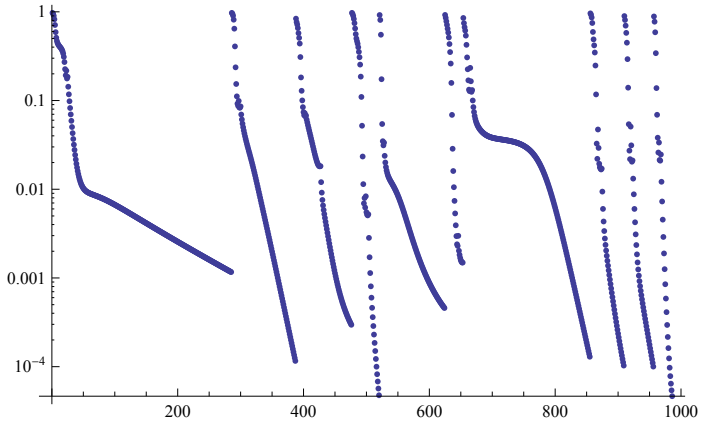
```
(* minimum set*)

ss1 = {{0, 1}, {1, 0}}; ss2 = {{0, -I}, {I, 0}}; ss3 = {{1, 0}, {0, -1}}; ss4 = {{1, 0}, {0, 1}};

QF3 = {{1/(2 Sqrt[2]), 1/(2 Sqrt[2]), 1/(2 Sqrt[2]), 1/(2 Sqrt[2]), 1/(2 Sqrt[2]), 1/(2 Sqrt[2]), 1/(2 Sqrt[2]), 1/(2 Sqrt[2])},
  {1/(2 Sqrt[2]), E^(I π/4)/(2 Sqrt[2]), I/(2 Sqrt[2]), E^(3 I π/4)/(2 Sqrt[2]), -1/(2 Sqrt[2]), E^(-3 I π/4)/(2 Sqrt[2]), -I/(2 Sqrt[2]), E^(-I π/4)/(2 Sqrt[2])},
  {1/(2 Sqrt[2]), I/(2 Sqrt[2]), -1/(2 Sqrt[2]), -I/(2 Sqrt[2]), 1/(2 Sqrt[2]), I/(2 Sqrt[2]), -1/(2 Sqrt[2]), -I/(2 Sqrt[2])},
  {1/(2 Sqrt[2]), E^(3 I π/4)/(2 Sqrt[2]), -I/(2 Sqrt[2]), E^(I π/4)/(2 Sqrt[2]), -1/(2 Sqrt[2]), E^(-I π/4)/(2 Sqrt[2]), I/(2 Sqrt[2]), E^(-3 I π/4)/(2 Sqrt[2])},
  {1/(2 Sqrt[2]), -1/(2 Sqrt[2]), 1/(2 Sqrt[2]), -1/(2 Sqrt[2]), 1/(2 Sqrt[2]), -1/(2 Sqrt[2]), 1/(2 Sqrt[2]), -1/(2 Sqrt[2])},
  {1/(2 Sqrt[2]), E^(-3 I π/4)/(2 Sqrt[2]), I/(2 Sqrt[2]), E^(-I π/4)/(2 Sqrt[2]), -1/(2 Sqrt[2]), E^(I π/4)/(2 Sqrt[2]), -I/(2 Sqrt[2]), E^(3 I π/4)/(2 Sqrt[2])},
  {1/(2 Sqrt[2]), -I/(2 Sqrt[2]), -1/(2 Sqrt[2]), I/(2 Sqrt[2]), 1/(2 Sqrt[2]), -I/(2 Sqrt[2]), -1/(2 Sqrt[2]), I/(2 Sqrt[2])},
  {1/(2 Sqrt[2]), E^(-I π/4)/(2 Sqrt[2]), -I/(2 Sqrt[2]), E^(-3 I π/4)/(2 Sqrt[2]), -1/(2 Sqrt[2]), E^(3 I π/4)/(2 Sqrt[2]), I/(2 Sqrt[2]), E^(I π/4)/(2 Sqrt[2])}};

B = N[ConjugateTranspose[QF3]];

VV3 = {{1, 4, 4}, {4, 2, 4}, {4, 3, 4}, {4, 1, 4}, {4, 4, 3},
    {4, 3, 3}, {4, 1, 3}, {1, 1, 4}, {1, 2, 4}, {3, 4, 2}, {1, 4, 3}};

(*Circuit*)

G[i_, j_, k_, x_] := N[MatrixExp[I x KroneckerProduct[ss_i, ss_j, ss_k]]]

Do[Do[Do[U = G[i, j, k, x]; A_{i,j,k}[x_] = N[U], {i, 1, 4}], {j, 1, 4}], {k, 1, 4}]

Ci[X_List] := A_{1,4,4}[X[[7]]].A_{4,3,3}[X[[1]]].A_{4,2,4}[X[[8]]].
  A_{1,1,4}[X[[3]]].A_{4,3,4}[X[[9]]].A_{1,2,4}[X[[4]]].A_{3,4,2}[X[[5]]].
  A_{1,4,3}[X[[6]]].A_{4,4,3}[X[[10]]].A_{4,1,3}[X[[2]]].A_{4,1,4}[X[[11]]]

Ci2[X_List] := A_{1,4,4}[X[[7]]].A_{4,3,3}[X[[1]]].A_{4,2,4}[X[[8]]].A_{1,1,4}[X[[3]]].
  A_{4,3,4}[X[[9]]].A_{1,2,4}[X[[4]]].A_{3,4,2}[X[[5]]].A_{1,4,3}[X[[6]]].A_{4,4,3}[X[[10]]].
  A_{4,1,3}[X[[2]]].A_{4,1,4}[X[[11]]].A_{1,4,4}[X[[12]]].A_{4,3,3}[X[[13]]].
  A_{4,2,4}[X[[14]]].A_{1,1,4}[X[[15]]].A_{4,3,4}[X[[16]]].A_{1,2,4}[X[[17]]].
  A_{3,4,2}[X[[18]]].A_{1,4,3}[X[[19]]].A_{4,4,3}[X[[20]]].A_{4,1,3}[X[[21]]].A_{4,1,4}[X[[22]]]
```

```
Ci3[X_List] := A1,4,4[X[[7]]].A4,3,3[X[[1]]].A4,2,4[X[[8]]].A1,1,4[X[[3]]].
  A4,3,4[X[[9]]].A1,2,4[X[[4]]].A3,4,2[X[[5]]].A1,4,3[X[[6]]].A4,4,3[X[[10]]].
  A4,1,3[X[[2]]].A4,1,4[X[[11]]].A1,4,4[X[[12]]].A4,3,3[X[[13]]].A4,2,4[X[[14]]].
  A1,1,4[X[[15]]].A4,3,4[X[[16]]].A1,2,4[X[[17]]].A3,4,2[X[[18]]].A1,4,3[X[[19]]].
  A4,4,3[X[[20]]].A4,1,3[X[[21]]].A4,1,4[X[[22]]].A1,4,4[X[[23]]].A4,3,3[X[[24]]].
  A4,2,4[X[[25]]].A1,1,4[X[[26]]].A4,3,4[X[[27]]].A1,2,4[X[[28]]].A3,4,2[X[[29]]]

Cost[X_List] := 1 - 1/64 Abs[Tr[Ci[X].B]]^2

Graa[W_List, NN_, dl_] := (
    uy = {}; Do[uO = {}; Do[uO = Append[uO, dl KroneckerDelta[i, j]], {j, 1, NN}];
     uy = Append[uy, {(Cost[W + uO] - Cost[W - uO]) / (2 dl)}], {i, 1, NN}];
    Return[
     uy]);

NN = 11; NM = 10; koko = {}; loko = {};
Do[L0 = {}; Do[If[i == NN, joy = {0, Pi}, joy = {0, Pi}];
  L0 = Append[L0, RandomReal[joy]], {i, 1, NN}];
 di = Cost[L0]; Print["Initial distance=", di];
 ko = di; stro = 0.5; stre = stro;
 Do[
  L = L0 - stre Transpose[Graa[L0, NN, 0.05]][[1]];
  ki = Cost[L]; koko = Append[koko, ki];
  If[ki < ko, (*Print["!"];*)L0 = L; If[Abs[ki - ko] < 10^-5, Goto[er]];
   ko = ki, (*If[Abs[ki-ko]< 10^-3,Goto[er]];*)stre = stre 0.8], {i, 1, 800}];
 (*Print[ko];*)Label[er]; loko = Append[loko, L0];
 KO = Min[koko]; Print[ko, L0], {kk, 1, NM}]
```

```
Initial distance=0.983891

0.603744{3.17462, 3.01117, 2.06972, 1.51696,
  0.735266, 0.980392, 1.23295, 0.581216, 1.29633, 3.69502, 1.16915}

Initial distance=0.98014

0.603745{0.0386507, 3.42592, -0.0256003, 1.76035,
  0.728867, 0.980404, 2.14179, 2.15115, 2.92385, 1.21777, 1.16822}

Initial distance=0.998593

0.603741{3.17556, 0.767755, 1.9788, 0.281275,
  2.41408, 2.55068, 0.130449, 2.15193, 1.55059, 3.22965, 1.1725}

Initial distance=0.992159

0.603741{0.0389802, 1.34989, 0.96508, 3.03201,
  0.734869, 0.980694, 0.6002, 2.15703, 1.8362, 1.1863, 2.74181}

Initial distance=0.951717

0.603742{0.0367053, 0.0821225, 0.250645, 3.22175,
  2.40636, 2.55124, 0.718628, 0.58917, 0.274024, 1.06775, 1.17546}

Initial distance=0.971804

0.60374{3.17566, 1.66861, -0.231604, 1.65791,
  0.733573, 0.980302, 1.53515, 2.15092, 2.8747, 3.39345, 2.73885}

Initial distance=0.955993

0.750094{0.7501, 0.132123, 1.10381, 2.9065,
  1.58287, 1.80456, 0.686189, 1.17561, 0.335331, 0.8844, 1.20322}

Initial distance=0.970809

0.603741{3.17598, 1.96106, 1.6483, 1.34598,
  0.730515, 0.980155, 0.849956, 2.14913, 0.172837, 2.50901, 1.16757}

Initial distance=0.890117

0.603742{1.60611, 2.3386, 0.411229, 0.279956,
  0.732242, 2.55112, 2.88319, -0.579023, 1.55559, 2.04591, 2.73771}

Initial distance=0.996032

0.603742{3.17497, 1.68479, -0.2136, -0.0957614,
  0.734316, 0.980343, 2.97101, 2.15136, 1.30713, 3.52811, 1.16858}
```

