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Cognitive mechanisms underlying subjective value of past and future events: Modeling systematic reversals of temporal value asymmetry

Peter D. Kvam
University of Florida

Matthew Baldwin
University of Florida

Erin C. Westgate
University of Florida

People discount both future outcomes that could happen and past outcomes that could have happened according to how far away they are in time. A common finding is that future outcomes are often preferred to past ones when the payoffs and temporal distance (how long ago / until they occur) are matched, referred to as temporal value asymmetry. In this paper, we examine the consistency of this effect by examining the effect of manipulating the magnitude and delays of past and future payoffs on participants' choices, and challenge the claim that differences in value are primarily due to differences in discounting rates for past and future events. We find reversals of the temporal value asymmetry when payoffs are low and when temporal distance is large, suggesting that people have different sensitivity to the magnitude of past and future payoffs. We show that these effects can be accommodated in an direct difference model of intertemporal choice but not in the most common discounting models (hyperboloid), suggesting that both temporal distance and payoff magnitude carry independent influences on the subjective value of past and future outcomes. Finally, we explore how these tendencies to represent past and future outcome values are related to one another and to individual differences in personality and psychological traits, showing how these measures cluster according to whether they measure processes related to past/future events, payoffs/delays, and whether they are behavioral/self-report measures.

Keywords: time perception, delay discounting, intertemporal choice, value representation, cognitive modeling

Intertemporal choice, and the tendency to value delayed rewards less than immediate ones referred to as *delay discounting*, is a cornerstone of research on impulsive behavior. The ability to put off immediately available rewards in favor of future ones is a capacity that improves outcomes across domains related to health, money, and politics (Berns et al., 2007). Furthermore, delay discounting is considered a transdiagnostic indicator of mental health disorders like depression, bipolar disorder, schizophrenia, and borderline personality; eating disorders like bulimia, anorexia, and binge eating (Amlung et al., 2019); and vulnerability to addiction such as substance dependence and compulsive gambling (Bickel et al., 2012, 2019). Treatments and interventions have even begun to target delay discounting processes with the goal of attenuating problems related to mental health and substance use (Bickel et al., 2011; Dixon & Holton, 2009; Lempert & Phelps, 2016; Rung & Madden, 2018). It should be possible to improve the efficacy of these interventions through a better understanding of the cognitive mechanisms governing intertemporal choice.

Typical intertemporal choice scenarios propose a decision between receiving a payoff in the future (larger, later) or receiving a payoff now or sometime nearer in the future (smaller, sooner). The classical theory in economics was that a sum received earlier will increase in value over time as it accumulates interest, and thus a payoff received now should be more valuable than the same payoff received later. Early work on intertemporal choice was driven by economic and rational considerations, suggesting that a decision maker should discount future payoffs at a constant rate, yielding an exponential function relating the time t at which a payoff $\$x$ is received to its subjective value $v(x,t)$ (Samuelson, 1937; Strotz, 1955). Although the original exponential form has since fallen out of favor, largely due to its inability to predict preference reversals when a constant time is added to immediate and delayed prospects (Berns et al., 2007; Thaler & Shefrin, 1981; Loewenstein & Thaler, 1989), nearly all theories still assume a discounting function where the value of a payoff decreases monotonically with the time until it is received.

Value of past events

A curious counterpoint to the observation that events occurring later are valued less than ones occurring sooner is work examining decisions between counterfactual events that could have occurred in the past (Yi et al., 2006), as opposed to ones that could occur in the future. On first consideration, these types of decisions can seem unintuitive: how could people have preferences among outcomes that should have already been realized? However, it is clear that people can not only consider counterfactual events (N. Roese, 1999; Ursu & Carter, 2005) and imagine their consequences (Byrne, 2016) but also form preferences among factual and counterfactual events. From simple musings about what could have been to full-blown stories about time travel, considering counterfactual sequences of events is a time-honored human ability that allows us to explore our own preferences, form expectations about the future, and anticipate how we will feel about the consequences of our own actions (Boninger et al., 1994; Zeelenberg, 1999; Coricelli et al., 2007). These counterfactual musings are not trivial; several studies have documented that thinking counterfactually about important events in one's own life (e.g., turning points, one's own birth) can positively influence judgments of life's meaning and significance (Heintzelman et al., 2013; Kray et al., 2010). The ability to assign value to counterfactual actions or events develops at a relatively early age (Harris et al., 1996; Rafetseder et al., 2013), and allows us to isolate the effects of specific changes to past events while reasoning about their consequences (N. J. Roese & Olson, 2014). Counterfactual thinking is arguably one of the more unique qualities of human life that sets us apart from other species.

Given our capacity to infer consequences and assign value to them, behavioral economic models are a natural way to quantify and evaluate comparisons among alternative timelines or courses of events (Bickel & Marsch, 2001). When people evaluate a preference for a counterfactual event (something that could have occurred) over one that occurred, they experience regret (Connolly & Zeelenberg, 2002; Bell, 1982). This experience of relative goodness among events that did and did not occur indicates that greater utility can be ascribed retroactively to counterfactual outcomes. As a result, a growing body of work has sought to study (counterfactual) past events in terms of the amount of utility they confer, and the effect of these utilities on decision-making (Yi et al., 2006, 2009). That is, people can not only consider the consequences of having won the lottery yesterday (or last week, or last year) – and the associated changes to their life at present – but they can assign value to these outcomes and relate them to their present and future resources. This enables comparisons between past, present, and future outcomes that we consider in this paper.

After evaluating how changing past events would affect the present and future, it becomes possible to consider how value is assigned to those counterfactual events as a function of how distant they are. Extrapolating the logic of discounting, at least as it relates to the accumulation of interest in financial terms, receiving a payoff in the past should be worth *more* than receiving the same payoff now. After all, if a person received $20 last month, they could have invested it such that it is worth more than $20 now, so long as their investment could out-pace inflation. This logic appears not to apply to decisions about past events, or at least it does not predict human behavior in these settings. In experiments examining choices between payoffs that could be received now and payoffs that could have been received in the past, participants show a pattern of "reverse discounting" where they prefer payoffs that are closer to the present to ones that are further in the past (Bickel et al., 2008; Yi et al., 2006; Molouki et al., 2019).

This apparent contradiction may be reconciled if we consider temporal discounting in terms of temporal *distance* – the length of time between the present and the time at which a payoff is received – as the mechanism underlying intertemporal choice, as opposed to the rate of return on investments (Bonato et al., 2012). In this view, a payoff attains its maximum value when received immediately and decreases in subjective value as it is moved backward or forward in time. This has been the dominant approach to intertemporal choice with past and future events, using a hyperbolic discounting function to predict how the subjective value of a payoff changes as it is displaced in time (Bickel et al., 2008; Yi et al., 2006; Molouki et al., 2019). Indeed, the rate at which a payoff decreases in subjective value as it is pushed into the past appears to be closely related to the rate at which it decreases in value as it is pushed into the future, leading some to posit that there is a common cognitive / neural mechanism underlying past and future discounting (He et al., 2012). Discounting of past and future events appears to be highly correlated, although people generally appear to favor future over past events, referred to as temporal value asymmetry (Caruso et al., 2008).

Past work on the temporal value asymmetry and on past discounting in general have assumed that discounting of past events follows the same (hyperbolic) function as future discounting, but that the subjective values of future and past events decay at different rates (Yi et al., 2006; Caruso et al., 2008; Caruso, 2010; Yi et al., 2009). The mechanism underlying this asymmetry is thought to be the sensitivity to differences in time, where future events seem subjectively closer than past events (often referred to as the "temporal Doppler" effect, Caruso et al., 2013; Suhler & Callender, 2012), modulated by whether attention is focused on the past or future (Guo et al., 2012).

Another possible explanation for this phenomenon is that people feel more connected to versions of themselves that are "nearby" in time (Zhang & Aggarwal, 2015). People seem to have some control over how subjectively close the past or future feels to their present selves. For instance, temporal self-appraisal theory suggests that people imagine being "far" from their past selves in order to contrast away from a negative past so that the present self appears better (Wilson & Ross, 2001; Baldwin et al., 2021). In general, people perceive past and present selves as "others" (Pronin & Ross, 2006) and will often use temporal landmarks (e.g., birthdays) to strategically distance the past from the present, if it serves a motivational or self-enhancement goal (Peetz & Wilson, 2013, 2014). But considering that people tend to imagine that they will improve over time (O'Brien & Kardas, 2016) and tend to use redemptive narratives when making sense of the life story (McAdams & McLean, 2013) it is possible that people strategically pull the future self closer, and push the past self further away from the present. It is also possible that it is easier to imagine how one's life would be improved by payoffs that are received near to the present. Predictions of the future, for instance, are heavily anchored in the present; people use their current desires to predict what they will want and feel in the future (or what they have wanted in the past; Gilbert et al., 2002; Gilbert & Wilson, 2007). Such predictions are based on initial hedonic reactions to an imagined event, which are then "corrected" to account for temporal distance. Further compounding this is the tendency for people to picture distant events more abstractly, while picturing events close in time more concretely (e.g., Construal Level Theory; Liberman et al., 2002). In general, there are individual differences in the extent to which one's current self is temporally extended into the past and future, and these differences may also hinge on the ease with which people can bring to mind concrete memories or images of their temporal selves (Grabowski & Broemer, 2015). We revisit these explanations in the Discussion.

In this paper, we evaluate possible explanations for the temporal value asymmetry, and temporal discounting more generally, through both empirical data and computational modeling. To do so, we asked participants to choose between pairs of payment options, manipulating both payoff (dollars), time (delay in days), and temporal direction (past vs future). To preview the results, our experiment uncovered both traditional temporal value asymmetries where future events were preferred to equivalent and equidistant past events, and reversals of the temporal value asymmetry where past events were preferred to equivalent future events. This reversal occurred with systematic manipulations of the magnitude of the payoff and temporal distance, such that past events were preferred for small payoffs and long delays and future events were preferred for large payoffs and short delays. This reversal places a strong constraint on models of intertemporal choice, categorically ruling out accounts based on hyperbolic discounting and even several more general hyperboloid discounting models.

Evaluating models of intertemporal choice

Traditional theories of intertemporal choice have been driven primarily by an "alternative-wise" approach (Berns et al., 2007), where the overall subjective value of a prospect is assessed as a function of its payoff (amount x) and the time at which it occurs (time t). The idea behind these models is that the subjective value $v(x,t)$ of a payoff at a given time should decrease as it is pushed further away in time. Formally, $v(x,t)$ peaks at the present, time $t = 0$, and decreases with $|t| > 0$. In the case of the temporal value asymmetry, it should be the case that $v(x,t) > v(x,-t)$ for any $t > 0$, indicating that future payoffs are universally preferred to equidistant (in time) past payoffs. There are a wide variety of discounting functions that could meet this criterion, simply by assigning a greater rate of discounting to past events $(-t)$ than future events $(+t)$.

However, if there are violations of temporal value asymmetry, the space of possible models or theories that can explain choice behavior gets much smaller. If preferences reverse as the temporal distance is manipulated, then there must be instances where $v(x,t) > v(x,-t)$ and $v(x,t+dt) < v(x,-(t+dt))$ for some value of dt. Similarly, if preferences reverse as the payoff x is manipulated, then the discounting function must allow for both $v(x,t) > v(x,-t)$ and $v(x+dx,t) < v(x+dx,-t)$. This turns out to be an extraordinarily difficult feat, and there appear to be no existing alternative-wise models that can produce both reversals.[1] Instead, we suggest that models indexing people's sensitivity to outcomes are critical to consider (Cheng & González-Vallejo, 2016; Dai & Busemeyer, 2014). We focus on one such attribute-wise model below, which computes a relative subjective value between a pair of response options as opposed to a separate subjective value for each option.

Hyperbolic discounting

The difficulty of accounting for reversals of the temporal value asymmetry is perhaps easier to understand when we examine a specific model. Suppose that we have a payoff x and a delay t, and are using a typical hyperbolic discounting model. If a person must choose whether they prefer to receive (resp., received) the same payoff x at t days in the future versus t days in the past, then they should choose whichever option (past or future) has a lower discounting rate. That is, they should pick past or future based on which temporal direction has a shallower rate of decay for the value of x. Thus, whichever option has the lower discounting rate, k_p for past or k_f for future, will be favored over the option with the higher discounting rate. In this model, the classical temporal

[1] We create and evaluate eight instances of a new alternative-wise model that can capture these effects later on, in addition to an attribute-wise account that provides the best account.

value asymmetry occurs because

$$\frac{x}{1+k_f t} > \frac{x}{1+k_p t} \tag{1}$$

for any $k_p > k_f$. The opposite would be true for all values of t if $k_f < k_p$. It is worth noting that this holds true for exponential discounting models as well (Chung, 1965). As in the hyperbolic model, a future option should be chosen if k is greater for past events, or a past option should be chosen if k is greater for future events.

