value is manager 5's return of 18 percent because of its location, which is in the middle of the range of returns. The median for an even number of returns is the arithmetic mean of the two middle returns. For example find the median of the following six managers:

Manager	Return
1	11%
2	18%
3	16%
4	9%
5	13%
6	12%

First we should arrange the returns from lowest to highest.

Manager	Return
4	9%
1	11%
6	12%
5	13%
3	16%
2	18%

The two middle returns are 12 percent and 13 percent. The median is the arithmetic mean of these two values, which is

$$\frac{12+13}{2} = 12.5\%$$

Mean

The mean simply is a computation of an "average." There are three types of means: 1) the arithmetic mean, 2) the dollar (weighted) mean, and 3) the geometric mean.

Arithmetic Mean

$$\text{Mean} = \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Equation 1-3

Where, x_i is the asset and n is the number of observations.

For example, what is the mean rate of return among investments A and B and the market M.

Year	Returns of A	Returns of B	Returns of M
1	2%	8%	6%
2	4%	11%	8%
3	–2%	14%	10%
4	–3%	16%	12%
5	4%	12%	13%
6	8%	8%	4%
7	9%	4%	1%
8	7%	–2%	–3%
9	6%	–7%	–4%
10	11%	–9%	3%

Using the formula:

$$\bar{x}_A = \frac{[2+4+(-2)+(-3)+4+8+9+7+6+11]}{10} = 4.6$$

$$\bar{x}_B = \frac{[8+11+14+16+12+8+4+(-2)+(-7)+(-9)]}{10} = 5.5$$

$$\bar{x}_M = \frac{[6+8+10+12+13+4+1+(-3)+(-4)+3]}{10} = 5.0$$

Ratios can be tricky when computing their means. For example:

If a \$50 stock has earnings of \$1.00 per share, the price-to-earnings (P/E) ratio is 50. If another \$50 stock has earnings of \$1.50 per share their P/E ratio is 33.33. The appropriate calculation is to divide the sum of the prices (\$50 + \$50) by the sum of each stock's earnings (\$1.00 + \$1.50). This will generate a P/E ratio of:

$$\frac{(\$50+\$50)}{(\$1.00+\$1.50)} = \frac{\$100}{\$2.50} = 40 \text{ to } 1$$

<u>Weighted Mean</u>

Using the mean doesn't work in all situations related to finance. In the example above we used returns to find the arithmetic mean of said returns, and that was fine for individual securities. But, if we are calculating a portfolio's rate of return, we must account for the fact that one asset class may be a larger part of the portfolio than another. Different dollar amounts have been invested in various assets, so each asset carries a different weight. Furthermore, each asset will have a different predicted return, so in order to find the return of a portfolio we must look at the *weighted* mean of the individual asset returns. There are two ways to approach this.

Weighting means how much the part contributes to the whole. If I have \$10,000 invested in stock A and my portfolio totals \$110,000, then the weight stock A brings to my portfolio is:

$$w_A = \$10{,}000 / \$110{,}000 = 0.09091 = 0.09091 \times 100 = 9.1\%$$

Mathematically the formula for weighting an investment is as follows:

Weight of an investment $= W_i = \dfrac{\text{Dollar Value Investment}_i}{\text{Total Dollars Invested}}$

Written using formal notation this is:

$$w_i = \frac{V_i}{\sum_{i=1}^{n} V_i}$$

Equation 1-4

V_i = Amount invested in asset i

$\sum_{i=1}^{n} V_i$ = Sum of the assets in the portfolio

Consider the table below that represents a portfolio of portfolios:

Client	Assets	Weight Calculation	Weight	Rate of Return
1	$600,000	600,000 /8,500,000	7.1%	13%
2	$1,00,000	1,000,000 /8,500,000	11.8%	10%
3	$1,300,000	1,300,000 /8,500,000	15.3%	9%
4	$1,600,000	1,600,000 /8,500,000	18.8%	7%
5	$1,900,000	1,900,000 /8,500,000	22.4%	6%
6	$2,100,000	2,100,000 / 8,500,000	24.7%	5%
Total	**$8,500,000**		**100.1%**	

From the table, we see that in order to find the weight of each client to the portfolio, we use equation 1-4. Due to rounding we don't quite arrive at 100 percent when summing the weights. We could be more accurate by rounding to two decimal places. However, for the purposes of keeping the example simple we can accept this minor error.

Let's now compute the weighted mean, which can be calculated by two methods:

Method 1:

$$\bar{x}\ \text{weighted} = \frac{\sum_{i=1}^{n} X_i V_i}{\sum_{i=1}^{n} V_i}$$

Equation 1-5

Where, X_i is the rate of return of an asset, V_i is the amount invested in the asset, and ΣV_i is the sum of the assets in the portfolio.

