

1. ΑΕΦΟΔΥΝΟΝ & ΜΕΤΡΙΚΕΣ

Confusion Matrix: $Acc = \frac{TP+TN}{N}$ | $Prec = \frac{TP}{TP+FP}$
 $Rec = \frac{TP}{TP+FN}$ | $Spec = \frac{TN}{TN+FP}$ | $F1 = \frac{2PR}{P+R}$.
Medical Bayes: $P(Disease|+)$ = $\frac{Sens \cdot Prev + (1-Spec)(1-Prev)}{Sens \cdot Prev + (1-Spec)(1-Prev)}$.
Errors: $MSE = \frac{1}{N} \sum (y - \hat{y})^2$. $R^2 = 1 - \frac{RSS}{TSS}$.
Bias-Var: $Err = Bias^2 + Var + Noise$.
• High Bias → Underfit (Simple). • High Var → Overfit (Complex).
ROC/AUC: Plot TPR vs FPR. Random=0.5, Perfect=1.
Cross-Val: k-Fold (Low Bias/Var est), Hold-out (Fast).

2. Probabilities & Info

Bayes: $P(h|d) \propto P(d|h)P(h)$ (Post Lik Prior).

Bayes Risk: Choose action w/ min exp. loss.

Gauss: $N \sim |\Sigma|^{-1/2} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)}$.

MLE: $\hat{\mu} = \bar{x}$, $\hat{\sigma}^2 = \frac{1}{N} \sum (x-\mu)^2$ (Biased). Unbiased: divide $N-1$.

Entropy: $H(P) = -\sum P \log P$. Max at uniform.

KL Div: $D_{KL} = \sum P \ln \frac{P}{Q}$. **CrossEnt:** $H(P, Q) = H(P) + D_{KL}$.

MLE: $\max \sum \ln P(x|\theta)$. $\nabla \ln L = 0$.

3. Γραμμικά Μοντέλα

LinReg: $w = (X^T X)^{-1} X^T y$. ($O(N^3)$).

LogReg: $P = \sigma(w^T x) = \frac{1}{1+e^{-z}}$. Convex Loss.

$\sigma' = \sigma(1-\sigma)$. Update: $w \leftarrow w + \eta(y - \hat{y})x$.

LDA: Max Sep $J(w) = \frac{w^T S_B w}{w^T S_W w}$.

$S_B = (m_2 - m_1)(m_2 - m_1)^T$. $S_W = \sum S_i$.

Opt: $w \propto S_W^{-1}(m_2 - m_1)$. Assumes equal Σ .

Geometry: Hyperplane $w^T x + w_0 = 0$.

Dist $r = y(x)/||w||$. $w \perp$ surface.

Regularization (Shrinkage):

L1 (Lasso): $\lambda|w|$ (Sparsity/Selection).

L2 (Ridge): λw^2 (Small weights).

4. SVM & Kernels

Primal: $\min \frac{1}{2} ||w||^2 + C \sum \xi_i$ s.t. $y_i(w^T x_i + b) \geq 1 - \xi_i$.

Convex: Global minimum guaranteed.

Support Vectors: Points on margin ($y(\cdot) = 1$) or errors. Only these affect w .

Margin: $2/||w||$. Larger $C \rightarrow$ Harder (Sm margin).

Slacks ξ : 0 (correct), $0 < \xi < 1$ (margin vio), $\xi > 1$ (error).

Kernel Trick: $x^T z \rightarrow K(x, z)$. Implicit high dim.

• Poly: $(x^T z + c)^d$. Dim $\approx d$ -order terms.

• RBF: $e^{-\gamma||x-z||^2}$. Dim ∞ . (Taylor exp).

Geometry: Max margin boundary is \perp bisector of closest points from classes.

5. Neural Networks

Non-Convex: Local optima possible.

Unit: $z = \sum w_i x_i + b \rightarrow a = g(z)$.

Actv: ReLU max(0, z) (Fast, no vanish grad).

Sigmoid (0,1), Tanh (-1,1) (Sat at limits).

Softmax: $y_k = e^{z_k} / \sum e^{z_j}$. For Multi-class.

Forward: $Z^{[l]} = W^{[l]} A^{[l-1]} + b^{[l]}$, $A^{[l]} = g(Z^{[l]})$.

Backprop: Chain rule $\frac{\partial J}{\partial W}$.

$\delta^L = (a^L - y)$, $\delta^l = (W^{l+1})^T \delta^{l+1} \cdot g'(z^l)$.

$\delta J / \partial W^l = \delta^l (a^{l-1})^T$.