This prediction is illustrated in the left panels of Figure 1. In these plots, we show the rate of discounting for future (blue) and past (orange) events based on the temporal distance to each even occurring (top row). The discounting rate for future events is shallower than for past events ($k_f = .10$ and $k_p = .15$), yielding subjective values that are always greater for future payoffs than equidistant past payoffs. Similarly, the effect of manipulating payoffs is shown at the bottom-left of Figure 1, where the magnitude of the payoff (x-axis) is related to the difference in subjective value between (x,t), and $(x,-t)$ (y-axis). As shown, this difference in subjective value never changes sign no matter how large the payoff gets, as it will stay perpetually above zero (always favoring the future payoff) or below zero (always favoring the past payoff). As a result, preferences between past and future events do not change based on manipulations of payoff x or time t, and therefore the hyperbolic discounting model always produces the classic temporal value asymmetry.

This is a proposition that we can address empirically. By creating pairs of options with the same payoff and temporal distance, one option in the past and one option in the future, we can examine whether manipulations of the delay t or the amount x change the pattern of decision behavior. If participants do not consistently choose past over future or future over past for all combinations of x and t, then the hyperbolic discounting model can be falsifiedy.

Hyperboloid model

Although it is the most common, the hyperbolic discounting model is not the only account of behavior on intertemporal choice tasks. A related but slightly more complex model is the hyperboloid model (Rachlin, 2006) (see also L. Green & Myerson, 2004, for a slightly different form of the two-parameter hyperboloid discounting model), which predicts that temporal discounting occurs as a function of two parameters rather than one. The first parameter describes the discounting rate (k) as in the hyperbolic model, but it adds an additional parameter describing time perception or sensitivity to delays (time sensitivity s). In some ways, this seems the more appropriate model to use based on past work on temporal value asymmetry, as the parameter s specifically describes time perception, which is thought to be the mechanism leading to differences in valuation between past and future outcomes. Specifically, temporal value asymmetry is thought to occur due to participants perceiving time in the future as being shorter than time that has already passed (Caruso et al., 2013; Suhler & Callender, 2012; Guo et al., 2012), i.e., positive values of t are thought to be longer than matching values $-t$.

In the hyperboloid discounting model, the subjective value of a delayed payoff (x,t) is given as

$$v(x,t) = \frac{x}{1+k\cdot t^s} \tag{2}$$

However, it is critical to note that this model suffers from the "muddled units" problem outlined by Vincent & Stewart (2020), because the units of t are known to be (e.g.) days. Normally, the units of k are $days^{-1}$, signifying a rate of decay, which makes the product $k\cdot t$ unitless. This allows values of k to be compared across participants because they are all in the same units, $days^{-1}$ (Falmagne & Narens, 1983). However, in the hyperboloid model, the units of k must be $days^{-s}$, which means that the scale of k depends on the s parameter for each person and thus that estimates of k cannot be directly compared across people. Fortunately, this problem is easily rectified by taking k to the power of s as well, giving us an alternative form for hyperboloid discounting (Vincent & Stewart, 2020):

$$v(x,t) = \frac{x}{1+(kt)^s}. \tag{3}$$

This slightly changes the meaning of the k parameter, as in the original formulation (Equation 2) it is the discounting rate for *subjective* time, i.e., the amount that a subjective period of time causes an option to decrease in value. However, in the formula provided in Equation 3, k is applied directly to the *objective* delay prior to subjective scaling. Formally, these two models are ultimately equivalent. In the supplementary materials, we provide a table for translating between estimates of k based on Equation 3 (estimated in this paper) and estimates of k that one would obtain by using 2.

It has not been established whether this model fits better than a hyperbolic discounting model for events that occur(red) in the past, but it typically appears to out-perform hyperbolic discounting for future events (McKerchar et al., 2009).

A key part of the hyperboloid model that distinguishes it from hyperbolic and exponential discounting models in this context is that it has two parameters that can affect the shape of discounting curves. In particular, it has both a power transformation (specified by s) and a linear transformation (specified by k) on the time delay t. As a result, this model could potentially switch between selecting a future option (x,t) and a past option $(x,-t)$ as the temporal distance $|t|$ increases, because there is a trade-off between the linear effect of k and the power effect of s as the delay increases.

For example, suppose we have a delayed prospect $x = \$40$, $t = 5$. As above, we assume that the discounting rate k for

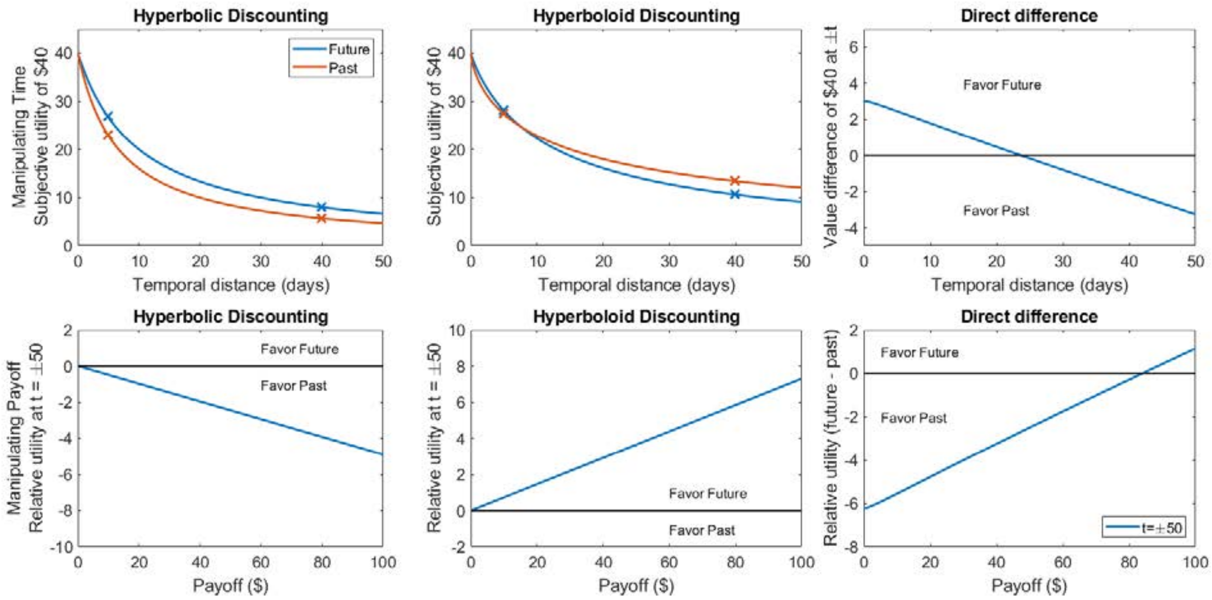

Figure 1

Model predictions for manipulations of temporal distance (top) and payoff amount (bottom). The hyperbolic discounting model (left panels) predicts no reversals, while the hyperboloid model (middle) predicts reversals based on time (top) but not payoff (bottom), and the direct difference model (right) predicts reversals based on both time and payoff.

future events is $k_f = 0.10$ and for past events is $k_p = 0.15$. If the time perception parameter for future delays is $s_f = .9$ and for past delays is $s_p = .6$, then we find that

$$\frac{x}{1+(k_p t)^{s_p}} < \frac{x}{1+(k_f t)^{s_f}} \tag{4}$$

However, if the temporal distance is increased to $t = 40$, for example, the sign flips so that

$$\frac{x}{1+(k_p t)^{s_p}} > \frac{x}{1+(k_f t)^{s_f}}. \tag{5}$$

This type of reversal is illustrated in the top middle panel of Figure 1, with x marks at $t = \pm 5$ and $t = \pm 40$ to illustrate the reversal. As with the hyperbolic model, the value of the future payoff (x,t) is given by the blue line and the value of the past payoff $(x,-t)$ is given by the orange line. Unlike the hyperbolic model, the linear effect of k_p being larger than k_f results in a preference for future events at short delays, but it is overtaken by the exponent s_f being larger than s_p, which results in a shift toward past events at longer delays. Naturally, the values of k_p, k_f, s_p, and s_f could be adjusted such that there is no reversal or so that the reversal occurs in the other direction. This is simply meant to be an illustrative example to show that the hyperboloid model is capable of predicting reversals of the temporal value asymmetry as delays are manipulated.

Despite capturing preference reversals between past and future events as the temporal distance t increases, the hyperboloid model is not able to capture reversals as the value of the payoff x increases or decreases. This is because x is a simple scalar in Equations 2-5, and will cancel out in any comparisons between the left and right sides of our inequalities (Equations 4 and 5). As shown in Figure 1 bottom-middle panel, manipulating the payoff (x-axis) can never cause the subjective value difference between past and future payoffs (y-axis) to change sign. Therefore, the hyperboloid model can predict reversals from past to future when the temporal distance changes (top middle panel of Figure 1), but not when payoffs change (bottom middle panel of Figure 1).

Hyperboloid-Utility model

One final variant of the hyperboloid model seeks to address past-future reversals by adding an additional parameter. In expected-utility models of decision making, an objective payoff x is assigned a subjective utility that is a nonlinear function of x. The addition of a parameter α transforms the objective payoff x into a subjective utility $u(x) = x^{\alpha}$, which is substituted for the value of x in Equation 3. Otherwise, it operates similarly to the hyperboloid model. The overall value of an option is then computed as

$$v(x,t) = \frac{x^{\alpha}}{1+k\cdot t^{s}}. \tag{6}$$

Although this model was not one of those that we preregistered, it was included in an effort to allow traditional alternative-wise models to account for the past-future reversals that occur in the data. This model is capable of producing preference reversals between past and future outcomes as the payoff x is manipulated thanks to the nonlinear shape of the utility function. Specifically, if there are separate utility functions for past and future events, corresponding to separate values for α_P (past) and α_F (future), then the model can produce differences as x is manipulated by virtue of asymmetries in the nonlinear shape of $u(x)$ produced by the different values of α. We therefore included it in our model comparisons to evaluate whether an alternative-wise approach could provide a viable explanation of the temporal value asymmetry and its reversals.

Direct difference model

The final model we examine predicts intertemporal choice in terms of trade-offs between payoffs and delays for pairs of prospects (Dai & Busemeyer, 2014; Scholten et al., 2014). This *attribute-wise* model suggests that people compare differences in subjective representations of time and weigh these against differences in subjective representations of payoffs. A decision maker choosing one option over another by balancing a desire for greater payoffs against a desire to receive them sooner (nearer) in time. Instead of computing the subjective value of an option as a whole, this model evaluates the relative value of each option by comparing their payoffs and delays against one another. Formally, the difference in subjective value between two delayed prospects (x_1,t_1) and (x_2,t_2) is given as

$$v(x_1,x_2,t_1,t_2) = w \cdot (x_1^{\alpha} - x_2^{\alpha}) - (1-w) \cdot (t_1^{v} - t_2^{v}) \quad (7)$$

There are two perception / sensitivity functions here. Sensitivity to gains / payoffs is described by the parameter α, while sensitivity to time is described by the parameter v. These allow a person to form a subjective impression of the increase in utility the payoff will confer and the loss of utility the delay will impose. The trade-off between delay and payoff is weighed according to the third parameter w $(0 \leq w \leq 1)$, which can be thought of as the relative amount of attention paid to payoffs (w) versus time $(1-w)$.

The direct difference is able to predict the temporal value asymmetry, and reversals thereof, when we allow α and v to depend on whether a choice option occurs in the past or future. This yields five total parameters: α_f, α_p, v_f, v_p, and w. Because w controls the trade-off between two options – which could be both in the past, both in the future, or one in the past and one in the future – there does not seem to be a sensible way to change it based on whether the choice options occur in the past or future.

As with the hyperbolic and hyperboloid models, we can examine what predictions the direct difference model makes for the temporal value asymmetry. However, because the direct difference model does not directly yield a subjective value for each option, we instead look at the *relative* value of a past versus future payoff. The results are shown in the right panels of Figure 1. Values greater than zero indicate support for a future payoff, while values less than zero indicate support for a past payoff.

We chose parameter values that are within the range of commonly-observed values payoff sensitivity (Tversky & Kahneman, 1992; Glöckner & Pachur, 2012; Busemeyer & Diederich, 2002) of $\alpha_f = .8$, $\alpha_p = .7$ (greater sensitivity to the value of future events), $v_f = .6$, $v_p = .8$ (greater sensitivity to past / experienced delays), and $w = .5$ (equal attention to payoff and delay). Similar to the hyperboloid model, there is a trade-off between the linear effect of w and the power effect of v, which allows for a future payoff to be favored for one value of time $t = 5$ (where $v(x,t) > 0$) and the past payoff to be favored at a greater value of time $t = 40$ (where $v(x,t) < 0$).

However, this model diverges from the traditional discounting models in another important way. Namely, the addition of payoff sensitivity parameter creates another trade-off between a linear effect (w) and nonlinear power function (specified by α). As a result, the difference in subjective value between options shifts as the payoffs of past and future options but time is held constant. This is shown in the bottom-right panel of Figure 1: as the payoff of the past and future options increase, their subjective value difference goes from negative values (favoring past options) to positive values (favoring future options). Therefore, this model is capable of creating reversals of the temporal value asymmetry as time is manipulated and as payoff is manipulated. This will be critical to accounting for our results.

