

# ECE 742 Final Project

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## 1 Theory

### 1.1 PML

Perfectly Matched Layer (PML) boundary conditions are absorbing boundary conditions. PML BCs decay the wave within a boundary layer at the edge of the simulation. The edge of the simulation BC can be implemented as PEC. Well-implemented PML BCs completely decay the wave from the time it enters the boundary layer to the time after it reflects and attempts to leave.

### 1.2 Graded Conductivity

Reflection Factor

$$R(\theta) = \exp^{-2\eta \cos(\theta) \int_0^d \sigma_x(x) dx}$$

Where  $\sigma_x$  is the graded conductivity of the PML material.  $\theta$  is the angle of incidence of the wave. So steeper angles of  $\theta$  will result in higher values of reflection error.

We want to minimize reflection R but also make sure the wave decays completely in the PML boundary layer.

We are going to compare the error for different types of grading profiles. And/or we can use different values in the grading profile

#### 1.2.1 Polynomial grading

Where the graded conductivity is:

$$\sigma_x = \left(\frac{x}{d}\right)^m \sigma_{x,max}$$

And the graded value for  $\kappa_x$  is:

$$\kappa_x = 1 + (\kappa_{x,max} - 1) \left(\frac{x}{d}\right)^m$$

Reflection factor simplifies to

### 1.2.2 Geometric grading

Where the graded conductivity is:

$$\sigma_x = (g^{\frac{1}{\Delta}})^x \sigma_{x,0}$$

$\sigma_{x,0}$  is the conductivity at the surface of the PML.  
g is a scaling factor. Nearly optimal:  $2 \leq g \leq 3$   
 $\Delta$  is spacing of FDTD lattice.

And the graded value for  $\kappa_x$  is:

$$\kappa_x = [(\kappa_{max})^{\frac{1}{a}} g^{\frac{1}{\Delta}}]^x$$

## 2 Code

## 3 Error Analysis

Insert Error Analysis Here

PMLs are exact for continuous functions, but error is introduced for discrete functions. Having a large step discontinuity can

### 3.0.1 Error of Polynomial Grading

### 3.0.2 Error of Geometric Grading

## 4 Fix me: Bibliography

Susan's book - third edition