ListLogPlot[koko, PlotRange → All]



```
dd = Dimensions[loko][[1]]; oups = {};
Do[L1 = loko[[i]]; ba = Cost[L1]; la = N[Count[ba, 0] / Dimensions[ba][[1]]];
 Print[i, "__", la]; oups = Append[oups, la], {i, 1, dd}]
```

```
(*29*)Min[koko]
```

```
0.0000464613
```

```
(*22*)Min[koko]
```

```
0.0380792
```

```
(*11*)Min[koko]
```

```
0.603733
```

```
(* let's take other gates*)
```

```
CiR[X_List] := A1,4,4[X[[7]]].A4,3,2[X[[1]]].A4,2,4[X[[8]]].
  A1,3,4[X[[3]]].A4,3,4[X[[9]]].A1,3,4[X[[4]]].A3,4,3[X[[5]]].
  A1,4,2[X[[6]]].A4,4,3[X[[10]]].A4,1,1[X[[2]]].A4,1,4[X[[11]]]
```

```
Ci3R[X_List] := A1,4,4[X[[7]]].A4,3,2[X[[1]]].A4,2,4[X[[8]]].A1,3,4[X[[3]]].
  A4,3,4[X[[9]]].A1,3,4[X[[4]]].A3,4,3[X[[5]]].A1,4,2[X[[6]]].A4,4,3[X[[10]]].
  A4,1,1[X[[2]]].A4,1,4[X[[11]]].A1,4,4[X[[12]]].A4,3,2[X[[13]]].A4,2,4[X[[14]]].
  A1,3,4[X[[15]]].A4,3,4[X[[16]]].A1,3,4[X[[17]]].A3,4,3[X[[18]]].A1,4,2[X[[19]]].
  A4,4,3[X[[20]]].A4,1,1[X[[21]]].A4,1,4[X[[22]]].A1,4,4[X[[23]]].A4,3,3[X[[24]]].
  A4,2,4[X[[25]]].A1,1,4[X[[26]]].A4,3,4[X[[27]]].A1,2,4[X[[28]]].A3,4,2[X[[29]]]
```