The additional complexity of the direct difference – although we note that it actually only has one more parameter than the hyperboloid model – is typically justified by its superior fits to intertemporal choice data (Cheng & González-Vallejo, 2016). The direct difference model also corresponds to observations that time and payoff exert separate (Scholten et al., 2014) and independent influences on choice (Amasino et al., 2019). We have the opportunity to evaluate qualitatively in addition to the quantitative fits: violations of the typical temporal value asymmetry constitute violations of the discounting models' predictions, providing support for the direct difference model and models of asymmetric payoff sensitivity more generally.

One final advantage of the direct difference model is that it predicts not just choice, but also the response times associated with these decisions (Dai & Busemeyer, 2014). We compare these predictions against a diffusion model instantiation of the best-fitting hyperboloid model. As an out of sample check on the predictions of the direct difference model and comparison against the hyperboloid-diffusion model, we use the drift rates (relative subjective utility) computed from the model of decision making to make predictions about response times, and show that the model accounts well for these data even though it is not directly fit to them. To preview our results, the direct difference model outperforms all competitors in both choice proportions and response time distributions.

Methods

To understand how people represent and compare delayed prospects, we ran an empirical study asking participants to make decisions between options that could occur in the future or past (past vs future) as well as short and long delays in the future (sooner vs later, both future) and short or long times ago (more vs less recent, both in the past). This included options that could be received the same day (present / 0 day delay). These intertemporal choice trials were supplemented by self-report measures aimed at understanding the individual differences that give rise to differences in choice behavior, described below. Participants were recruited through Prolific, and were paid \$10 per hour for participating in the experiment.

A total of 70 participants took part in the experiment, which was sufficient for hierarchical estimation of model parameters as well as for detection of reasonably strong relationships between self-report and model-based measures. This allowed us to detect correlations between measures of $\sim$.15 with $1-\beta > .8$ power, allowing for some drop-out and the slightly more conservative Bayesian approach to estimation (using uninformative priors; Kruschke, 2014). Of these participants, three failed more than two attention checks (out of six) and so were removed from further analyses. These attention checks were instances where one option dominated the other one by offering both a greater sum of money and a shorter delay – participants were judged to have failed the attention check if they chose the smaller, more delayed option on these trials. Of the remaining 67 participants, 25 were women (42 men) with an average age of 27.4 (SD = 8.9) years. Because Prolific uses an international participant pool, there were a wide range of nationalities included in the study. These included participants from the UK (18 participants), USA (9 participants), EU countries (37 participants from Poland, Portugal, Spain, Germany, Ireland, Greece, Italy, Bulgaria, Hungary, Finland, and the Netherlands), Canada (2 participants), and Chile (1 participant). All participants were screened through Prolific and required to be fluent English speakers.

Intertemporal choice experiment

The study consisted of two main parts: a series of 224 intertemporal choice trials (experimental part) and a set of 9 self-report measures related to motivation, time perception, imagination, impulsiveness, and sense of self as it changes over time (self-report part). These two parts are described in more detail below. The study was administered entirely through Qualtrics. Upon being directed to the study from Prolific, participants completed informed consent (approved by the University of Florida IRB, #IRB201902817) followed by the experimental part of the study and then the self-report measures. The experimental / intertemporal choice portion of the study was implemented in javascript and embedded in a jframe in Qualtrics, while the self-report / survey measures were administered directly on the Qualtrics platform. Once participants had completed both parts of the study, they were automatically redirected back to Prolific to complete payment.

In the intertemporal choice component of the study, participants were presented with pairs of options, one on each side of the screen. Prior to the start of each trial, participants were presented with a fixation (+) in the middle of the screen. They then pressed the space bar to begin the trial, and the stimuli appeared on screen after a 750 ms delay. Each stimulus was composed of two response options, where option consisted of an amount of money (e.g., \$15) and a delay (e.g. 5 days). Delays were presented as X days *from now* for future outcomes, and as X days *ago* for past outcomes. The past outcomes were explained to participants by asking them to imagine that someone had given them \$D X days ago, and that they could have spent or saved that money since then however they wished. They were asked to consider all of the changes and consequences in their life that \$D would have altered, and use that to evaluate how much better (or worse) their life would be if they had experienced that outcome at that point in time. Similarly, the future outcomes were explained to participants by asking them to imagine that someone would give them \$D in X days from now, and that they could spend or save that money however they wish. They were asked to consider all of the changes and consequences that money would have for their life if they received the \$D, and use that to evaluate how much better (or worse) their life would be if they experienced that outcome at that point in time.

Participants pressed the 'F' key to indicate that they preferred the option on the left, or the 'J' key to indicate that they preferred the option on the right. Response times were recorded from the time at which the stimuli appeared on the screen to the time at which they pressed one of these buttons. Participants were informed that they should make their responses carefully, taking as much time as they needed to decide, but that they should make their responses as soon as they have made up their mind about the options presented to them. In informed consent, they were informed that their response times would be recorded and would be used to test theories of the decision processes underlying their choices.

In total, participants saw 224 pairs of response options, including combinations of past, future, and present options. A list of all stimuli presented during the task is provided on the Open Science Framework page at osf.io/zwv6m. Negative delays correspond to options that could have occurred in the past, whereas positive delays correspond to options that could occur in the future. Which option was presented on the right or left side of the screen was randomized from trial to trial, and the order in which the stimuli were presented was shuffled across participants.

Within the 224 choice pairs, we included 64 *matched* trials, where participants were presented with a past and a future outcome that had the same payoff and same temporal distance. For example, participants might see a choice between "$35, 21 days from now" vs "$35, 21 days ago." We included a complete factorial design within participants so that each participant saw matched trials for 8 levels of payoffs ($11, $20, $35, $60, $80, $100, $10,000, and $1,000,000) and 8 levels of delays (7 days, 14 days, 21 days, 60 days, 85 days, 112 days, 365 days, and 730 days). This allowed us to examine directly how the value of past and future payoffs changed based on manipulations of payoff and temporal distance, by examining the proportion of choices favoring the matched past and future outcomes.

Self-report measures

In addition to the intertemporal choice task, we explored individual differences that are potentially related to people's propensities to favor past or future outcomes. After completing all 224 trials of the intertemporal choice experiment, participants completed an additional 9 self-report measures. These measures were designed to examine how well participants could imagine the consequences of the choice options they were presented with, measure the degree to which they perceived themselves to be the same or different in the past and future, and evaluate trait impulsivity in each participant.

Zimbardo Time Perspective Inventory (ZTPI). The original ZTPI (Keough et al., 1999) is a 56-item scale assessing a person's views on their past, present, and future at a given time. Those views are represented by five temporal orientations: Past Positive (PP), Past Negative (PN), Present Hedonistic (PH), Present Fatalistic (PF), and Future (F). In the current study, we used the brief 18-item scale from (Košt'ál et al., 2016) that assess the orientations from above but separates the Future orientation into Future Positive (FP) and Future Negative (FN).Participants responded to items such as "Familiar childhood sights, sounds, smells often bring back a flood of wonderful memories" using a 5-point scale (1 = "Strongly Disagree"; 7 = "Strongly Agree").

Future Time Perspective. The 10-item Future Time Perspective scale assesses an individual's perceptions of their remaining time in life (Lang & Carstensen, 2002). Participants responded to items such as "Many opportunities await me in the future" using a 7-point scale (1 = "Very Untrue"; 7 = "Very True").

Temporal Self Extension. The temporal self extension scale (Grabowski & Broemer, 2015) measures individual's perceptions of the amount of time they have existed as their current self (i.e., how long they have been who they are now). The first item uses a sliding scale with anchors on "Birth" and "Present" and participants moved the slider to answer the question: "How long have you perceived yourself to be as you are now?" Then participants responded to two additional items. The first was: "How long have you held this representation of your current self?" and participants responded on a 7-point scale (1 = "A very short time", 7 = "Always"). The second item was: "How much does your current self-perception date back into the past?" and participants responded on a 7-point scale (1 = "Just a little bit", 7 = "Very much indeed").

We also adapted these items to form a measure of future temporal self extension. The sliding scale was changed to include anchors on "Present" and "Death" and participants moved the slider to answer the question: "How long do you believe you will stay the same as you are now?" Then participants responded to the other two additional items. The first was: "How long do you think you will continue to have this representation of your current self?" and participants responded on a 7-point scale (1 = "A very short time", 7 = "Always"). The second item was: "How much does your current self-perception continue into the future" and participants responded on a 7-point scale (1 = "Just a little bit", 7 = "Very much indeed").

Narrative immersion. The Transport Narrative Questionnaire (M. C. Green & Brock, 2002) is a 12-item measure of the extent to which participants were absorbed or psychologically immersed in the scenarios they were imagining. We amended instructions to refer specifically to the decision task participants had just completed ("When I was completing the task, I could easily picture the events in it taking place"). Participants responded using a 7-point unipolar Likert scale, ranging from 0 = "Not at all" to 7 = "Very much."

Daydreaming. The Short Imaginal Processes Inventory (SIPI; Huba et al., 1981) is a 45-item measure of people's inner mental life and daydreaming, consisting of three subscales: positive constructive daydreaming (15 items; "My fantasies usually provide me with pleasant thoughts"), guilt and fear-of-failure daydreaming (15 items; "I imagine myself failing those I love"), and poor attentional control (15 items; "I am the kind of person whose thoughts often wander"). Participants responded using a 5-item bipolar Likert scale (1 = "definitely untrue or strongly uncharactertic of me", 2 = "moderately untrue or uncharacteristic of me", 3 = "neither particularly characteristic nor uncharacteristic of me", 4 = "moderately true or characteristic of me", 5 = "very true or strongly characteristic of me").

In addition to temporal self-perception and participants' ability (or willingness) to entertain the possibilities they were shown, it is clear that motivational processes should play a role in intertemporal choice (Loewenstein & Thaler, 1989; Loewenstein & Elster, 1992). In decisions between sooner and later rewards, participants must balance competing motivations to receive immediate rewards (impulsivity) against their desires to receive large rewards (gain sensitivity). Choices between past and future options are certainly impacted by motivation, but the influence of impulsivity is not so clear-cut. Is it impulsive to select an outcome that would have occurred in the near past as opposed to one that would have occurred a long time ago? To us at least, the answer is not clear. A proclivity to choose smaller sooner over larger later options appears to be related to a proclivity to choose near-past over far-past options (He et al., 2012; Xu et al., 2009), suggesting that impulsivity could be related to both types of decisions. We therefore assess trait impulsivity in the hopes of better understanding how motivation and value change as a function of temporal distance. To do so, we examine the results of three measures of impulsivity and sensitivity to positive outcomes: the sensation seeking scale (Zuckerman et al., 1964; Zuckerman, 2007), the UPPS impulsive behavior scale (Whiteside & Lynam, 2001), and the Barratt Impulsiveness Scale (Patton et al., 1995; Stanford et al., 2009).

Finally, a great deal of work has been devoted to understanding the relationship between intertemporal choice and risky choice, connecting impulsivity to a propensity for risk-taking (Baumann & Odum, 2012; Johnson et al., 2020). Single-process theories suggest that both risks and delays cause monetary amounts to be discounted, and that discounting occurs at the same rate when defined in terms of temporal distance / delay discounting or odds against / probability discounting (Benzion et al., 1989; Rachlin et al., 1986; Stevenson, 1986). More recent work has suggested that the two are dissociable into separate processes, one related to risks and another to delays (Johnson et al., 2020; Sun & Li, 2010). However, we remain open to the possibility that patterns of behavior in intertemporal choice are at least predictive of risk-taking. To examine the relationship between risk taking and past / future discounting, we also include a self-report measure of risk taking across domains, the DOSPERT (Blais & Weber, 2006).

Results

In reporting the results, we focus on three main parts: first, we report the basic behavioral results of people's decisions on the intertemporal choice experiment. Second we present a model comparison of four theories that might account for the intertemporal choice behavior observed (hyperbolic discounting, hyperboloid discounting, modified hyperboloid with an outcome-utility component, and direct differences models). Finally, we report differences on the self-report measures, including how self-reports compared to model-based metrics as predictors of intertemporal choice behavior. As with the stimuli used in the experiment, the data and model code used for these analyses are available on the Open Science Framework at osf.io/zwv6m.

Behavioral results

Overall, participants showed a general tendency to choose outcomes that could occur in the future over ones that could have occurred in the past, with 59% of their response favoring a future option when it was paired against a past option. Other pairs of choices showed less consistent patterns of behavior: participants chose present and future options approximately evenly with 50.8% of choices favoring present; they chose present over past options 49.5% of the time; they chose near future over far future options 49.6% of the time; and they chose near past options over far past options 54.5% of the time.

Of course, these choice proportions depend heavily on the particular stimuli (e.g., specific monetary payouts; time delays) that we used in our experiment. They are therefore much less informative than the model parameter estimates we present in the next section, which let us quantitatively describe and compare the cognitive and motivational processes that determine value for past and future outcomes.

One set of behavioral results that we can meaningfully examine in a model-free way are the matched trials, where the past and future outcomes had the same monetary value and same temporal distance. In other words, they varied only on whether the payoff would occur in the past vs the future. A summary of these results is presented in Figure 2: for each combination of $\$x$ and y days, this figure shows the proportion of participants choosing the past option. Larger, bluer circles indicate more participants choosing the past options, while smaller, yellower circles indicate more participants choosing the equal-payoff future option.