It is not important to remember the equation. What's important is to understand how to calculate it using the following steps:

1) Multiply the rates of return times each client's asset amount for a dollar return for each client, then sum the dollar returns.

Client	Assets	Rate of Return	Weight Calculations	Dollar Return
1	$600,000	13%	600,000 × .13	$78,000
2	$1,00,000	10%	1,000,000 × .10	$100,000
3	$1,300,000	9%	1,300,000 × .09	$117,000
4	$1,600,000	7%	1,600,000 × .07	$112,000
5	$1,900,000	6%	1,900,000 × .06	$114,000
6	$2,100,000	5%	2,100,000 × .05	$105,000
Total	**$8,500,000**			**$626,000**

2) Divide this sum by the total amount invested.

The weighted mean return is $\frac{\$626{,}000}{\$8{,}500{,}000} = .0736 = 7.4\%$

Method 2:

$$\bar{x}\ \text{weighted} = \sum_{i=1}^{n} X_i W_i$$

Equation 1-6

Where, X_i is the rate of return of an asset, and W_i is the weight of an asset in a portfolio.

The important point, as with method 1, is to understand the steps. The calculation is shown in two steps:

1) Multiply the weight of each client (expressed in decimal forms) to the portfolio times each client's rate of return (see equation 1-4 for the calculation of W_i, weight of an investment).

Client	Weight	Rate of Return	Weight × Rate
1	7.1%	13%	0.9%
2	11.8%	10%	1.2%
3	15.3%	9%	1.4%
4	18.8%	7%	1.3%
5	22.4%	6%	1.3%
6	24.7%	5%	1.2%
Total	**100.1%**		

2) Sum the weight × rate for each client.

Client	Weight	Rate of Return	Weight × Rate
1	7.1%	13%	0.9%
2	11.8%	10%	1.2%
3	15.3%	9%	1.4%
4	18.8%	7%	1.3%
5	22.4%	6%	1.3%
6	24.7%	5%	1.2%
Total	**100.1%**		**7.4%**

Notice that the **weighted mean** return in method 2 (after rounding) is the same as method 1. If we were to calculate an **arithmetic** mean of the returns from this portfolio, we would have:

$$\bar{X}_{Portfolio} = \frac{(13+10+9+7+6+5)}{6} = \frac{50}{6} = 8.3\%$$

We find that the weighted mean is different than the arithmetic mean. Why? This results because the arithmetic mean does not calculate the impact of weighting within the portfolio.

Geometric Mean

The process by which money today (present value) earns interest over time and grows to a larger amount (future value) is called *compounding*. Neither the arithmetic mean nor the weighted mean consider the effect of compounding. The *geometric mean* does. To calculate, we take the following steps:

1) Take the return for each year in decimal format and add one (12% = 0.12 + 1 = 1.12).
2) Multiply the results together.
3) Take this result by the root of the number of values in your calculation. If you are calculating the geometric mean for three years of annual returns, then the third root is used ($\sqrt[3]{?}$), a four-year calculation would use the fourth root, etc.
4) Finally, subtract one from the root result for the geometric mean.

It may help to look at the formula:

$$\bar{X}_{geometric} = \sqrt[n]{(1+R_1)(1+R_2)\times \ldots \times (1+R_n)} - 1$$

Equation 1-7

This also can be written as:

$$\bar{X}_{geometric} = \sqrt[n]{\prod_{i=1}^{n}(1+R_i)} - 1$$

Where, R_i is the rate of return of an asset, and n is the number of observations.

The following example shows annual rates of return over a ten-year period.

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Returns	–6.4%	33.2%	5.4%	11.0%	2.3%	38.5%	23.1%	35.8%	26.5%	37.4%

Step 1. Add one to the decimal form of each of the annual returns as illustrated (1 + return)

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Returns	–6.4%	33.2%	5.4%	11.0%	2.3%	38.5%	23.1%	35.8%	26.5%	37.4%
1+ Return	0.936%	1.332%	1.054%	1.11%	1.023%	1.385%	1.231%	1.358%	1.265%	1.374%

Step 2. Multiply all the answers from step 1.

$$0.936 \times 1.332 \times 1.054 \times 1.11 \times 1.023 \times 1.385 \times 1.231 \times 1.358 \times 1.265 \times 1.374$$
$$= 6.004877 = 6.005$$

Step 3. Take the root of the number, which is equal to the number of values (returns). In this example, it is the 10th root.

$$\sqrt[10]{6.005} = 6.005^{\frac{1}{10}} = 1.1963\%$$

This is where a calculator is helpful. The $1/x$ key and the y^x key on your calculator will help to solve this problem. Please see your calculator's instruction manual for how to calculate using these two function keys.