$$\text{Cost[X\_List]} := 1 - \frac{1}{64}\,\text{Abs[Tr[Ci3R[X].B]]}^2$$

```
NN = 29; NM = 10; koko = {}; loko = {};
Do[L0 = {}; Do[If[i == NN, joy = {0, Pi}, joy = {0, Pi}];
  L0 = Append[L0, RandomReal[joy]], {i, 1, NN}];
 di = Cost[L0]; Print["Initial distance=", di];
 ko = di;  stro = 0.5; stre = stro;
 Do[
  L = L0 - stre Transpose[Graa[L0, NN, 0.05]][[1]];
  ki = Cost[L]; koko = Append[koko, ki];
  If[ki < ko, (*Print["!"];*)L0 = L; If[Abs[ki - ko] < 10^-5, Goto[er]];
   ko = ki,  (*If[Abs[ki-ko]< 10^-3,Goto[er]];*)stre = stre 0.8], {i, 1, 800}];
 (*Print[ko];*)Label[er]; loko = Append[loko, L0];
 KO = Min[koko]; Print[ko, L0], {kk, 1, NM}]
```

```
Initial distance=0.990025
```

```
0.0746958{2.35619, 1.5653, 1.93031, 2.38938, 3.92699, 0.785238,
  0.19635, 0.000670122, 2.74889, 0.785398, 0.78662, 3.13795, 2.35619, 1.41135,
  2.32442, 2.35663, 2.38795, 2.74887, 0.785398, 2.35619, 2.35533, -0.159448,
  2.36153, 0.785398, 0.785399, 1.57098, 2.74889, 0.000183405, 2.36156}
```

```
Initial distance=0.992711
```