There are a few important effects to note here. First, there is a general tendency to select the future option over the past option (as in the other trials where past and future options were pitted against one another). Second, participants tended to shift toward choosing the *future* option as payoffs increased. Third, and somewhat less visibly, participants tended to shift toward choosing the *past* option as temporal distance increased.

We quantified these effects using a Bayesian logistic regression model, where payoff and temporal distance were standardized and used to predict the probability of selecting the future option. Before looking at estimates of effects, we used JASP (JASP Team, 2020) to model probabilities, evaluating the posterior probability of models with and without a main effect of payoff, within and without a main effect of delay, and with and without their interaction. The priors on each model (intercepts, main effects, interactions) were lowered according to how many factors were involved, compensating for model flexibility as is the JASP default (Consonni

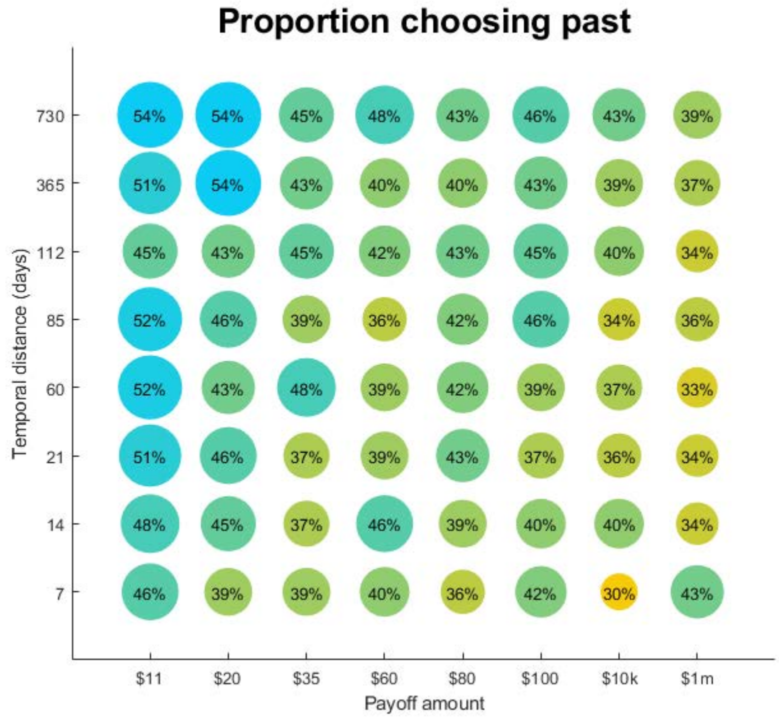

Figure 2

Proportion of participants who preferred a payoff ($x) in the past (y days ago) relative to the same payoff in the future (y days from now) based on the amount they could receive (x-axis) and the temporal distance of the events (y-axis). Lighter colors / larger circles indicate a greater choice proportion.

et al., 2018). The best model was one with main effects of delay and payoff, but no interaction (model probability = .98), which was strongly favored in model ranks, log likelihoods, and Bayes factors.

To examine the magnitude of these effects, we report the standardized effects of each manipulation along with the 95% highest density interval [HDI], indicating the 95% most likely values for the parameters of the best model, i.e., one with main effects but no interaction (Kruschke, 2014). This resulted in an intercept of $M(b_0) = 0.32$, 95% HDI = $[0.27, 0.39]$ indicating a general tendency toward selecting the future option (a negative intercept would indicate an average preference for past options), a positive effect of payoff $M(b_1) = 0.14$, 95% HDI = $[0.08, 0.20]$ indicating a tendency to select the future option more as payoff increased, and a negative effect of delay $M(b_0) = -0.08$, 95% HDI = $[-0.14, -0.01]$ indicating a tendency to select the past option more as temporal distance increased.

It is worth noting that a linear regression, even on standardized values, will be strongly influenced by the longest delays (e.g., 730) and the largest payoffs (e.g., $1 million). To reduce the influence of these extreme values, we repeated the analyses on ranked payoffs and ranked delays, so that payoffs of $11, $20, ..., $1m and delays of 7, 14, ... 730 days were transformed to ranks of 1, 2, ..., 8. Looking at the order rather than the magnitude of payoffs and delays, the size of the effects change slightly, but not the conclusions. The resulting intercept is still positive $M(b_0) = 0.32$, 95% HDI = $[0.26, 0.39]$ indicating a tendency to favor future options, the effect of payoff is still positive $M(b_0) = 0.09$, 95% HDI = $[0.02, 0.15]$ indicating a tendency to favor future options as payoffs increased, and the effect of delay was still negative $M(b_0) = -0.08$, 95% HDI = $[-0.15, -0.02]$, indicating a tendency to favor past options as temporal distance increased. As before, the model with main effects of payoff and temporal distance, but no interactions, was favored (posterior model probability = .97).

The consistency of these effects across methods of analysis indicate quite conclusive evidence that preferences for past and future options can be influenced, and often reversed, by changing the payoffs and delays of the options. Naturally, this does not bode well for the hyperbolic or hyperboloid models of delay discounting, which we examine next.

Model comparison

The empirical results certainly do not favor the alternative-wise models of intertemporal choice, as the hyperboloid models (which predict temporal reversals based on time but not payoffs) qualitatively cannot account for the effect of manipulating payoffs on the probability of choosing past versus future prospects, and the hyperbolic model (which predict no temporal reversals) qualitatively cannot account for the effects of either manipulation. Nevertheless, they could potentially still provide better quantitative accounts of patterns of choice on the task than competing accounts of intertemporal choice behavior. To test this possibility, we implemented the hyperbolic, hyperboloid, and direct difference models described in the introduction – as well as an additional hyperboloid-utility model described in Appendix B – and used them to account for choice behavior on our task.

All models were implemented in a hierarchical Bayesian way to avoid the estimation and recovery errors that are associated with non-hierarchical delay discounting models (Molloy et al., 2020; P. D. Kvam et al., 2021). Unless otherwise specified, each model was implemented in JAGS (Plummer, 2003) and used 4 chains of 5000 samples with 500 burn-ins. Chain convergence was assessed using r-hat statistics and visual inspection. All models successfully converged, thanks in part to the hierarchical priors that allowed for individual choice data to be constrained by group-level trends. Model code is provided on the OSF site.

For the hyperbolic discounting model, we implemented two versions of the model. Each version of the model predicted the probability of choosing one option $Pr((\text{Choose } 1)$ as a function of the payoffs for the two options x_1 and x_2 and the delays for the options t_1 and t_2. These choice probabilities were obtained by using a logistic choice function to map the subjective values of each option onto a choice probability between 0 and 1.

$$Pr(\text{Choose } 1) = \frac{1}{1+e^{-m \cdot d}} \tag{8}$$

where the subjective value difference d is determined as

$$d = \frac{x_1}{1+k|t_1|} - \frac{x_2}{1+k|t_2|} \tag{9}$$

Note that the addition of the logistic choice rule and the choice variability parameter m is in part a practical matter, as leaving the hyperbolic and hyperboloid models alone would result in deterministic predictions that would lead to model likelihoods of zero. It is generally more desirable to do a likelihood-based comparison than other approaches based on the number of correct predictions or squared error (see P. Kvam & Pleskac, 2017, , Appendix C), so we allow a transformation between subjective / discounted value differences and choice probabilities. Note that this confers some additional flexibility to the hyperbolic and hyperboloid models that is not afforded to the direct difference model (whose choice variability arises out of a cognitive mechanism specified directly by the model, below), in some ways making the direct difference model a more theoretically constrained approach to modeling intertemporal choice probabilities.

The basic version of the hyperbolic model has two parameters, k and m, which describe discounting rate and choice variability respectively. The second version of the model allowed for k to vary as a function of whether the choice options were in the past or future, resulting in three free parameters. This corresponds to a a logistic choice rule mapping the difference in subjective value between options A and B onto a choice probability. [2]

For the hyperboloid model, we simply added a sensitivity parameter that transformed the temporal distance t according to a nonlinear perception of time, as in Equation 3. As we suggested in the introduction, this model needs to be modified to account for the muddled units problem (Vincent & Stewart, 2020) so that we can meaningfully compare estimates across participants and across conditions. We allowed both the discounting rate k and time sensitivity s parameters to vary between past and future outcomes, and compared this saturated model against one where k was fixed across past and future, s was fixed across past and future, or both.

We tested one final version of the hyperboloid model, which we refer to as the hyperboloid utility or hyper-utility model. This model adds a utility component to the outcome x using a power transformation x^{α} as in models of expected utility, reflecting the assumption that participants experience diminishing marginal benefits to increasing sums of money (Savage, 1954). We tested eight versions of this model, assuming that α, k, and s could all change as a function of whether an option was in the past or future. Because each one could either be fixed across past and future or vary, this created a 2 (fixed α or α_P/α_F) $\times$ 2 (fixed k or k_P/k_F) $\times$ 2 (fixed s or s_P/s_F) factorial combination of model parameters, each of which we fit. The intuition behind whether α could change for past or future options is that the utility of an option could be altered based on whether it corresponded to an past event that could be changed or a future event that had not occurred yet. For example, participants might be better able to imagine how they would have used a payoff x in the past, but have greater difficulty (and therefore assign lower utility) when this payoff could occur in the future, where events are less predictable or easy to imagine.

Finally, we compared these three alternative-wise models against the attribute-wise direct difference model (Dai & Busemeyer, 2014; Cheng & González-Vallejo, 2016). The mean difference in expected value is given by first transforming both the payoffs and delays according to a subjective utility parameter α and time perception parameter v as in Equa-

[2] We also implemented "random utility" implementation of the models that used a cumulative normal to map differences in subjective value to differences in choice probability, reflecting a normal distribution of subjective values around the mean given by Equation 9. However, this was not meaningfully different in performance or interpretation relative to the models using a logistic choice rule, so we do not report the results here.

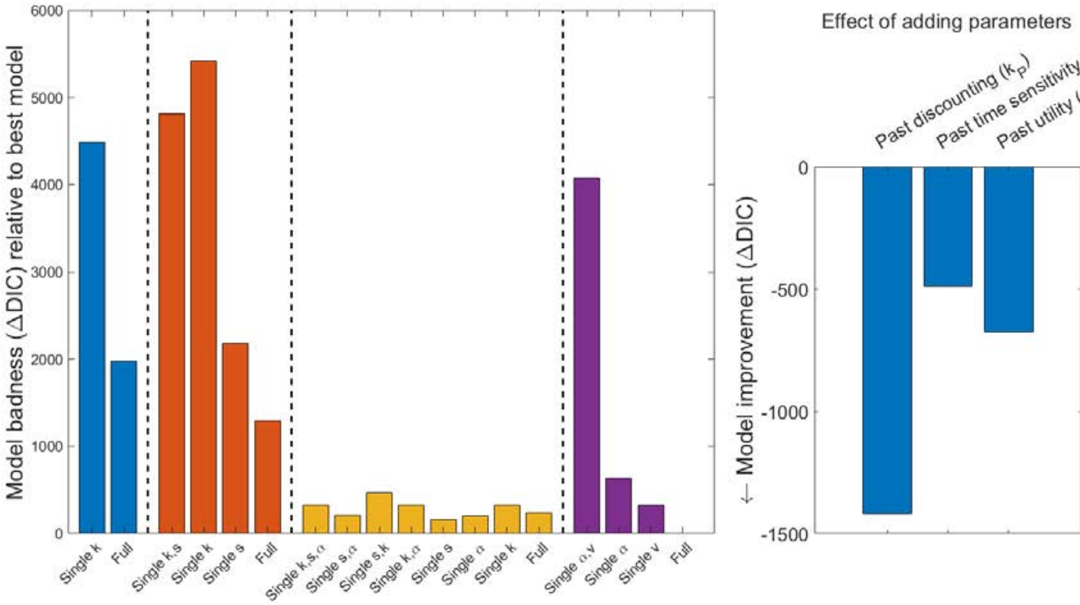

Figure 3

*Comparison between the different models tested (*blue = hyperbolic, orange = hyperboloid, yellow = hyper-utility, purple = direct difference*). The left panel shows the basic performance of each model in terms of their DIC values (*lower = better*), while the right panel shows the improvement in model performance (decrease in DIC) that is conferred by adding each of the cognitive mechanisms (past / future discount rates, past / future sensitivity to time, and past / future sensitivity to payoffs) to the model. Lower bars indicate greater improvement in model fit.*

tion 7, and then weighting the differences in subjective utility and subjective time by the attention weight w.

The choice probability from the direct difference model is computed as a function of the mean value difference ($v(x,t)$ from Equation 7) and the variance in the values of the outcomes, σ^2:

$$\sigma^2 = w \cdot (x_1^\alpha - x_2^\alpha)^2 + (1-w) \cdot (t_1^v - t_2^v)^2 - v(x_1, x_2, t_1, t_2)^2 \tag{10}$$

The probability of choosing Option 1 is simply $\Phi(v(x,t), \sigma^2)$, where Φ is the cumulative density function for the normal distribution.