Params: FC: $(N_{in} + 1)N_{out}$.

CNN: Out = $\lfloor \frac{W-K+2P}{S} \rfloor + 1$.

Params: $F \times F \times C_{in} \times C_{out} + C_{out}$.

Optim: SGD (Noisy), Batch (Slow), Mini-batch.

Momentum (Velocity), Adam (Adapt LR).

6. Ensembles

Bagging: Bootstrap (replace) + Parallel Models.

Reduces Variance. Ex: Random Forest.

Random Forest: Bagging + Feature subset (\sqrt{p}).

Decorrelates trees → Better reduction of var.

Boosting: Sequential. Fix prev errors.

Reduces Bias (and Var). Ex: AdaBoost, XGB.

AdaBoost: Weights $\alpha_t = \frac{1}{2} \ln \frac{1-\epsilon}{\epsilon}$.

Data weights $D_{t+1} \propto D_t e^{-\alpha_t y y}$ (Focus Hard).

Trees: Split: Max InfoGain ($H_{pre} - H_{post}$).

Entropy = $-\sum p \log p$, Gini = $1 - \sum p^2$.

7. Unsupervised & PCA

PCA: Max Variance directions (Ortho).

1. Center ($x - \mu$). 2. Cov $\Sigma = \frac{1}{N} X^T X$. 3. Eigendecom UAU T .

Proj $z = U_k^T x$. Var ratio $\sum_1^k \lambda_i / \sum \lambda_{all}$.

Included '1' feature doesn't change PCA (const var=0).

K-Means: Iterate: Assign closest, Update centroids. Converges finite steps. Spherical clusters. Sensitive to K/Init.

GMM: Soft K-means. Probabilistic (π, μ, Σ).

EM Algo: Max Likelihood (local max).

E-step: Calc responsibilities γ . M-step: Update params.

DBSCAN: Density, core/border/noise. Args: eps, minPts.

Hierarchical: Agglomerative.

Linkage: Single (min dist), Complete (max), Avg.

Silhouette: $\frac{b-a}{\max(a,b)}$. +1 Good, 0 Border, -1 Wrong.

8. KNN & Dim Reduction

KNN: Lazy. Distances (L_2).

Small k → High Var (Noise). Large k → High Bias (Smooth).

Curse of Dim: Vol grows exp. Points equidistant.

Need exp data. Solutions: PCA, Feature Sel.

9. True/False Exam Bible (1/2)

-- SVM Objective: Convex Quadratic (Global Min).

-- NN Objective: Non-Convex (Local Min).

-- EM Algo: Maximizes Likelihood (Local Max).

-- K-Means: Loss decreases monotonically. Converges.

-- RBF Feature Space: Infinite dimensional.

-- PCA: Adding const feature → No change.

-- Perceptron: Oscillates if not lin sep.

-- Hard Margin SVM: Requires lin sep.

-- Bootstrapping: Sampling WITH replacement.

10. True/False Exam Bible (2/2)

-- L1 Reg: Sparsity (Feat Sel). L2: Small Weights.

-- Bias-Var: 1-NN (High Var), K-NN (High Bias).

-- Kernel Trick: w is lin comb of data (Representer).

-- Softmax: Used for Multi-class Output.

-- Linear Activ in Hidden: Collapse to linear model.

-- Generative: Naive Bayes, GMM, LDA.

-- Discriminative: LogReg, SVM, NN, KNN.

-- Decision Trees: Pruning reduces overfitting.

11. Quick Math/Formulas

Logarithms: $\ln(xy) = \ln x + \ln y$, $\ln(e^x) = x$.

Derivatives: $\frac{d}{dx} \sigma(x) = \sigma(1-\sigma)$.

Matrix: $(AB)^T = B^T A^T$. $\nabla_x w^T x = w$.

$\nabla_x x^T Ax = (A + A^T)x$.

Distances: L_1 Manhattan $\sum |x|$, L_2 Euclidean $\sqrt{\sum x^2}$.

Eigen: $Av = \lambda v$. $\det(A - \lambda I) = 0$.

Normal: 68% (1σ), 95% (2σ), 99.7% (3σ).

12. Algorithm Summary

Naive Bayes: $P(y|x) \propto P(y) \prod P(x_i|y)$. Fast, High Bias.

LogReg: Linear boundary. Interpretable. P-values.

SVM: Max Margin. Robust. Kernels → Non-Lin.

DT: Non-parametric. Interp. Orthogonal splits.

RF/GBM: Black-box. High Acc. Robust.

NN: Univ approx. Needs data/compute. Non-convex.

K-Means: Simple. Scalable. Convex clusters only.