0.410159{0.347644, 1.57509, 1.66607, 1.86815, 0.785416, 1.56705, 1.76722, 1.57141, 0.856406,
1.32522, 1.57131, 1.556, 2.35618, 0.797348, 0.521718, 3.21303, 1.04933, 2.67337, 1.57445,
1.5708, 2.9887, 2.03107, 1.58078, 3.14173, 3.07365, 2.32097, 1.96285, 1.52235, 0.791793}

Initial distance=0.973176

0.0746973{2.35619, 0.785398, 1.72503, 2.59466, 2.35619, 2.35619,
3.33794, 4.21752×10^{-10}, 3.53429, 3.14159, 2.35619, 3.13776, 3.13579,
3.13774, 0.567314, 0.785398, 1.00348, 0.392729, 2.35619, 0.785398, 0.785398,
2.35982, 0.791015, 2.35619, 0.785398, 1.1781, 1.5708, 0.785398, 0.779731}

Initial distance=0.947157

0.43256{0.785424, 3.08863, 1.39153, 2.92717, 0.782428, 2.43684,
1.76708, 3.14074, 0.783761, 3.14186, 0.00251461, 2.09411, 2.35619,
3.14387, 3.17058, 3.05682, 0.781379, 1.5719, 0.705058, 1.57194, 1.56869,
2.35619, 0.523601, 1.62386, 1.26441, 3.11055, 1.96277, 3.10154, 3.92624}

Initial distance=0.993584

0.519082{3.14005, 2.0661, 0.696601, -0.120664, -0.000110698, 0.784426,
2.4819, 2.15683, 2.8223, -0.000472813, 2.04822, 2.53743, -0.000211799,
1.84843, 2.79195, 2.55284, 0.612086, 0.000188107, 0.784547, 2.15984, 1.5691,
2.02025, 2.2456, 2.45103, 0.522298, 2.18243, 0.208901, 0.791239, 3.927}

Initial distance=0.978945

0.431735{0.782884, 1.49279, 0.972963, 2.55004, 0.783016, 2.20025,
3.33812, 1.56967, 2.35407, -0.000409466, 1.58468, 1.86518, 0.78553, 1.56979,
1.27443, -0.0200895, 0.862757, 1.56383, 2.19929, 0.000978286, 1.59053,
0.78436, 0.294297, 1.65727, 0.375917, 1.76842, 1.91825, 0.28752, 0.785669}

Initial distance=0.973424

0.0747174{0.785398, 1.57067, 2.85026, -0.101374, 0.785399, 0.784515,
0.19635, -0.000238488, 2.74889, 2.35619, -0.785498, 3.14168, 0.785398, 2.016,
0.516768, 0.785438, 1.04801, 1.1781, 2.35619, 0.785398, 3.92665, 2.69642,
2.35631, 0.785398, 0.785398, 8.51948×10^{-6}, 2.74889, -3.19401×10^{-6}, 0.785278}

Initial distance=0.984058

0.308707{2.35619, 3.14156, 2.56566, 3.32483, 0.785398, 1.5708,
3.33793, 3.14158, 1.88372, 2.35619, 3.14162, 0.785313, 0.785398, 2.35619,
2.66832, 1.5708, 0.473288, 2.3564, 1.5708, 1.5708, 0.785398, 0.8731,
0.981748, 3.14159, 1.96358, 2.36836, 0.392699, 1.55609, 0.00022428}

Initial distance=0.969477

0.410022{0.0285093, -0.0000949964, 1.37265, 2.94701, 2.35619, 1.54874,
1.76715, -0.000021987, 0.883364, 1.37496, 1.57082, 1.57106, 0.785396, 1.90678,
2.42094, 1.04039, 2.29119, -0.757112, 1.5487, 1.57081, 1.56723, 2.07991,
0.0000202506, 0.0000667498, 1.48659, 0.405191, 0.109868, 2.24355, 0.785394}

Initial distance=0.97152

0.432326{0.784897, 3.148, 0.587886, 1.37088, 2.35627, 0.753207,
3.3381, 1.57181, 1.18572, 3.12776, -0.000310069, 1.92543, 0.785803, 1.01583,
1.76898, 1.83094, 0.267483, 0.000825555, 2.3883, 1.57081, -0.000940502,
2.34712, 2.78731, 1.55064, 1.82693, 1.44059, 0.956929, 1.09129, 2.35718}

(*11*)Min[koko]

0.774026

```
(*29*)Min[koko]
```

```
0.0746887
```