In each of the models, we allowed some of the parameters to vary based on whether the choice options were in the past or future. We theorized that both time perception and subjective value parameters might change based on whether payoffs occurred in the past or future, and thus allowed the α and v parameters to change based on whether a choice option was in the past or future. We compared this full model – with one w value and two values for α and v, totalling 5 parameters – against models constraining α, v, or both to be the same across conditions.

In total, we tested eighteen models: two variants of the hyperbolic discounting model, four variants of the hyperboloid model, eight variants of the hyperboloid-utility model, and four variants of the direct difference model. This was done for two reasons. First, we wanted to identify which theory provided the best account of the data in order to get a handle on the cognitive mechanisms underlying intertemporal choice. Second, we wanted to identify which parts of the model were most important to include. In particular, we wanted to investigate what changed when people considered outcomes that could occur in the past versus future. As we suggested, this was done by testing whether events being in the past or future affected discounting rates k, perception of time s / v, or the perceived value of the payoffs α.

The results of this model comparison are shown in Figure 3. In the left panel, hyperbolic discounting models are shown in blue, hyperboloid models in orange, the hyperboloid-utility models in yellow and direct difference models in purple. The height of each bar corresponds to the difference between that model and the best model in terms of deviance information criterion [DIC] values, which describe the model performance corrected for model flexibility. A lower DIC indicates a better model. The best overall model was the full direct difference model, which allowed α and v to change between past and future outcomes. Even the best-fitting hyperbolic and hyperboloid models were far behind the best direct difference model by over 100, indicating that there is strong evidence for the direct difference model over its competitors. Thus, it is not only the qualitative effects but also the

Table 1
Group-level mean estimates (single hyperparameter), standard deviation of individual-level mean estimates across participants (SD), and 95% HDI of the group-level mean for each parameter of the (full) hyperbolic discounting model (posterior of single hyperparameter). F = future, P = past.

Parameter	Mean	SD	95% HDI (Mean)
$\log(k_F)$	-4.31	3.17	[-4.86, -3.74]
$\log(k_P)$	-3.86	3.87	[-4.41, -3.27]
m	2.22	8.44	[1.71, 2.67]

Table 2
Group-level mean estimates, standard deviation of individual-level mean estimates across participants (SD), and 95% HDI of the group-level mean parameter for the hyperboloid model. F = future, P = past.

Parameter	Mean	SD	95% HDI (Mean)
$\log(k_F)$	-4.91	3.53	[-5.10, -4.72]
$\log(k_P)$	-4.16	3.88	[-4.36, -3.96]
s_F	0.82	0.29	[0.79, 0.89]
s_P	0.92	0.43	[0.90, 1.11]
m	2.12	7.98	[1.56, 2.66]

quantitative fits that favor the direct difference model over alternative-wise approaches.

Beyond the comparisons between models, we can also evaluate what model mechanisms are important to account for differences in valuation for past and future outcomes by evaluating model performance when particular parameters are included or removed. This is shown in the right panel of Figure 3 for three parameters: larger decreases in DIC (bars that go lower) indicate greater model improvement when a particular parameter / process is added to the models.

For the hyperbolic, hyperboloid, and hyperboloid-utility models, we can compare the effect of allowing k to vary between past and future outcomes ("Adding past discounting") – this clearly improves both models, illustrated in both the average change in DIC when adding this parameter as well as the change in model fit (Single k vs Full for hyperbolic, and Single k vs matching non-single k models for hyperboloid).

The model parameters for the hyperbolic, hyperboloid, and hyperboloid-utility models are shown in Tables 1, 2, and 3, respectively. If we choose to take an alternative-wise approach to modeling intertemporal choice, then it certainly seems to be the case that discounting rates differ for past and future outcomes. On average, log k values (log transformed to be closer to normally distributed) in the hyperbolic discounting model were $M(\log(k_{future})) = -4.31$ for future outcomes and $M(\log(k_{past})) = -3.86$ for past outcomes. Similarly, in the best-fitting hyperboloid model these average parameter values were $M(\log(k_{future})) = -4.91$ for future outcomes and $M(\log(k_{past})) = -4.16$ for past outcomes. In both

Table 3
Mean estimates, standard deviation across participants, and 95% HDI on the group-level mean parameters for the hyperboloid-utility model. P = past, F = future.

Parameter	Mean	SD	95% HDI (Mean)
$\log(k_F)$	-8.37	1.45	[-8.78, -7.94]
$\log(k_P)$	-8.81	1.75	[-9.22, -8.18]
s	0.68	0.18	[0.59, 0.74]
α_F	0.31	0.25	[0.28, 0.35]
α_P	0.3	0.25	[0.28, 0.34]
m	10.74	10.77	[8.86, 12.28]
b	0.02	0.01	[.018, .023]
θ	3.65	0.9	[3.63, 3.68]
τ	0.24	0.07	[0.23, 0.25]

cases, the steeper discounting rate for past outcomes means that outcomes in the distant past should be devalued more than those in the distant future. In terms of empirical effects, this corresponds to the observation that participants generally favored choice options that would occur in the future over ones in the past.

Interestingly, the discounting rates in the hyperboloid-utility model are much lower than the corresponding values in the hyperbolic and hyperboloid models (Tables 1 and 2). This appears to be because it could offload the discounting of outcomes to a concave utility as opposed to a hyperboloid function of time. As a result, the α values were substantially lower than those typically found in risky decision making studies, which are normally closer to around .6-1 (Tversky & Kahneman, 1992; Glöckner & Pachur, 2012). This may be how participants truly discount past and future outcomes, but it seems likely that the effect of temporal discounting (which the hyperboloid model claims there is almost none) is instead being absorbed into the utility of the payoff independent of time. This may have occurred because there is a built-in relationship between payoff and time in all discounting experiments because SS options have lower payoffs and delays than LL options.

The DIC values for each variant of this model that we tested are shown in Figure 3. In each case, the hyperboloid-utility model fit better than its pure hyperbolic and hyperboloid discounting models, but failed to surpass the full direct difference model. The direct difference model was strongly or overwhelmingly favored in DIC comparisons to the hyperboloid-utility model (all difference > 30), indicating that the attribute-wise account appears to be superior to all of the alternative-wise accounts we considered, even those that could theoretically account for the temporal value asymmetry reversals.

Although the hyperboloid model and direct difference model do not appear to have much in common, they do share a parameter that transforms the objective delays into subjec-

Table 4

Parameter estimates from the best-fitting direct difference model. Parameters in the top part of the table were used for fitting choice data, while those in the bottom part of the table were added to account for response time data. F = future, P = past. Columns correspond to the group-level mean estimate (single hyperparameter), standard deviation of individual-level mean estimates across participants (SD), and 95% HDI of the group-level mean parameter (single hyperparameter as in the Mean column).

Parameter	Mean	SD	95% HDI (Mean)
α_F	0.97	0.12	[0.93, 1.02]
α_P	0.99	0.13	[0.95, 1.03]
v_F	0.68	0.35	[0.62, 0.75]
v_P	0.76	0.58	[0.71, 0.83]
w	0.63	0.21	[0.61, 0.65]
θ	4.18	1.97	[4.13, 4.23]
τ	0.75	0.55	[0.64, 0.80]

tive ones: s in the hyperboloid, and v in the direct difference model (although we note in the next section that the effects of the s parameters may not be exactly consistent with their intended meaning). Although allowing the time sensitivity parameters to vary between past and future outcomes seems to confer a relatively small advantage relative to other model modifications, as shown in Figure 3, it does still seem to improve model fit. On average, time perception appeared to be more nonlinear for future than for past outcomes: $M(s_{future}) = 0.84$ vs $M(s_{past}) = 1.00$ in the hyperboloid model, and $M(v_{future}) = 0.68$ vs $M(v_{past}) = 0.76$ for the direct difference model. Note that values close to 1 indicate subjective time that is close to objective time, while values less than 1 indicate subjective time representations that are less sensitive and less accurate relative to the objective times. Interpreting the average parameters suggests that participants' representations of time that has already passed are relatively accurate and close to the true objective values, while they are less sensitive to differences in time to future events. This seems sensible – participants have experienced time that has already passed, and so we might expect them to accurately represent this time, while future time has not been experienced yet and so might not be represented with such high accuracy.

Finally, the direct difference and hyperboloid-utility models added a third parameter α that could change between past and future, which corresponds to the utility assigned to objective payoffs. As with discount rates and time perception, allowing this parameter to vary between past and future substantially improved model performance, but mainly for the direct difference model. Despite this, the average values for past and future payoffs were similar in the direct difference model, $M(\alpha_{future}) = 0.97$ versus $M(\alpha_{past}) = 0.99$. In the hyperboloid-utility model, these values were much lower, at $M(\alpha_{future}) = 0.31$ versus $M(\alpha_{past}) = 0.30$, but still similar to one another. Although the model comparisons all provided support for differentiating α between past and future, the mean values of these parameters did not appear to systematically differ between past and future in either model. The reason that they were retained in the model is that there were substantial individual differences across participants in terms of which one was larger: participants ranged from past α being greater than future α by 0.49, to past α being less than future α by 0.45. These individual differences between past and future parameter values are likely why the model improved by allowing α to vary, as it permitted the model to characterize these large individual differences rather than constraining $\alpha_{past} = \alpha_{future}$.

Individual choice problems

Clues as to why the direct difference model out-performed the others can potentially be found by examining how well each one accounted for individual choice problems or types of choice problems. We therefore divided the choices that participants made into five types: past vs past outcomes, past vs present outcomes, past vs future outcomes, present vs future outcomes, and future vs future outcomes. Patterns of decisions across these five types of choice problems were highly correlated (see Table S1), but between models we can see patterns emerge in terms of how well each model accounted for different types of decisions.

In general, the areas where the direct difference model succeeded but hyperbolic and hyperboloid models fell flat were decisions between past options. For example, switching from a hyperbolic or hyperboloid discounting model to a direct difference model produced the greatest improvement in model fit for the choice pair (in format $[x_1,t_1; x_2,t_2]$) [\$63, 2 days; \$63, -14 days], with the squared error of the hyperbolic model being over 200 times that of the direct difference model. This problem and similar past vs future choice pairs consistently show up as problems that hyperbolic / hyperboloid models struggle the most with compared to the direct difference model.

A summary of the mean squared error for each category of choice pair is shown in Table 5. The hyperbolic discounting model had the greatest error in its predictions for the past-past pairs of choice options followed by past-future pairs, indicating a general tendency to struggle to predict the outcomes of choices involving past outcomes. The hyperboloid and even hyperboloid-utility models had similar trouble with past-future decisions, which were the most common type due to the 64 pairs shown in Figure 2. Conversely, these pairs were the ones that the direct difference model did the best with; although it generally performed well on all types of choices, its *best* performance was on the past-future choice pairs. This emphasizes the importance of the temporal value

Table 5
Mean squared error between model predictions (columns) and five different categories of choice problems (rows). The total number of problems in each category is shown in the right column, and the total squared error (total number of problems) is shown in the bottom row.

	Model				**Average**	Number of
Choice type	Hyperbolic	Hyperboloid	Hyper-Utility	Direct Difference	(All Models)	choice pairs
Past-Past	0.0391	0.0362	0.0320	0.0111	0.0296	26
Past-Present	0.0154	0.0531	0.0368	0.0213	0.0316	36
Past-Future	0.0156	0.0604	0.0463	0.0060	0.0321	103
Present-Future	0.0126	0.0348	0.0494	0.0133	0.0275	32
Future-Future	0.0079	0.0245	0.0159	0.0190	0.0168	27
Average/Total	0.0169	0.0484	0.0399	0.0117	0.0275	224

asymmetry in determining model fit: the biggest factor favoring the direct difference over other models was its ability to handle past versus future choices. It was able to keep its mean squared error so low by virtue of being able to produce reversals as outcome and temporal distance were manipulated.

A complete list of participants' choice proportions for each type of choice pair is provided in the supplementary material. For each model, we also generated an ordered list of all the choice pairs according to how well the model could account for decisions on that particular choice problem. These are provided on the OSF site (named by "Model_Error.csv"). Readers are free to explore the exact problems that produced the differences in model performance, but they tend to be choices involving past options. As shown in Table 5, the direct difference model substantially outpaced its competitors in past-past and past-future choice pairs.

For interested readers, the parameter values for each model and participant are provided on the OSF site. We have tried to summarize the most interesting findings here and in the self-report section, but there are certainly additional individual differences to explore in the raw parameter estimates.

Response times

One additional benefit of the direct difference model is that it can account for distributions of response times associated with each decision that participants made in the task. The direct difference model suggests that support for one option or another during intertemporal choice is represented as a balance between the two that changes as a decision maker considers the attributes of the alternatives. In traditional intertemporal choice, thinking about the payoffs provides support for choosing an LL (larger-but-later) option over an SS (sooner-but-smaller) option, whereas thinking about the delays provides support for choosing an SS option over an LL option. The attention weight parameter w controls the frequency with which delay or outcome is considered. As the decision maker considers both attributes, the balance shifts toward one option or the other over time until it reaches a threshold $\pm\theta$. Higher values of θ result in slower but more consistent decisions, while lower values result in faster but less consistent decisions (choice proportions closer to 50%).