Step 4. Subtract 1 from the results of the previous step.

$$1.1963 - 1 = 0.1963 = 19.63\%$$

This result is the average compounded **rate of return** over the ten-year period. In dollar terms, if $1 were invested on January 1 of year 1, it would have compounded to approximately $6.005 by December 31 of year 10 ($[1 + 0.1963]^{10}$). Note that the $6.005 is the result of step 2.

Arithmetic Mean vs. Geometric Mean

The arithmetic and geometric mean produce very different results. For example, assume a stock is bought at $100, and at the end of the year it is selling for $125. It is later sold in the second year at $100. In the first year the stock gains 25 percent (([125/100] – 1) × 100). In the second year, the stock loses 20 percent (([100/125] – 1) × 100). The arithmetic mean is

$$\bar{x} = \frac{25 + (-20)}{2} = 2.5$$

Thus, the arithmetic mean shows that the investor earned 2.5 percent even though the stock was sold at the same price it was bought. The geometric mean is

$$\bar{x}_{geometric} = \sqrt[n]{(1 + R_1)(1 + R_2)} - 1$$
$$= \sqrt{(1.25)(0.80)} - 1 = 0$$

Thus, the geometric average more accurately shows that the investor had no gain from the stock trade. It was bought at $100 and later sold at $100. In fact, the geometric return always will be less than the arithmetic return.

Graphical Form

Form is a graphical interpretation of data and for our purposes describes a shape: either a symmetrical or asymmetrical (skewed) bell-shaped curve as illustrated in exhibits 1, 2, and 3.

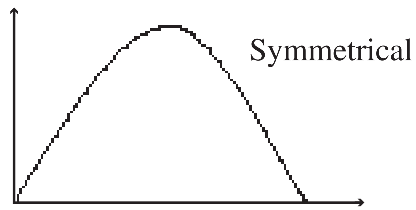
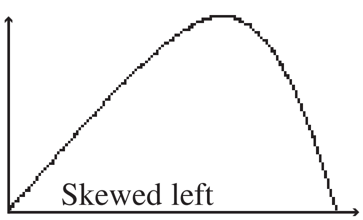


Exhibit 1

Asymmetrical



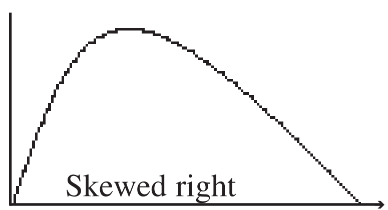


Exhibit 2

Exhibit 3

Variability is dispersion, and so it is a measure of how spread out the data points are. In that sense it describes a distance from some location on the graphs. If you have a volatile investment, then it has a high variability of return, both positive and negative from its average. Think of it in terms of how the returns of an investment or group of investments deviate from either the average or a target return we would like to achieve. Variability is measured by variance and denoted by the symbol σ^2, which is the Greek letter sigma (σ), raised to the 2nd power, or "squared." The standard deviation of returns is sigma (σ), which is the square root of the variance. We can consider variance and standard deviation as measures of consistency or randomness. We will learn more about standard deviation and variance later.

The mean, median, and mode have the same value when the data is symmetric (ehibit 1) and have different values when the data is skewed (exhibits 2 and 3):

- Positively skewed (skewed right): mean > median > mode
- Negatively skewed (skewed left): mean < median < mode

For our purposes a left skew signifies a large probability of a downside return. A right skew would be the opposite.

Mode is not as important to the investment consultant as are the mean and the median. The median discounts the presence of "uneven tails" and extreme values in a distribution of data. Consider the following distribution of incomes for prospective clients in your practice:

Annual Income Data from Two Texas Towns

	Texas 1	Texas 2
	$650,000	$2,800,000
	590,000	900,000
	570,000	320,000
	420,000	80,000
	380,000	50,000
	370,000	35,000
	270,000	20,000
Median	$420,000	$80,000
Mean (average)	$464,286	$600,714

You can notice that four of the seven incomes in Texas 2 are $80,000 or less. The "uneven tail" in Texas 2 is the $2,800,000, which dramatically increases the value of the arithmetic mean. Your practice is more likely to grow faster if you prospect from the median standpoint rather than from the mean (arithmetic average). It is the arithmetic mean, however, that will be most fundamental to our discussion of variance and standard deviation because many investors may be described as "mean-variance" investors.