Formally, response times in the model are given by the time t it takes a random walk $s(t)$ to cross the choice boundary θ for one option or the other. Because options were randomly assigned to the left and right sides and systematic biases toward one side or the other were negligible, we assume that this process starts at $s(0) = 0$ (although post-stimulus biases can occasionally be gainfully assigned to start points Zhao et al., 2020). As a decision maker considers the potential payoffs of options, they use payoff information to update their preferences with probability w and delay information to update their preferences with probability $1-w$. On average, this will result in a "drift" toward the favored option, such that the average change of the state μ is described by the expected value of $v(x,t)$ from Equation 7. However, because the accumulation process is stochastic, there will be some variability in the decision maker's representation of the subjective values of the choice options, and therefore noise in how the preference state changes over time. This variability can actually be directly calculated from Equation 10, giving the *diffusion rate* σ^2.

The rate of accumulation toward one option or another is represented by a drift rate $d = \mu/\sigma^2$, which indexes the "signal to noise" ratio of preference change (Busemeyer & Townsend, 1993; Dai & Busemeyer, 2014). Specifying the drift rate of a diffusion model in this way also sets the scale of the model (Donkin et al., 2009; Ratcliff et al., 2016) by making the variability (diffusion rate) equal to one (by virtue of $\sigma^2/\sigma^2 = 1$), allowing the other parameters (α, v, w) to be estimated freely. Direct difference model predictions for response times can then be computed with two additional free parameters: θ, the threshold; and τ, the amount of time devoted to non-decision processes like encoding the stimuli and entering a desired response by pressing a button on the keyboard / mouse.

If the parameters have been estimated well, then the α, v, and w parameters should be able to predict the choice proportions and distributions of response times quite well. We

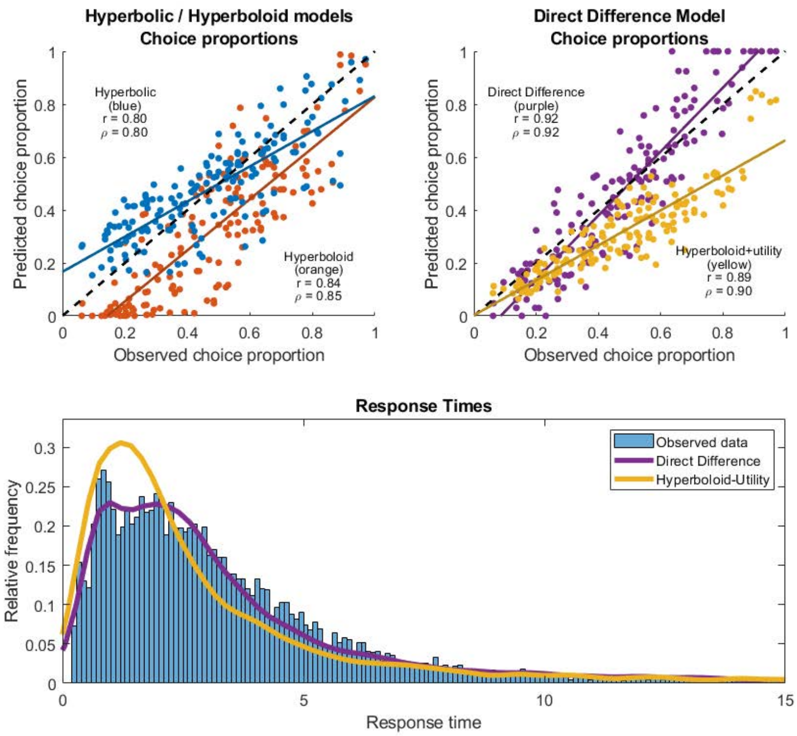

Figure 4

Plot of model predictions for choice proportions (top panels) for hyperbolic (blue, left) / hyperboloid (orange, left), hyperboloid plus utility (yellow, right), and direct difference (purple, right) models, and response time distributions from the direct difference model (bottom panel) compared to the empirical choice proportions and response times.

therefore fixed the value of the drift rate in a diffusion decision model (Ratcliff et al., 2016) according to these parameters for each participant, and estimated the threshold θ and non-decision time τ in a hierarchical Bayesian manner, using the *dwiener* package in JAGS (Wabersich & Vandekerckhove, 2014).

The direct difference model's predictions for response times were compared against those of the best-fitting alternative-wise model, the hyperboloid-utility model. To generate response time predictions for this model, we linearly scaled the value difference between alternatives on each choice problem (calculated using k, s, and α parameters) into a drift rate using a free parameter b. This was then fed into the same type of diffusion decision model as the direct difference model, adding a threshold θ and non-decision time τ to the drifts, with the key differences being that the hyperboloid-utility did not predict any differences in σ (momentary variability). The resulting predictions from this model are shown in yellow in the bottom panel of Figure 4.

As with the other parameters, individual-level estimates for all models of response times are provided on the OSF site. The resulting distribution of response times from the direct difference model is also shown in the bottom panel of Figure 4 for the aggregate response times across all participants. As shown, the model does a good job of accounting for both the choice proportions (top-left of Figure 4) as well as the overall shape of the response time distribution especially when compared against the hyperboloid-utility model. As a result, the DIC value for the hyperboloid model (121250) was much higher than that of the direct difference model (71380) indicating worse performance. We suspect that this was due to its inability to handle differences in diffusion rates across choice options, as the only thing that could change across pairs of choice options was the drift rate. The ability of the direct different model to mechanistically predict the *variance* in the accumulation process (Equation 10) appears to be a major advantage of this model, as it can predict fast response times to high-variance pairs of choice options even if the mean difference between them is relatively small. This resulted in the small "hump" at the leading edge of the direct difference model (and possibly the data) that was not present in the hyperboloid-utility model, and thus the improvements in fits to the response time distribution. This adds further credibility to the performance of the direct difference model: it is able to accurately reconstruct the pattern of choice RTs above and beyond the choice proportions, as well as outcompete all of the other models on both choice proportions and response time fits.

Self-report measures

Given that the model parameters are relatively robust and appear to succeed reasonably well on an out-of-sample test, we might cautiously hope that they also index stable individual differences related to valuation and time perception. It is not yet clear whether these individual differences in behavior are related to self-report measures of psychological traits. To test this possibility, we examined the posterior parameter estimates from all four models and how they compared to our self-report measures of traits related to perceptions of the past and future, self and time, imagination, and trait impulsivity.

In raw behavior, the only variable that predicted people's overall tendency to select the future payoff, as opposed to past payoffs, was the Zimbardo Time Perspective Inventory (ZTPI) Present-Fatalistic scale. Greater scores on this scale indicated a greater propensity to select future outcomes, $M_{corr} = .27$ (95% HDI = $[.02, .51]$). The ZTPI Present-Fatalistic scale also predicted whether people would flip between selecting past outcomes and selecting future outcomes as the temporal distances (length of time between present and past/future events) increased, $M_{corr} = .32$ (95% HDI = $[.10, .54]$). The Barratt Impulsiveness scores also predicted this tendency to favor future over past outcomes as the time increased, $M_{corr} = .28$, (95% HDI = $[.03, .50]$).

Fortunately, model parameters tend to be much more closely related to the various self-report measures. This happens both because meaningful cognitive model parameters are more reliable than simple behavioral metrics like choice proportions and because they are more valid measures of individual differences in cognition (Haines et al., 2020). Rather than reporting all of the correlations between self-report measures and model parameters, we present a visual representation of their relationships in Figure 5. To construct this figure, we first estimated the full correlation matrix between each subscale of the self-report measures and each of the parameters from the best-fitting models of each type (i.e., best hyerbolic, hyperboloid, and direct difference models). This was done in a Bayesian way using a diffuse prior (uniform on $[-1, 1]$) to estimate each entry in a correlation matrix, where each row/column corresponded to a different self-report or model-based measure.

The resulting correlation matrix $\mathbf{R}$ was then used to do two things. First, it was used in a multidimensional scaling procedure, where the "distance" between different measures (model parameters, self-report scores) was computed as one minus the absolute value of the correlation between them, giving a distance matrix $D = 1 - |\mathbf{R}|$ that was based on the degree of relatedness between measures. The distance matrix was then used to compute a (nonclassical, stress-based) multidimensional scaling solution in two dimensions (Shepard, 1982). In this MDS solution, the position of each individual difference measure described its relationship to the other measures. This gave the positions of the pink and blue circles shown in Figure 5. Measures that are further apart are typically less related to one another, whereas measures that are closer together are more closely related.

Second, the correlation matrix was used to create a network of individual differences, where edges connecting nodes correspond to a credibly nonzero correlation between them. Thus, every connection between measures in Figure 5 corresponds to a "significant" / credible (95% HDI excluding zero) relationship between measures. Thicker lines indicate stronger relationships between measures, black lines indicate a positive correlation, and red lines indicate a negative correlation between measures. This is similar to the network analysis approach proposed by Borsboom et al. (2021), where psychological measurements can be connected and arranged by their relationship to one another.

As in previous work comparing a large number of behavioral metrics with self-report scales (Frey et al., 2017; Hertwig et al., 2018), there appears to be a gap between these two types of measures. Despite this, there is a stronger relationship between model parameters and self-report scales than there is between mean choice probabilities and self-report scales, where there are no significant relationships between behavioral and self-report measures. Thanks to the use of our modeling approaches, there are some connections that can be drawn and as a result the outcome of the multidimensional scaling and network approach is not entirely uninterpretable.

Broadly, the organization of this MDS solution and network can broken into three parts: behavioral model parameters, behavioral parameters related to payoffs, self-report measures related to time / impulsivity, and self-report measures related to payoffs and self-representation.

We start first by discussing the behavioral model parameters. The k parameters in the hyperbolic (k_1) and hyperboloid (k_2) models do most of the work in temporal discounting, relating the delay of an option to its value. They seem to straddle the time-payoff line of the self-report measures, as they relate time to subjective value and are thus related to both constructs. Despite being clearly distinguishable and both having noticeable effects on model fit (Figure 3), the past (k^P) and future (k^F) discounting rates are highly correlated, indicating that perception of the past and perception of the future may indeed overlap greatly even though they are not entirely redundant.

The direct difference model, which was favored in all of our comparisons (Figures 3 & 1), can also be related to self-report measures. As with the discounting rates, past (α^P / ν^P) and future (α^F / ν^F) parameters are highly correlated and seem to reflect overlapping cognitive processes. The atten-

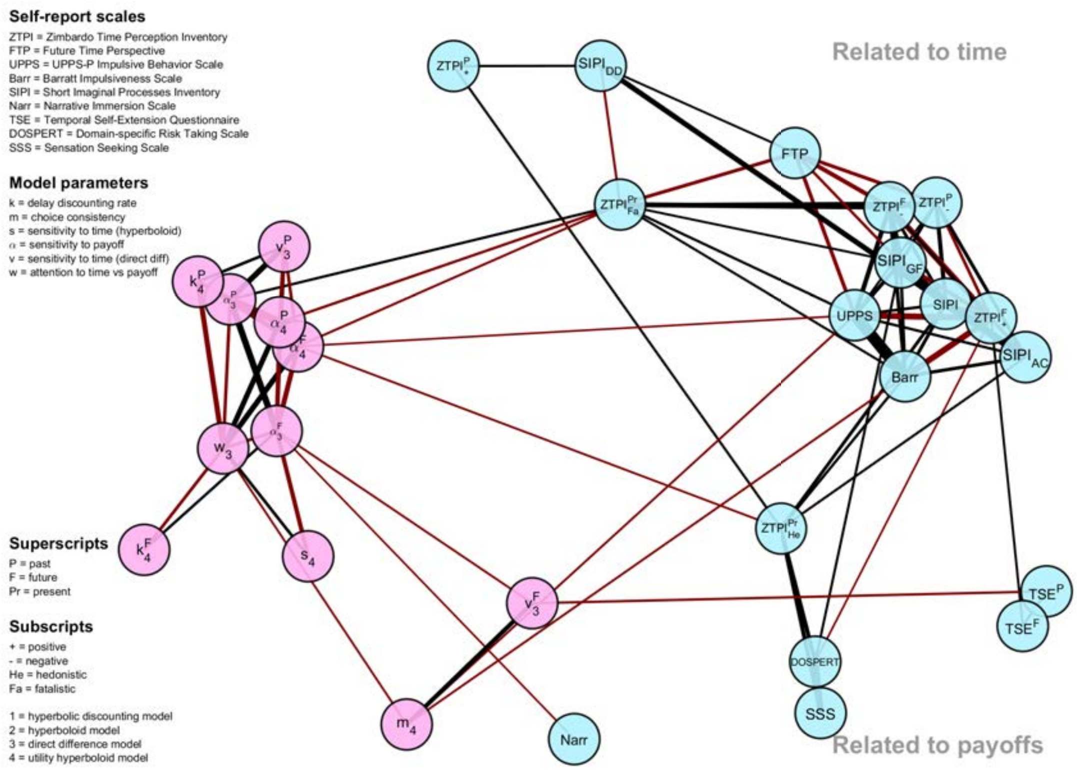

Figure 5

Network of relationships between self-report (blue) and model-based behavioral (pink) measures. Meaning of each node is given by the key on the left.

tion parameter, being related to both payoffs (w) and delays ($1-w$) in the model (Equation 7), appears centrally among the behavioral model parameters as it can trade off with either α or v.

Although the model parameters are tightly packed, showing high correlations with one another, there are relatively scant relationships between behavioral and self-report measures.[3] The only measures that appear to show consistent relationships with behavior are the ZTPI subscales. In particular, high scores on the present-hedonistic and the past-positive subscales often predict future discounting rates, while the present-fatalistic subscale generally appears to predict higher discounting of past outcomes. Several of the impulsivity measures (UPPS, Barratt) also negatively predict choice variability in the hyperbolic / hyperboloid models (m_1 / m_2), suggesting that participants high in choice impulsivity tend to make relatively inconsistent selections.

Among the self-report measures, there are much stronger relationships. In the top-right of Figure 5, there is a cluster of related measures all pertaining to impulsivity and time perception / attitudes. The main measures of impulsivity (UPPS, Barratt) and the SIPI scale (which has not traditionally be interpreted as impulsivity, although here it appears that poor attentional control is closely related to impulsivity) are all closely related to one another, positively related to the future-positive subscale of the ZTPI, and negatively related to past-negative and future-negative subscales of the ZTPI. Interestingly, the SIPI guilt/fear-of-failure (GF) and attentional control (AC) subscales were closely related to measures of impulsivity, while the daydreaming subscale was more closely related to the ZTPI past-positive, future time perception, and (negatively) to the ZTPI present-fatalistic subscale. The former grouping of SIPI subscales seems to indicate that there are common processes in attentional control, fear / guilt, and impulsivity; working memory or cognitive control seem the most likely culprits, as they are known to affect all three (Finn et al., 1999; McVay & Kane, 2009; Kane et al., 2001; Schmidt et al., 2002). The latter grouping is also somewhat interesting, suggesting that our participants were more likely

[3] Behavior is quantified by model parameters here because they are more valid indicators of psychological processes (Haines et al., 2020). Using raw behavioral metrics like choice proportions results in weak relationships with self-report measures because they are contaminated with structural error from being a more distal measure of the cognitive mechanisms we are trying to measure than the cognitive model parameters.

to report daydreaming when they felt positively about the past and present, and when they were able to project themselves into the future (positive relationship with FTP).

More generally, participants that were positive about the future (high on ZTPI future-positive / low on ZTPI future-negative) were less negative about the past (lower on ZTPI past-negative) and more willing to wait for positive events that could occur in the future, indicating that a degree of optimism about future events and positive perspective on past outcomes may dispose people toward lower levels of impulsivity.

A final cluster of self-report measures appears near the bottom of Figure 5. These include measures of the propensity to seek out rewarding sensations (SSS; Zuckerman et al., 1964; Zuckerman, 2007) and risks (DOSPERT Blais & Weber, 2006). These are both strongly related to the present-hedonistic subscale of the ZTPI, constituting a set of scales that all seem to measure sensitivity to payoffs that are currently available or that offer greater benefits in the present than in the future. Finally, the Temporal Self Extension (TSE) subscales were clustered near the bottom-right. The future self-extension scale correlated with more positive views about the future, indicating that participants who expected themselves to stay the same also had a relatively positive outlook on the future.

Overall, the structure and layout of Figure 5 indicates two main distinctions that we can draw: (1) between model parameters / behavior and self-reports, indicating that behavior and self-report measures are assessing separate but both meaningful individual differences; and (2) between payoff-relevant and time-relevant self-report measures, lending strength to the claim that processing of the payoffs and the delays are distinct psychological processes (Amasino et al., 2019). A further distinction can be drawn between past and future processes payoff / time representation based on the model comparisons (Figure 3), although the network suggests that these processes are still highly related.

Discussion

The ability to project ourselves forward and backward in time, as well as into imagined events that may or may not happen provides an enormous amount of utility for planning, maintaining a sense of self, predicting outcomes, and simply experiencing well-being. Looking forward to experiencing something, enjoying the experience itself, and looking back on the things we have experienced can all confer positive feelings (Baucells & Bellezza, 2017; Loewenstein & Elster, 1992). While events that could occur in the future offer the prospect of all three, the subjective value of these experiences decreases as they are pushed further away in time (Myerson et al., 2011). Despite this, previous work has suggested that future events retain their value as they are displaced in time more than past events do (Caruso et al., 2008).

Mechanisms of the temporal value asymmetry

Several explanations have been proposed for this temporal value asymmetry, including suggestions that past events retain only the value conferred by reflection / looking back while future events retain the opportunity to experience anticipation and the event itself occurring (Mitchell et al., 1997). Alternatively, it has been proposed that the time to future events is perceived as shorter because the waiting time is theoretical rather than known (Caruso et al., 2013). In either case, the asymmetry has been cast in terms of discounting rates, suggesting that the rate at which future events decrease in value is slower than the rate at which past events decreased in value as they are displaced in time(Bickel et al., 2008).

Our work provides a direct counterpoint to both the empirical phenomenon and this theoretical explanation for the temporal value asymmetry. First, we show systematic reversals of the temporal value asymmetry where participants sometimes preferred past outcomes. There were multiple instances where participants indicated that they preferred payoffs that could have occurred in the past to ones that could occur in the future in perfectly matched pairs – where the same dollar amount could be received in either the past or future at the same temporal distance (X days ago vs X days from now). Second, these reversals occurred as both the magnitude of and distance to the past / future payoffs were manipulated. Participants favored past events when payoffs were small and temporal distance was large ($11, 2 years ago / from now), and favored future events when payoffs were large or temporal distance was small ($10k, 7 days ago / from now). This may explain apparent replication failures related to the temporal value asymmetry (El Halabi et al., 2021) – not because the phenomenon is not real, but because different stimuli can cause it to reverse, and thus on average, fail to appear. Third and finally, a model comparison showed that framing the temporal value asymmetry in terms of hyperbolic (or even the more general hyperboloid) discounting is insufficient to account for the patterns of behavior we observed. Instead, an alternative-wise direct difference model (Dai & Busemeyer, 2014) thoroughly out-performed the discounting models and was able to predict the reversals of the temporal value asymmetry that we uncovered.

Within each of the models, there are several cognitive processes that could create a temporal value asymmetry, but only a few of the models include mechanisms that can account for the reversal. The overall pattern of results suggests that participants have a general preference for future outcomes over past ones, which is reflected in shallower discounting rates (k) for future events relative to past events in all three of the hyperbolic, hyperboloid, and hyper-utility models. This is consistent with past work that attributed the asymmetry to discounting (Bickel et al., 2008; Yi et al., 2006). However, the picture of the temporal value asymmetry given by the models is more nuanced. In addition to discounting rates being shallower for future events, the hyperboloid and direct discounting models also estimated that sensitivity / perception of time was flatter for future events, indicated by $s_F < s_P$ (hyperboloid) and $v_F < v_P$ (direct difference). These findings seem to lend credibility to the claim that time that will pass in the future is not perceived to be as long as time that has already passed, referred to as a "temporal doppler effect" (Caruso et al., 2013). This finding would also produce a temporal value asymmetry on its own (without discounting) as equidistant past and future events will result in the future event seeming closer. Looking across models, both time sensitivity and discounting appear to contribute to the typical temporal value asymmetry effect.

Much more difficult to account for is the reversal of the temporal value asymmetry we observed in our data. As we showed, only the hyperboloid-utility and direct difference models are even capable of predicting this effect. However, they appear to operate mainly at the individual level – the average sensitivity to payoffs α did not differ by more than 0.01 in either model. However, the wide variability between individuals in these parameters indicates that some people will reverse their preferences between past and future events when the payoffs are manipulated due to asymmetries in the utilities assigned to either set of payoffs. Put together, a general tendency to perceive future time as shorter and discount future payoffs less steeply leads to an overall pattern consistent with the temporal value asymmetry; however, this is qualified by a flatter utility function for future payoffs for some individuals, which leads them to undervalue future payoffs in terms of utility relative to past payoffs.

Remaining questions and extensions

Somewhat unusual within our data is that no participant appeared to completely follow the economically rational approach where large payoffs further in the past were always chosen over smaller payoffs that occurred in the most recent past, present, or future. Assuming a positive rate of return on an investment, a more distant and larger past payoff would theoretically provide the greatest amount of money at any point in time. However, even the participants who favored far-past events in general did not do so completely: most participants (93%) chose the far-past payoff around 30-70% of the time, with only a single participant selecting far-past payoffs more than 90% of the time. Instead, it seems that participants are willing to forego the investment potential of far-past options in favor of the salient or recent positive experience of receiving a smaller, temporally closer payoff.

In light of economic considerations, the fact that choices between past and future reverse between short and long delays is perhaps not too surprising. Even if we consider only how the value of money changes over time with inflation, it is typically the case that $1 in the past is more valuable than $1 in the future because positive inflation reduces the purchasing power of the dollar over time. If we add to this the difference in time perception between past and future events (Caruso et al., 2013), it is reasonable to expect some reversal of the temporal value asymmetry as delays / temporal distances are manipulated. However, the fact that the asymmetry also reverses with payoff magnitude is much more problematic for these accounts of intertemporal choice. It suggests that our sensitivity to changes in payoffs that are in the future versus changes in payoffs in the past must diverge. This divergence may represent a sort of temporal value 'blind spot' whereby people value a small payoff in the distant past, without recognizing that taking that same payoff in the present would (eventually) come to represent that distant payoff that they currently value more. People may place too high value on large and relatively near future amounts (which may be rather unlikely to payoff), without recognizing that as time goes on, they tend to shift toward favoring smaller payoffs that could have occurred in the past. In other words, people may wish, with time, that they had followed the rule to take "small wins often" but are instead caught by the allure of (unlikely) large payoffs in the future. Whatever the case, the direct difference model offers a mechanism for this asymmetry reversal in terms of separate utility parameters for past and future outcomes (α_P / α_F), allowing it to explain why this phenomenon occurs.

The model-based approach also allowed us to generate new insights into how the values of past and future payoffs are represented. It suggests that there is a meaningful distinction between discounting rates for past and future events (Figure 3), even though the two are highly correlated (Figure 5. This may explain the failure of certain episodic thinking paradigms in modifying intertemporal choice behavior, where episodic thinking about the future seems to help future discounting rates but episodic thinking about the past does not help future discounting rates (Dassen et al., 2016). Ultimately, these processes appear to be dissociable, so there is no guarantee that modifying past discounting rates will have an effect on choice impulsivity.

Naturally, there are many extensions of this type of work that would deepen our understanding of subjective value in past and future events. Payoffs or rewards other than money are commonly employed to understand disordered intertemporal decisions in eating disorders (Amlung et al., 2016), alcohol and drug abuse (Locey & Dallery, 2009), or sexual activity related to STIs (Wongsomboon & Robles, 2017). Interventions to reduce impulsivity in these domains could leverage assessments of discounting rates and payoff sensitivity to design decision strategies or choice architectures that reduce discounting or re-frame the available options (Peters & Büchel, 2010; Rung & Epstein, 2020). For instance, a variant of episodic future thinking interventions might ask participants to re-consider the question "Would you rather have a drink now, or remain sober for the rest of the week?"

as “A week from now, do you imagine you would prefer to have had a drink a week ago, or to have remained sober for the past week?” The model-based approach we have used here can provide guidance – for example, it suggests that the latter may be more successful for large payoffs and short delays than small payoffs and long delays. More broadly, the efficacy of any intervention for a given individual could be assessed by utilizing individual-level estimates of parameters related to time sensitivity, payoff sensitivity, and attention / choice variability.

Declarations

The authors have no relevant financial or non-financial conflicts of interest to disclose.

Ethics

All participants completed informed consent and the study procedures were approved by the University of Florida Institutional Review Board (#IRB201902817).

Open Science

The study, accompanying self-report measures, and simple model based analyses were preregistered on AsPredicted (aspredicted.org/blind.php?x=kw3tc9). Data and model analysis code are available on the Open Science Framework at osf.io/zwv6m. However, the utility-hyperboloid model was not included in the preregistered analyses, and instead was tested post hoc at the suggestion of a reviewer.

Constraints on generality

The study was carried out using participants recruited through Prolific and included an international sample with substantial demographic variability, albeit only participants who were fluent in English. It was carried out during the COVID-19 pandemic. While we expect that these circumstances will not have an overall effect on the generality of our conclusions, it is possible that the composition of the sample and the circumstances in which the study was run could have affected temporal discounting behavior. We expect that the results could be replicated as long as it includes sufficient variability in delays and payoff magnitudes (to allow for reversals on these dimensions).

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Supplementary Materials for "Cognitive mechanisms underlying subjective value of past and future events: Modeling systematic reversals of temporal value asymmetry"

Peter D. Kvam
University of Florida

Matthew Baldwin
University of Florida

Erin C. Westgate
University of Florida

Raw choice proportions

In the main text, we chose to focus on the relationships between model parameters and the self-report outcomes, as opposed to the relationships between raw behavioral measures like choice proportions. The reason for this is that generative model parameters are better measures of individual differences than summary statistics like choice proportions or mean response times, showing consistently greater reliability (Haines et al., 2020) and validity (Molloy et al., 2020; Kvam et al., 2021; Busemeyer & Diederich, 2002; Romeu et al., 2019) than behavioral summaries. This occurs because the latent parameters quantified by cognitive models are "closer" to the true generative processes than simple statistical summaries, and as a result are less contaminated by systematic error and bias due to model mis-specification.

Despite this, some readers may be interested in the the patterns of raw choice proportions among past, present, and future outcomes as well as how these choice patterns were related to the measures we gathered. We divide these responses into five categories: decisions among past outcomes (26 pairs), decisions among future outcomes (27 pairs), decisions between past and future outcomes (103 pairs, including all of the pairs from main text Figure 2), decisions between past and immediate outcomes (36 pairs), and decisions between future and immediate outcomes (32 pairs). For each of these categories and each participant, we quantified the number of times they choose the option that was furthest in the past (i.e., chose far > near past, past > future, past > present, present > future, and near future > far future). In general, participants preferred future options in all cases except present vs future decisions: they chose far past over near past options 45.50% of the time, past over future options 41.05% of the time, chose near future over far future options 49.64% of the time, chose past over present options 49.54% of the time, and chose present over future outcomes 50.84% of the time.

These choice proportions were also used as individual differences by quantifying the proportion of times each participant chose past, present, and future options in each of these categories. The correlations among these choice proportions, as well as their correlations with self-report scales, are shown in Table S1. In general, the choice proportions in each category were strongly related.

Those participants who tended to prefer far past options to near past options were the same participants who preferred past to future, near-future to far-future, past to present, and present to future outcomes. This suggests that participants had largely consistent valuations of past to future (or present) events. Furthermore, it indicates that they made sense of the options that were shown to them whether they were possible future outcomes or counterfactual past outcomes, as evidenced by extremely systematic choices in each of the categories they were presented.

Table S1

*Estimated correlations between self-report measures (rows) and choice proportions (columns). Correlations excluding zero are shown in **bold**. A key for all abbreviations is in main text Figure 5.*

	Choice proportions				
Self-report	Near >Far Past	Past >Future	SS >LL	Present >Past	Present >Future
Barratt	0.00	-0.08	0.01	0.09	0.03
DOSPERT	-0.04	0.05	-0.10	-0.08	-0.15
SSS	-0.03	-0.04	-0.10	0.04	-0.02
UPPS-P	-0.05	-0.16	0.00	0.14	0.01
SIPI	-0.15	-0.12	-0.21	0.01	-0.12
FTP	0.01	0.11	0.03	-0.13	-0.09
TS P-	**-0.25**	-0.17	-0.20	-0.12	-0.15
TS P+	0.15	0.09	0.08	0.09	0.12
ZTPI P-F	0.07	-0.16	0.17	0.23	0.21
ZTPI P-H	0.17	0.08	0.05	0.13	0.17
ZTPI F-	-0.08	-0.11	-0.06	0.05	-0.04
ZTPI F+	-0.01	0.03	-0.01	-0.08	0.06
Narrative Imm.	-0.22	**-0.27**	-0.08	0.06	-0.13
TSE - Past	-0.08	-0.03	-0.10	-0.01	-0.11
TSE - Future	0.03	0.02	-0.01	0.12	-0.05
Near >Far Past	**1.00**	**0.68**	**0.82**	**0.45**	**0.79**
Past >Future	**0.68**	**1.00**	**0.35**	**-0.27**	**0.32**
Near >Far Future	**0.82**	**0.35**	**1.00**	**0.44**	**0.78**
Present >Past	**0.45**	**-0.27**	**0.44**	**1.00**	**0.46**
Present >Future	**0.79**	**0.32**	**0.78**	**0.46**	**1.00**

Conversely, the relationships between raw choice proportions and self-report measures were extremely weak. Out of 90 correlations we computed, only three relationships were credible, and even then they were all below .30. This is approximately the number of credible correlations we would expect to find simply by chance. This reinforces the idea that summary statistics of behavior like choice proportions are poor measures of individual differences, and that model parameters should be used instead when possible (Haines et al., 2020).

Table S2

Choice proportions for each participant (rows) for each of the five types of choice pairs. P = past, Pr = present, F = future, FP = far past, NP = near past, FF = far future, NF = near future.

	Choice proportions				
Participant	FP>NP	P>F	NF>FF	Pr>P	Pr>F
1	0.50	0.46	0.44	0.58	0.50

2	0.14	0.08	0.07	0.19	0.22
3	0.51	0.70	0.63	0.03	0.69
4	0.34	0.62	0.15	0.03	0.09
5	0.65	0.41	0.63	1.00	1.00
6	0.44	0.47	0.15	0.47	0.66
7	0.49	0.53	0.44	0.53	0.31
8	0.50	0.22	0.52	0.97	0.63
9	0.38	0.20	0.22	0.86	0.38
10	0.31	0.12	0.52	0.50	0.44
11	0.34	0.20	0.37	0.56	0.41
12	0.25	0.20	0.30	0.36	0.34
13	0.71	0.61	0.85	0.78	0.72
14	0.52	0.40	0.63	0.58	0.56
15	0.52	0.25	0.81	0.75	0.63
16	0.70	0.83	0.59	0.61	0.63
17	0.20	0.23	0.15	0.11	0.34
18	0.33	0.24	0.33	0.53	0.31
19	0.58	0.74	0.74	0.19	0.78
20	0.44	0.20	0.70	0.67	0.56
21	0.50	0.22	0.70	0.83	0.66
22	0.48	0.20	0.56	0.81	0.78
23	0.52	0.43	0.67	0.53	0.53
24	0.38	0.63	0.22	0.03	0.34
25	0.35	0.27	0.37	0.31	0.53
26	0.47	0.50	0.52	0.39	0.41
27	0.43	0.22	0.59	0.92	0.09
28	0.90	0.88	0.96	0.97	0.84
29	0.41	0.56	0.30	0.11	0.53
30	0.41	0.29	0.37	0.69	0.38
31	0.54	0.78	0.59	0.00	0.66
32	0.68	0.70	0.70	0.56	0.69
33	0.72	0.52	0.89	0.78	0.94
34	0.40	0.35	0.37	0.56	0.34
35	0.40	0.64	0.22	0.08	0.31
36	0.61	0.20	0.85	0.97	0.97
37	0.34	0.15	0.37	0.50	0.50
38	0.56	0.45	0.44	0.83	0.78
39	0.53	0.68	0.56	0.33	0.47
40	0.39	0.20	0.26	1.00	0.31
41	0.27	0.13	0.33	0.44	0.38
42	0.46	0.67	0.56	0.03	0.50
43	0.32	0.18	0.37	0.61	0.38
44	0.05	0.01	0.07	0.06	0.09
45	0.58	0.68	0.44	0.58	0.47
46	0.73	0.60	0.85	0.81	0.81

47	0.37	0.26	0.56	0.47	0.41
48	0.19	0.09	0.33	0.25	0.28
49	0.44	0.34	0.59	0.47	0.63
50	0.46	0.19	0.70	0.81	0.56
51	0.33	0.29	0.22	0.47	0.41
52	0.60	0.77	0.89	0.08	0.78
53	0.49	0.36	0.48	0.69	0.63
54	0.43	0.25	0.41	0.81	0.41
55	0.52	0.74	0.56	0.17	0.50
56	0.67	0.73	0.78	0.39	0.72
57	0.54	0.53	0.52	0.56	0.56
58	0.22	0.11	0.26	0.36	0.34
59	0.37	0.64	0.15	0.11	0.13
60	0.60	0.33	0.89	0.81	0.72
61	0.47	0.68	0.63	0.08	0.44
62	0.51	0.30	0.74	0.61	0.75
63	0.44	0.35	0.44	0.61	0.56
64	0.50	0.25	0.78	0.69	0.59
65	0.34	0.61	0.15	0.08	0.09
66	0.06	0.02	0.15	0.08	0.09
67	0.65	0.77	0.63	0.56	0.59

For completeness, we provide the individual-level choice proportions for each of these categories: near vs far past, past vs future, near vs far future, present vs past, and present vs future. These are provided in Table S2. As we noted, however, these individual differences in choice proportions are not closely related to any of the self-report measures (Table S1.

Reliability of self-report measures

Although each of the self-report measures we used in the text has been previously validated, it is worth checking that our sample showed reasonable internal consistency and reproduced relationships between measures that are commonly observed. To address this, we calculated the linear correlation matrix between measures and the internal consistency of each one using Cronbach's alpha.

The results are shown in Table S3. In general, each of the self-report measures showed high reliability ($> .7$), with the exception of the ZTPI measures. Part of this is because the subscales of the ZPTI are very different than one another; there is no reason to expect (for example) that responses to the present-fatalistic and past-negative scales would necessarily be highly related to one another.

To ensure due diligence on the ZTPI, we therefore looked at the reliability of each subscale. The resulting values for Cronbach's alpha were .73 (Past-Negative), .75 (Past-Positive), .62 (Present-Fatalistic), .50 (Present-Hedonistic), .67 (Future-Negative), and .69 (Future-Positive). Although not as high as some of the other measures, these are not too bad for subscales with only 3 items each. Overall, the reliability of the self-report measures appears within acceptable ranges.

Table S3

Correlations between summed scale scores. The main diagonal corresponds to reliability of each scale, quantified using Cronbach's alpha (Cronbach, 1951). NI = Narrative Immersion, BIS = Barratt Impulsiveness Scale, DOSPERT = Domain-specific Risk Taking scale, ZTPI = Zimbardo Time Perception inventory, FTP = Future Time Perception scale, SIPI = Short Imaginal Processes Inventory, TSE = Temporal Self-Extension (past and future combined), SSS = Sensation Seeking Scale, and UPPS = UPPS-P Impulsive Behavior Scale.

	NI	BIS	DOSPERT	ZTPI	FTP	SIPI	TSE	SSS	UPPS
NI	0.73								
BIS	-0.06	0.84							
DOSPERT	0.12	0.09	0.84						
ZTPI	-0.15	0.21	0.24	0.47					
FTP	0.15	0.04	-0.35	-0.22	0.89				
SIPI	0.10	0.13	0.39	0.40	-0.16	0.77			
TSE	0.20	-0.02	0.03	0.11	-0.23	-0.03	0.73		
SSS	0.11	0.13	-0.10	0.09	0.02	0.13	0.02	0.78	
UPPS	-0.13	0.17	0.24	0.81	-0.43	0.35	0.16	0.01	0.91

Converting between hyperboloid models

In the main text, we introduced an alternative form of the classic hyperboloid model (Rachlin, 2006), on the basis of the "muddled units" problem associated with the scale of its parameters (Vincent & Stewart, 2020). In the revised version of the model, the k parameter is applied to the time t *before* being subjectively scaled, yielding $v(x,t) = \frac{x}{1+(kt)^s}$ rather than $v(x,t) = \frac{x}{1+k \cdot t^s}$. This creates a slightly different interpretation of the discounting rate k, as it describes discounting of the *objective* time t as opposed to discounting the *subjective* time t^s.

The problem with the original formulation is that values of k, when a single value of k is used and different values of s are used for past / future events, will be on different scales. For example, if one option is in the future, then the discounting rate will be in units of $days^{-s_F}$ while the discounting rate for past events will be in units of $days^{-s_P}$. Therefore, the same discounting rate will have different units for different response options, even within the same choice. This is a potentially major problem, especially when comparing values of k across conditions. The simplest way to fix this is to use the corrected version of the hyperboloid model provided by Vincent & Stewart (2020).

Despite this, many researchers may prefer the interpretation of k as a discounting factor that is applied to subjective time. To facilitate this, we provide a conversion table in Table S4.

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Table S4

Hyperboloid discounting functions for the classic Rachlin (2006) (see also Green & Myerson, 2004) versus the adjusted model proposed by Vincent & Stewart (2020), along with formulas for converting from objective to subjective discounting rates (right column).

Parameters	Rachlin (2006)	Vincent & Stewart (2020)	Conversion (R $\leftarrow$ V&S)
s, k (Single s,k)	$\frac{x}{1+k\cdot t^s}$	$\frac{x}{1+(kt)^s}$	$k \leftarrow k^s$
s_F, s_P, k (Single k)	$\frac{x}{1+k\cdot t^{s_P}}$	$\frac{x}{1+(kt)^{s_P}}$	$k \leftarrow k^{s_P}$
	$\frac{x}{1+k\cdot t^{s_F}}$	$\frac{x}{1+(kt)^{s_F}}$	$k \leftarrow k^{s_F}$
s, k_P, k_F (Single s)	$\frac{x}{1+k_P\cdot t^s}$	$\frac{x}{1+(k_P t)^s}$	$k_P \leftarrow k_P^s$
	$\frac{x}{1+k_F\cdot t^s}$	$\frac{x}{1+(k_F t)^s}$	$k_F \leftarrow k_F^s$
s_F, s_P, k_P, k_F (Full)	$\frac{x}{1+k_P\cdot t^{s_P}}$	$\frac{x}{1+(k_P t)^{s_P}}$	$k_P \leftarrow k_P^{s_P}$
	$\frac{x}{1+k_F\cdot t^{s_F}}$	$\frac{x}{1+(k_F t)^{s_F}}$	$k_F \leftarrow k_P^{s_F}$

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